Pearson New International Edition

Principles of Electric Circuits
Conventional Current Version

Thomas L. Floyd

Ninth Edition

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QUANTITIES AND UNITS

CHAPTER OUTLINE

1 Units of Measurement
2 Scientific Notation
3 Engineering Notation and Metric Prefixes
4 Metric Unit Conversions
5 Measured Numbers

CHAPTER OBJECTIVES

- Discuss the SI standard
- Use scientific notation (powers of ten) to represent quantities
- Use engineering notation and metric prefixes to represent large and small quantities
- Convert from one unit with a metric prefix to another
- Express measured data with the proper number of significant digits

KEY TERMS

- SI
- Scientific notation
- Power of ten
- Exponent
- Engineering notation
- Metric prefix
- Error
- Accuracy
- Precision
- Significant digits
- Round off

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

You must be familiar with the units used in electronics and know how to express electrical quantities in various ways using metric prefixes. Scientific notation and engineering notation are indispensable tools whether you use a computer, a calculator, or do computations the old-fashioned way.

SAFETY NOTE

When you work with electricity, you must always consider safety first. Safety notes remind you of the importance of safety and provide tips for a safe workplace.

From Chapter 1 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 UNITS OF MEASUREMENT

In the 19th century, the principal weight and measurement units dealt with commerce. As technology advanced, scientists and engineers saw the need for international standard measurement units. In 1875, at a conference called by the French, representatives from eighteen nations signed a treaty that established international standards. Today, all engineering and scientific work use an improved international system of units, Le Système International d'Unités, abbreviated **SI***.

After completing this section, you should be able to

- **Discuss the SI standard**
  - Specify the fundamental SI units
  - Specify the supplementary units
  - Explain what derived units are

Fundamental and Derived Units

The SI system is based on seven fundamental units (sometimes called *base units*) and two supplementary units. All measurements can be expressed as some combination of fundamental and supplementary units. Table 1 lists the fundamental units, and Table 2 lists the supplementary units.

The fundamental electrical unit, the ampere, is the unit for electrical current. Current is abbreviated with the letter I (for intensity) and uses the symbol A (for ampere). The ampere is unique in that it uses the fundamental unit of time (t) in its definition (second). All other electrical and magnetic units (such as voltage, power, and magnetic flux) use various combinations of fundamental units in their definitions and are called *derived units*.

For example, the derived unit of voltage, which is the volt (V), is defined in terms of fundamental units as $\mathrm{m}^2 \cdot \mathrm{kg} \cdot \mathrm{s}^{-3} \cdot \mathrm{A}^{-1}$. As you can see, this combination of fundamental units is very cumbersome and impractical. Therefore, volt is used as the derived unit.

▶ TABLE 1

SI fundamental units.

QUANTITY	UNIT	SYMBOL
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Luminous intensity	Candela	cd
Amount of substance	Mole	mol

▶ TABLE 2

SI supplementary units.

QUANTITY	UNIT	SYMBOL
Plane angle	Radian	r
Solid angle	Steradian	sr

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

***The bold terms in color are key terms and are defined at the end of the chapter.**

Letter symbols are used to represent both quantities and their units. One symbol is used to represent the name of the quantity, and another symbol is used to represent the unit of measurement of that quantity. For example, italic P stands for *power,* and nonitalic W stands for *watt,* which is the unit of power. Another example is voltage, where the same letter stands for both the quantity and its unit. Italic V represents voltage and nonitalic V represents *volt*, which is the unit of voltage. As a rule, italic letters stand for the quantity and nonitalic (roman) letters represent the unit of that quantity.

Table 3 lists the most important electrical quantities, along with their derived SI units and symbols. Table 4 lists magnetic quantities, along with their derived SI units and symbols.

QUANTITY	SYMBOL	SI UNIT	SYMBOL
Capacitance	C	Farad	F
Charge	Q	Coulomb	C
Conductance	G	Siemens	S
Energy (work)	W	Joule	J
Frequency	f	Hertz	Hz
Impedance	Z	Ohm	Ω
Inductance	L	Henry	H
Power	P	Watt	W
Reactance	X	Ohm	Ω
Resistance	R	Ohm	Ω
Voltage	V	Volt	V

◀ **TABLE 3**

Electrical quantities and derived units with SI symbols.

QUANTITY	SYMBOL	SI UNIT	SYMBOL
Magnetic field intensity	H	Ampere-turns/meter	At/m
Magnetic flux	ϕ	Weber	Wb
Magnetic flux density	B	Tesla	T
Magnetomotive force	F_m	Ampere-turn	At
Permeability	μ	Webers/ampere-turn · meter	Wb/At · m
Reluctance	$\mathcal{R}$	Ampere-turns/weber	At/Wb

◀ **TABLE 4**

Magnetic quantities and derived units with SI symbols.

In addition to the common electrical units shown in Table 3, the SI system has many other units that are defined in terms of certain fundamental units. In 1954, by international agreement, *meter, kilogram, second, ampere, degree Kelvin*, and *candela* were adopted as the basic SI units (*degree Kelvin* was later changed to just *kelvin*). These units form the basis of the mks (for meter-kilogram-second) units that are used for derived quantities and have become the preferred units for nearly all scientific and engineering work. An older metric system, called the cgs system, was based on the centimeter, gram, and second as fundamental units. There are still a number of units in common use based on the cgs system; for example, the gauss is a magnetic flux unit in the cgs system and is still in common usage. In keeping with preferred practice, we use mks units, except when otherwise noted.

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. How does a fundamental unit differ from a derived unit?
2. What is the fundamental electrical unit?
3. What does *SI* stand for?
4. Without referring to Table 3, list as many electrical quantities as possible, including their symbols, units, and unit symbols.
5. Without referring to Table 4, list as many magnetic quantities as possible, including their symbols, units, and unit symbols.

2 SCIENTIFIC NOTATION

In electrical and electronics fields, both very small and very large quantities are commonly used. For example, it is common to have electrical current values of only a few thousandths or even a few millionths of an ampere and to have resistance values ranging up to several thousand or several million ohms.

After completing this section, you should be able to

- **Use scientific notation (powers of ten) to represent quantities**
    - Express any number using a power of ten
    - Perform calculations with powers of ten

Scientific notation provides a convenient method to represent large and small numbers and to perform calculations involving such numbers. In scientific notation, a quantity is expressed as a product of a number between 1 and 10 and a power of ten. For example, the quantity 150,000 is expressed in scientific notation as 1.5×10^5, and the quantity 0.00022 is expressed as 2.2×10^{-4}.

Powers of Ten

Table 5 lists some powers of ten, both positive and negative, and the corresponding decimal numbers. The **power of ten** is expressed as an exponent of the base 10 in each case (10^x). An **exponent** is a number to which a base number is raised. It indicates the number

TABLE 5

Some positive and negative powers of ten.

$10^6 = 1{,}000{,}000$	$10^{-6} = 0.000001$
$10^5 = 100{,}000$	$10^{-5} = 0.00001$
$10^4 = 10{,}000$	$10^{-4} = 0.0001$
$10^3 = 1{,}000$	$10^{-3} = 0.001$
$10^2 = 100$	$10^{-2} = 0.01$
$10^1 = 10$	$10^{-1} = 0.1$
$10^0 = 1$	

of places that the decimal point is moved to the right or left to produce the decimal number. For a positive power of ten, move the decimal point to the right to get the equivalent decimal number. For example, for an exponent of 4,

$$10^4 = 1 \times 10^4 = 1\underset{\curvearrowright}{.0000}. = 10{,}000$$

For a negative the power of ten, move the decimal point to the left to get the equivalent decimal number. For example, for an exponent of −4,

$$10^{-4} = 1 \times 10^{-4} = \underset{\curvearrowleft}{.0001.} = 0.0001$$

EXAMPLE 1

Express each number in scientific notation.

(a) 200 **(b)** 5000 **(c)** 85,000 **(d)** 3,000,000

Solution In each case, move the decimal point an appropriate number of places to the left to determine the positive power of ten. Notice that the result is always a number between 1 and 10 times a power of ten.

(a) $200 = \mathbf{2 \times 10^2}$ **(b)** $5000 = \mathbf{5 \times 10^3}$

(c) $85{,}000 = \mathbf{8.5 \times 10^4}$ **(d)** $3{,}000{,}000 = \mathbf{3 \times 10^6}$

*Related Problem** Express 4750 in scientific notation.

---

*Answers are at the end of the chapter.

EXAMPLE 2

Express each number in scientific notation.

(a) 0.2 **(b)** 0.005 **(c)** 0.00063 **(d)** 0.000015

Solution In each case, move the decimal point an appropriate number of places to the right to determine the negative power of ten.

(a) $0.2 = \mathbf{2 \times 10^{-1}}$ **(b)** $0.005 = \mathbf{5 \times 10^{-3}}$

(c) $0.00063 = \mathbf{6.3 \times 10^{-4}}$ **(d)** $0.000015 = \mathbf{1.5 \times 10^{-5}}$

Related Problem Express 0.00738 in scientific notation.

EXAMPLE 3

Express each of the following as a regular decimal number:

(a) 1×10^5 **(b)** 2×10^3 **(c)** 3.2×10^{-2} **(d)** 2.50×10^{-6}

Solution Move the decimal point to the right or left a number of places indicated by the positive or the negative power of ten respectively.

(a) $1 \times 10^5 = \mathbf{100{,}000}$ **(b)** $2 \times 10^3 = \mathbf{2000}$

(c) $3.2 \times 10^{-2} = \mathbf{0.032}$ **(d)** $2.5 \times 10^{-6} = \mathbf{0.0000025}$

Related Problem Express 9.12×10^3 as a regular decimal number.

Calculations with Powers of Ten

The advantage of scientific notation is in addition, subtraction, multiplication, and division of very small or very large numbers.

Addition The steps for adding numbers in powers of ten are as follows:

1. Express the numbers to be added in the same power of ten.
2. Add the numbers without their powers of ten to get the sum.
3. Bring down the common power of ten, which is the power of ten of the sum.

EXAMPLE 4 Add 2×10^6 and 5×10^7 and express the result in scientific notation.

Solution

1. Express both numbers in the same power of ten: $(2 \times 10^6) + (50 \times 10^6)$.
2. Add $2 + 50 = 52$.
3. Bring down the common power of ten (10^6); the sum is $52 \times 10^6 = \mathbf{5.2 \times 10^7}$.

Related Problem Add 3.1×10^3 and 5.5×10^4.

Subtraction The steps for subtracting numbers in powers of ten are as follows:

1. Express the numbers to be subtracted in the same power of ten.
2. Subtract the numbers without their powers of ten to get the difference.
3. Bring down the common power of ten, which is the power of ten of the difference.

EXAMPLE 5 Subtract 2.5×10^{-12} from 7.5×10^{-11} and express the result in scientific notation.

Solution

1. Express each number in the same power of ten: $(7.5 \times 10^{-11}) - (0.25 \times 10^{-11})$.
2. Subtract $7.5 - 0.25 = 7.25$.
3. Bring down the common power of ten (10^{-11}); the difference is $\mathbf{7.25 \times 10^{-11}}$.

Related Problem Subtract 3.5×10^{-6} from 2.2×10^{-5}.

Multiplication The steps for multiplying numbers in powers of ten are as follows:

1. Multiply the numbers directly without their powers of ten.
2. Add the powers of ten algebraically (the exponents do not have to be the same).

EXAMPLE 6 Multiply 5×10^{12} and 3×10^{-6} and express the result in scientific notation.

Solution Multiply the numbers, and algebraically add the powers.

$$(5 \times 10^{12})(3 \times 10^{-6}) = (5)(3) \times 10^{12+(-6)} = 15 \times 10^6 = \mathbf{1.5 \times 10^7}$$

Related Problem Multiply 3.2×10^6 and 1.5×10^{-3}.

Division The steps for dividing numbers in powers of ten are as follows:

1. Divide the numbers directly without their powers of ten.
2. Subtract the power of ten in the denominator from the power of ten in the numerator (the powers do not have to be the same).

EXAMPLE 7 Divide 5.0×10^8 by 2.5×10^3 and express the result in scientific notation.

Solution Write the division problem with a numerator and denominator as

$$\frac{5.0 \times 10^8}{2.5 \times 10^3}$$

Divide the numbers and subtract the powers of ten (3 from 8).

$$\frac{5.0 \times 10^8}{2.5 \times 10^3} = 2 \times 10^{8-3} = \mathbf{2 \times 10^5}$$

Related Problem Divide 8×10^{-6} by 2×10^{-10}.

SECTION 2 CHECKUP

1. Scientific notation uses powers of ten. (True or False)
2. Express 100 as a power of ten.
3. Express the following numbers in scientific notation:
   (a) 4350 **(b)** 12,010 **(c)** 29,000,000
4. Express the following numbers in scientific notation:
   (a) 0.760 **(b)** 0.00025 **(c)** 0.000000597
5. Do the following operations:
   (a) $(1 \times 10^5) + (2 \times 10^5)$ **(b)** $(3 \times 10^6)(2 \times 10^4)$
   (c) $(8 \times 10^3) \div (4 \times 10^2)$ **(d)** $(2.5 \times 10^{-6}) - (1.3 \times 10^{-7})$

3 ENGINEERING NOTATION AND METRIC PREFIXES

Engineering notation, a specialized form of scientific notation, is used widely in technical fields to represent large and small quantities. In electronics, engineering notation is used to represent values of voltage, current, power, resistance, capacitance, inductance, and time, to name a few. Metric prefixes are used in conjunction with engineering notation as a "short hand" for the certain powers of ten that are multiples of three.

After completing this section, you should be able to

- **Use engineering notation and metric prefixes to represent large and small quantities**
  - List the metric prefixes
  - Change a power of ten in engineering notation to a metric prefix
  - Use metric prefixes to express electrical quantities
  - Convert one metric prefix to another

Engineering Notation

Engineering notation is similar to scientific notation. However, in **engineering notation** a number can have from one to three digits to the left of the decimal point and the power-of-ten exponent must be a multiple of three. For example, the number 33,000 expressed in engineering notation is 33×10^3. In scientific notation, it is expressed as 3.3×10^4. As another example, the number 0.045 expressed in engineering notation is 45×10^{-3}. In scientific notation, it is expressed as 4.5×10^{-2}.

EXAMPLE 8

Express the following numbers in engineering notation:

(a) 82,000 **(b)** 243,000 **(c)** 1,956,000

Solution In engineering notation,

(a) 82,000 is expressed as $\mathbf{82 \times 10^3}$.

(b) 243,000 is expressed as $\mathbf{243 \times 10^3}$.

(c) 1,956,000 is expressed as $\mathbf{1.956 \times 10^6}$.

Related Problem Express 36,000,000,000 in engineering notation.

EXAMPLE 9

Convert each of the following numbers to engineering notation:

(a) 0.0022 **(b)** 0.000000047 **(c)** 0.00033

Solution In engineering notation,

(a) 0.0022 is expressed as $\mathbf{2.2 \times 10^{-3}}$.

(b) 0.000000047 is expressed as $\mathbf{47 \times 10^{-9}}$.

(c) 0.00033 is expressed as $\mathbf{330 \times 10^{-6}}$.

Related Problem Express 0.0000000000056 in engineering notation.

Metric Prefixes

In engineering notation **metric prefixes** represent each of the most commonly used powers of ten. These metric prefixes are listed in Table 6 with their symbols and corresponding powers of ten.

Metric prefixes are used only with numbers that have a unit of measure, such as volts, amperes, and ohms, and precede the unit symbol. For example, 0.025 amperes can be expressed in engineering notation as 25×10^{-3} A. This quantity expressed using a metric prefix is 25 mA, which is read 25 milliamps. Note that the metric prefix *milli* has replaced 10^{-3}. As another example, 100,000,000 ohms can be expressed as $100 \times 10^6\ \Omega$. This quantity expressed using a metric prefix is 100 MΩ, which is read 100 megohms. The metric prefix *mega* has replaced 10^6.

METRIC PREFIX	SYMBOL	POWER OF TEN	VALUE
femto	f	10^{-15}	one-quadrillionth
pico	p	10^{-12}	one-trillionth
nano	n	10^{-9}	one-billionth
micro	μ	10^{-6}	one-millionth
milli	m	10^{-3}	one-thousandth
kilo	k	10^{3}	one thousand
mega	M	10^{6}	one million
giga	G	10^{9}	one billion
tera	T	10^{12}	one trillion

TABLE 6

Metric prefixes with their symbols and corresponding powers of ten and values.

EXAMPLE 10

Express each quantity using a metric prefix:

(a) 50,000 V **(b)** 25,000,000 Ω **(c)** 0.000036 A

Solution

(a) $50{,}000\text{ V} = 50 \times 10^{3}\text{ V} = \mathbf{50\text{ kV}}$

(b) $25{,}000{,}000\ \Omega = 25 \times 10^{6}\ \Omega = \mathbf{25\text{ M}\Omega}$

(c) $0.000036\text{ A} = 36 \times 10^{-6}\text{ A} = \mathbf{36\ \mu\text{A}}$

Related Problem

Express using metric prefixes:

(a) 56,000,000 Ω **(b)** 0.000470 A

Calculator Tip

All scientific and graphing calculators provide features for entering and displaying numbers in various formats. Scientific and engineering notation are special cases of exponential (power of ten) notation. Most calculators have a key labeled EE (or EXP) that is used to enter the exponent of numbers. To enter a number in exponential notation, enter the base number first, including the sign, and then press the EE key, followed by the exponent, including the sign.

Scientific and graphing calculators have displays for showing the power of ten. Some calculators display the exponent as a small raised number on the right side of the display.

47.0 ^{03}

Other calculators display the number with a small E followed by the exponent.

47.0E03

Notice that the base 10 is not generally shown, but it is implied or represented by the E. When you write out the number, you need to include the base 10. The displayed number shown above is written out as 47.0×10^{3} in engineering notation.

Some calculators are placed in the scientific or engineering notation mode using a secondary or tertiary function, such as SCI or ENG. Then numbers are entered in regular decimal form. The calculator automatically converts them to the proper format. Other calculators provide for mode selection using a menu.

Always check the owner's manual for your particular calculator to determine how to use the exponential notation features.

SECTION 3 CHECKUP

1. Express the following numbers in engineering notation:
   (a) 0.0056 **(b)** 0.0000000283 **(c)** 950,000 **(d)** 375,000,000,000
2. List the metric prefix for each of the following powers of ten:
   10^6, 10^3, 10^{-3}, 10^{-6}, 10^{-9}, and 10^{-12}
3. Use an appropriate metric prefix to express 0.000001 A.
4. Use an appropriate metric prefix to express 250,000 W.

4 METRIC UNIT CONVERSIONS

It is sometimes necessary or convenient to convert a quantity from one unit with a metric prefix to another, such as from milliamperes (mA) to microamperes (μA). Moving the decimal point in the number an appropriate number of places to the left or to the right, depending on the particular conversion, results in a metric unit conversion.

After completing this section, you should be able to

- **Convert from one unit with a metric prefix to another**
  - Convert between milli, micro, nano, and pico
  - Convert between kilo and mega

The following rules apply to metric unit conversions:

1. When converting from a larger unit to a smaller unit, move the decimal point to the right.
2. When converting from a smaller unit to a larger unit, move the decimal point to the left.
3. Determine the number of places to move the decimal point by finding the difference in the powers of ten of the units being converted.

For example, when converting from milliamperes (mA) to microamperes (μA), move the decimal point three places to the right because there is a three-place difference between the two units (mA is 10^{-3} A and μA is 10^{-6} A). The following examples illustrate a few conversions.

EXAMPLE 11 Convert 0.15 milliampere (0.15 mA) to microamperes (μA).

Solution Move the decimal point three places to the right.

$$0.15 \text{ mA} = 0.15 \times 10^{-3} \text{ A} = 150 \times 10^{-6} \text{ A} = \mathbf{150\ \mu A}$$

Related Problem Convert 1 mA to microamperes.

EXAMPLE 12 Convert 4500 microvolts (4500 μV) to millivolts (mV).

Solution Move the decimal point three places to the left.

$$4500\ \mu\text{V} = 4500 \times 10^{-6} \text{ V} = 4.5 \times 10^{-3} \text{ V} = \mathbf{4.5\ mV}$$

Related Problem Convert 1000 μV to millivolts.

EXAMPLE 13 Convert 5000 nanoamperes (5000 nA) to microamperes (μA).

Solution Move the decimal point three places to the left.

$$5000\text{ nA} = 5000 \times 10^{-9}\text{ A} = 5 \times 10^{-6}\text{ A} = \mathbf{5\ \mu A}$$

Related Problem Convert 893 nA to microamperes.

EXAMPLE 14 Convert 47,000 picofarads (47,000 pF) to microfarads (μF).

Solution Move the decimal point six places to the left.

$$47{,}000\text{ pF} = 47{,}000 \times 10^{-12}\text{ F} = 0.047 \times 10^{-6}\text{ F} = \mathbf{0.047\ \mu F}$$

Related Problem Convert 10,000 pF to microfarads.

EXAMPLE 15 Convert 0.00022 microfarad (0.00022 μF) to picofarads (pF).

Solution Move the decimal point six places to the right.

$$0.00022\ \mu\text{F} = 0.00022 \times 10^{-6}\text{ F} = 220 \times 10^{-12}\text{ F} = \mathbf{220\ pF}$$

Related Problem Convert 0.0022 μF to picofarads.

EXAMPLE 16 Convert 1800 kilohms (1800 kΩ) to megohms (MΩ).

Solution Move the decimal point three places to the left.

$$1800\text{ k}\Omega = 1800 \times 10^{3}\ \Omega = 1.8 \times 10^{6}\ \Omega = \mathbf{1.8\ M\Omega}$$

Related Problem Convert 2.2 kΩ to megohms.

When adding (or subtracting) quantities with different metric prefixes, first convert one of the quantities to the same prefix as the other quantity.

EXAMPLE 17 Add 15 mA and 8000 μA and express the sum in milliamperes.

Solution Convert 8000 μA to 8 mA and add.

$$\begin{aligned}15\text{ mA} + 8000\ \mu\text{A} &= 15 \times 10^{-3}\text{ A} + 8000 \times 10^{-6}\text{ A}\\ &= 15 \times 10^{-3}\text{ A} + 8 \times 10^{-3}\text{ A} = 15\text{ mA} + 8\text{ mA} = \mathbf{23\ mA}\end{aligned}$$

Related Problem Add 2873 mA to 10,000 μA; express the sum in milliamperes.

SECTION 4 CHECKUP

1. Convert 0.01 MV to kilovolts (kV).
2. Convert 250,000 pA to milliamperes (mA).
3. Add 0.05 MW and 75 kW and express the result in kW.
4. Add 50 mV and 25,000 μV and express the result in mV.

5 MEASURED NUMBERS

Whenever a quantity is measured, there is uncertainty in the result due to limitations of the instruments used. When a measured quantity contains approximate numbers, the digits known to be correct are called significant digits. When reporting measured quantities, the number of digits that should be retained are the significant digits and no more than one uncertain digit.

After completing this section, you should be able to

- **Express measured data with the proper number of significant digits**
  - Define *accuracy, error,* and *precision*
  - Round numbers properly

Error, Accuracy, and Precision

Data taken in experiments are not perfect because the accuracy of the data depends on the accuracy of the test equipment and the conditions under which the measurement was made. In order to properly report measured data, the error associated with the measurement should be taken into account. Experimental error should not be thought of as a mistake. All measurements that do not involve counting are approximations of the true value. The difference between the true or best-accepted value of some quantity and the measured value is the **error**. A measurement is said to be accurate if the error is small. **Accuracy** is an indication of the range of error in a measurement. For example, if you measure thickness of a 10.00 mm gauge block with a micrometer and find that it is 10.8 mm, the reading is not accurate because a gauge block is considered to be a working standard. If you measure 10.02 mm, the reading is accurate because it is in reasonable agreement with the standard.

Another term associated with the quality of a measurement is *precision*. **Precision** is a measure of the repeatability (or consistency) of a measurement of some quantity. It is possible to have a precise measurement in which a series of readings are not scattered, but each measurement is inaccurate because of an instrument error. For example, a meter may be out of calibration and produce inaccurate but consistent (precise) results. However, it is not possible to have an accurate instrument unless it is also precise.

Significant Digits

The digits in a measured number that are known to be correct are called **significant digits**. Most measuring instruments show the proper number of significant digits, but some instruments can show digits that are not significant, leaving it to the user to determine what should be reported. This may occur because of an effect called *loading*. A meter can change the actual reading in a circuit by its very presence. It is important to recognize when a reading may be inaccurate; you should not report digits that are known to be inaccurate.

Another problem with significant digits occurs when you perform mathematical operations with numbers. The number of significant digits should never exceed the number in the original measurement. For example, if 1.0 V is divided by 3.0 Ω, a calculator will show 0.33333333. Since the original numbers each contain 2 significant digits, the answer should be reported as 0.33 A, the same number of significant digits.

The rules for determining if a reported digit is significant are

1. Nonzero digits are always considered to be significant.
2. Zeros to the left of the first nonzero digit are never significant.
3. Zeros between nonzero digits are always significant.

4. Zeros to the right of the decimal point for a decimal number are significant.
5. Zeros to the left of the decimal point with a whole number may or may not be significant depending on the measurement. For example, the number 12,100 Ω can have 3, 4, or 5 significant digits. To clarify the significant digits, scientific notation (or a metric prefix) should be used. For example, 12.10 kΩ has 4 significant digits.

When a measured value is reported, one uncertain digit may be retained but other uncertain digits should be discarded. To find the number of significant digits in a number, ignore the decimal point, and count the number of digits from left to right starting with the first nonzero digit and ending with the last digit to the right. All of the digits counted are significant except zeros to the right end of the number, which may or may not be significant. In the absence of other information, the significance of the right-hand zeros is uncertain. Generally, zeros that are placeholders, and not part of a measurement, are considered to be not significant. To avoid confusion, numbers should be shown using scientific or engineering notation if it is necessary to show the significant zeros.

EXAMPLE 18

Express the measured number 4300 with 2, 3, and 4 significant digits.

Solution Zeros to the right of the decimal point in a decimal number are significant. Therefore, to show two significant digits, write

$$\mathbf{4.3 \times 10^3}$$

To show three significant digits, write

$$\mathbf{4.30 \times 10^3}$$

To show four significant digits, write

$$\mathbf{4.300 \times 10^3}$$

Related Problem How would you show the number 10,000 showing three significant digits?

EXAMPLE 19

Underline the significant digits in each of the following measurements:

(a) 40.0 **(b)** 0.3040 **(c)** 1.20×10^5 **(d)** 120,000 **(e)** 0.00502

Solution

(a) $\underline{40.0}$ has three significant digits; see rule 4.

(b) $0.\underline{3040}$ has four significant digits; see rules 2 and 3.

(c) $\underline{1.20} \times 10^5$ has three significant digits; see rule 4.

(d) $\underline{12}0{,}000$ has at least two significant digits. Although the number has the same value as in (c), zeros in this example are uncertain; see rule 5. This is *not* a recommended method for reporting a measured quantity; use scientific notation or a metric prefix in this case. See Example 18.

(e) $0.00\underline{502}$ has three significant digits; see rules 2 and 3.

Related Problem What is the difference between a measured quantity of 10 and 10.0?

Rounding Off Numbers

Since they always contain approximate numbers, measurements should be shown only with those digits that are significant plus no more than one uncertain digit. The number of digits shown is indicative of the precision of the measurement. For this reason, you should

round off a number by dropping one or more digits to the right of the last significant digit. Use only the most significant dropped digit to decide how to round off. The rules for rounding off are

1. If the most significant digit dropped is greater than 5, increase the last retained digit by 1.
2. If the digit dropped is less than 5, do not change the last retained digit.
3. If the digit dropped is 5, increase the last retained digit *if* it makes it an even number, otherwise do not. This is called the "round-to-even" rule.

EXAMPLE 20 Round of the following numbers to three significant digits:

(a) 10.071 **(b)** 29.961 **(c)** 6.3948 **(d)** 123.52 **(e)** 122.52

Solution **(a)** 10.071 rounds to **10.1**. **(b)** 29.961 rounds to **30.0**.

(c) 6.3948 rounds to **6.39**. **(d)** 123.52 rounds to **124**.

(e) 122.52 round to **122**.

Related Problem Round 3.2850 to three significant digits using the round-to-even rule.

In most electrical and electronics work, components have tolerances greater than 1% (5% and 10% are common). Most measuring instruments have accuracy specifications better than this, but it is unusual for measurements to be made with higher accuracy than 1 part in 1000. For this reason, three significant digits are appropriate for numbers that represent measured quantities in all but the most exacting work. If you are working with a problem with several intermediate results, keep all digits in your calculator, but round the answers to three when reporting a result.

SECTION 5 CHECKUP

1. What is the rule for showing zeros to the right of the decimal point?
2. What is the round-to-even rule?
3. On schematics, you will frequently see a 1000 Ω resistor listed as 1.0 kΩ. What does this imply about the value of the resistor?
4. If a power supply is required to be set to 10.00 V, what does this imply about the accuracy needed for the measuring instrument?
5. How can scientific or engineering notation be used to show the correct number of significant digits in a measurement?

SUMMARY

- SI is an abbreviation for Le Système International d'Unités and is a standardized system of units.
- A fundamental unit is an SI unit from which other SI units are derived. There are seven fundamental units.
- Scientific notation is a method for representing very large and very small numbers as a number between one and ten (one digit to left of decimal point) times a power of ten.
- Engineering notation is a form of scientific notation in which quantities are represented with one, two, or three digits to the left of the decimal point times a power of ten that is a multiple of three.
- Metric prefixes represent powers of ten in numbers expressed in engineering notation.

- The uncertainty of a measured quantity depends on the accuracy and precision of the measurement.
- The number of significant digits in the result of a mathematical operation should never exceed the significant digits in the original numbers.

KEY TERMS

Accuracy An indication of the range of error in a measurement.

Engineering notation A system for representing any number as a one-, two-, or three-digit number times a power of ten with an exponent that is a multiple of 3.

Error The difference between the true or best-accepted value of some quantity and the measured value.

Exponent The number to which a base number is raised.

Metric prefix An affix that represents a power-of-ten number expressed in engineering notation.

Power of ten A numerical representation consisting of a base of 10 and an exponent; the number 10 raised to a power.

Precision A measure of the repeatability (or consistency) of a series of measurements.

Round off The process of dropping one or more digits to the right of the last significant digit in a number.

Scientific notation A system for representing any number as a number between 1 and 10 times an appropriate power of ten.

SI Standardized internationalized system of units used for all engineering and scientific work; abbreviation for French *Le Système International d'Unités*.

Significant digit A digit known to be correct in a number.

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The number 3300 is written as 3.3×10^3 in both scientific and engineering notation.
2. A negative number that is expressed in scientific notation will always have a negative exponent.
3. When you multiply two numbers written in scientific notation, the exponents need to be the same.
4. When you divide two numbers written in scientific notation, the exponent of the denominator is subtracted from the exponent of the numerator.
5. The metric prefix *micro* has an equivalent power of ten equal to 10^6.
6. To express 56×10^6 with a metric prefix, the result is 56 M.
7. 0.047 μF is equal to 47 nF.
8. The number of significant digits in the number 0.0102 is three.
9. When you apply the *round-to-even* rule to round off 26.25 to three digits, the result is 26.3.
10. In general, three significant digits are appropriate for most measured quantities.

SELF-TEST

Answers are at the end of the chapter.

1. Which of the following is not an electrical quantity?

   (a) current **(b)** voltage **(c)** time **(d)** power

2. The unit of current is

   (a) volt **(b)** watt **(c)** ampere **(d)** joule

3. The unit of voltage is

   (a) ohm **(b)** watt **(c)** volt **(d)** farad

4. The unit of resistance is

   (a) ampere **(b)** henry **(c)** hertz **(d)** ohm

5. Hertz is the unit of

(a) power **(b)** inductance **(c)** frequency **(d)** time

6. 15,000 W is the same as

(a) 15 mW **(b)** 15 kW **(c)** 15 MW **(d)** 15 μW

7. The quantity 4.7×10^3 is the same as

(a) 470 **(b)** 4700 **(c)** 47,000 **(d)** 0.0047

8. The quantity 56×10^{-3} is the same as

(a) 0.056 **(b)** 0.560 **(c)** 560 **(d)** 56,000

9. The number 3,300,000 can be expressed in engineering notation as

(a) 3300×10^3 **(b)** 3.3×10^{-6} **(c)** 3.3×10^6 **(d)** either answer (a) or (c)

10. Ten milliamperes can be expressed as

(a) 10 MA **(b)** 10 μA **(c)** 10 kA **(d)** 10 mA

11. Five thousand volts can be expressed as

(a) 5000 V **(b)** 5 MV **(c)** 5 kV **(d)** either answer (a) or (c)

12. Twenty million ohms can be expressed as

(a) 20 mΩ **(b)** 20 MW **(c)** 20 MΩ **(d)** 20 $\mu\Omega$

13. The number of significant digits in 0.1050 is

(a) two **(b)** three **(c)** four **(d)** five

PROBLEMS

Answers to odd-numbered problems are at the end of the chapter.

SECTION 2 Scientific Notation

1. Express each of the following numbers in scientific notation:

(a) 3000 **(b)** 75,000 **(c)** 2,000,000

2. Express each fractional number in scientific notation:

(a) 1/500 **(b)** 1/2000 **(c)** 1/5,000,000

3. Express each of the following numbers in scientific notation:

(a) 8400 **(b)** 99,000 **(c)** 0.2×10^6

4. Express each of the following numbers in scientific notation:

(a) 0.0002 **(b)** 0.6 **(c)** 7.8×10^{-2}

5. Express each of the following numbers in scientific notation:

(a) 32×10^3 **(b)** 6800×10^{-6} **(c)** 870×10^8

6. Express each of the following as a regular decimal number:

(a) 2×10^5 **(b)** 5.4×10^{-9} **(c)** 1.0×10^1

7. Express each of the following as a regular decimal number:

(a) 2.5×10^{-6} **(b)** 5.0×10^2 **(c)** 3.9×10^{-1}

8. Express each number in regular decimal form:

(a) 4.5×10^{-6} **(b)** 8×10^{-9} **(c)** 4.0×10^{-12}

9. Add the following numbers:

(a) $(9.2 \times 10^6) + (3.4 \times 10^7)$ **(b)** $(5 \times 10^3) + (8.5 \times 10^{-1})$

(c) $(5.6 \times 10^{-8}) + (4.6 \times 10^{-9})$

10. Perform the following subtractions:

(a) $(3.2 \times 10^{12}) - (1.1 \times 10^{12})$ **(b)** $(2.6 \times 10^8) - (1.3 \times 10^7)$

(c) $(1.5 \times 10^{-12}) - (8 \times 10^{-13})$

11. Perform the following multiplications:

(a) $(5 \times 10^3)(4 \times 10^5)$ **(b)** $(1.2 \times 10^{12})(3 \times 10^2)$

(c) $(2.2 \times 10^{-9})(7 \times 10^{-6})$

12. Divide the following:

(a) $(1.0 \times 10^{3}) \div (2.5 \times 10^{2})$ **(b)** $(2.5 \times 10^{-6}) \div (5.0 \times 10^{-8})$

(c) $(4.2 \times 10^{8}) \div (2 \times 10^{-5})$

13. Perform the indicated operations:

(a) $(8 \times 10^{4} + 4 \times 10^{3}) \div 2 \times 10^{2}$ **(b)** $(3 \times 10^{7})(5 \times 10^{5}) - 9 \times 10^{12}$

(c) $(2.2 \times 10^{2} \div 1.1 \times 10^{2})(5.5 \times 10^{4})$

SECTION 3 Engineering Notation and Metric Prefixes

14. List the powers of ten used in engineering notation.

15. Express each of the following numbers in engineering notation:

(a) 89,000 **(b)** 450,000 **(c)** 12,040,000,000,000

16. Express each number in engineering notation:

(a) 2.35×10^{5} **(b)** 7.32×10^{7} **(c)** 1.333×10^{9}

17. Express each number in engineering notation:

(a) 0.000345 **(b)** 0.025 **(c)** 0.00000000129

18. Express each number in engineering notation:

(a) 9.81×10^{-3} **(b)** 4.82×10^{-4} **(c)** 4.38×10^{-7}

19. Add the following numbers and express each result in engineering notation:

(a) $(2.5 \times 10^{-3}) + (4.6 \times 10^{-3})$ **(b)** $(68 \times 10^{6}) + (33 \times 10^{6})$

(c) $(1.25 \times 10^{6}) + (250 \times 10^{3})$

20. Multiply the following numbers and express each result in engineering notation:

(a) $(32 \times 10^{-3})(56 \times 10^{3})$ **(b)** $(1.2 \times 10^{-6})(1.2 \times 10^{-6})$

(c) $100(55 \times 10^{-3})$

21. Divide the following numbers and express each result in engineering notation:

(a) $50 \div (2.2 \times 10^{3})$ **(b)** $(5 \times 10^{3}) \div (25 \times 10^{-6})$

(c) $560 \times 10^{3} \div (660 \times 10^{3})$

22. Express each number in Problem 15 in ohms using a metric prefix.

23. Express each number in Problem 17 in amperes using a metric prefix.

24. Express each of the following as a quantity having a metric prefix:

(a) 31×10^{-3} A **(b)** 5.5×10^{3} V **(c)** 20×10^{-12} F

25. Express the following using metric prefixes:

(a) 3×10^{-6} F **(b)** $3.3 \times 10^{6}\ \Omega$ **(c)** 350×10^{-9} A

26. Express the following using metric prefixes:

(a) 2.5×10^{-12} A **(b)** 8×10^{9} Hz **(c)** $4.7 \times 10^{3}\ \Omega$

27. Express each quantity by converting the metric prefix to a power-of-10:

(a) 7.5 pA **(b)** 3.3 GHz **(c)** 280 nW

28. Express each quantity in engineering notation:

(a) 5 μA **(b)** 43 mV **(c)** 275 kΩ **(d)** 10 MW

SECTION 4 Metric Unit Conversions

29. Perform the indicated conversions:

(a) 5 mA to microamperes **(b)** 3200 μW to milliwatts

(c) 5000 kV to megavolts **(d)** 10 MW to kilowatts

30. Determine the following:

(a) The number of microamperes in 1 milliampere

(b) The number of millivolts in 0.05 kilovolt

(c) The number of megohms in 0.02 kilohm

(d) The number of kilowatts in 155 milliwatts

31. Add the following quantities:

(a) 50 mA + 680 μA **(b)** 120 kΩ + 2.2 MΩ **(c)** 0.02 μF + 3300 pF

32. Do the following operations:

(a) 10 kΩ ÷ (2.2 kΩ + 10 kΩ) **(b)** 250 mV ÷ 50 μV **(c)** 1 MW ÷ 2 kW

SECTION 5 Measured Numbers

33. How many significant digits are in each of the following numbers:

(a) 1.00×10^3 **(b)** 0.0057 **(c)** 1502.0

(d) 0.000036 **(e)** 0.105 **(f)** 2.6×10^2

34. Round each of the following numbers to three significant digits. Use the "round-to-even" rule.

(a) 50,505 **(b)** 220.45 **(c)** 4646

(d) 10.99 **(e)** 1.005

ANSWERS

SECTION CHECKUPS

SECTION 1 Units of Measurement

1. Fundamental units define derived units.
2. Ampere
3. SI is the abbreviation for Système International.
4. Refer to Table 3 after you have compiled your list of electrical quantities.
5. Refer to Table 4 after you have compiled your list of magnetic quantities.

SECTION 2 Scientific Notation

1. True
2. 10^2
3. **(a)** 4.35×10^3 **(b)** 1.201×10^4 **(c)** 2.9×10^7
4. **(a)** 7.6×10^{-1} **(b)** 2.5×10^{-4} **(c)** 5.97×10^{-7}
5. **(a)** 3×10^5 **(b)** 6×10^{10} **(c)** 2×10^1 **(d)** 2.37×10^{-6}

SECTION 3 Engineering Notation and Metric Prefixes

1. **(a)** 5.6×10^{-3} **(b)** 28.3×10^{-9} **(c)** 950×10^3 **(d)** 375×10^9
2. Mega (M), kilo (k), milli (m), micro (μ), nano (n), and pico (p)
3. 1 μA (one microampere)
4. 250 kW (250 kilowatts)

SECTION 4 Metric Unit Conversions

1. 0.01 MV = 10 kV
2. 250,000 pA = 0.00025 mA
3. 0.05 MW + 75 kW = 50 kW + 75 kW = 125 kW
4. 50 mV + 25,000 μV = 50 mV + 25 mV = 75 mV

SECTION 5 Measured Numbers

1. Zeros should be retained only if they are significant because if they are shown, they are considered significant.
2. If the digit dropped is 5, increase the last retained digit *if* it makes it even, otherwise do not.
3. A zero to the right of the decimal point implies that the resistor is accurate to the nearest 100 Ω (0.1 kΩ).

4. The instrument must be accurate to four significant digits.

5. Scientific and engineering notation can show any number of digits to the right of a decimal. Numbers to the right of the decimal are always considered significant.

RELATED PROBLEMS FOR EXAMPLES

1 4.75×10^3
2 7.38×10^{-3}
3 9120
4 5.81×10^4
5 1.85×10^{-5}
6 4.8×10^3
7 4×10^4
8 36×10^9
9 5.6×10^{-12}
10 **(a)** 56 MΩ **(b)** 470 μA
11 1000 μA
12 1 mV
13 0.893 μA
14 0.01 μF
15 2200 pF
16 0.0022 MΩ
17 2883 mA
18 10.0×10^3
19 The number 10 has two significant digits; the number 10.0 has three.
20 3.28

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** F
6. T **7.** T **8.** T **9.** F **10.** T

SELF-TEST

1. (c) **2.** (c) **3.** (c) **4.** (d) **5.** (c) **6.** (b) **7.** (b)
8. (a) **9.** (d) **10.** (d) **11.** (d) **12.** (c) **13.** (c)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. **(a)** 3×10^3 **(b)** 7.5×10^4 **(c)** 2×10^6

3. **(a)** 8.4×10^3 **(b)** 9.9×10^4 **(c)** 2×10^5

5. **(a)** 3.2×10^4 **(b)** 6.8×10^{-3} **(c)** 8.7×10^{10}

7. **(a)** 0.0000025 **(b)** 500 **(c)** 0.39

9. **(a)** 4.32×10^7 **(b)** 5.00085×10^3 **(c)** 6.06×10^{-8}

11. **(a)** 2.0×10^9 **(b)** 3.6×10^{14} **(c)** 1.54×10^{-14}

13. **(a)** 4.20×10^2 **(b)** 6×10^{12} **(c)** 11×10^4

15. **(a)** 89×10^3 **(b)** 450×10^3 **(c)** 12.04×10^{12}

17. **(a)** 345×10^{-6} **(b)** 25×10^{-3} **(c)** 1.29×10^{-9}

19. **(a)** 7.1×10^{-3} **(b)** 101×10^6 **(c)** 1.50×10^6

21. **(a)** 22.7×10^{-3} **(b)** 200×10^6 **(c)** 848×10^{-3}

23. **(a)** 345 μA **(b)** 25 mA **(c)** 1.29 nA

25. **(a)** 3 μF **(b)** 3.3 MΩ **(c)** 350 nA

27. **(a)** 7.5×10^{-12} A **(b)** 3.3×10^9 Hz **(c)** 2.8×10^{-7} W

29. **(a)** 5000 μA **(b)** 3.2 mW **(c)** 5 MV **(d)** 10,000 kW

31. **(a)** 50.68 mA **(b)** 2.32 MΩ **(c)** 0.0233 μF

33. **(a)** 3 **(b)** 2 **(c)** 5 **(d)** 2 **(e)** 3 **(f)** 2

VOLTAGE, CURRENT, AND RESISTANCE

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VOLTAGE, CURRENT, AND RESISTANCE

CHAPTER OUTLINE

1 Atomic Structure
2 Electrical Charge
3 Voltage
4 Current
5 Resistance
6 The Electric Circuit
7 Basic Circuit Measurements
8 Electrical Safety
Application Activity

CHAPTER OBJECTIVES

- Describe the basic structure of atoms
- Explain the concept of electrical charge
- Define *voltage* and discuss its characteristics
- Define *current* and discuss its characteristics
- Define *resistance* and discuss its characteristics
- Describe a basic electric circuit
- Make basic circuit measurements
- Recognize electrical hazards and practice proper safety procedures

KEY TERMS

- Atom
- Electron
- Free electron
- Conductor
- Semiconductor
- Insulator
- Charge
- Coulomb's law
- Coulomb
- Voltage
- Volt
- Voltage source
- Power supply
- Current
- Ampere
- Current source
- Resistance
- Ohm
- Conductance
- Siemens
- Resistor
- Potentiometer
- Rheostat
- Circuit
- Load
- Closed circuit
- Open circuit
- AWG
- Ground
- Voltmeter
- Ammeter
- Ohmmeter
- DMM
- Electrical shock

APPLICATION ACTIVITY PREVIEW

In this application activity, you will see how the theory presented in this chapter is applied to a practical circuit that simulates part of a car's instrument panel lighting system. An automobile's lights are examples of simple types of electric circuits. When you turn on the headlights and taillights, you are connecting the lamps to the battery, which provides the voltage and produces current through each lamp. The current causes the lamps to emit light. The lamps themselves have resistance that limits the amount of current. The instrument panel lamp in most cars can be adjusted for brightness. By turning a knob, you change the resistance in the circuit, thereby causing the current to change. The amount of current through the lamp determines its brightness.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

The theoretical concepts of electrical current, voltage, and resistance are introduced in this chapter. You will learn how to express each of these quantities in the proper units and how each quantity is measured. The essential elements that form a basic electric circuit and how they are put together are covered.

Types of devices that generate voltage and current are introduced. Also, you will see a variety of components that are used to introduce resistance into electric circuits. The operation of protective devices such as fuses and circuit breakers are discussed, and mechanical switches commonly used in electric circuits are introduced. You will learn how to control and measure voltage, current, and resistance using laboratory instruments.

Voltage is essential in any kind of electric circuit. Voltage is the potential energy of electrical charge required to make the circuit work. Current is also necessary for electric circuits to operate, but it takes voltage to produce the current. Current is the movement of electrons through a circuit. Resistance in a circuit limits the amount of current. A water system can be used as an analogy for a simple circuit. Voltage can be considered analogous to the pressure required to force water through the pipes. Current through wires can be thought of as analogous to the water moving through the pipes. Resistance can be thought of as analogous to the restriction on the water flow produced by adjusting a valve.

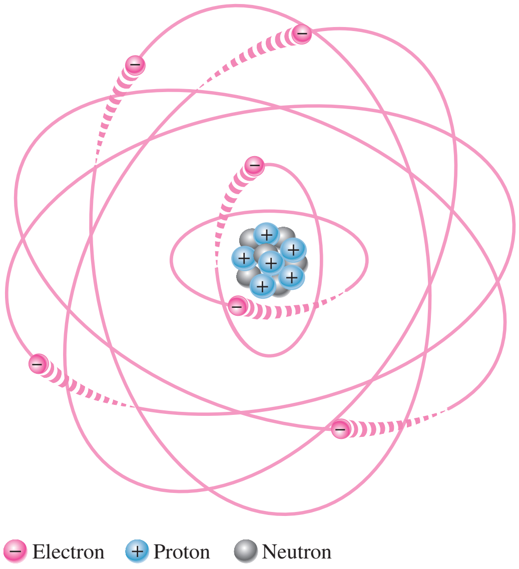
1 ATOMIC STRUCTURE

All matter is made of atoms; and all atoms consist of electrons, protons, and neutrons. In this section, you will learn about the structure of an atom, including electron shells and orbits, valence electrons, ions, and energy levels. The configuration of certain electrons in an atom is the key factor in determining how well a given conductive or semiconductive material conducts electric current.

After completing this section, you should be able to

- **Describe the basic structure of atoms**
  - Define *nucleus, proton, neutron,* and *electron*
  - Define *atomic number*
  - Define *shell*
  - Explain what a valence electron is
  - Describe ionization
  - Explain what a free electron is
  - Define *conductor, semiconductor,* and *insulator*

An **atom** is the smallest particle of an **element** that retains the characteristics of that element. Each of the known 110 elements has atoms that are different from the atoms of all other elements. This gives each element a unique atomic structure. According to the classic Bohr model, an atom is visualized as having a planetary type of structure that consists of a central nucleus surrounded by orbiting electrons, as illustrated in Figure 1. The **nucleus** consists of positively charged particles called **protons** and uncharged particles called **neutrons**. The basic particles of negative charge are called **electrons**, which orbit the nucleus.

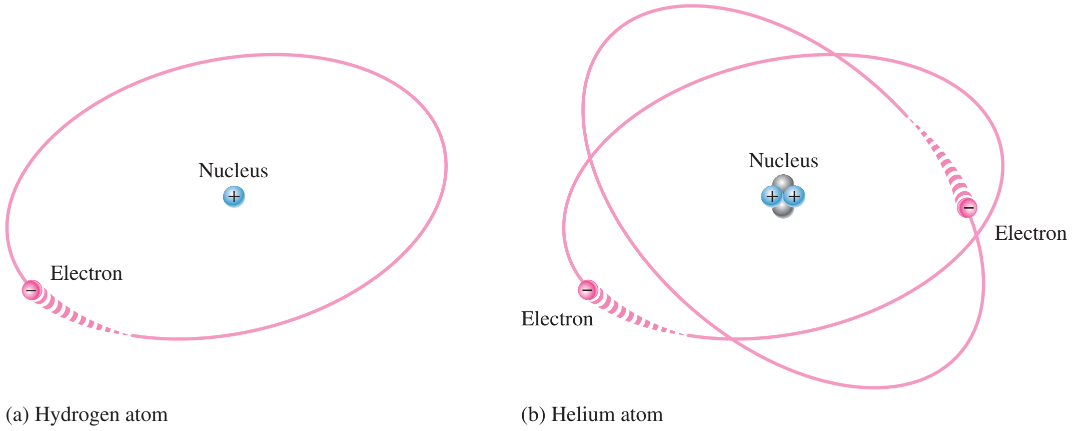


▲ **FIGURE 1**

The Bohr model of an atom showing electrons in circular orbits around the nucleus. The “tails” on the electrons indicate they are in motion.

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Each type of atom has a certain number of protons that distinguishes it from the atoms of all other elements. For example, the simplest atom is that of hydrogen, which has one proton and one electron, as pictured in Figure 2(a). As another example, the helium atom, shown in Figure 2(b), has two protons and two neutrons in the nucleus and two electrons orbiting the nucleus.



▲ FIGURE 2

The two simplest atoms, hydrogen and helium.

Atomic Number

All elements are arranged in the periodic table of the elements in order according to their **atomic number**. The atomic number equals the number of protons in the nucleus. For example, hydrogen has an atomic number of 1 and helium has an atomic number of 2. In their normal (or neutral) state, all atoms of a given element have the same number of electrons as protons; the positive charges cancel the negative charges, and the atom has a net charge of zero, making it electrically balanced.

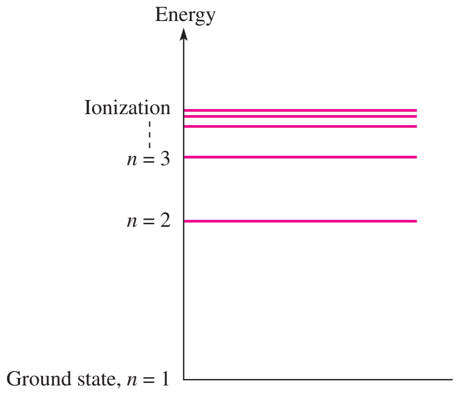
Shells, Orbits, and Energy Levels

As you have seen in the Bohr model, electrons orbit the nucleus of an atom at certain distances from the nucleus and are restricted to these specific orbits. Each orbit corresponds to a different energy level within the atom known as a **shell**. The shells are designated 1, 2, 3, and so on, with 1 being closest to the nucleus. Electrons further from the nucleus are at higher energy levels.

The line spectrums of hydrogen from the Bohr model of the atom shows that the electrons can only absorb or emit a specific amount of energy that represents the exact difference between the levels. Figure 3 shows the energy levels within the hydrogen atom. The

▶ FIGURE 3

Energy levels in hydrogen.



lowest level ($n = 1$) is called the *ground state* and represents the most stable atom with a single electron in the first shell. If this electron acquires a specific amount of energy by absorbing a photon, it can be raised to one of the higher energy levels. In this higher state, it can emit a photon with exactly the same energy and return to the ground state. Transitions between the levels account for various phenomena we see in electronics, such as the color of light from a light-emitting diode.

After Bohr's work, Erin Schroedinger (1887–1961) proposed a mathematical theory for the atom that explained more complicated atoms. He suggested that the electron has a wavelike property, and he considered the simplest case as having a three-dimensional standing wave pattern due to vibrations. Schroedinger theorized the standing wave of an electron with a spherical shape could have only certain wavelengths. This wave-mechanics model of the atom gave the same equation for the electron energy in hydrogen as Bohr's model, but in the wave-mechanics model, more complicated atoms could be explained by involving shapes other than spheres and adding a designation for the orientation of a given shape within the atom. In both models, electrons near the nucleus have less energy than those further out, which was the basic concept of the energy levels.

The idea of discrete energy levels within the atom is still a foundation for understanding the atom, and the wave-mechanics model has been very successful at predicting the energy levels for various atoms. The wave-mechanics model of the atom used the shell number, called the *principal quantum number,* in the energy equation. Three other quantum numbers describe each electron within the atom. All electrons in an atom have a unique set of quantum numbers.

When an atom is part of a large group, as in a crystal, the discrete energy levels broaden into energy bands, which is an important idea in solid-state electronics. The bands also differentiate between conductors, semiconductors, and insulators.

Valence Electrons

Electrons that are in orbits farther from the nucleus have higher energy and are less tightly bound to the atom than those closer to the nucleus. This is because the force of attraction between the positively charged nucleus and the negatively charged electron decreases with increasing distance from the nucleus. Electrons with the highest energy levels exist in the outermost shell of an atom and are relatively loosely bound to the atom. This outermost shell is known as the **valence** shell, and electrons in this shell are called **valence electrons**. These valence electrons contribute to chemical reactions and bonding within the structure of a material, and they determine the material's electrical properties.

Energy Levels and Ionization Energy

If an electron absorbs a photon with sufficient energy, it escapes from the atom and becomes a **free electron**. This is indicated by the ionization energy level in Figure 3. Any time an atom or group of atoms is left with a net charge, it is called an **ion**. When an electron escapes from the neutral hydrogen atom (designated H), the atom is left with a net positive charge and becomes a *positive ion* (designated H^+). In other cases, an atom or group of atoms can acquire an electron, in which case it is called a *negative ion*.

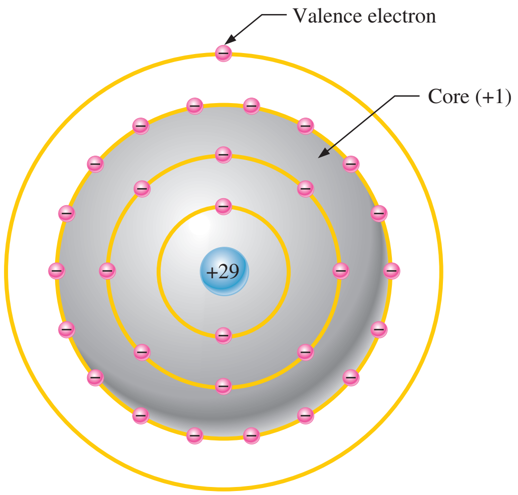
The Copper Atom

Copper is the most commonly used metal in **electrical** applications. The copper atom has 29 electrons that orbit the nucleus in four shells. The number of electrons in each shell follows a predictable pattern according to the formula, $2N^2$, where N is the number of the shell. The first shell of any atom can have up to two electrons, the second shell up to eight electrons, the third shell up to 18 electrons, and the fourth shell up to 32 electrons.

A copper atom is represented in Figure 4. Notice that the fourth or outermost shell, the valence shell, has only one valence electron. The inner shells are called the *core*. When the valence electron in the outer shell of the copper atom gains sufficient thermal energy, it

▶ **FIGURE 4**

The copper atom.



can break away from the parent atom and become a free electron. In a piece of copper at room temperature, a "sea" of these free electrons is present. These electrons are not bound to a given atom but are free to move in the copper material. Free electrons make copper an excellent conductor and make electrical current possible.

Categories of Materials

Three categories of materials are used in electronics: conductors, semiconductors, and insulators.

Conductors **Conductors** are materials that readily allow current. They have a large number of free electrons and are characterized by one to three valence electrons in their structure. Most metals are good conductors. Silver is the best conductor, and copper is next. Copper is the most widely used conductive material because it is less expensive than silver. Copper wire is commonly used as a conductor in electric circuits.

Semiconductors **Semiconductors** are classed below the conductors in their ability to carry current because they have fewer free electrons than do conductors. Semiconductors have four valence electrons in their atomic structures. However, because of their unique characteristics, certain semiconductor materials are the basis for **electronic** devices such as the diode, transistor, and integrated circuit. Silicon and germanium are common semiconductive materials.

Insulators **Insulators** are nonmetallic materials that are poor conductors of electric current; they are used to prevent current where it is not wanted. Insulators have no free electrons in their structure. The valence electrons are bound to the nucleus and not considered "free." Most practical insulators used in electrical and electronic applications are compounds such as glass, porcelain, teflon, and polyethylene, to name a few.

SECTION 1
CHECKUP
Answers are at the end of the chapter.

1. What is the basic particle of negative charge?
2. Define *atom*.
3. What does an atom consist of?
4. Define *atomic number*.
5. Do all elements have the same types of atoms?
6. What is a free electron?
7. What is a shell in the atomic structure?
8. Name two conductive materials.

2 ELECTRICAL CHARGE

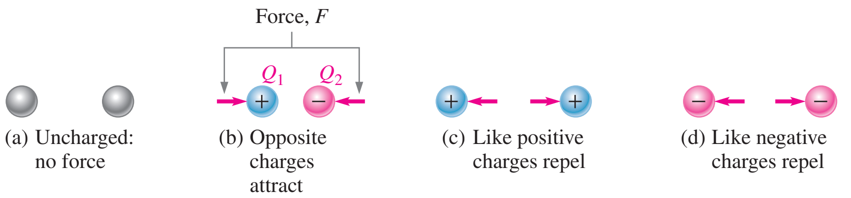
As you know, an electron is the smallest particle that exhibits negative electrical charge. When an excess of electrons exists in a material, there is a net negative electrical charge. When a deficiency of electrons exists, there is a net positive electrical charge.

After completing this section, you should be able to

- **Explain the concept of electrical charge**
    - Name the unit of charge
    - Name the types of charge
    - Discuss attractive and repulsive forces
    - Determine the amount of charge on a given number of electrons

The charge of an electron and that of a proton are equal in magnitude. Electrical **charge** is an electrical property of matter that exists because of an excess or deficiency of electrons. Charge is symbolized by the letter Q. Static electricity is the presence of a net positive or negative charge in a material. Everyone has experienced the effects of static electricity from time to time, for example, when attempting to touch a metal surface or another person or when the clothes in a dryer cling together.

Materials with charges of opposite polarity are attracted to each other, and materials with charges of the same polarity are repelled, as indicated in Figure 5. A force acts between charges, as evidenced by the attraction or repulsion. This force, called an *electric field*, is represented by imaginary lines, as represented in Figure 6.



(a) Uncharged: no force (b) Opposite charges attract (c) Like positive charges repel (d) Like negative charges repel

FIGURE 5

Attraction and repulsion of electrical charges.

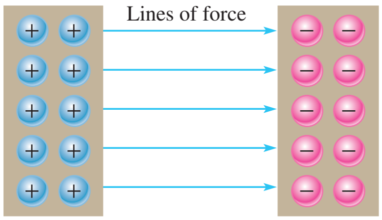


FIGURE 6

Electric field between two oppositely charged surfaces as represented by lines of force.

Coulomb's law states

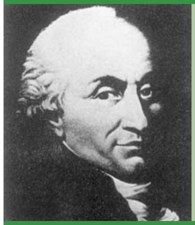
A force (F) exists between two point-source charges (Q_1, Q_2) that is directly proportional to the product of the two charges and inversely proportional to the square of the distance (d) between the charges.

Coulomb: The Unit of Charge

Electrical charge (Q) is measured in coulombs, symbolized by C.

One coulomb is the total charge possessed by 6.25×10^{18} electrons.

HISTORY NOTE



Charles Augustin Coulomb, 1736–1806

Coulomb, a Frenchman, spent many years as a military engineer. When bad health forced him to retire, he devoted his time to scientific research. He is best known for his work on electricity and magnetism due to his development of the inverse square law for the force between two charges. The unit of electrical charge is named in his honor. (Photo credit: Courtesy of the Smithsonian Institution. Photo number 52,597.)

A single electron has a charge of 1.6×10^{-19} C. The total charge Q, expressed in coulombs, for a given number of electrons is stated in the following formula:

Equation 1

$$Q = \frac{\text{number of electrons}}{6.25 \times 10^{18}\ \text{electrons/C}}$$

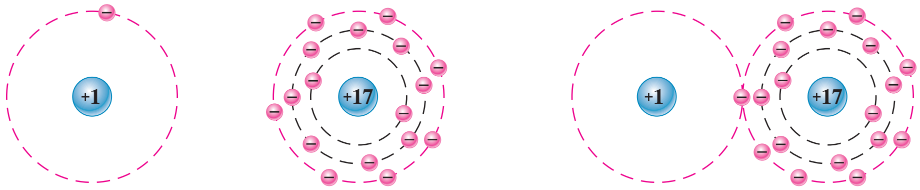
Positive and Negative Charge

Consider a neutral atom—that is, one that has the same number of electrons and protons and thus has no net charge. As you know, when a valence electron is pulled away from the atom by the application of energy, the atom is left with a net positive charge (more protons than electrons) and becomes a *positive ion*. If an atom acquires an extra electron in its outer shell, it has a net negative charge and becomes a *negative ion*.

The amount of energy required to free a valence electron is related to the number of electrons in the outer shell. An atom can have up to eight valence electrons. The more complete the outer shell, the more stable the atom and thus the more energy is required to remove an electron. Figure 7 illustrates the creation of a positive ion and a negative ion when a hydrogen atom gives up its single valence electron to a chlorine atom, forming gaseous hydrogen chloride (HCl). When the gaseous HCl is dissolved in water, hydrochloric acid is formed.

▶ FIGURE 7

Example of the formation of positive and negative ions.

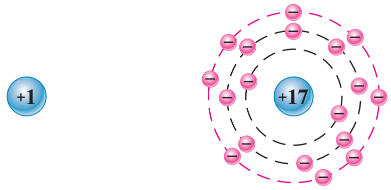


Hydrogen atom
(1 proton, 1 electron)

Chlorine atom
(17 protons, 17 electrons)

(a) The neutral hydrogen atom has a single valence electron.

(b) The atoms combine by sharing the valence electron to form gaseous hydrogen chloride (HCl).



Positive hydrogen ion
(1 proton, no electrons)

Negative chloride ion
(17 protons, 18 electrons)

(c) When dissolved in water, hydrogen chloride gas separates into positive hydrogen ions and negative chloride ions. The chlorine atom retains the electron given up by the hydrogen atom forming both positive and negative ions in the same solution.

EXAMPLE 1

How many coulombs do 93.8×10^{16} electrons represent?

Solution

$$Q = \frac{\text{number of electrons}}{6.25 \times 10^{18}\ \text{electrons/C}} = \frac{93.8 \times 10^{16}\ \text{electrons}}{6.25 \times 10^{18}\ \text{electrons/C}} = 15 \times 10^{-2}\ \text{C} = \mathbf{0.15\ C}$$

*Related Problem**

How many electrons does it take to have 3 C of charge?

---

*Answers are at the end of the chapter.

SECTION 2 CHECKUP

1. What is the symbol for charge?
2. What is the unit of charge, and what is the unit symbol?
3. What causes positive and negative charge?
4. How much charge, in coulombs, is there in 10×10^{12} electrons?

3 VOLTAGE

As you have seen, a force of attraction exists between a positive and a negative charge. A certain amount of energy must be exerted, in the form of work, to overcome the force and move the charges a given distance apart. All opposite charges possess a certain potential energy because of the separation between them. The difference in potential energy per charge is the potential difference or **voltage**. Voltage is the driving force, sometimes called electromotive force or emf, in electric circuits and is what establishes current.

After completing this section, you should be able to

- **Define *voltage* and discuss its characteristics**
    - State the formula for voltage
    - Name and define the unit of voltage
    - Describe the basic sources of voltage

Voltage, symbolized by V, is defined as energy or work per unit charge.

$$V = \frac{W}{Q}$$

Equation 2

where V is voltage in volts (V), W is energy in joules (J), and Q is charge in coulombs (C). Some sources use E instead of V to symbolize voltage, but we use V.

As an analogy, consider a water tank that is supported several feet above the ground. A given amount of energy must be exerted in the form of work to pump water up to fill the tank. Once the water is stored in the tank, it has a certain potential energy which, if released, can be used to perform work.

The Volt

The unit of voltage is the volt, symbolized by V.

One volt is the potential difference (voltage) between two points when one joule of energy is used to move one coulomb of charge from one point to the other.

HISTORY NOTE

Alessandro Volta 1745–1827

Volta, an Italian, invented a device to generate static electricity and he also discovered methane gas. Volta investigated reactions between dissimilar metals and developed the first battery in 1800. Electrical potential, more commonly known as voltage, and the unit of voltage, the volt, are named in his honor. (Photo credit: AIP Emilio Segrè Visual Archives, Lande Collection.)

EXAMPLE 2

If 50 J of energy are required to move 10 C of charge, what is the voltage?

Solution

$$V = \frac{W}{Q} = \frac{50\text{ J}}{10\text{ C}} = \mathbf{5\ V}$$

Related Problem How much energy is required to move 50 C from one point to another when the voltage between the two points is 12 V?

The Voltage Source

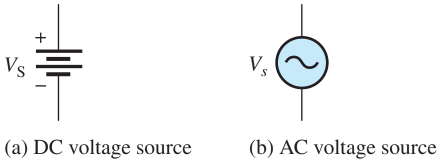
A **voltage source** provides electrical energy or electromotive force (emf), more commonly known as *voltage*. Voltage is produced by means of chemical energy, light energy, and magnetic energy combined with mechanical motion.

The Ideal Voltage Source An ideal voltage source can provide a constant voltage for any current required by a circuit. The ideal voltage source does not exist but can be closely approximated in practice. For purposes of analysis the ideal source is assumed unless otherwise specified.

Voltage sources can be either dc or ac. A common symbol for a dc voltage source is shown in Figure 8(a) and one for an ac voltage source is shown in part (b).

▶ FIGURE 8

Symbols for voltage sources.

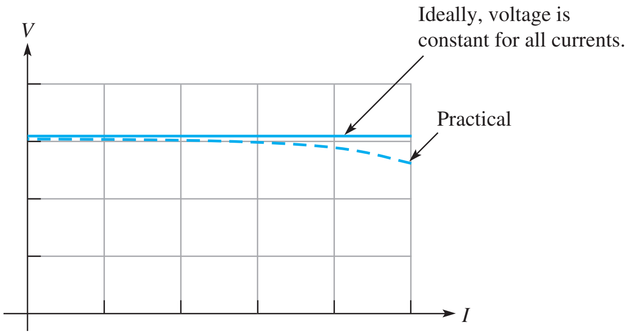


(a) DC voltage source (b) AC voltage source

A graph showing voltage versus current for an ideal dc voltage source is illustrated in Figure 9. As you can see, the voltage is constant for any current (within limits) from the source. For a practical voltage source connected in a circuit, the voltage decreases slightly as the current increases, as shown by the dashed curve. Current is always drawn from a voltage source when a load such as a resistance is connected to it.

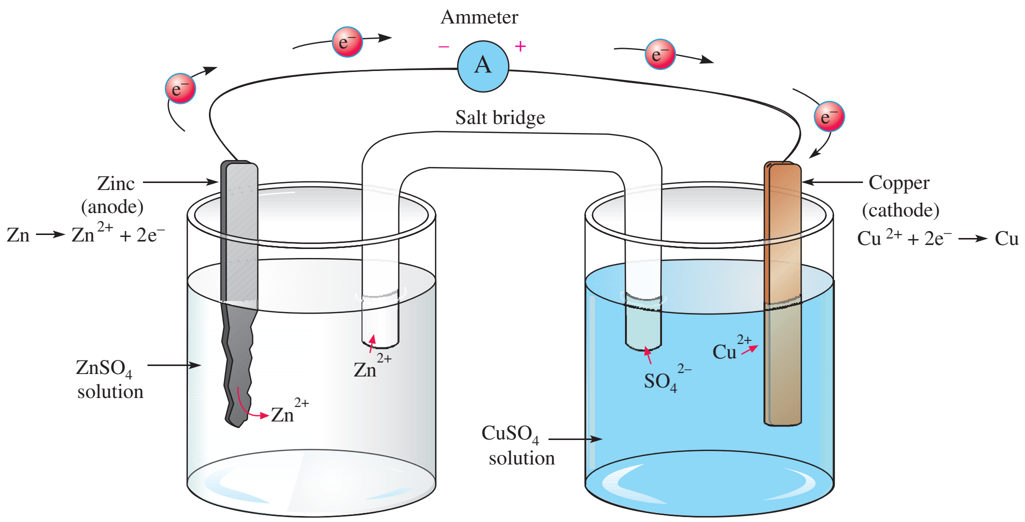
▶ FIGURE 9

Voltage source graph.



Types of DC Voltage Sources

Batteries A **battery** is a type of voltage source that converts chemical energy directly into electrical energy. As you know, work (or energy) per charge is the basic unit for voltage, and a battery adds energy to each unit of charge. It is something of a misnomer to talk about "charging a battery" because a battery does not store charge but rather stores chemical potential energy. All batteries use a specific type of chemical reaction called an *oxidation-reduction reaction*. In this type of reaction, electrons are transferred from one reactant to the other. If the chemicals used in the reaction are separated, it is possible to cause the electrons to travel in the external circuit, creating current. As long as there is an external path for the electrons, the reaction can proceed, and stored chemical energy is converted to electrical current. If the path is broken, the reaction stops and the battery is said to be in equilibrium. In a battery, the terminal that supplies electrons has a surplus of electrons and is the negative electrode or **anode**. The electrode that acquires electrons has a positive potential and is the **cathode**.



▲ **FIGURE 10**

A copper-zinc battery. The reaction can only occur if an external path is provided for the electrons. As the reaction proceeds, the Zn anode is eaten away and Cu^{2+} ions combine with electrons to form copper metal on the cathode.

Figure 10 shows a nonrechargeable single-cell copper-zinc battery that we will use for illustration of battery operation. The copper-zinc cell is simple to construct and illustrates concepts common to all nonrechargeable batteries. A zinc electrode and a copper electrode are immersed in solutions of zinc sulfate ($ZnSO_4$) and copper sulfate ($CuSO_4$), which are separated by a salt bridge that prevents the Cu^{2+} ions from reacting directly with the Zn metal. The zinc metal electrode supplies Zn^{2+} ions to the solution and electrons to the external circuit, so this electrode is constantly eaten away as the reaction proceeds. The salt bridge allows ions to pass through it to maintain charge balance in the cell. There are no free electrons in the solutions, so an external path for electrons is provided through an ammeter (in our case) or other load. On the cathode side, the electrons that were given up by the zinc combine with copper ions from the solution to form copper metal, which deposits on the copper electrode. The chemical reactions (shown in the diagram) occur at the electrode. Different types of batteries have different reactions, but all involve transfer of electrons in the external circuit.

A single cell will have a certain fixed voltage. In the copper-zinc cell, the voltage is 1.1 V. In a lead-acid cell, the kind used in car batteries, a potential difference of about 2.1 V is between the anode and cathode. The voltage of any cell depends on the cell chemistry. Nickel-cadmium cells are about 1.2 V and lithium cells can be as high as almost 4 V. Cell chemistry also determines the shelf life and discharge characteristics for a battery. For example, a lithium-MnO_2 battery typically has five times the shelf life of a comparable carbon-zinc battery.

Although the voltage of a battery cell is fixed by its chemistry, the capacity is variable and depends on the quantity of materials in the cell. Essentially, the *capacity* of a cell is the number of electrons that can be obtained from it and is measured by the amount of current that can be supplied over time.

Batteries normally consist of multiple cells that are electrically connected together internally. The way that the cells are connected and the type of cells determine the voltage and current capacity of the battery. If the positive electrode of one cell is connected to the negative electrode of the next and so on, as illustrated in Figure 11(a), the battery voltage is the sum of the individual cell voltages. This is called a series connection. To increase battery current capacity, the positive electrodes of several cells are connected together and all the negative electrodes are connected together, as illustrated in Figure 11(b). This is called a parallel connection. Also, by using larger cells, which have a greater quantity of material, the ability to supply current can be increased but the voltage is not affected.

SAFETY NOTE

Lead-acid batteries can be dangerous because sulfuric acid is highly corrosive and battery gases (primarily hydrogen) are explosive. The acid in the battery can cause serious eye damage if it contacts the eye and can cause skin burns or destroy clothing. You should always wear eye protection when working on or around batteries and wash well after handling batteries.

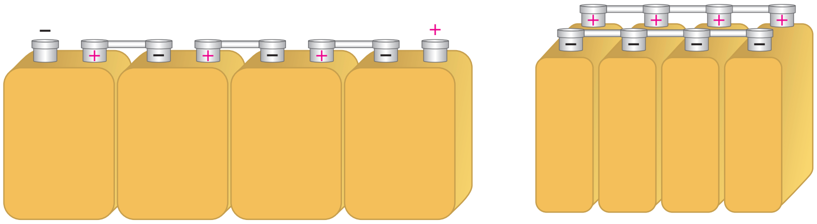
When removing a battery from service, make sure the switch is off. When a cable is removed, a spark may be created and ignite the explosive battery gases.

TECH NOTE

To store a lead-acid battery for an extended period, it should be fully charged and placed in a cool, dry location that is protected from freezing or excessive heat. A battery will self-discharge over time, so it needs to be periodically checked and recharged when it is less than 70% fully charged. Battery manufacturers will have specific recommendations for storage on their websites.

▶ FIGURE 11

Cells connected to form batteries.



(a) Series-connected cells increase voltage.

(b) Parallel-connected cells increase current capacity.

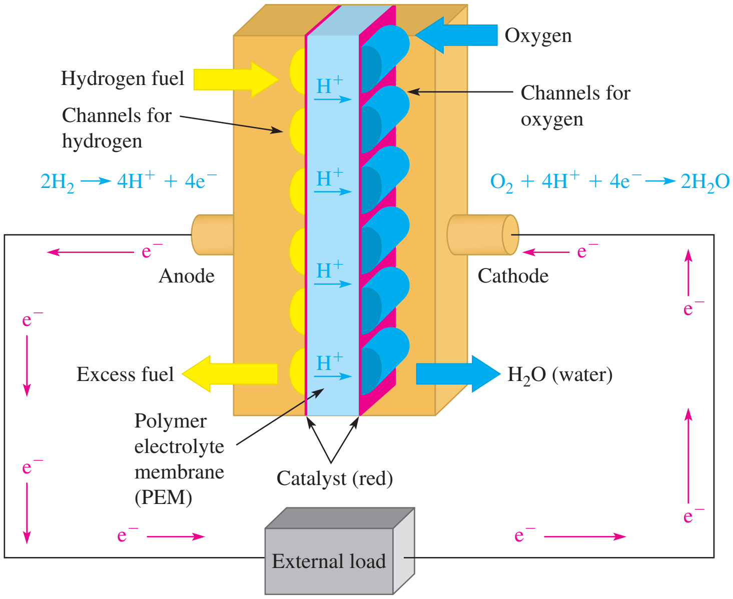
Batteries are divided into two major classes, *primary* and *secondary*. Primary batteries are used once and discarded because their chemical reactions are irreversible. Secondary batteries can be recharged and reused many times because they are characterized by reversible chemical reactions.

Primary and secondary batteries are available in a variety of shapes and sizes. Some of the sizes that you are most familiar with are AAA, AA, C, D, and 9 V but there are many others that are less common. Batteries are also typed according to their chemical makeup. A few common types of primary and secondary batteries are listed below.

- *Alkaline-*MnO_2 This is a primary battery that is commonly used in palm-type computers, photographic equipment, toys, radios, and recorders.
- *Carbon-zinc* This is a primary battery used in flashlights and small appliances.
- *Lead-acid* This is a secondary (rechargable) battery that is commonly used in automotive, marine, and other similar applications.
- *Lithium-ion* This is a secondary battery that is commonly used in all types of portable electronics. This type of battery is increasingly being used in defense, aerospace, and automotive applications.
- *Lithium-*MnO_2 This is a primary battery that is commonly used in photographic and electronic equipment, smoke alarms, personal organizers, memory backup, and communications equipment.
- *Nickel-metal hydride* This is a secondary (rechargable) battery that is commonly used in portable computers, cell phones, camcorders, and other portable consumer electronics.
- *Silver oxide* This is a primary battery that is commonly used in watches, photographic equipment, hearing aids, and electronics requiring high-capacity batteries.
- *Zinc air* This is a primary battery that is commonly used in hearing aids, medical monitoring instruments, pagers, and other frequency-use applications.

Fuel Cells A **fuel cell** is a device that converts electrochemical energy into dc voltage directly. Fuel cells combine a fuel (usually hydrogen) with an oxidizing agent (usually oxygen). In the hydrogen fuel cell, hydrogen and oxygen react to form water, which is the only by-product. The process is clean, quiet, and more efficient than burning. Fuel cells and batteries are similar in that they both are electrochemical devices that produce electricity using an oxidation-reduction reaction. However, a battery is a closed system with all its chemicals stored inside, whereas in a fuel cell, the chemicals (hydrogen and oxygen) constantly flow into the cell where they combine and produce electricity.

Hydrogen fuel cells are usually classified by their operating temperature and the type of electrolyte they use. Some types work well for use in stationary power generation plants. Others may be useful for small portable applications or for powering cars. For example, the



▲ **FIGURE 12**

Simplified diagram of a fuel cell.

type that holds the most promise for automotive applications is the polymer exchange membrane fuel cell (PEMFC), which is a type of hydrogen fuel cell. A simplified diagram is shown in Figure 12 to illustrate the basic operation.

The channels disperse pressurized hydrogen gas and oxygen gas equally over the surface of the catalyst, which facilitates the reaction of the hydrogen and oxygen. When an H_2 molecule comes in contact with the platinum catalyst on the anode side of the fuel cell, it splits into two H^+ ions and two electrons (e^-). The hydrogen ions are passed through the polymer electrolyte membrane (PEM) onto the cathode. The electrons pass through the anode and into the external circuit to create current.

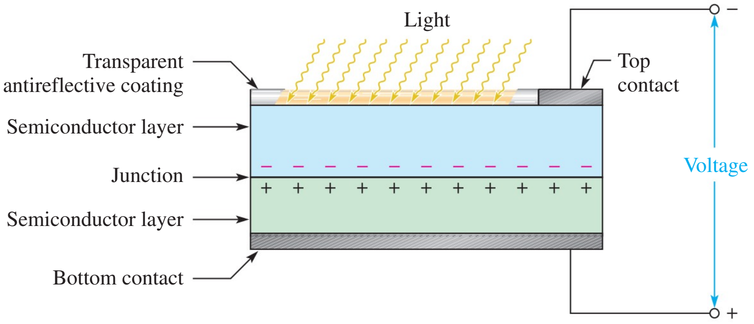
When an O_2 molecule comes in contact with the catalyst on the cathode side, it breaks apart, forming two oxygen ions. The negative charge of these ions attracts two H^+ ions through the electrolyte membrane and together they combine with electrons from the external circuit to form a water molecule (H_2O), which is passed from the cell as a by-product. In a single fuel cell, this reaction produces only about 0.7 V. To get higher voltages, multiple fuel cells are connected in series.

Current research on fuel cells is ongoing and is focused on developing reliable, smaller, and cost-effective components for vehicles and other applications. The conversion to fuel cells also requires research on how best to obtain and provide hydrogen fuel where it is needed. Potential sources for hydrogen include using solar, geothermal, or wind energy to break apart water. Hydrogen can also be obtained by breaking down coal or natural gas molecules, which are rich in hydrogen.

Solar Cells The operation of solar cells is based on the **photovoltaic effect**, which is the process whereby light energy is converted directly into electrical energy. A basic solar cell consists of two layers of different types of semiconductive materials joined together to form a junction. When one layer is exposed to light, many electrons acquire enough energy to break away from their parent atoms and cross the junction. This process forms negative ions on one side of the junction and positive ions on the other, and thus a potential difference (voltage) is developed. Figure 13 shows the construction of a basic solar cell.

► FIGURE 13

Construction of a basic solar cell.

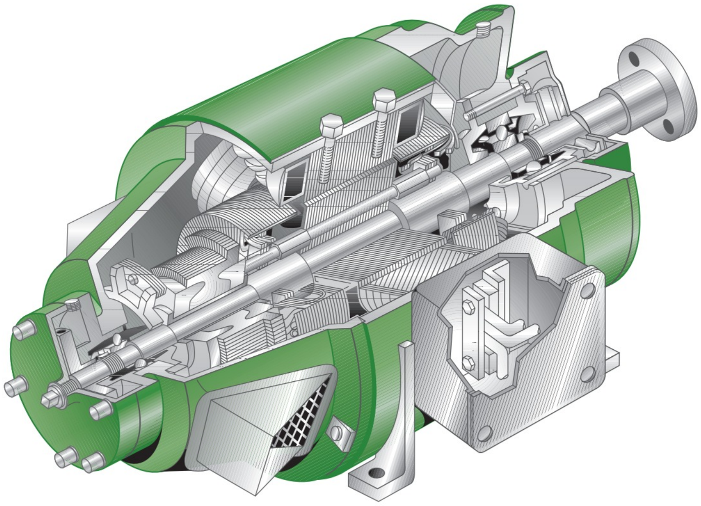


Although solar cells can be used in room light for powering a calculator, research is focusing more on converting sunlight to electricity. There is considerable research in increasing the efficiency of solar cells and photovoltaic (PV) modules today because they are a very clean source of energy using sunlight. A complete system for continuous power generally requires a battery backup to provide energy when the sun is not shining. Solar cells are well suited for remote locations where energy sources are unavailable and are used in providing power to satellites.

DC Generator Electrical **generators** convert mechanical energy into electrical energy using a principle called *electromagnetic induction*. A conductor is rotated through a magnetic field, and a voltage is produced across the conductor. A typical generator is pictured in Figure 14.

► FIGURE 14

Cutaway view of a dc voltage generator.



The Electronic Power Supply Electronic **power supplies** convert the ac voltage from a wall outlet to a dc voltage that can be varied over a specified range. Typical laboratory power supplies are shown in Figure 15.

Thermocouples The **thermocouple** is a thermoelectric type of voltage source that is commonly used to sense temperature. A thermocouple is formed by the junction of two dissimilar metals, and its operation is based on the **Seebeck effect** that describes the voltage generated at the junction of the metals as a function of temperature.

Standard types of thermocouple are characterized by the specific metals used. These standard thermocouples produce predictable output voltages for a range of temperatures. The most common is type K, made of chromel and alumel. Other types are also designated by letters as E, J, N, B, R, and S. Most thermocouples are available in wire or probe form.

Piezoelectric Sensors These sensors act as voltage sources and are based on the **piezoelectric effect** where a voltage is generated when a piezoelectric material is mechanically

BK PRECISION
9124 0-72V/0-1.2A Single Output Programmable DC Power Supply
OUTPUT
POWER
1745A DC POWER SUPPLY
COARSE
FINE
VOLTAGE
CURRENT

FIGURE 15

Electronic power supplies. (Courtesy of B+K Precision)

deformed by an external force. Quartz and ceramic are two types of piezoelectric material. Piezoelectric sensors are used in applications such as pressure sensors, force sensors, accelerometers, microphones, ultrasonic devices, and many others.

SECTION 3 CHECKUP

1. Define *voltage*.
2. What is the unit of voltage?
3. What is the voltage when 24 J of energy are required to move 10 C or charge?
4. List six sources of voltage.
5. What types of chemical reaction occurs in both batteries and fuel cells?

4 CURRENT

Voltage provides energy to electrons, allowing them to move through a circuit. This movement of electrons is the current, which results in work being done in an electrical circuit.

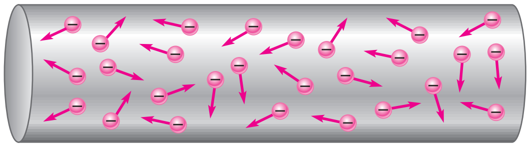
After completing this section, you should be able to

- **Define *current* and discuss its characteristics**
  - Explain the movement of electrons
  - State the formula for current
  - Name and define the unit of current

As you have learned, free electrons are available in all conductive and semiconductive materials. These outer-shell electrons drift randomly in all directions, from atom to atom, within the structure of the material, as indicated in Figure 16. These electrons are loosely bound to the positive metal ions in the material, but because of thermal energy, they are free to move about the crystalline structure of the metal.

▶ FIGURE 16

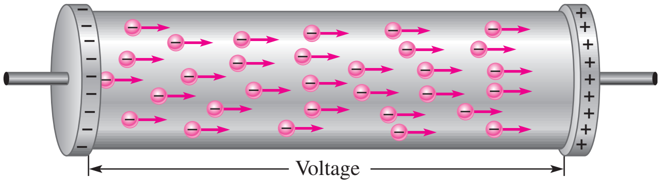
Random motion of free electrons in a material.



If a voltage is placed across a conductive or semiconductive material, one end becomes positive and the other negative, as indicated in Figure 17. The repulsive force produced by the negative voltage at the left end causes the free electrons (negative charges) to move toward the right. The attractive force produced by the positive voltage at the right end pulls the free electrons to the right. The result is a net movement of the free electrons from the negative end of the material to the positive end, as shown in Figure 17.

▶ FIGURE 17

Electrons flow from negative to positive when a voltage is applied across a conductive or semiconductive material.



The movement of these free electrons from the negative end of the material to the positive end is the electrical current, symbolized by *I*.

Electrical current is the rate of flow of charge.

Current in a conductive material is determined by the number of electrons (amount of charge) that flow past a point in a unit of time.

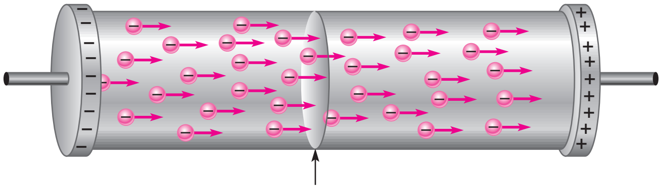
Equation 3

$$I = \frac{Q}{t}$$

where I is current in amperes (A), Q is charge in coulombs (C), and t is time in seconds (s).

One ampere (1 A) is the amount of current that exists when a number of electrons having a total charge of one coulomb (1 C) move through a given cross-sectional area in one second (1 s).

See Figure 18. Remember, one coulomb is the charge carried by 6.25×10^{18} electrons.



When a number of electrons having a total charge of 1 C pass through a cross-sectional area in 1 s, there is 1 A of current.

▲ FIGURE 18

Illustration of 1 A of current (1 C/s) in a material.

HISTORY NOTE

André Marie Ampère
1775–1836

In 1820 Ampère, a Frenchman, developed a theory of electricity and magnetism that was fundamental for 19th century developments in the field. He was the first to build an instrument to measure charge flow (current). The unit of electrical current is named in his honor. (Photo credit: AIP Emilio Segrè Visual Archives.)

EXAMPLE 3 Ten coulombs of charge flow past a given point in a wire in 2 s. What is the current in amperes?

Solution

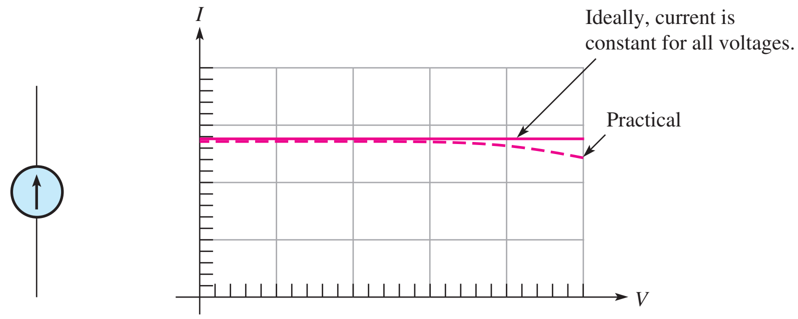
$$I = \frac{Q}{t} = \frac{10\text{ C}}{2\text{ s}} = \mathbf{5\ A}$$

Related Problem If there are 8 A of current through the filament of a lamp, how many coulombs of charge move through the filament in 1.5 s?

The Current Source

The Ideal Current Source As you know, an ideal voltage source can provide a constant voltage for any load. An ideal **current source** can provide a constant current in any load. Just as in the case of a voltage source, the ideal current source does not exist but can be approximated in practice. We will assume ideal unless otherwise specified.

The symbol for a current source is shown in Figure 19(a). The graph for an ideal current source is a horizontal line as illustrated in Figure 19(b). This is called the *IV* characteristic. Notice that the current is constant for any voltage across the current source. In a practical current source, the current decreases slightly with voltage, as shown by the dashed line.



(a) Symbol (b) *IV* characteristic

▲ **FIGURE 19**

The current source.

SAFETY NOTE

Current sources change the output voltage in order to supply a constant current to the load. For example, a meter calibrator can have a different output voltage that depends on the meter under test. You should never touch the leads from a current source; the voltage can be high, and a shock will result, particularly if the load is a high resistance load or the load is disconnected when the current source is turned on.

Practical Current Sources Power supplies are normally thought of as voltage sources because they are the most common type of source in the laboratory. However, current sources are another type of energy source. Current sources may be "stand-alone" instruments or may be combined with other instruments, such as a voltage source, DMM, or function generator. Examples of combination instruments are the source-measurement units shown in Figure 20.

◀ **FIGURE 20**

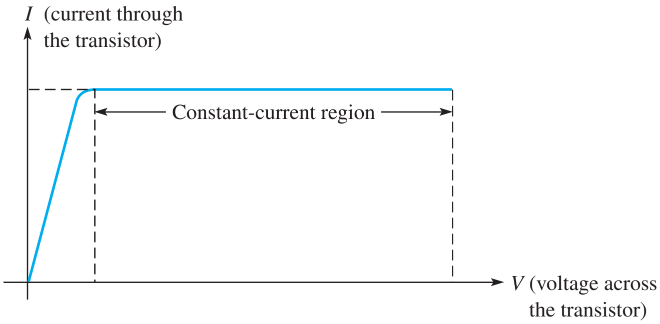
Typical source-measurement instruments. (Courtesy of Keithley Instruments).

These units can be set up as voltage or current sources and include a built-in DMM, as well as other instruments. They are used primarily for testing transistors and other semiconductors.

In most transistor circuits, the transistor acts as a current source because part of the *IV* characteristic curve is a horizontal line as shown by the transistor characteristic in Figure 21. The flat part of the graph indicates where the transistor current is constant over a range of voltages. The constant-current region is used to form a constant-current source.

▶ FIGURE 21

Characteristic curve of a transistor showing the constant-current region.



SECTION 4 CHECKUP

1. Define *current* and state its unit.
2. How many electrons make up one coulomb of charge?
3. What is the current in amperes when 20 C flow past a point in a wire in 4 s?

5 RESISTANCE

When there is current through a material, the free electrons move through the material and occasionally collide with atoms. These collisions cause the electrons to lose some of their energy, and thus their movement is restricted. The more collisions, the more the flow of electrons is restricted. This restriction varies and is determined by the type of material. The property of a material that restricts the flow of electrons is called *resistance*, designated with an *R*.

After completing this section, you should be able to

- **Define *resistance* and discuss its characteristics**
  - Name and define the unit of resistance
  - Describe the basic types of resistors
  - Determine resistance value by color code or labeling

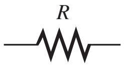
When there is current through a material, the free electrons move through the material and occasionally collide with atoms. These collisions cause the electrons to lose some of their energy, thus restricting their movement. The more collisions, the more the flow of electrons is restricted. This restriction varies and is determined by the type of material. The property of a material to restrict or oppose the flow of electrons is called resistance, *R*.

Resistance is the opposition to current.

Resistance is expressed in ohms, symbolized by the Greek letter omega (Ω).

One ohm (1 Ω) of resistance exists if there is one ampere (1 A) of current in a material when one volt (1 V) is applied across the material.

The schematic symbol for resistance is shown in Figure 22.



▲ FIGURE 22

Resistance symbol.

Conductance The reciprocal of resistance is **conductance**, symbolized by G. It is a measure of the ease with which current is established. The formula is

$$G = \frac{1}{R}$$

Equation 4

The unit of conductance is the **siemens**, abbreviated S. For example, the conductance of a 22 kΩ resistor is

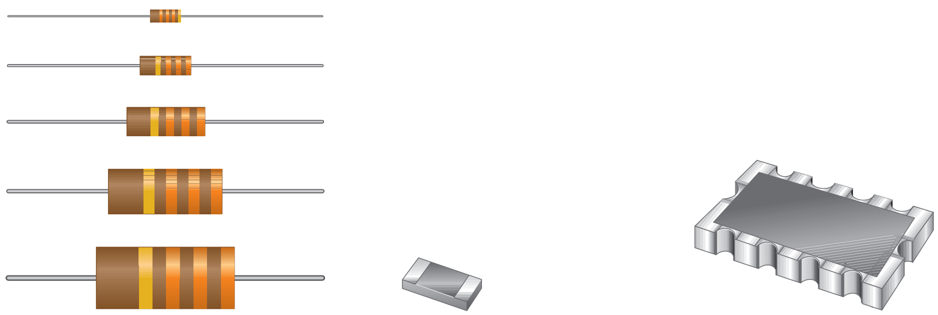
$$G = \frac{1}{22\ \text{k}\Omega} = 45.5\ \mu\text{S}$$

The obsolete unit of *mho* (ohm spelled backwards) was previously used for conductance.

Resistors

A component that is specifically designed to have a certain amount of resistance is called a **resistor**. The principal applications of resistors are to limit current in a circuit, to divide voltage, and, in certain cases, to generate heat. Although resistors come in many shapes and sizes, they can all be placed in one of two main categories: fixed or variable.

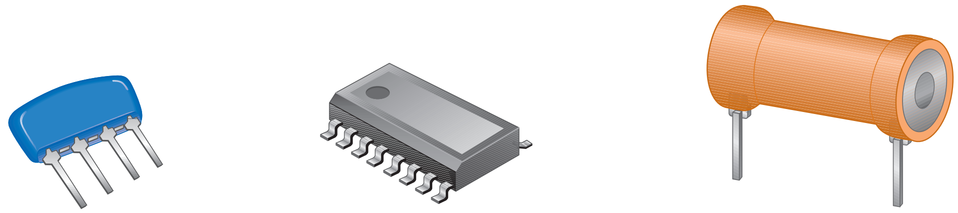
Fixed Resistors Fixed resistors are available with a large selection of resistance values that are set during manufacturing and cannot be changed easily. They are constructed using various methods and materials. Figure 23 shows several common types.



(a) Carbon-composition resistors with various power ratings

(b) Metal film chip resistor

(c) Chip resistor array



(d) Resistor network (simm)

(e) Resistor network (surface mount)

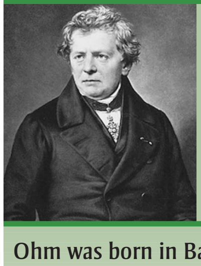
(f) Radial-lead for PC board insertion

▲ FIGURE 23

Typical fixed resistors.

One common fixed resistor is the carbon-composition type, which is made with a mixture of finely ground carbon, insulating filler, and a resin binder. The ratio of carbon to insulating filler sets the resistance value. The mixture is formed into rods, and conductive lead connections are made. The entire resistor is then encapsulated in an insulated coating for protection. Figure 24(a) shows the construction of a typical carbon-composition resistor.

HISTORY NOTE



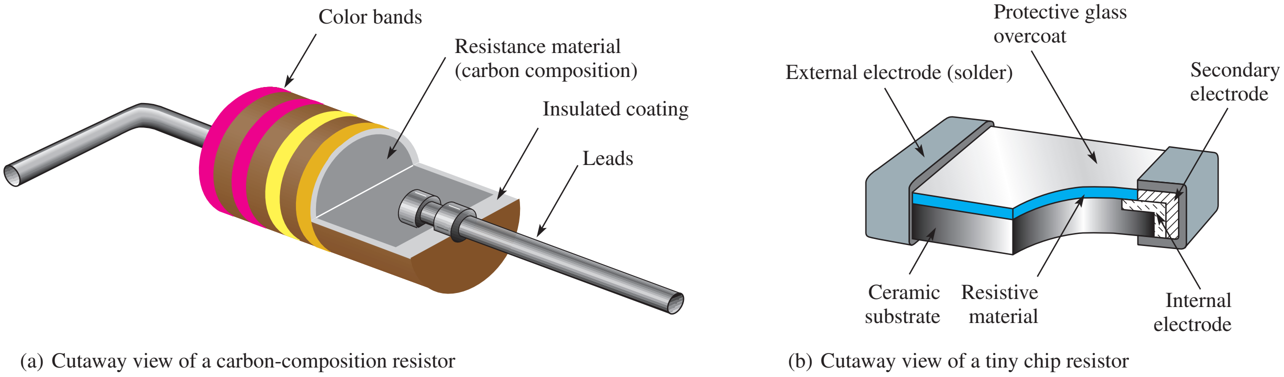
Georg Simon Ohm 1787–1854

Ohm was born in Bavaria and struggled for years to gain recognition for his work in formulating the relationship of current, voltage, and resistance. This mathematical relationship is known today as Ohm's law and the unit of resistance is named in his honor. (Photo credit: Library of Congress, LC-USZ62-40943.)

HISTORY NOTE

Ernst Werner von Siemens 1816–1872

Siemens was born in Prussia. While in prison for acting as a second in a duel, he began to experiment with chemistry, which led to his invention of the first electroplating system. In 1837, Siemens began making improvements in the early telegraph and contributed greatly to the development of telegraphic systems. The unit of conductance is named in his honor. (Photo credit: AIP Emilio Segrè Visual Archives, E. Scott Barr Collection.)


Color bands
Resistance material (carbon composition)
Insulated coating
Leads
Protective glass overcoat
External electrode (solder)
Secondary electrode
Ceramic substrate
Resistive material
Internal electrode

(a) Cutaway view of a carbon-composition resistor

(b) Cutaway view of a tiny chip resistor

▲ FIGURE 24

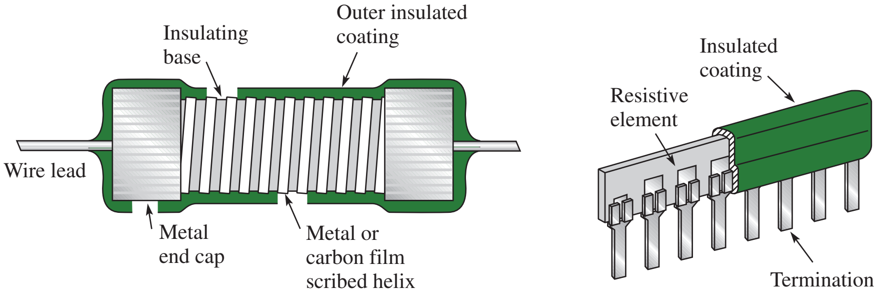
Two types of fixed resistors (not to scale).

The chip resistor is another type of fixed resistor and is in the category of SMT (surface mount technology) components. It has the advantage of a very small size for compact assemblies. Figure 24(b) shows the construction of a chip resistor.

Other types of fixed resistors include carbon film, metal film, and wirewound. In film resistors, a resistive material is deposited evenly onto a high-grade ceramic rod. The resistive film may be carbon (carbon film) or nickel chromium (metal film). In these types of resistors, the desired resistance value is obtained by removing part of the resistive material in a helical pattern along the rod using a spiraling technique, as shown in Figure 25(a). Very close **tolerance** can be achieved with this method. Film resistors are also available in the form of resistor networks, as shown in Figure 25(b).

► FIGURE 25

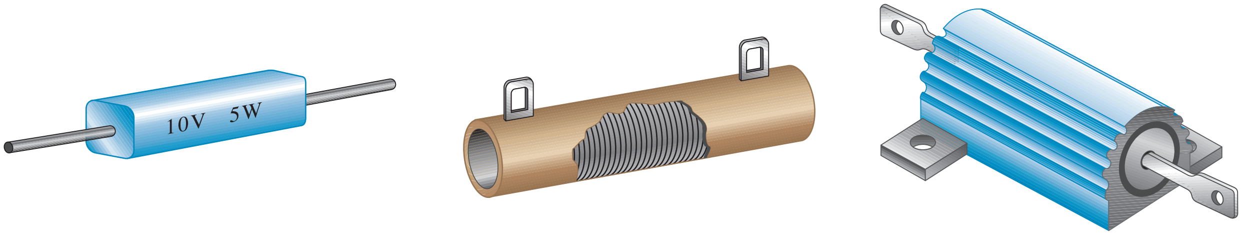
Construction views of typical film resistors.


Insulating base
Outer insulated coating
Wire lead
Metal end cap
Metal or carbon film scribed helix
Insulated coating
Resistive element
Termination

(a) Film resistor showing spiraling technique

(b) Resistor network

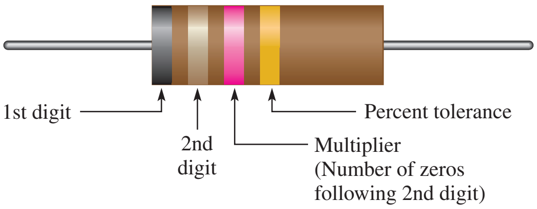
Wirewound resistors are constructed with resistive wire wound around an insulating rod and then sealed. Normally, wirewound resistors are used in applications that require higher power ratings. Since they are constructed with a coil of wire, wirewound resistors have significant inductance and are not used at higher frequencies. Some typical wirewound resistors are shown in Figure 26.



▲ FIGURE 26

Typical wirewound power resistors.

Resistor Color Codes Fixed resistors with value tolerances of 5% or 10% are color coded with four bands to indicate the resistance value and the tolerance. This color-code band system is shown in Figure 27, and the color code is listed in Table 1. The bands are always closer to one end.



◀ **FIGURE 27**

Color-code bands on a 4-band resistor.

The color code is read as follows:

1. Start with the band closest to one end of the resistor. The first band is the first digit of the resistance value. If it is not clear which is the banded end, start from the end that does not begin with a gold or silver band.
2. The second band is the second digit of the resistance value.
3. The third band is the number of zeros following the second digit, or the multiplier.
4. The fourth band indicates the percent tolerance and is usually gold or silver.

For example, a 5% tolerance means that the *actual* resistance value is within ±5% of the color-coded value. Thus, a 100 Ω resistor with a tolerance of ±5% can have an acceptable range of values from a minimum of 95 Ω to a maximum of 105 Ω.

▼ **TABLE 1**

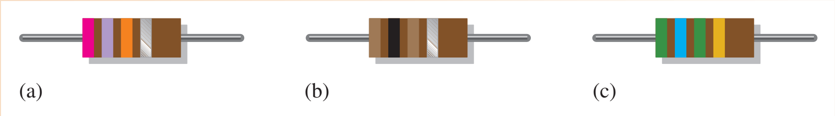
Resistor 4-band color code.

	Digit	Color	
Resistance value, first three bands: First band—1st digit Second band—2nd digit Third band—multiplier (number of zeros following the 2nd digit)	0	⬛	Black
	1	🟫	Brown
	2	🟥	Red
	3	🟧	Orange
	4	🟨	Yellow
	5	🟩	Green
	6	🟦	Blue
	7	🟪	Violet
	8	▩	Gray
	9	⬜	White
Fourth band—tolerance	±5%	🟨	Gold
	±10%	▨	Silver

For resistance values less than 10 Ω, the third band is either gold or silver. Gold represents a multiplier of 0.1, and silver represents 0.01. For example, a color code of red, violet, gold, and silver represents 2.7 Ω with a tolerance of ±10%.

EXAMPLE 4

Find the resistance value in ohms and the percent tolerance for each of the color-coded resistors shown in Figure 28.



▲ **FIGURE 28**

Solution **(a)** First band is red = 2, second band is violet = 7, third band is orange = 3 zeros, fourth band is silver = 10% tolerance.

$$R = \mathbf{27{,}000\ \Omega \pm 10\%}$$

(b) First band is brown = 1, second band is black = 0, third band is brown = 1 zero, fourth band is silver = 10% tolerance.

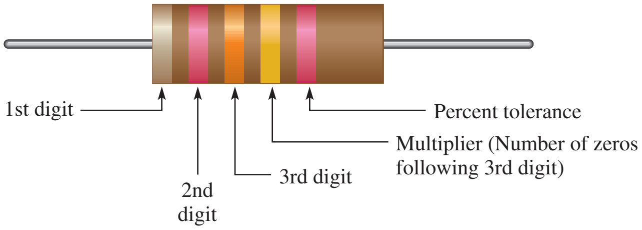
$$R = \mathbf{100\ \Omega \pm 10\%}$$

(c) First band is green = 5, second band is blue = 6, third band is green = 5 zeros, fourth band is gold = 5% tolerance.

$$R = \mathbf{5{,}600{,}000\ \Omega \pm 5\%}$$

Related Problem A certain resistor has a yellow first band, a violet second band, a red third band, and a gold fourth band. Determine its value in ohms and its percent tolerance.

Five-Band Color Code Certain precision resistors with tolerances of 2%, 1%, or less are generally color coded with five bands, as shown in Figure 29. Begin at the band closest to one end. The first band is the first digit of the resistance value, the second band is the second digit, the third band is the third digit, the fourth band is the multiplier (number of zeros after the third digit), and the fifth band indicates the percent tolerance. Table 2 shows the 5-band color code.



▲ **FIGURE 29**

Color-code bands on a 5-band resistor.

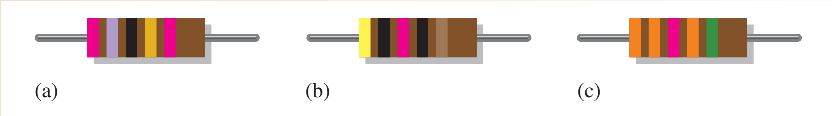
	DIGIT	COLOR
Resistance value, first	0	Black
three bands:	1	Brown
	2	Red
First band—1st digit	3	Orange
Second band—2nd digit	4	Yellow
Third band—3rd digit	5	Green
Fourth band—multiplier	6	Blue
(number of zeros	7	Violet
following 3rd digit)	8	Gray
	9	White
Fourth band—multiplier	0.1	Gold
	0.01	Silver
Fifth band—tolerance	±2%	Red
	±1%	Brown
	±0.5%	Green
	±0.25%	Blue
	±0.1%	Violet

◄ TABLE 2

Resistor 5-band color code.

EXAMPLE 5

Find the resistance value in ohms and the percent tolerance for each of the color-coded resistors shown in Figure 30.



▲ FIGURE 30

Solution

(a) First band is red = 2, second band is violet = 7, third band is black = 0, fourth band is gold = ×0.1, fifth band is red = ±2% tolerance.

$$R = 270 \times 0.1 = \mathbf{27\ \Omega \pm 2\%}$$

(b) First band is yellow = 4, second band is black = 0, third band is red = 2, fourth band is black = 0, fifth band is brown = ±1% tolerance.

$$R = \mathbf{402\ \Omega \pm 1\%}$$

(c) First band is orange = 3, second band is orange = 3, third band is red = 2, fourth band is orange = 3, fifth band is green = ±0.5% tolerance.

$$R = \mathbf{332{,}000\ \Omega \pm 0.5\%}$$

Related Problem

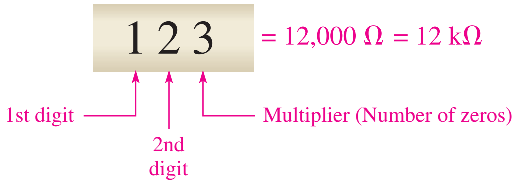
A certain resistor has a yellow first band, a violet second band, a green third band, a gold fourth band, and a red fifth band. Determine its value in ohms and its percent tolerance.

Resistor Label Codes Not all types of resistors are color coded. Many, including surface-mount resistors, use typographical marking to indicate the resistance value and tolerance. These label codes consist of either all numbers (numeric) or a combination of numbers and letters (alphanumeric). In some cases when the body of the resistor is large enough, the entire resistance value and tolerance are stamped on it in standard form.

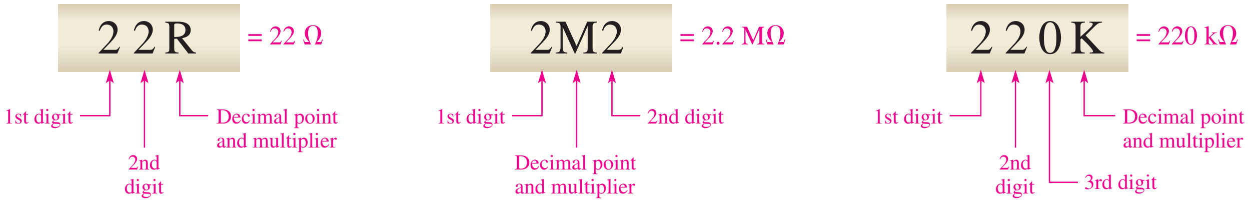
Numeric labeling uses three digits to indicate the resistance value, as shown in Figure 31 using a specific example. The first two digits give the first two digits of the resistance value, and the third digit gives the multiplier or number of zeros that follow the first two digits. This code is limited to values of 10 Ω or greater.

▶ **FIGURE 31**

Example of three-digit labeling for a resistor.


1 2 3
= 12,000 Ω = 12 kΩ
1st digit
2nd digit
Multiplier (Number of zeros)

Another common type of marking is a three- or four-character label that uses both digits and letters. An alphanumeric label typically consists of only three digits or two or three digits and one of the letters R, K, or M. The letter is used to indicate the multiplier, and the position of the letter indicates the decimal point placement. The letter R indicates a multiplier of 1 (no zeros after the digits), the K indicates a multiplier of 1000 (three zeros after the digits), and the M indicates a multiplier of 1,000,000 (six zeros after the digits). In this format, values from 100 to 999 consist of three digits and no letter to represent the three digits in the resistance value. Figure 32 shows three examples of this type of resistor label.


2 2 R
= 22 Ω
1st digit
2nd digit
Decimal point and multiplier
2M2
= 2.2 MΩ
1st digit
Decimal point and multiplier
2nd digit
2 2 0 K
= 220 kΩ
1st digit
2nd digit
3rd digit
Decimal point and multiplier

▲ **FIGURE 32**

Examples of the alphanumeric resistor label.

EXAMPLE 6

Interpret the following alphanumeric resistor labels:

(a) 470 **(b)** 5R6 **(c)** 68K **(d)** 10M **(e)** 3M3

Solution

(a) 470 = **470 Ω** **(b)** 5R6 = **5.6 Ω** **(c)** 68K = **68 kΩ**

(d) 10M = **10 MΩ** **(e)** 3M3 = **3.3 MΩ**

Related Problem

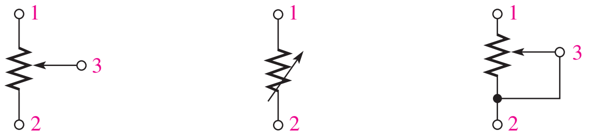
What is the resistance indicated by 1K25?

One system of labels for resistance tolerance values uses the letters F, G, and J:

$$F = \pm 1\% \qquad \text{G} = \pm 2\% \qquad \text{J} = \pm 5\%$$

For example, 620F indicates a 620 Ω resistor with a tolerance of ±1%, 4R6G is a 4.6 Ω ±2% resistor, and 56KJ is a 56 kΩ ±5% resistor.

Variable Resistors Variable resistors are designed so that their resistance values can be changed easily. Two basic uses for variable resistors are to divide voltage and to control current. The variable resistor used to divide voltage is called a **potentiometer**. The variable resistor used to control current is called a **rheostat**. Schematic symbols for these types are shown in Figure 33. The potentiometer is a three-terminal device, as indicated in part (a). Terminals 1 and 2 have a fixed resistance between them, which is the total resistance. Terminal 3 is connected to a moving contact (**wiper**). You can vary the resistance between 3 and 1 or between 3 and 2 by moving the contact.

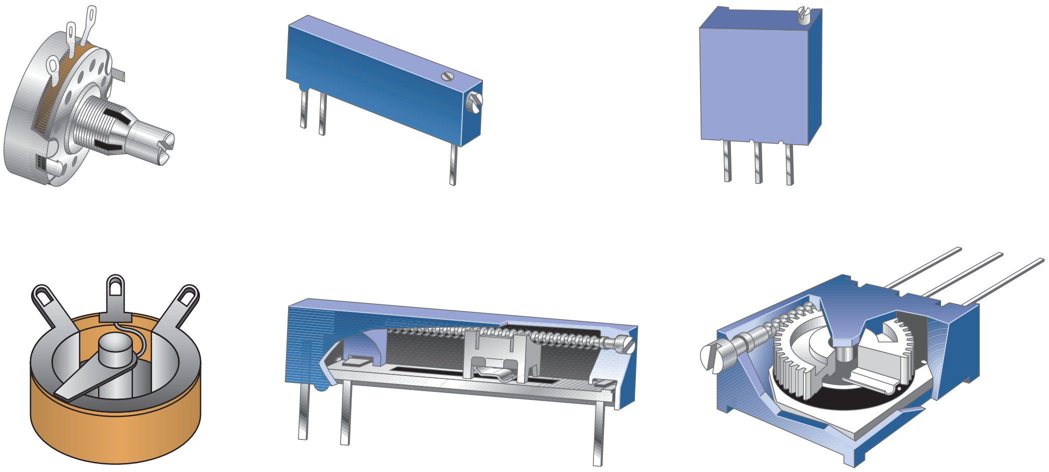


(a) Potentiometer (b) Rheostat (c) Potentiometer connected as a rheostat

◀ FIGURE 33

Potentiometer and rheostat symbols.

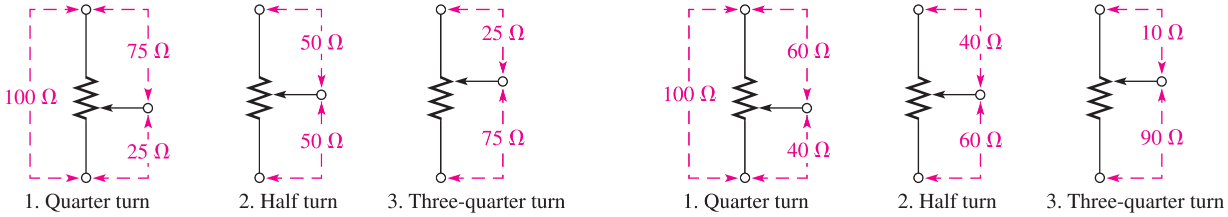
Figure 33(b) shows the rheostat as a two-terminal variable resistor. Part (c) shows how you can use a potentiometer as a rheostat by connecting terminal 3 to either terminal 1 or terminal 2. Parts (b) and (c) are equivalent symbols. Some typical potentiometers are pictured in Figure 34.



▲ FIGURE 34

Typical potentiometers and construction views.

Potentiometers and rheostats can be classified as linear or tapered, as shown in Figure 35, where a potentiometer with a total resistance of 100 Ω is used as an example. As shown in part (a), in a linear potentiometer, the resistance between either terminal and the moving contact



(a) Linear (b) Tapered (nonlinear)

▲ FIGURE 35

Examples of linear and tapered potentiometers.

varies linearly with the position of the moving contact. For example, one-half of the total contact movement results in one-half the total resistance. Three-quarters of the total movement results in three-quarters of the total resistance between the moving contact and one terminal, or one-quarter of the total resistance between the other terminal and the moving contact.

In the **tapered** potentiometer, the resistance varies nonlinearly with the position of the moving contact, so that one-half of a turn does not necessarily result in one-half the total resistance. This concept is illustrated in Figure 35(b), where the nonlinear values are arbitrary.

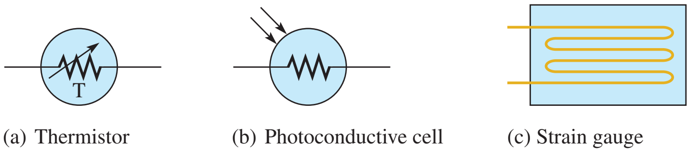
The potentiometer is used as a voltage-control device because when a fixed voltage is applied across the end terminals, a variable voltage is obtained at the wiper contact with respect to either end terminal. The rheostat is used as a current-control device because the current can be changed by changing the wiper position.

Variable Resistance Sensors Many sensors operate on the concept of a variable resistance, in which a physical quantity alters the electrical resistance. Depending on the sensor and the measurement requirements, the change in resistance may be determined directly or indirectly using the resistance change to alter a voltage or current.

Examples of resistance sensors include **thermistors** that change resistance as a function of temperature, **photoconductive cells** that change resistance as a function of light, and strain gauges that change resistance when a force is applied to them. Strain gauges are widely used in scales and applications where mechanical motion needs to be sensed. The measuring instruments need to be very sensitive because the change in resistance is very small. Figure 36 shows symbols for these various resistance sensors.

▶ **FIGURE 36**

Symbols for resistance sensors.



(a) Thermistor (b) Photoconductive cell (c) Strain gauge

SECTION 5 CHECKUP

1. Define *resistance* and name its unit.
2. What are the two main categories of resistors? Briefly explain the difference between them.
3. In the 4-band resistor color code, what does each band represent?
4. Determine the resistance and percent tolerance for each of the following color codes:
   (a) yellow, violet, red, gold **(b)** blue, red, orange, silver
   (c) brown, gray, black, gold **(d)** red, red, blue, red, green
5. What resistance value is indicated by each alphanumeric label:
   (a) 33R **(b)** 5K6 **(c)** 900 **(d)** 6M8
6. What is the basic difference between a rheostat and a potentiometer?
7. Name three resistance sensors and the physical quantity that affects their resistance.

6 THE ELECTRIC CIRCUIT

A basic electric circuit is an arrangement of physical components that use voltage, current, and resistance to perform some useful function.

After completing this section, you should be able to

- **Describe a basic electric circuit**
  - Relate a schematic to a physical circuit
  - Define *open circuit* and *closed circuit*

- Describe various types of protective devices
- Describe various types of switches
- Explain how wire sizes are related to gauge numbers
- Define *ground* or *common*

Direction of Current

For a few years after the discovery of electricity, people assumed all current consisted of moving positive charges. However, in the 1890s, the electron was identified as the charge carrier in solid conductors.

Today, there are two accepted conventions for the direction of electrical current. *Electron flow direction*, preferred by many in the fields of electrical and electronics technology, assumes for analysis purposes that current is out of the negative terminal of a voltage source, through the circuit, and into the positive terminal of the source. *Conventional current direction* assumes for analysis purposes that current is out of the positive terminal of a voltage source, through the circuit, and into the negative terminal of the source. By following the direction of conventional current, there is a rise in voltage across a source (negative to positive) and a drop in voltage across a resistor (positive to negative).

Since you cannot actually see current, only its effects, it actually makes no difference which direction of current is assumed as long as it is used *consistently.* The results of electric circuit analysis are not affected by the direction of current that is assumed for analytical purposes. The direction used for analysis is largely a matter of preference, and there are many proponents for each approach. We use conventional current direction.

Conventional current direction is also used in electronics technology and is used almost exclusively at the engineering level.

SAFETY NOTE

To avoid electrical shock, never touch a circuit while it is connected to a voltage source. If you need to handle a circuit, remove a component, or change a component, first make sure the voltage source is disconnected.

The Basic Circuit

Basically, an electric **circuit** consists of a voltage source, a load, and a path for current between the source and the load. Figure 37 shows in pictorial form an example of a simple electric circuit: a battery connected to a lamp with two conductors (wires). The battery is the voltage source, the lamp is the **load** on the battery because it draws current from the battery, and the two wires provide the current path from the positive terminal of the battery to the lamp and back to the negative terminal of the battery. Current goes through the filament of the lamp (which has a resistance), causing it to emit visible light. Current through the battery occurs by chemical action.

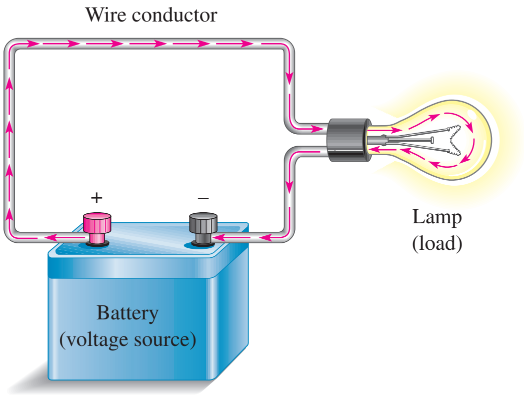
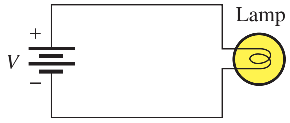


FIGURE 37

A simple electric circuit.

In many practical cases, one terminal of the battery is connected to a common or ground point. For example, in most automobiles, the negative battery terminal is connected to the metal chassis of the car. The chassis is the ground for the automobile electrical system and acts as a conductor that completes the circuit.



▲ FIGURE 38

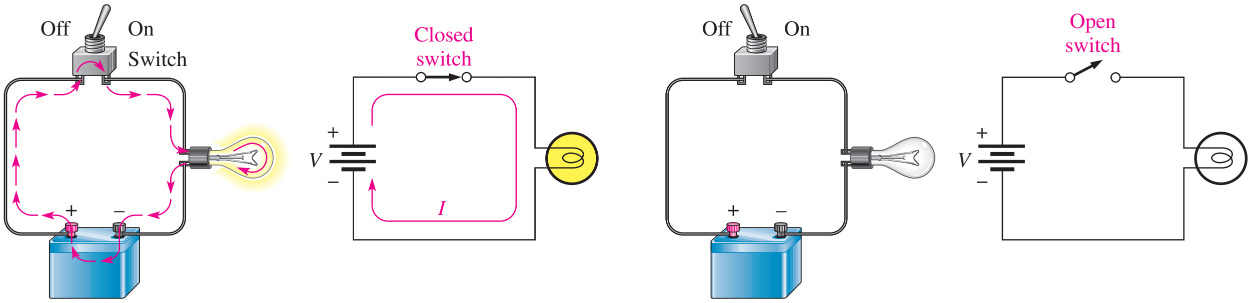
Schematic for the circuit in Figure 37.

The Electric Circuit Schematic An electric circuit can be represented by a **schematic** using standard symbols for each element, as shown in Figure 38 for the simple circuit in Figure 37. A schematic, in an organized manner, shows how the various components in a given circuit are interconnected so that the operation of the circuit can be determined.

Current Control and Protection

The example circuit in Figure 37 illustrated a **closed circuit**—that is, a circuit in which the current has a complete path. When the current path is broken, the circuit is called an **open circuit**.

Mechanical Switches **Switches** are commonly used for controlling the opening or closing of circuits. For example, a switch is used to turn a lamp on or off, as illustrated in Figure 39. Each circuit pictorial is shown with its associated schematic. The type of switch indicated is a single-pole–single-throw (SPST) toggle switch. The term *pole* refers to the movable arm in a switch, and the term *throw* indicates the number of contacts that are affected (either opened or closed) by a single switch action (a single movement of a pole).



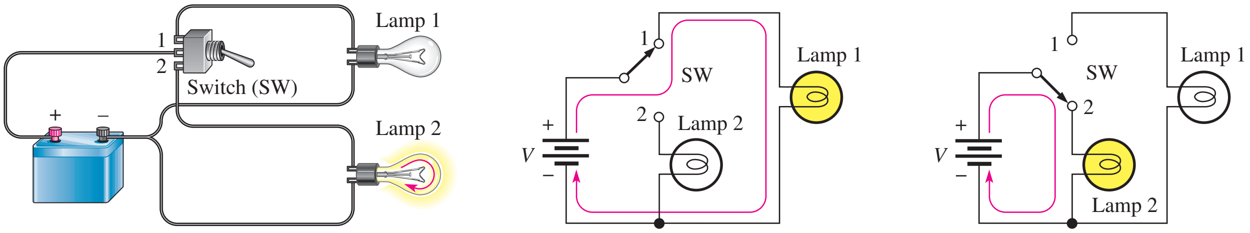
(a) There is current in a *closed* circuit because there is a complete current path (switch is ON or in the *closed* position). We almost always indicate current by a red arrow.

(b) There is no current in an *open* circuit because the path is broken (switch is OFF or in the *open* position).

▲ FIGURE 39

Illustration of closed and open circuits using an SPST switch for control.

Figure 40 shows a somewhat more complicated circuit using a single-pole–double-throw (SPDT) type of switch to control the current to two different lamps. When one lamp is on, the other is off, and vice versa, as illustrated by the two schematics in parts (b) and (c), which represent each of the switch positions.



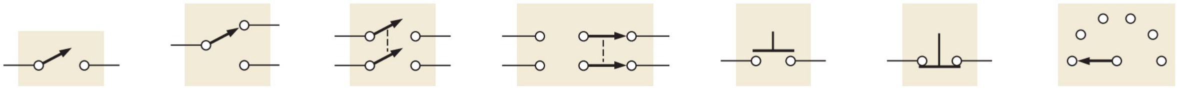
(a) Pictorial

(b) A schematic showing Lamp 1 on and Lamp 2 off

(c) A schematic showing Lamp 2 on and Lamp 1 off

▲ FIGURE 40

An example of an SPDT switch controlling two lamps.


(a) SPST (b) SPDT (c) DPST (d) DPDT (e) NOPB (f) NCPB (g) Single-pole rotary (6-position)

▲ **FIGURE 41**

Switch symbols.

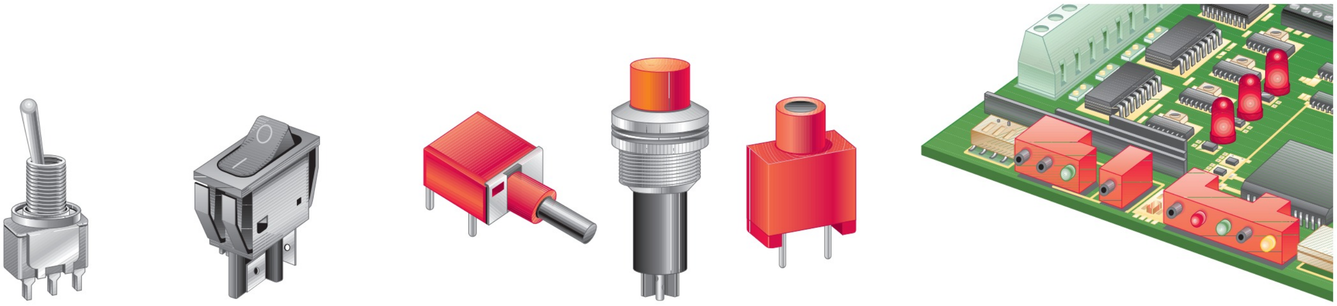
In addition to the SPST and the SPDT switches (symbols are shown in Figure 41(a) and (b)), the following other types are important:

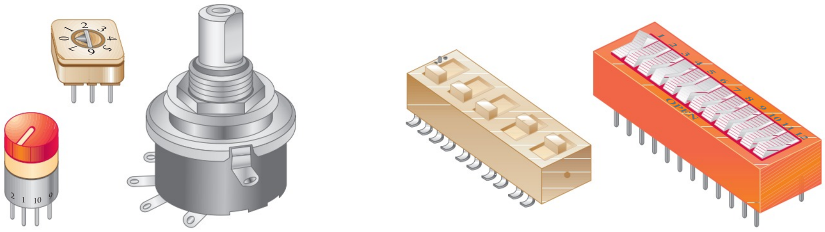
- *Double-pole–single-throw (DPST)* The DPST switch permits simultaneous opening or closing of two sets of contacts. The symbol is shown in Figure 41(c). The dashed line indicates that the contact arms are mechanically linked so that both move with a single switch action.
- *Double-pole–double-throw (DPDT)* The DPDT switch provides connection from one set of contacts to either of two other sets. The schematic symbol is shown in Figure 41(d).
- *Push-button (PB)* In the normally open push-button switch (NOPB), shown in Figure 41(e), connection is made between two contacts when the button is depressed, and connection is broken when the button is released. In the normally closed push-button switch (NCPB), shown in Figure 41(f), connection between the two contacts is broken when the button is depressed.
- *Rotary* In a rotary switch, connection between one contact and any one of several others is made by turning a knob. A symbol for a simple six-position rotary switch is shown in Figure 41(g).

TECH NOTE

Small electronic parts can be damaged easily if you apply too much heat to them when soldering. Small switches frequently are constructed with plastic that can melt and render the switch useless. Manufacturers usually supply a maximum time and temperature that can be applied to a part without damage. A small heat sink can be temporarily connected between the point where solder is applied and the sensitive region of the component.

Figure 42 shows several varieties of mechanical switches, and Figure 43 shows the construction view of a typical toggle switch.


Toggle switch Rocker switch Push-button switches PC board mounted push-button switches

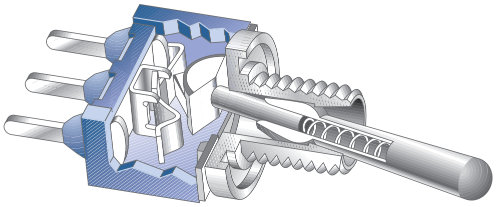

Rotary switches DIP switches for mounting on PC boards

▲ **FIGURE 42**

Typical mechanical switches.

▶ FIGURE 43

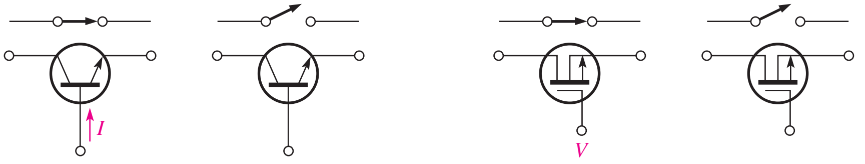
Construction view of a typical toggle switch.



Semiconductor Switches Transistors are widely used as switches in many applications. The transistor can be used as the equivalent of a single-pole–single-throw switch. You can open and close a circuit path by controlling the state of the transistor. Two types of transistor symbols are shown in Figure 44 with their mechanical switch equivalents.

▶ FIGURE 44

Transistor switches.



Current creates a closed switch.

No current creates an open switch.

(a) Bipolar transistor

Voltage creates a closed switch.

No voltage creates an open switch.

(b) Field-effect transistor

Here is a greatly simplified description of operation. One type, called the *bipolar transistor,* is controlled by current. When there is current at a specific terminal, the transistor acts as a closed switch; when there is no current at that terminal, the transistor acts as an open switch, as illustrated in Figure 44(a). Another type, called the *field-effect transistor,* is controlled by voltage. When there is voltage at a specific terminal, the transistor acts as a closed switch; when there is no voltage at that terminal, the transistor acts as an open switch, as illustrated in part (b).

SAFETY NOTE

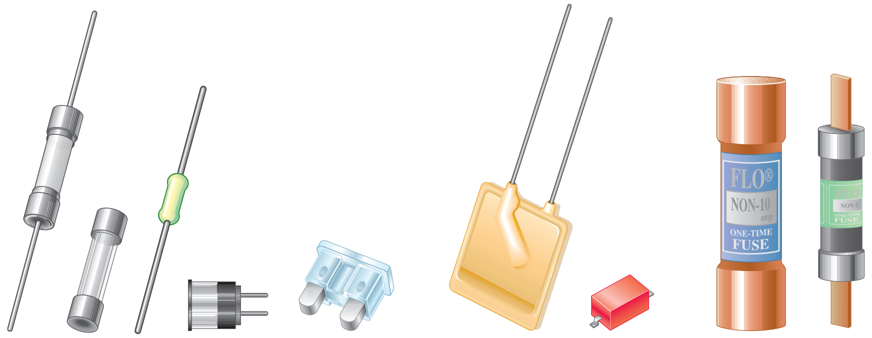
Always use fully insulated fuse pullers to remove and replace fuses in an electrical box. Even if the disconnect switch is in the off position, line voltage is still present in the box. Never use metal tools to remove and replace fuses.

Protective Devices **Fuses** and **circuit breakers** are used to deliberately create an open circuit when the current exceeds a specified number of amperes due to a malfunction or other abnormal condition in a circuit. For example, a 20 A fuse or circuit breaker will open a circuit when the current exceeds 20 A.

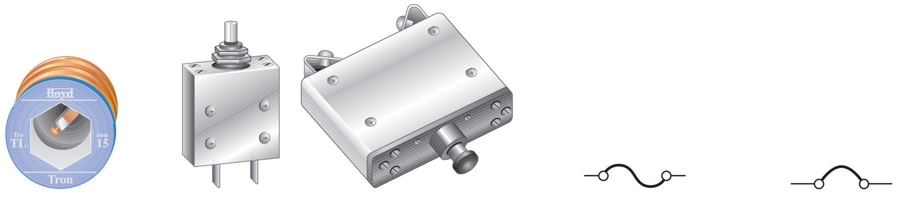
The basic difference between a fuse and a circuit breaker is that when a fuse is "blown," it must be replaced; but when a circuit breaker opens, it can be reset and reused repeatedly. Both of these devices protect against damage to a circuit due to excess current or prevent a hazardous condition created by the overheating of wires and other components when the current is too great. Several typical fuses and circuit breakers, along with their schematic symbols, are shown in Figure 45.

Two basic categories of fuses in terms of their physical configuration are cartridge type and plug type (screw in). Cartridge-type fuses have various-shaped housings with leads or other types of contacts, as shown in Figure 45(a). A typical plug-type fuse is shown in part (b). Fuse operation is based on the melting temperature of a wire or other metal element. As current increases, the fuse element heats up and when the rated current is exceeded, the element reaches its melting temperature and opens, thus removing power from the circuit.

Two common types of fuses are the fast-acting and the time-delay (slow-blow). Fast-acting fuses are type F and time-delay fuses are type T. In normal operation, fuses are often subjected to intermittent current surges that may exceed the rated current, such as when power to a circuit is turned on. Over time, this reduces the fuse's ability to withstand short surges or even current at the rated value. A slow-blow fuse can tolerate greater and longer duration surges of current than the typical fast-acting fuse. A fuse symbol is shown in Figure 45(d).



(a) Cartridge fuses



(b) Plug fuse (c) Circuit breakers (d) Fuse symbol (e) Circuit breaker symbol

▲ FIGURE 45

Typical fuses and circuit breakers and their symbols.

Typical circuit breakers are shown in Figure 45(c) and the symbol is shown in part (e). Generally, a circuit breaker detects excess current either by the heating effect of the current or by the magnetic field it creates. In a circuit breaker based on the heating effect, a bimetallic spring opens the contacts when the rated current is exceeded. Once opened, the contact is held open by mechanical means until manually reset. In a circuit breaker based on a magnetic field, the contacts are opened by a sufficient magnetic force created by excess current and must be mechanically reset.

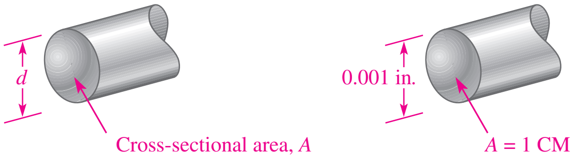
Wires

Wires are the most common form of conductive material used in electrical applications. They vary in diameter and are arranged according to standard gauge numbers, called **AWG** (American Wire Gauge) sizes. As the gauge number increases, the wire diameter decreases. The size of a wire is also specified in terms of its cross-sectional area, as illustrated in Figure 46. A unit of cross-sectional area used for wires is the **circular mil**, abbreviated CM. One circular mil is the area of a wire with a diameter of 0.001 inch (1 mil). You can find the cross-sectional area by expressing the diameter in thousandths of an inch (mils) and squaring it, as follows:

$$A = d^2$$

Equation 5

where A is the cross-sectional area in circular mils and d is the diameter in mils. Table 3 lists the AWG sizes with their corresponding cross-sectional area and resistance in ohms per 1000 ft at 20°C.



◀ FIGURE 46

Cross-sectional area of a wire.

TABLE 3

American Wire Gauge (AWG) sizes and resistances for solid round copper.

AWG #	AREA (CM)	RESISTANCE (Ω/1000 FT AT 20°C)	AWG #	AREA (CM)	RESISTANCE (Ω/1000 FT AT 20°C)
0000	211,600	0.0490	19	1,288.1	8.051
000	167,810	0.0618	20	1,021.5	10.15
00	133,080	0.0780	21	810.10	12.80
0	105,530	0.0983	22	642.40	16.14
1	83,694	0.1240	23	509.45	20.36
2	66,373	0.1563	24	404.01	25.67
3	52,634	0.1970	25	320.40	32.37
4	41,742	0.2485	26	254.10	40.81
5	33,102	0.3133	27	201.50	51.47
6	26,250	0.3951	28	159.79	64.90
7	20,816	0.4982	29	126.72	81.83
8	16,509	0.6282	30	100.50	103.2
9	13,094	0.7921	31	79.70	130.1
10	10,381	0.9989	32	63.21	164.1
11	8,234.0	1.260	33	50.13	206.9
12	6,529.0	1.588	34	39.75	260.9
13	5,178.4	2.003	35	31.52	329.0
14	4,106.8	2.525	36	25.00	414.8
15	3,256.7	3.184	37	19.83	523.1
16	2,582.9	4.016	38	15.72	659.6
17	2,048.2	5.064	39	12.47	831.8
18	1,624.3	6.385	40	9.89	1049.0

EXAMPLE 7 What is the cross-sectional area of a wire with a diameter of 0.005 inch?

Solution

$$d = 0.005 \text{ in.} = 5 \text{ mils}$$

$$A = d^2 = 5^2 = \mathbf{25\ CM}$$

Related Problem What is the cross-sectional area of a 0.0015 in. diameter wire?

Wire Resistance Although copper wire conducts electricity extremely well, it still has some resistance, as do all conductors. The resistance of a wire depends on three physical characteristics: (a) type of material, (b) length of wire, and (c) cross-sectional area. In addition, temperature can also affect the resistance.

Each type of conductive material has a characteristic called its *resistivity,* ρ. For each material, ρ is a constant value at a given temperature. The formula for the resistance of a wire of length l and cross-sectional area A is

Equation 6

$$R = \frac{\rho l}{A}$$

This formula shows that resistance increases with an increase in resistivity and length and decreases with an increase in cross-sectional area. For resistance to be calculated in ohms, the length must be in feet, the cross-sectional area in circular mils, and the resistivity in CM-Ω/ft.

EXAMPLE 8 Find the resistance of a 100 ft length of copper wire with a cross-sectional area of 810.1 CM. The resistivity of copper is 10.37 CM-Ω/ft.

Solution

$$R = \frac{\rho l}{A} = \frac{(10.37\text{ CM-}\Omega/\text{ft})(100\text{ ft})}{810.1\text{ CM}} = \mathbf{1.280\ \Omega}$$

Related Problem Use Table 3 to determine the resistance of 100 ft of copper wire with a cross-sectional area of 810.1 CM. Compare with the calculated result.

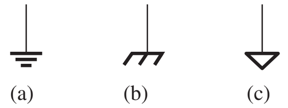
As mentioned, Table 3 lists the resistance of the various standard wire sizes in ohms per 1000 feet at 20°C. For example, a 1000 ft length of 14 gauge copper wire has a resistance of 2.525 Ω. A 1000 ft length of 22 gauge wire has a resistance of 16.14 Ω. For a given length, the smaller wire has more resistance. Thus, for a given voltage, larger wires can carry more current than smaller ones.

Ground

Ground is the reference point in electric circuits. The term *ground* originated from the fact that one conductor of a circuit was typically connected with an 8-foot long metal rod driven into the earth itself. Today, this type of connection is referred to as an *earth ground*. In household wiring, earth ground is indicated with a green or bare copper wire. Earth ground is normally connected to the metal chassis of an appliance or a metal electrical box for safety. Unfortunately, there have been exceptions to this rule, which can present a safety hazard if a metal chassis is not at earth ground. It is a good idea to confirm that a metal chassis is actually at earth ground potential before doing any work on an instrument or appliance.

Another type of ground is called a *reference ground*. Voltages are always specified with respect to another point. If that point is not stated explicitly, the reference ground is understood. Reference ground defines 0 V for the circuit. The reference ground can be at a completely different potential than the earth ground. Reference ground is also called **common** and labeled COM or COMM because it represents a common conductor. When you are wiring a protoboard in the laboratory, you will normally reserve one of the bus strips (a long line along the length of the board) for this common conductor.

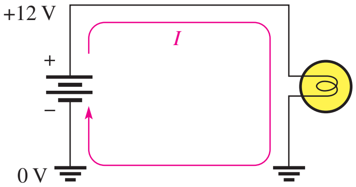
Three ground symbols are shown in Figure 47. Unfortunately, there is not a separate symbol to distinguish between earth ground and reference ground. The symbol in (a) represents either an earth ground or a reference ground, (b) shows a chassis ground, and (c) is an alternate reference symbol typically used when there is more than one common connection (such as analog and digital ground in the same circuit). We use the symbol in part (a).



▲ FIGURE 47

Commonly used ground symbols.

Figure 48 illustrates a simple circuit with ground connections. The current is from the positive terminal of the 12 V source, through the lamp, and back to the negative terminal of the source through the ground connection. Ground provides a return path for the current back to the source because all of the ground points are electrically the same point. The voltage at the top of the circuit is +12 V with respect to ground.



◀ FIGURE 48

A simple circuit with ground connections.

SECTION 6 CHECKUP

1. What are the basic elements of an electric circuit?
2. What is an open circuit?
3. What is a closed circuit?
4. What is the difference between a fuse and a circuit breaker?
5. Which wire is larger in diameter, AWG 3 or AWG 22?
6. What is ground (common) in an electric circuit?

7 Basic Circuit Measurements

In working on electrical or electronic circuits, you will frequently need to measure voltage, current, or resistance using meters safely and correctly.

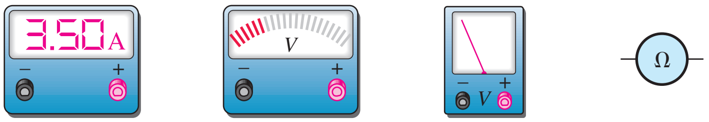
After completing this section, you should be able to

- **Make basic circuit measurements**
    - Properly measure voltage in a circuit
    - Properly measure current in a circuit
    - Properly measure resistance
    - Set up and read basic meters

Voltage, current, and resistance measurements are commonly required in electronics work. The instrument used to measure voltage is a **voltmeter**, the instrument used to measure current is an **ammeter**, and the instrument used to measure resistance is an **ohmmeter**. Commonly, all three instruments are combined into a single instrument called a **multimeter**, in which you can choose what specific quantity to measure by selecting the appropriate function with a switch.

Meter Symbols

Certain symbols can be used in circuits to represent meters, as shown in Figure 49. You may see any of four types of symbols for voltmeters, ammeters, or ohmmeters, depending on which symbol most effectively conveys the information required. The digital meter symbol is used when specific values are to be indicated in a circuit. The bar graph meter symbol and sometimes the analog meter symbol are used to illustrate the operation of a circuit when *relative* measurements or changes in quantities, rather than specific values, need to be depicted. A changing quantity may be indicated by an arrow in the display showing an increase or decrease. The generic symbol is used to indicate placement of meters in a circuit when no values or value changes need to be shown.



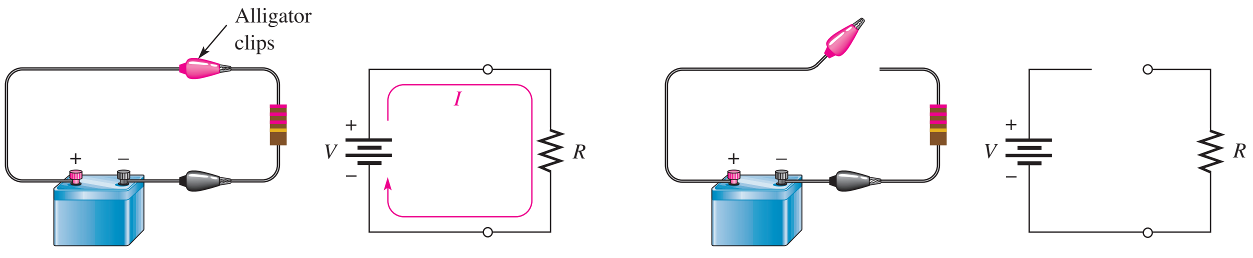
(a) Digital (b) Bar graph (c) Analog (d) Generic

▲ **FIGURE 49**

Examples of meter symbols. Each of the symbols can be used to represent either an ammeter (A), a voltmeter (V), or an ohmmeter (Ω).

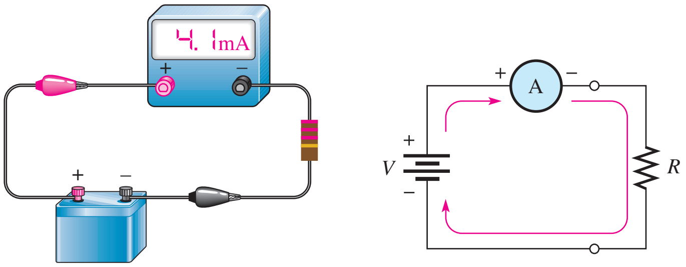
Measuring Current

Figure 50 illustrates how to measure current with an ammeter. Part (a) shows a simple circuit in which the current through the resistor is to be measured. First make sure the range setting of the ammeter is greater than the expected current and then connect the ammeter in the current path by first opening the circuit, as shown in part (b). Then insert the meter as shown in part (c). Such a connection is a series connection. The polarity of the meter must be such that the current is in at the positive terminal and out at the negative terminal.



(a) Circuit in which the current is to be measured

(b) Open the circuit either between the resistor and the positive terminal or between the resistor and the negative terminal of source.



(c) Install the ammeter in the current path with polarity as shown (negative to negative, positive to positive).

▲ **FIGURE 50**

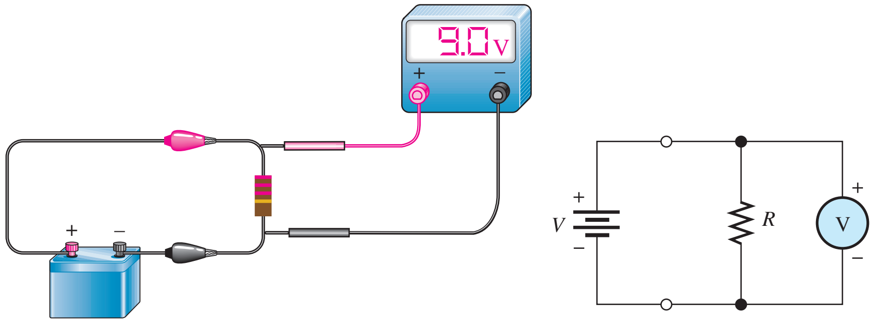
Example of an ammeter connection in a simple circuit to measure current.

Measuring Voltage

To measure voltage, connect the voltmeter across the component for which the voltage is to be found. Such a connection is a parallel connection. The negative terminal of the meter must be connected to the negative side of the circuit, and the positive terminal of the meter must be connected to the positive side of the circuit. Figure 51 shows a voltmeter connected to measure the voltage across the resistor.

SAFETY NOTE

Never wear rings or any type of metallic jewelry while working on a circuit. These items may accidentally come in contact with the circuit, causing shock and/or damage to the circuit. With high-energy sources, such as a car battery, a short across jewelry (watch or ring) can become hot instantly, causing severe burns.



▲ **FIGURE 51**

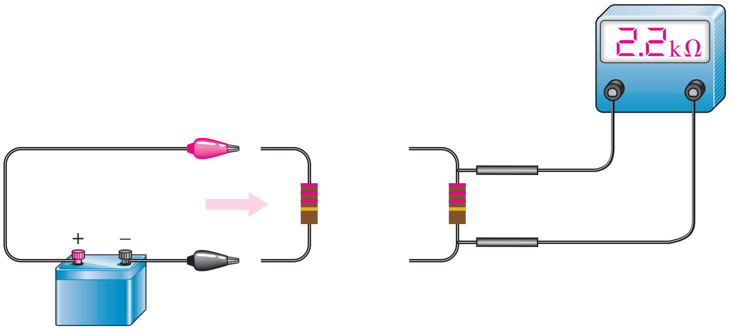
Example of a voltmeter connection in a simple circuit to measure voltage.

Measuring Resistance

To measure resistance, first turn off the power and disconnect one end or both ends of the resistor from the circuit; then connect the ohmmeter across the resistor. This procedure is shown in Figure 52.

▶ FIGURE 52

Example of using an ohmmeter to measure resistance.



(a) Disconnect the resistor from the circuit to avoid damage to the meter and/or incorrect measurement.

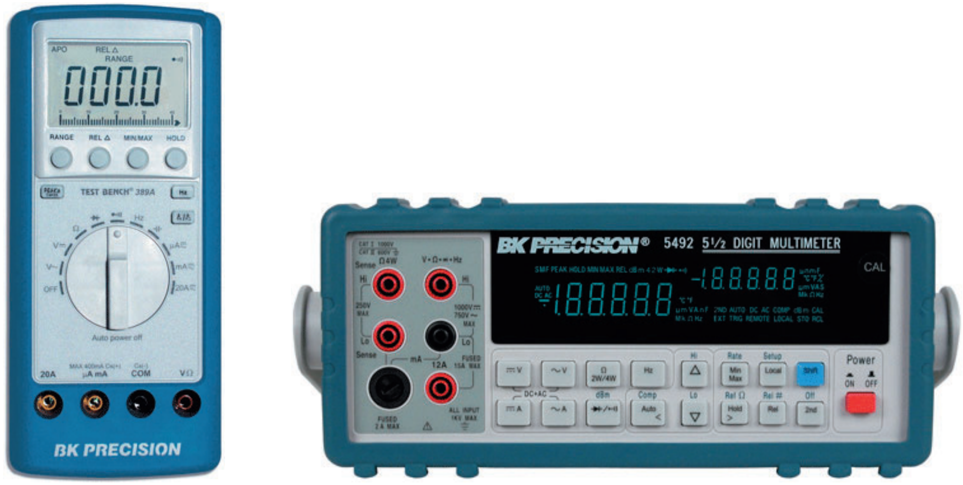
(b) Measure the resistance. (Polarity is not important.)

Digital Multimeters

A **DMM** (digital multimeter) is a multifunction electronic instrument that can measure voltage, current, and resistance. DMMs are the most widely used type of electronic measuring instrument. Generally, DMMs provide more functions, better accuracy, greater ease of reading, and greater reliability than do many analog meters. Analog meters have at least one advantage over DMMs, however. They can track short-term variations and trends in a measured quantity that many DMMs are too slow to respond to. Figure 53 shows typical DMMs.

▶ FIGURE 53

Typical digital multimeters (DMMs). (Courtesy of B+K Precision)



(a)

(b)

DMM Functions The basic functions found on most DMMs include the following:

- Ohms
- DC voltage and current
- AC voltage and current

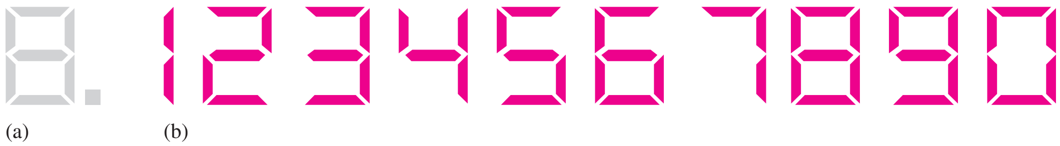
Some DMMs provide special functions such as transistor or diode tests, power measurement, and decibel measurement for audio amplifier tests. Some meters require manual selection of the ranges for the functions. Many meters provide automatic range selection and are called *autoranging.*

SAFETY NOTE

Manufacturers use the CE mark to indicate that a product meets all the essential requirements of the European Directives for health and safety. For example, DMMs that meet the directive IEC 61010-1 are in compliance and can use the CE mark on the DMM. This mark is used on many products in various countries much like the UL (underwriters lab) mark that originated in the U.S. It also shows compliance with safety standards.

DMM Displays DMMs are available with either LCD (liquid-crystal display) or LED (light-emitting diode) readouts. The LCD is the most commonly used readout in battery-powered instruments because it requires only very small amounts of current. A typical battery-powered DMM with an LCD readout operates on a 9 V battery that will last from a few hundred hours to 2000 hours and more. The disadvantages of LCD readouts are that (a) they are difficult or impossible to see in low-light conditions and (b) they are relatively slow to respond to measurement changes. LEDs, on the other hand, can be seen in the dark and respond quickly to changes in measured values. LED displays require much more current than LCDs, and, therefore, battery life is shortened when LEDs are used in portable equipment.

Both LCD and LED DMM displays are in a 7-segment format. Each digit in a display consists of seven separate segments as shown in Figure 54(a). Each of the ten decimal digits is formed by the activation of appropriate segments, as illustrated in Figure 54(b). In addition to the seven segments, there is also a decimal point.



▲ FIGURE 54

Seven-segment display.

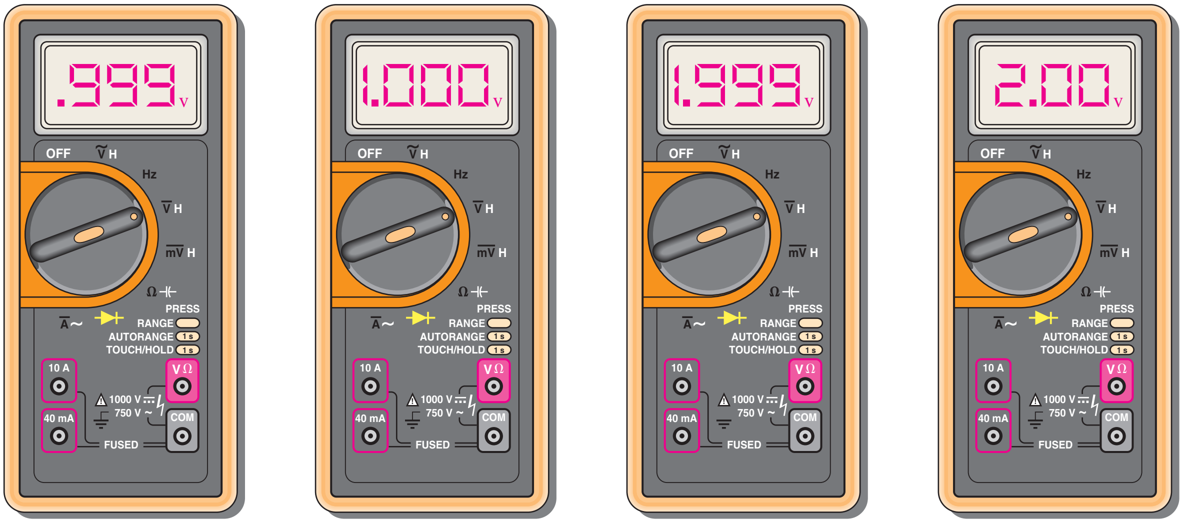
Resolution The **resolution** of a DMM is the smallest increment of a quantity that the meter can measure. The smaller the increment, the better the resolution. One factor that determines the resolution of a meter is the number of digits in the display.

Because many DMMs have 3½ digits in their display, we will use this case for illustration. A 3½-digit multimeter has three digit positions that can indicate from 0 through 9, and one digit position that can indicate only a value of 0 or 1. This latter digit, called the *half-digit,* is always the most significant digit in the display. For example, suppose that a DMM is reading 0.999 V, as shown in Figure 55(a). If the voltage increases by 0.001 V to 1 V, the display correctly shows 1.000 V, as shown in part (b). The “1” is the half-digit. Thus, with 3½ digits, a variation of 0.001 V, which is the resolution, can be observed.

Now, suppose that the voltage increases to 1.999 V. This value is indicated on the meter as shown in Figure 55(c). If the voltage increases by 0.001 V to 2 V, the half-digit cannot display the “2,” so the display shows 2.00. The half-digit is blanked and only three digits are active, as indicated in part (d). With only three digits active, the resolution is 0.01 V rather than 0.001 V as it is with 3½ active digits. The resolution remains 0.01 V up to 19.99 V. The resolution goes to 0.1 V for readings of 20.0 V to 199.9 V. At 200 V, the resolution goes to 1 V, and so on.

The resolution capability of a DMM is also determined by the internal circuitry and the rate at which the measured quantity is sampled. DMMs with displays of 4½ through 8½ digits are also available.

Accuracy The **accuracy** is the degree to which a measured value represents the true or accepted value of a quantity. The accuracy of a DMM is established strictly by its internal circuitry and calibration. For typical DMMs, accuracies range from 0.01% to 0.5%, with some precision laboratory-grade DMMs going to 0.002%.


.999 V
1.000 V
1.999 V
2.00 V
OFF
Hz
mV H
PRESS
RANGE
AUTORANGE
TOUCH/HOLD
10 A
40 mA
VΩ
COM
1000 V
750 V
FUSED

(a) Resolution: 0.001 V (b) Resolution: 0.001 V (c) Resolution: 0.001 V (d) Resolution: 0.01 V

▲ **FIGURE 55**

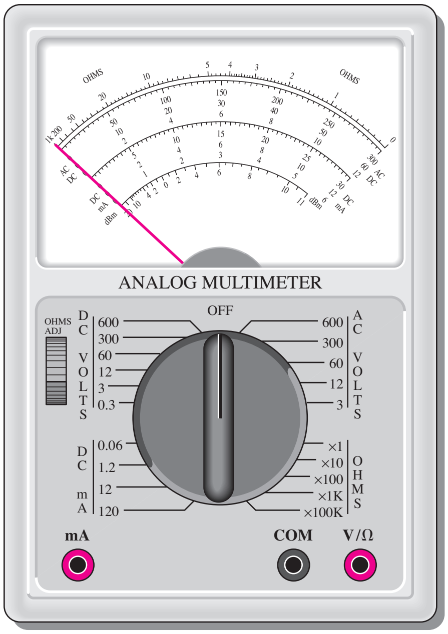
A 3½-digit DMM illustrates how the resolution changes with the number of digits in use.

Reading Analog Multimeters

Although the DMM is the predominate type of multimeter, you may occasionally have to use an analog meter. A representation of a typical analog multimeter is shown in Figure 56. This particular instrument can be used to measure both direct current (dc) and alternating current (ac) quantities as well as resistance values. It has four selectable functions: dc volts (DC VOLTS), dc milliamperes (DC mA), ac volts (AC VOLTS), and OHMS. Most analog multimeters are similar to this one.

► **FIGURE 56**

A typical analog multimeter.


OHMS
ANALOG MULTIMETER
OFF
OHMS ADJ
DC VOLTS
600
300
60
12
3
0.3
AC VOLTS
600
300
60
12
3
DC mA
0.06
1.2
12
120
OHMS
×1
×10
×100
×1K
×100K
mA
COM
V/Ω

Within each function there are several ranges, as indicated by the brackets around the selector switch. For example, the DC VOLTS function has 0.3 V, 3 V, 12 V, 60 V, 300 V, and 600 V ranges. Thus, dc voltages from 0.3 V full-scale to 600 V full-scale can be measured. On the DC mA function, direct currents from 0.06 mA full-scale to 120 mA full-scale can be measured. On the ohm scale, the settings are ×1, ×10, ×100, ×1000, and ×100,000.

The Ohm Scale Ohms are read on the top scale of the meter. This scale is nonlinear; that is, the values represented by each division (large or small) vary as you go across the scale. In Figure 56, notice how the scale becomes more compressed as you go from right to left.

To read the actual value in ohms, multiply the number on the scale as indicated by the pointer by the factor selected by the switch. For example, when the switch is set at ×100 and the pointer is at 20, the reading is 20 × 100 = 2000 Ω.

As another example, assume that the switch is at ×10 and the pointer is at the seventh small division between the 1 and 2 marks, indicating 17 Ω (1.7 × 10). Now, if the meter remains connected to the same resistance and the switch setting is changed to ×1, the pointer will move to the second small division between the 15 and 20 marks. This, of course, is also a 17 Ω reading, illustrating that a given resistance value can often be read at more than one range switch setting. However, the meter should be *zeroed* each time the range is changed by touching the leads together and adjusting the needle.

The AC-DC and DC mA Scales The second, third, and fourth scales from the top, labeled “AC” and “DC,” are used in conjunction with the DC VOLTS and AC VOLTS functions. The upper ac-dc scale ends at the 300 mark and is used with range settings, such as 0.3, 3, and 300. For example, when the switch is at 3 on the DC VOLTS function, the 300 scale has a full-scale value of 3 V; at the range setting of 300, the full-scale value is 300 V. The middle ac-dc scale ends at 60. This scale is used in conjunction with range settings, such as 0.06, 60, and 600. For example, when the switch is at 60 on the DC VOLTS function, the full-scale value is 60 V. The lower ac-dc scale ends at 12 and is used in conjunction with switch settings, such as 1.2, 12, and 120. The three DC mA scales are used in a similar way to measure current.

EXAMPLE 9

Determine the quantity (voltage, current, or resistance) that is being measured and its value for each specified switch setting on the meter in Figure 57.

(a) The switch is set on the DC VOLTS function and the 60 V range.

(b) The switch is set on the DC mA function and the 12 mA range.

(c) The switch is set on the OHMS function and the ×1K range.

Solution

(a) The reading taken from the middle AC-DC scale is **18 V**.

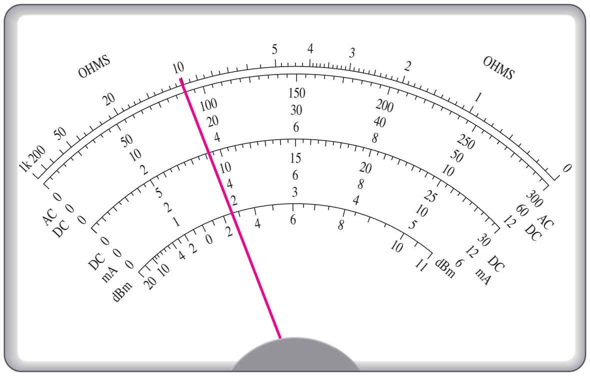
(b) The reading taken from the lower AC-DC scale is **3.8 mA**.

(c) The reading taken from the ohms scale (top) is **10 kΩ**.

Related Problem

In Figure 57 the switch is moved to the ×100 ohms setting. Assuming that the same resistance is being measured as in part (c), what will the needle do?

▶ FIGURE 57



SECTION 7 CHECKUP

1. Name the meters for measurement of (a) current, (b) voltage, and (c) resistance.
2. Show how to place two ammeters in the circuit of Figure 40 to measure the current through either lamp (be sure to observe the polarities). How can the same measurements be accomplished with only one ammeter?
3. Show how to place a voltmeter to measure the voltage across Lamp 2 in Figure 40.
4. List two common types of DMM displays, and discuss the advantages and disadvantages of each.
5. Define *resolution* in a DMM.
6. The analog multimeter in Figure 56 is set on the 3 V range to measure dc voltage. The pointer is at 150 on the upper ac-dc scale. What voltage is being measured?

8 Electrical Safety

Safety is a major concern when working with electricity. The possibility of an electric shock or a burn is always present, so caution should always be used. You provide a current path when voltage is applied across two points on your body, and current produces electrical shock. Electrical components often operate at high temperatures, so you can sustain skin burns when you come in contact with them. Also, the presence of electricity creates a potential fire hazard.

After completing this section, you should be able to

- **Recognize electrical hazards and practice proper safety procedures**
    - Describe the cause of electrical shock
    - List the groups of current paths through the body
    - Discuss the effects of current on the human body
    - List the safety precautions that you should observe when you work with electricity

Electric Shock

Current through your body, not the voltage, is the cause of **electrical shock**. Of course, it takes voltage across a resistance to produce current. When a point on your body comes in contact with a voltage and another point comes in contact with a different voltage or with ground, such as a metal chassis, there will be current through your body from one point to the other. The path of the current depends on the points across which the voltage occurs. The severity of the resulting electrical shock depends on the amount of voltage and the path that the current takes through your body. The current path through the body determines which tissues and organs will be affected.

Effects of Current on the Human Body The amount of current is dependent on voltage and resistance. The human body has resistance that depends on many factors, which include body mass, skin moisture, and points of contact of the body with a voltage potential. Table 4 shows the effects for various values of current in milliamperes.

Body Resistance Resistance of the human body is typically between 10 kΩ and 50 kΩ and depends on the two points between which it is measured. The moisture of the skin also affects the resistance between two points. The resistance determines the amount of voltage

CURRENT (mA)	PHYSICAL EFFECT
0.4	Slight sensation
1.1	Perception threshold
1.8	Shock, no pain, no loss of muscular control
9	Painful shock, no loss of muscular control
16	Painful shock, let-go threshold
23	Severe painful shock, muscular contractions, breathing difficulty
75	Ventricular fibrillation, threshold
235	Ventricular fibrillation, usually fatal for duration of 5 seconds or more
4,000	Heart paralysis (no ventricular fibrillation)
5,000	Tissue burn

◀ **TABLE 4**

Physical effects of electrical current. Values vary depending on body mass.

required to produce each of the effects listed in Table 4. For example, if you have a resistance of 10 kΩ between two given points on your body, 90 V across those two points will produce enough current (9 mA) to cause painful shock.

Utility Voltages

We tend to take utility voltages for granted, but they can be and have been lethal. It is best to be careful around any source of voltage (even low voltages can present a serious burn hazard). As a general rule, you should avoid working on any energized circuit, and check that the power is off with a known good meter. Most work in educational labs uses low voltages, but you should still avoid touching any energized circuit. If you are working on a circuit that is connected to utility voltages, the service should be disconnected, a notice should be placed on the equipment or place where the service is disconnected, and a padlock should be used to prevent someone from accidentally turning on the power. This procedure is called *lockout/tagout* and is widely used in industry. There are specific OSHA and industry standards for lockout/tagout.

Most laboratory equipment is connected to the utility line ("ac") and in North America, this is 120 Vrms. A faulty piece of equipment can cause the "hot" lead to inadvertently become exposed. You should inspect cords for exposed wires and check equipment for missing covers or other potential safety problems. The single-phase utility lines in homes and electrical laboratories use three insulated wires that are referred to as the "hot" (black or red wire), neutral (white wire), and safety ground (green wire). The hot and neutral wires will have current, but the green safety line should never have current in normal operation. The safety wire is connected to the metal exterior of encased equipment and is also connected to conduit and the metal boxes for housing receptacles. Figure 58(a) shows the location of these conductors on a standard receptacle. Notice that the neutral line is larger than the hot line.

The safety ground should be connected to the neutral at the service panel. The metal chassis of an instrument or appliance is also connected to ground. In the event that the hot wire is accidentally in contact with ground, the resulting high current should trip the circuit breaker or open a fuse to remove the hazard. However, a broken or missing ground lead may not have high current until it is contacted by a person. This danger is one obvious reason for ensuring that line cords have not been altered by removing the ground pin.

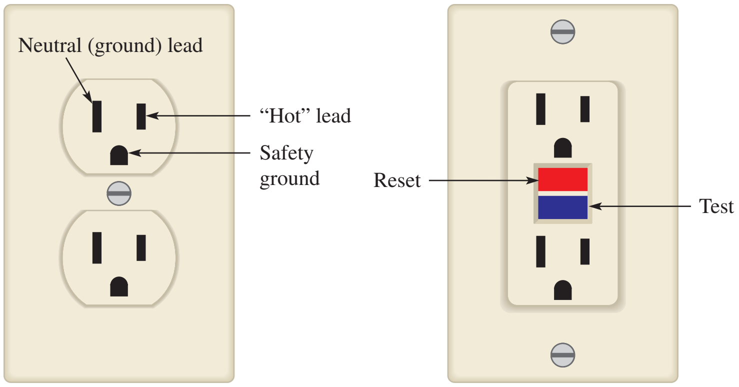
Many circuits are further protected with a special device called a ground-fault circuit interrupter (GFCI, which is sometimes called just GFI). If a fault occurs in a GFCI circuit, a sensor detects that the current in the hot line and the neutral line are not equal as they should be and trips the circuit breaker. The GFCI breaker is very fast acting and can trip faster than the breaker on the main panel. GFCI breakers are required in areas where a

TECH TIP

Receptacle testers are designed for use with specific receptacle types including specialized outlets. They can pinpoint problems such as open lines, faulty wiring, or reversed polarity; they show results with a lighted LED or neon bulb. Some testers are designed to test ground fault circuit interrupters (GFCI) for proper operation.

▶ **FIGURE 58**

Standard electrical outlets.



(a) Standard receptacle and connections

(b) GFCI receptacle

shock hazard exists such as wherever there is water or moisture. Pools, bathrooms, kitchens, basements, and garages should all have GFCI outlets. Figure 58(b) shows a ground-fault receptacle with reset and test buttons. When the test button is pressed, the circuit should immediately be opened. The reset button restores power.

SAFETY NOTE

A GFCI outlet does not prevent shock or injury in all cases. If you are touching the hot and neutral wires without being grounded, no ground fault is detected and the GFCI breaker will not trip. In another case, the GFCI may prevent electrocution but not the initial electric shock before it interrupts the circuit. The initial shock could cause a secondary injury, such as a fall.

Safety Precautions

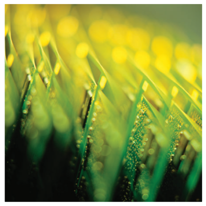
There are many practical things that you should do when you work with electrical and electronic equipment. Some important precautions are listed here.

- Avoid contact with any voltage source. Turn power off before you work on circuits when touching circuit parts is required.
- Do not work alone. A telephone should be available for emergencies.
- Do not work when tired or taking medications that make you drowsy.
- Remove rings, watches, and other metallic jewelry when you work on circuits.
- Do not work on equipment until you know proper procedures and are aware of potential hazards.
- Make sure power cords are in good condition and grounding pins are not missing or bent.
- Keep your tools properly maintained. Make sure the insulation on metal tool handles is in good condition.
- Handle tools properly and maintain a neat work area.
- Wear safety glasses when appropriate, particularly when soldering and clipping wires.
- Always shut off power and discharge capacitors before you touch any part of a circuit with your hands.
- Know the location of the emergency power-off switch and emergency exits.
- Never try to override or tamper with safety devices such as an interlock switch.
- Always wear shoes and keep them dry. Do not stand on metal or wet floors when working on electrical circuits.
- Never handle instruments when your hands are wet.
- Never assume that a circuit is off. Double-check it with a reliable meter before handling.
- Set the limiter on electronic power supplies to prevent currents larger than necessary to supply the circuit under test.

- Some devices such as capacitors can store a lethal charge for long periods after power is removed. They must be properly discharged before you work with them.
- When making circuit connections, always make the connection to the point with the highest voltage as your last step.
- Avoid contact with the terminals of power supplies.
- Always use wires with insulation and connectors or clips with insulating shrouds.
- Keep cables and wires as short as possible. Connect polarized components properly.
- Be aware of and follow all workplace and laboratory rules. Do not have drinks or food near equipment.
- If another person cannot let go of an energized conductor, switch the power off immediately. If that is not possible, use any available nonconductive material to try to separate the body from the contact.
- Use a lockout/tagout procedure to avoid someone turning power on while you are working on a circuit.

SECTION 8 CHECKUP

1. What causes physical pain and/or damage to the body when electrical contact is made?
2. It's OK to wear a ring when working on an electrical circuit. (T or F)
3. Standing on a wet floor presents no safety hazard when working with electricity. (T or F)
4. A circuit can be rewired without removing the power if you are careful. (T or F)
5. Electrical shock can be extremely painful or even fatal. (T or F)
6. What does GFCI stand for?

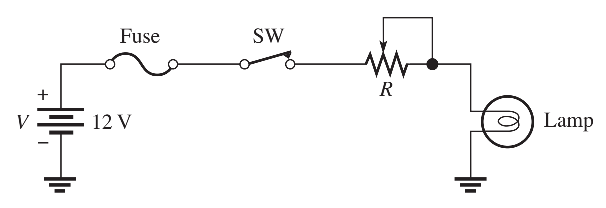


Application Activity

In this application activity, a dc voltage is applied to a circuit in order to produce current through a lamp to provide light. You will see how the current is controlled by resistance. The circuit simulates an instrument panel illumination circuit, which allows you to increase or decrease the amount of light on the instruments.

The instrument panel illumination circuit operates from a 12 V battery that is the voltage source for the circuit. The circuit uses a potentiometer connected as a rheostat, controlled by a knob on the instrument panel, which is used to set the amount of current through the lamp to back-light the instruments. The brightness of the lamp is proportional to the amount of current through the lamp. The switch used to turn the lamp on and off is the same one used for the headlights. There is a fuse for circuit protection in case of a short circuit.

Figure 59 shows the schematic for the illumination circuit, and Figure 60 shows the breadboarded circuit. A laboratory dc power supply is used in the place of an actual battery. The protoboard in Figure 60 is a type that commonly used for constructing circuits on the test bench.



▲ FIGURE 59

Basic panel illumination circuit schematic.

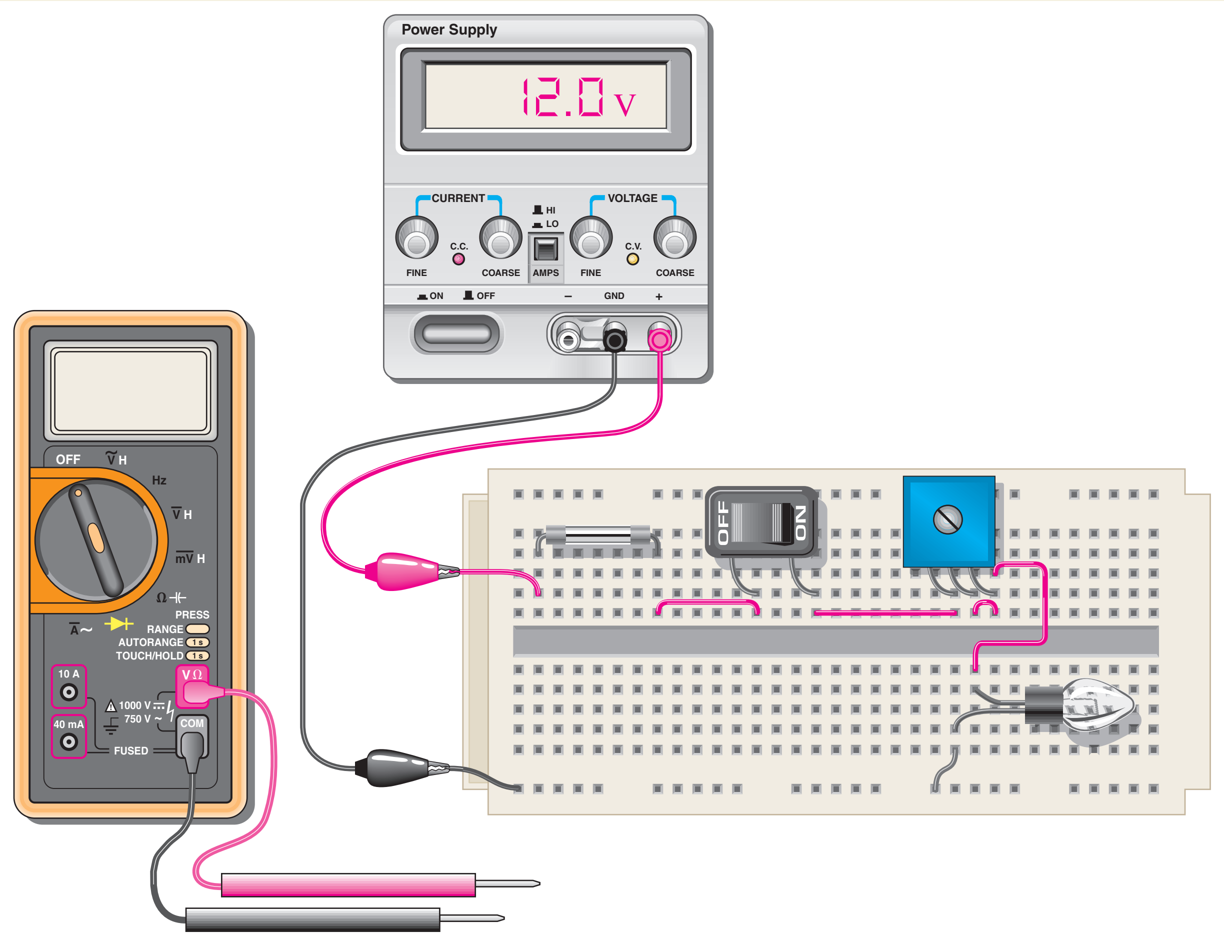

Power Supply
12.0 V
CURRENT
VOLTAGE
HI
LO
C.C.
C.V.
FINE
COARSE
AMPS
FINE
COARSE
ON
OFF
−
GND
+
OFF
ON
OFF
V H
Hz
V H
mV H
Ω
PRESS
A~
RANGE
AUTORANGE 1 s
TOUCH/HOLD 1 s
10 A
40 mA
VΩ
1000 V
750 V ~
COM
FUSED

FIGURE 60

Test bench setup for simulating the panel illumination circuit.

The Test Bench

Figure 60 shows the breadboarded circuit, a dc power supply, and a digital multimeter. The power supply is connected to provide 12 V to the circuit. The multimeter is used to measure current, voltage, and resistance in the circuit.

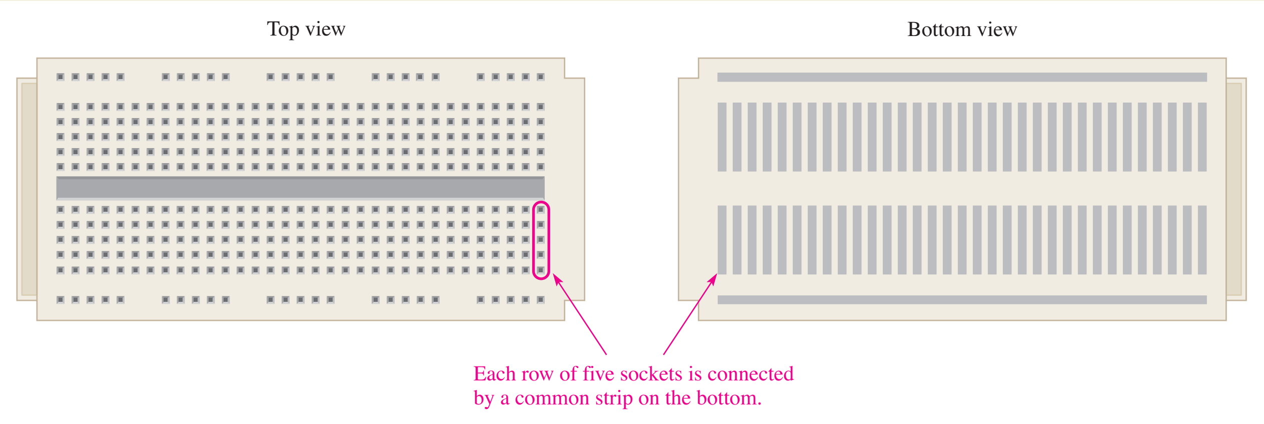
1. Identify each component in the circuit and check the breadboarded circuit in Figure 60 to make sure it is connected as the schematic in Figure 59 indicates.
2. Explain the purpose of each component in the circuit.

As shown in Figure 61, the typical protoboard consists of rows of small sockets into which component leads and wires are inserted. In this particular configuration, all five sockets in each row are connected together and are effectively one electrical point as shown in the bottom view. All sockets arranged on the outer edges of the board are typically connected together as shown.

Measuring Current with the Multimeter

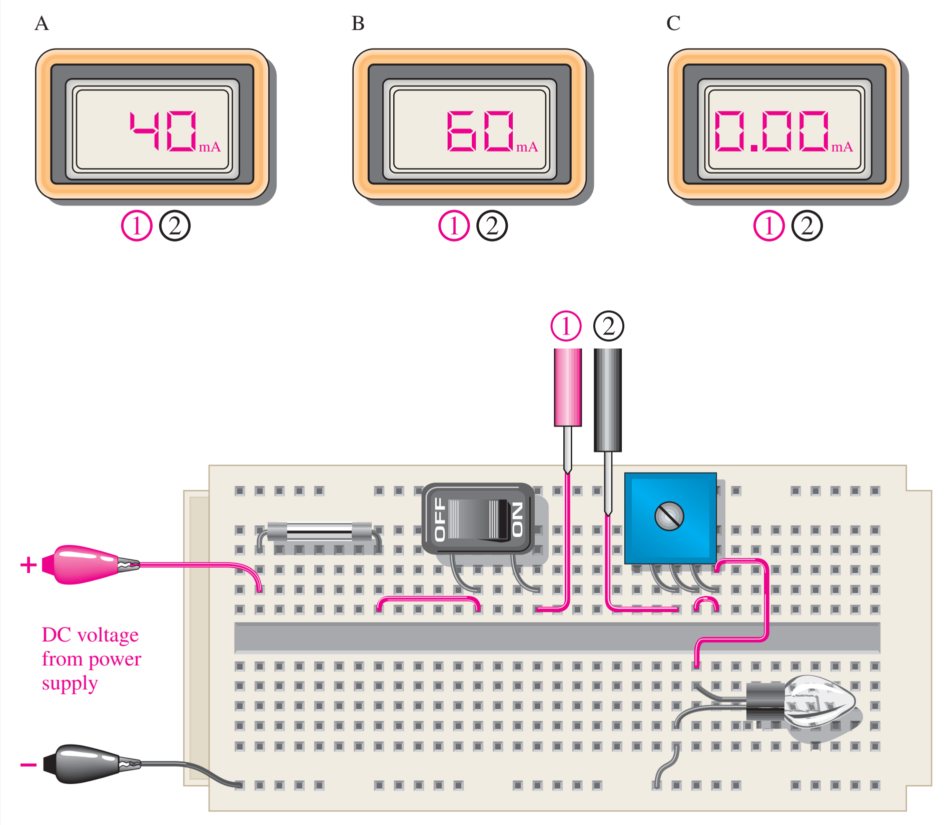
Set the DMM to the ammeter function to measure current. You must break the circuit in order to connect the ammeter in series to measure current. Refer to Figure 62.

3. Redraw the schematic in Figure 59 to include the ammeter.
4. For which measurement (A, B, or C) is the lamp brightest? Explain.
5. List the change(s) in the circuit that can cause the ammeter reading to go from A to B.
6. List the circuit condition(s) that will produce the ammeter reading in C.


Top view
Bottom view
Each row of five sockets is connected
by a common strip on the bottom.

▲ **FIGURE 61**

A typical protoboard used for breadboarding.


A
B
C
40 mA
60 mA
0.00 mA
① ②
① ②
① ②
① ②
OFF
ON
+
DC voltage
from power
supply
−

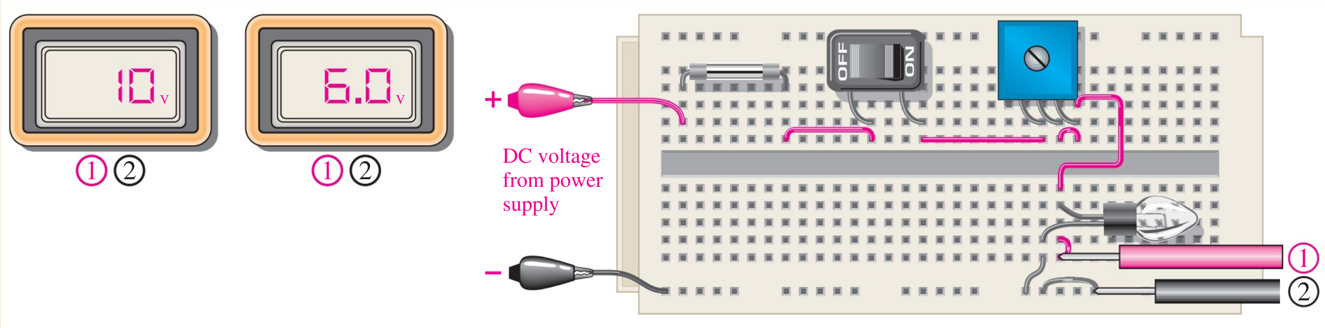
◀ **FIGURE 62**

Current measurements. Only the meter displays are shown.

Measuring Voltage with the Multimeter

Set the DMM to the voltmeter function to measure voltage. You must connect the voltmeter to the two points across which you are measuring the voltage. Refer to Figure 63.

7. Across which component is the voltage measured?
8. Redraw the schematic in Figure 59 to include the voltmeter.
9. For which measurement (A or B) is the lamp brighter? Explain.
10. List the change(s) in the circuit that can cause the voltmeter reading to go from A to B.



▲ FIGURE 63

Voltage measurements.

Measuring Resistance with the Multimeter

Set the DMM to the ohmmeter function to measure resistance. Before you connect the ohmmeter, you must disconnect the resistance to be measured from the circuit. Before you disconnect any component, first turn the power supply off. Refer to Figure 64.

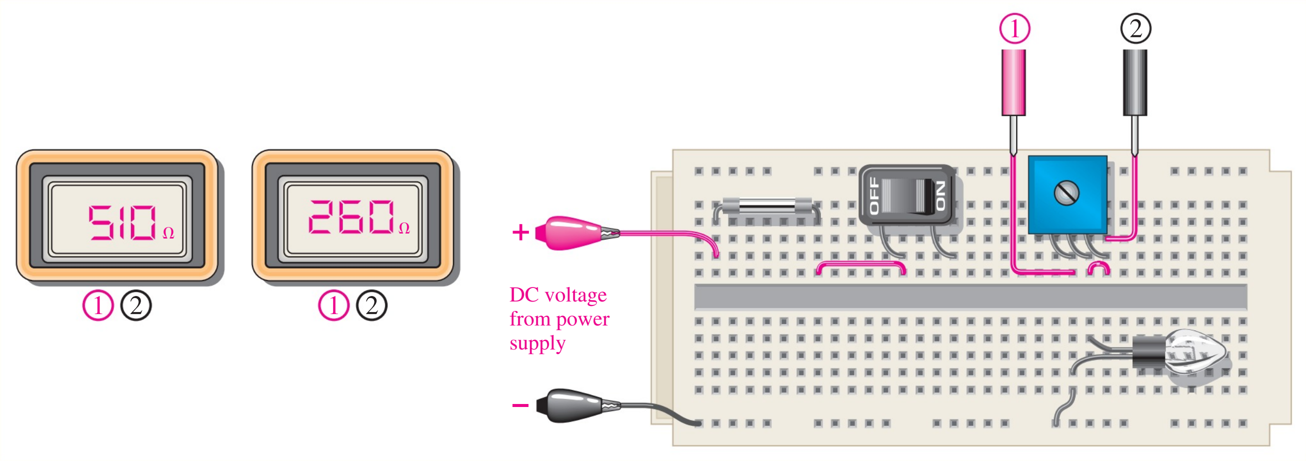
11. For which component is the resistance measured?
12. For which measurement (A or B) will the lamp be brighter when the circuit is reconnected and the power turned on? Explain.

Review

13. If the dc supply voltage in the panel illumination circuit is reduced, how is the amount of light produced by the lamp affected? Explain.
14. Should the potentiometer be adjusted to a higher or lower resistance for the circuit to produce more light?

Multisim Analysis

Open multisim and create the instrument panel illumination circuit. Run the simulation; operate the switch and rheostat to observe operation.



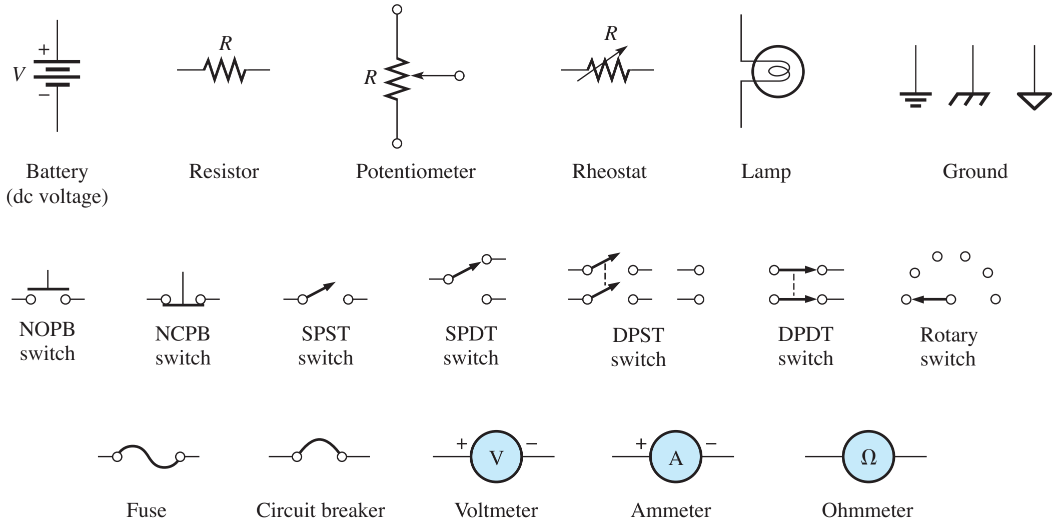
▲ FIGURE 64

Resistance measurements.

SUMMARY

- An atom is the smallest particle of an element that retains the characteristics of that element.
- When electrons in the outer orbit of an atom (valence electrons) break away, they become free electrons.
- Free electrons make current possible.

- Like charges repel each other, and opposite charges attract each other.
- Voltage must be applied to a circuit to produce current.
- Resistance limits the current.
- Basically, an electric circuit consists of a source, a load, and a current path.
- An open circuit is one in which the current path is broken.
- A closed circuit is one which has a complete current path.
- An ammeter is connected in line with the current path.
- A voltmeter is connected across the current path.
- An ohmmeter is connected across a resistor (resistor must be disconnected from circuit).
- One coulomb is the charge of 6.25×10^{18} electrons.
- One volt is the potential difference (voltage) between two points when one joule of energy is used to move one coulomb from one point to the other.
- One ampere is the amount of current that exists when one coulomb of charge moves through a given cross-sectional area of a material in one second.
- One ohm is the resistance when there is one ampere of current in a material with one volt applied across the material.
- Figure 65 shows the electrical symbols introduced in this chapter.
- Standard connections to electrical outlets include a hot wire, a neutral, and a safety ground.
- GFCI breakers sense the currents in the hot wire and neutral wire and trip the breaker if the currents are different, indicating a ground fault.



▲ FIGURE 65

KEY TERMS

Ammeter An electrical instrument used to measure current.

Ampere (A) The unit of electrical current.

Atom The smallest particle of an element possessing the unique characteristics of that element.

AWG American wire gauge; a standardization based on wire diameter.

Charge An electrical property of matter that exists because of an excess or a deficiency of electrons. Charge can be either positive or negative.

Circuit An interconnection of electrical components designed to produce a desired result. A basic circuit consists of a source, a load, and an interconnecting current path.

Closed circuit A circuit with a complete current path.

Conductance The ability of a circuit to allow current. The unit is the siemens (S).

Conductor A material in which electric current is easily established. An example is copper.

Coulomb (C) The unit of electrical charge; the total charge possessed by 6.25×10^{18} electrons.

Coulomb's law A law that states a force exists between two charged bodies that is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

Current The rate of flow of charge (electrons).

Current source A device that produces a constant current for a varying load.

DMM Digital multimeter; an electronic instrument that combines meters for measurement of voltage, current, and resistance.

Electrical shock The physical sensation resulting from electrical current through the body.

Electron A basic particle of electrical charge in matter. The electron possesses negative charge.

Free electron A valence electron that has broken away from its parent atom and is free to move from atom to atom within the atomic structure of a material.

Ground The common or reference point in a circuit.

Insulator A material that does not allow current under normal conditions.

Load An element connected across the output terminals of a circuit that draws current from the source and upon which work is done.

Ohm (Ω) The unit of resistance.

Ohmmeter An instrument for measuring resistance.

Open circuit A circuit in which there is not a complete current path.

Potentiometer A three-terminal variable resistor.

Power supply An electronic device that converts ac voltage to dc voltage.

Resistance Opposition to current. The unit is the ohm (Ω).

Resistor An electrical component specifically designed to have a certain amount of resistance.

Rheostat A two-terminal variable resistor.

Semiconductor A material that has a conductance value between that of a conductor and an insulator. Silicon and germanium are examples.

Siemens (S) The unit of conductance.

Volt The unit of voltage or electromotive force.

Voltage The amount of energy per charge available to move electrons from one point to another in an electric circuit.

Voltage source A device that produces a constant voltage for a varying load.

Voltmeter An instrument used to measure voltage.

FORMULAS

1	$Q = \dfrac{\text{number of electrons}}{6.25 \times 10^{18}\ \text{electrons/C}}$	Charge
2	$V = \dfrac{W}{Q}$	Voltage equals energy divided by charge.
3	$I = \dfrac{Q}{t}$	Current equals charge divided by time.
4	$G = \dfrac{1}{R}$	Conductance is the reciprocal of resistance.
5	$A = d^2$	Cross-sectional area equals the diameter squared.
6	$R = \dfrac{\rho l}{A}$	Resistance is resistivity times length divided by cross-sectional area.

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The number of neutrons in the nucleus is the atomic number of that element.
2. The unit of charge is the ampere.
3. Energy in a battery is stored in the form of chemical energy.
4. A volt can be defined in terms of energy per charge.
5. In a five-band precision resistor, the fourth band is the tolerance band.
6. A rheostat performs the same function as a potentiometer.
7. A strain gauge changes resistance in response to an applied force.
8. All circuits must have a complete path for current.
9. A circular mil is a unit of area.
10. The three basic measurements that can be done by a DMM are voltage, current, and power.

SELF-TEST

Answers are at the end of the chapter.

1. A neutral atom with an atomic number of three has how many electrons?

   (a) 1 **(b)** 3 **(c)** none **(d)** depends on the type of atom

2. Electron orbits are called

   (a) shells **(b)** nuclei **(c)** waves **(d)** valences

3. Materials in which there is no current when voltage is applied are called

   (a) filters **(b)** conductors **(c)** insulators **(d)** semiconductors

4. When placed close together, a positively charged material and a negatively charged material will

   (a) repel **(b)** become neutral **(c)** attract **(d)** exchange charges

5. The charge on a single electron is

   (a) 6.25×10^{-18} C **(b)** 1.6×10^{-19} C **(c)** 1.6×10^{-19} J **(d)** 3.14×10^{-6} C

6. *Potential difference* is another term for

   (a) energy **(b)** voltage **(c)** distance of an electron from the nucleus **(d)** charge

7. The unit of energy is the

   (a) watt **(b)** coulomb **(c)** joule **(d)** volt

8. Which one of the following is not a type of energy source?

   (a) battery **(b)** solar cell **(c)** generator **(d)** potentiometer

9. Which one of the following is not a possible condition in an electric circuit?

   (a) voltage and no current **(b)** current and no voltage
   (c) voltage and current **(d)** no voltage and no current

10. Electrical current is defined as

    (a) free electrons
    (b) the rate of flow of free electrons
    (c) the energy required to move electrons
    (d) the charge on free electrons

11. There is no current in a circuit when

    (a) a switch is closed **(b)** a switch is open **(c)** there is no voltage
    (d) answers (a) and (c) **(e)** answers (b) and (c)

12. The primary purpose of a resistor is to

    (a) increase current **(b)** limit current
    (c) produce heat **(d)** resist current change

13. Potentiometers and rheostats are types of

(a) voltage sources **(b)** variable resistors

(c) fixed resistors **(d)** circuit breakers

14. The current in a given circuit is not to exceed 22 A. Which value of fuse is best?

(a) 10 A **(b)** 25 A **(c)** 20 A **(d)** a fuse is not necessary

PROBLEMS

Answers to odd-numbered problems are at the end of the chapter.

SECTION 2 Electrical Charge

1. What is the charge in coulombs of the nucleus of a copper atom?

2. What is the charge in coulombs of the nucleus of a chlorine atom?

3. How many coulombs of charge do 50×10^{31} electrons possess?

4. How many electrons does it take to make 80 μC (microcoulombs) of charge?

SECTION 3 Voltage

5. Determine the voltage in each of the following cases:

(a) 10 J/C **(b)** 5 J/2 C **(c)** 100 J/25 C

6. Five hundred joules of energy are used to move 100 C of charge through a resistor. What is the voltage across the resistor?

7. What is the voltage of a battery that uses 800 J of energy to move 40 C of charge through a resistor?

8. How much energy does a 12 V battery use to move 2.5 C through a circuit?

9. If a resistor with a current of 2 A through it converts 1000 J of electrical energy into heat energy in 15 s, what is the voltage across the resistor?

10. List four common sources of voltage.

11. Upon what principle is electrical generators based?

12. How does and electronic power supply differ from the other sources of voltage?

SECTION 4 Current

13. A certain current source provides 100 mA to a 1 kΩ load. If the resistance is decreased to 500 Ω, what the current in the load?

14. Determine the current in each of the following cases:

(a) 75 C in 1 s **(b)** 10 C in 0.5 s **(c)** 5 C in 2 s

15. Six-tenths coulomb passes a point in 3 s. What is the current in amperes?

16. How long does it take 10 C to flow past a point if the current is 5 A?

17. How many coulombs pass a point in 0.1 s when the current is 1.5 A?

18. 5.74×10^{17} electrons flow through a wire in 250 ms. What is the current in amperes?

SECTION 5 Resistance

19. Find the conductance for each of the following resistance values:

(a) 5 Ω **(b)** 25 Ω **(c)** 100 Ω

20. Find the resistance corresponding to the following conductances:

(a) 0.1 S **(b)** 0.5 S **(c)** 0.02 S

21. Determine the resistance values and tolerance for the following 4-band resistors:

(a) red, violet, orange, gold **(b)** brown, gray, red, silver

22. Find the minimum and the maximum resistance within the tolerance limits for each resistor in Problem 21.

23. Determine the color bands for each of the following 4-band, 5% values: 330 Ω, 2.2 kΩ, 56 kΩ, 100 kΩ, and 39 kΩ.

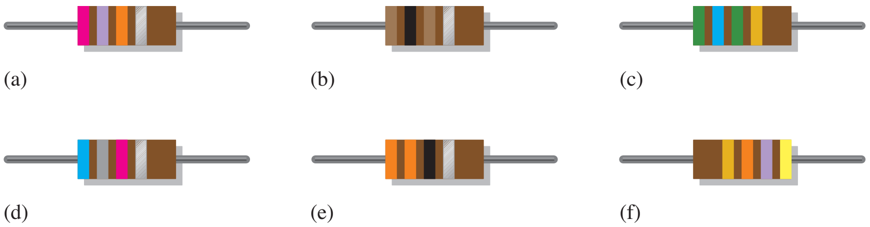
24. Determine the resistance and tolerance of each of the following 4-band resistors:

(a) brown, black, black, gold

(b) green, brown, green, silver

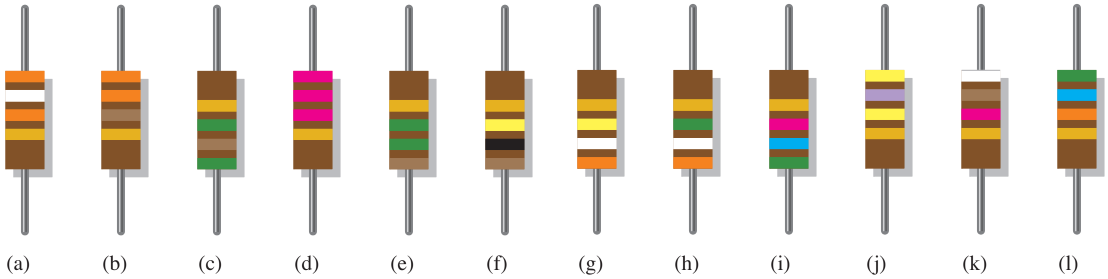
(c) blue, gray, black, gold

25. Determine the resistance and percent tolerance for each of the resistors in Figure 66.



▲ FIGURE 66

26. From the selection of resistors in Figure 67, select the following values: 330 Ω, 2.2 kΩ, 56 kΩ, 100 kΩ, and 39 kΩ.



▲ FIGURE 67

27. Determine the color bands for each of the following 4-band resistors. Assume each has a 5% tolerance.

(a) 0.47 Ω **(b)** 270 kΩ **(c)** 5.1 MΩ

28. Determine the resistance and tolerance of each of the following 5-band resistors:

(a) red, gray, violet, red, brown

(b) blue, black, yellow, gold, brown

(c) white, orange, brown, brown, brown

29. Determine the color bands for each of the following 5-band resistors. Assume each has a 1% tolerance.

(a) 14.7 kΩ **(b)** 39.2 Ω **(c)** 9.76 kΩ

30. The adjustable contact of a linear potentiometer is set at the mechanical center of its adjustment. If the total resistance is 1000 Ω, what is the resistance between each end terminal and the adjustable contact?

31. What resistance is indicated by 4K7?

32. Determine the resistance and tolerance of each resistor labeled as follows:

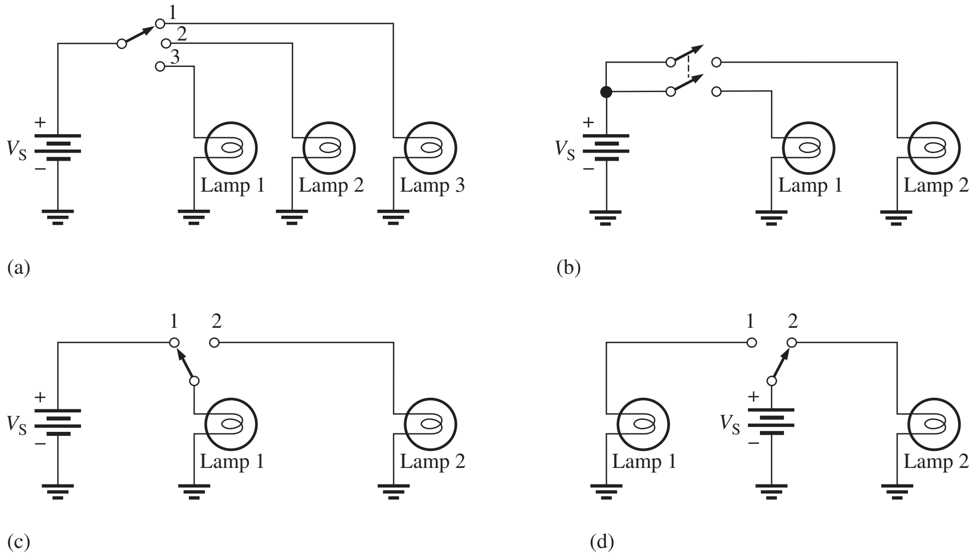
(a) 4R7J **(b)** 5602M **(c)** 1501F

SECTION 6 The Electric Circuit

33. Trace the current path in Figure 68(a) with the switch in position 2.

34. With the switch in either position, redraw the circuit in Figure 68(d) with a fuse connected to protect the circuit against excessive current.

▶ **FIGURE 68**

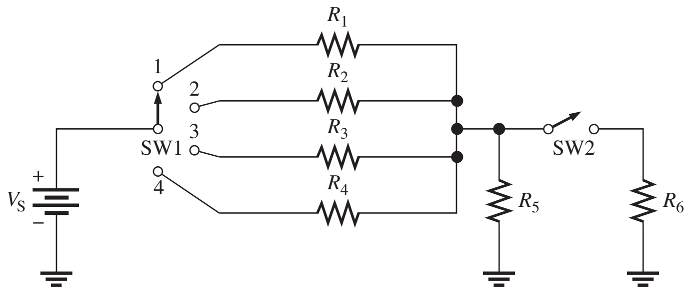


(a) (b) (c) (d)

35. There is only one circuit in Figure 68 in which it is possible to have all lamps on at the same time. Determine which circuit it is.

36. Through which resistor in Figure 69 is there always current, regardless of the position of the switches?

▶ **FIGURE 69**



37. Devise a switch arrangement whereby two voltage sources (V_{S1} and V_{S2}) can be connected simultaneously to either of two resistors (R_1 and R_2) as follows:

$$V_{S1} \text{ connected to } R_1 \text{ and } V_{S2} \text{ connected to } R_2$$

or

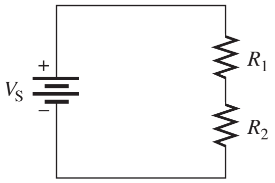
$$V_{S1} \text{ connected to } R_2 \text{ and } V_{S2} \text{ connected to } R_1$$

38. Show how a single switch can be used to connect a phonograph, a CD (compact disk) player, a tape deck, an AM tuner, or an FM tuner to an amplifier by a single knob control. Only one unit can be connected to the amplifier at any time.

SECTION 7 Basic Circuit Measurements

39. Show the placement of an ammeter and a voltmeter to measure the current and the source voltage in Figure 70.

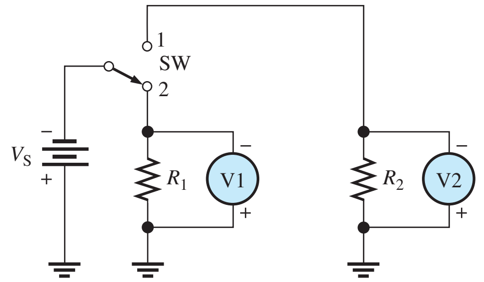
▶ **FIGURE 70**



40. Explain how you would measure the resistance of R_2 in Figure 70.

41. In Figure 71, how much voltage does each meter indicate when the switch is in position 1? In position 2?

▶ FIGURE 71


1
SW
2
V_S
−
+
R_1
V1
−
+
R_2
V2
−
+

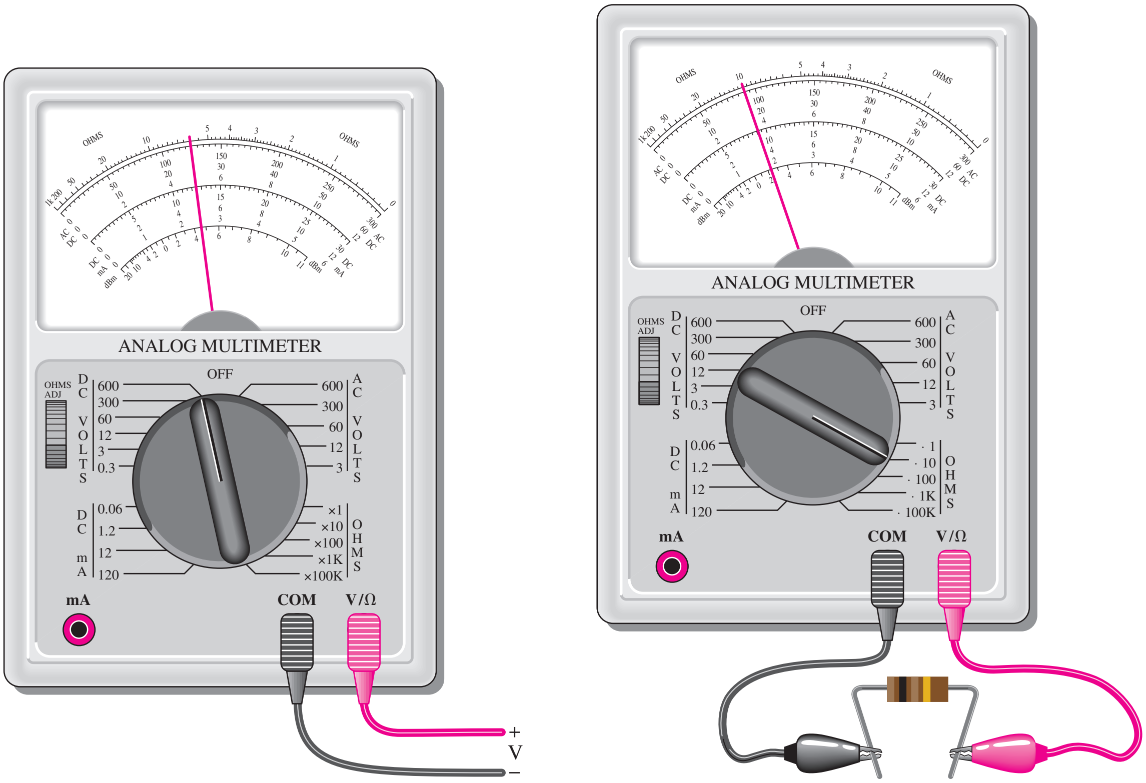
42. In Figure 71, indicate how to connect an ammeter to measure the current from the voltage source regardless of the switch position.

43. In Figure 69, show the proper placement of ammeters to measure the current through each resistor and the current out of the battery.

44. Show the proper placement of voltmeters to measure the voltage across each resistor in Figure 69.

45. What is the voltage reading of the meter in Figure 72(a)?

46. How much resistance is the ohmmeter in Figure 72(b) measuring?


ANALOG MULTIMETER
OFF
OHMS ADJ
DC VOLTS
600
300
60
12
3
0.3
AC VOLTS
DC mA
0.06
1.2
12
120
OHMS
×1
×10
×100
×1K
×100K
mA
COM
V/Ω
+
V
−

(a)

(b)

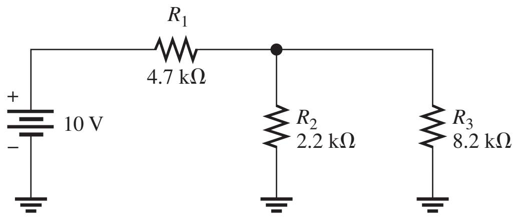
▲ FIGURE 72

47. Determine the resistance indicated by each of the following ohmmeter readings and range settings:
  (a) pointer at 2, range setting at ×10
  (b) pointer at 15, range setting at ×100,000
  (c) pointer at 45, range setting at ×100

48. What is the maximum resolution of a 4½-digit DMM?

49. Indicate how you would connect the multimeter in Figure 72(b) to the circuit in Figure 73 to measure each of the following quantities. In each case indicate the appropriate function and range.
  (a) I_1 **(b)** V_1 **(c)** R_1

► FIGURE 73



ANSWERS

SECTION CHECKUPS

SECTION 1 Atomic Structure

1. The electron is the basic particle of negative charge.
2. An atom is the smallest particle of an element that retains the unique characteristics of the element.
3. An atom is a positively charged nucleus surrounded by orbiting electrons.
4. Atomic number is the number of protons in a nucleus.
5. No, each element has a different type of atom.
6. A free electron is an outer-shell electron that has drifted away from the parent atom.
7. Shells are energy bands in which electrons orbit the nucleus of an atom.
8. Copper and silver

SECTION 2 Electrical Charge

1. Q = charge
2. Unit of charge is the coulomb; C
3. Positive or negative charge is caused by the loss or acquisition respectively of an outer-shell (valence) electron.
4. $Q = \dfrac{10 \times 10^{12}\ \text{electrons}}{6.25 \times 10^{18}\ \text{electrons/C}} = 1.6 \times 10^{-6}\ \text{C} = 1.6\ \mu\text{C}$

SECTION 3 Voltage

1. Voltage is energy per unit charge.
2. Volt is the unit of voltage.
3. $V = W/Q = 24\ \text{J}/10\ \text{C} = 2.4\ \text{V}$
4. Battery, fuel cell, solar cell, generator, power supply, thermocouple
5. Oxidation-reduction

SECTION 4 Current

1. Current is the rate of flow of electrons; its unit is the ampere (A).
2. electrons/coulomb $= 6.25 \times 10^{18}$
3. $I = Q/t = 20\ \text{C}/4\ \text{s} = 5\ \text{A}$

SECTION 5 Resistance

1. Resistance is opposition to current and its unit is the ohm.
2. Two resistor categories are fixed and variable. The value of a fixed resistor cannot be changed, but that of a variable resistor can.
3. *First band:* first digit of resistance value. *Second band:* second digit of resistance value. *Third band:* multiplier (number of zeros following the second digit). *Fourth band:* % tolerance.
4. **(a)** 4700 Ω ± 5% **(b)** 62 kΩ ± 10%
   (c) 18 Ω ± 10% **(d)** 22.6 kΩ ± 0.5%
5. **(a)** 33R = 33 Ω **(b)** 5K6 = 5.6 kΩ
   (c) 900 = 900 Ω **(d)** 6M8 = 6.8 MΩ
6. A rheostat has two terminals; a potentiometer has three terminals.
7. Thermistor: temperature
   photoconductive cell: light
   strain gauge: mechanical force

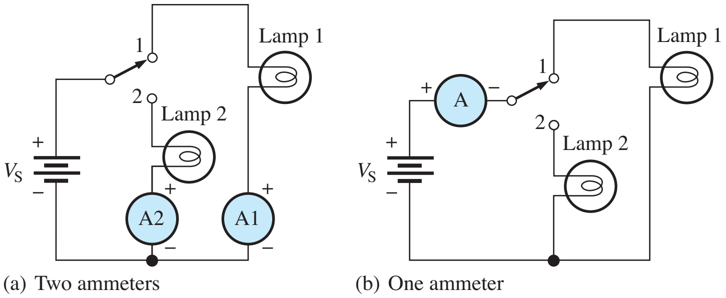
SECTION 6 The Electric Circuit

1. An electric circuit consists of a source, load, and current path between source and load.
2. An open circuit is one that has no path for current.
3. A closed circuit is one that has a complete path for current.
4. A fuse is not resettable, a circuit breaker is.
5. AWG 3 is larger.
6. Ground is the common or reference point.

SECTION 7 Basic Circuit Measurements

1. **(a)** An ammeter measures current.
   (b) A voltmeter measures voltage.
   (c) An ohmmeter measures resistance.
2. See Figure 74.

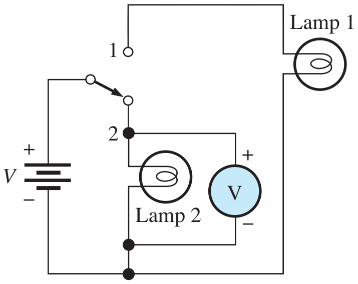
FIGURE 74



(a) Two ammeters (b) One ammeter

3. See Figure 75.

FIGURE 75



4. Two types of DMM displays are liquid-crystal display (LCD) and light-emitting display (LED). The LCD requires little current, but it is difficult to see in low light and is slow to respond. The LED can be seen in the dark, and it responds quickly. However, it requires much more current than does the LCD.

5. Resolution is the smallest increment of a quantity that the meter can measure.

6. 1.5 V

SECTION 8 Electrical Safety

1. Current **2.** F **3.** F **4.** F **5.** T

6. GFCI: ground fault circuit interrupter

RELATED PROBLEMS FOR EXAMPLES

1 1.88×10^{19} electrons

2 600 J

3 12 C

4 4700 Ω ± 5%

5 47.5 Ω ± 2%

6 1.25 kΩ

7 2.25 CM

8 1.280 Ω; same as calculated result

9 The needle will move left to the “100” mark.

TRUE/FALSE QUIZ

1. F **2.** F **3.** T **4.** T **5.** F **6.** F **7.** T **8.** T **9.** T **10.** F

SELF-TEST

1. (b) **2.** (a) **3.** (c) **4.** (c) **5.** (b) **6.** (b) **7.** (c)

8. (d) **9.** (b) **10.** (b) **11.** (e) **12.** (b) **13.** (b) **14.** (c)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. 4.64×10^{-18} C

3. 80×10^{12} C

5. **(a)** 10 V **(b)** 2.5 V **(c)** 4 V

7. 20 V

9. 33.3 V

11. Electromagnetic induction

13. 100 mA

15. 0.2 A

17. 0.15 C

19. **(a)** 200 mS **(b)** 40 mS **(c)** 10 mS

21. **(a)** 27 kΩ ± 5% **(b)** 1.8 kΩ ± 10%

23. 330 Ω: orange, orange, brown, gold
2.2 kΩ: red, red, red, gold
56 kΩ: green, blue, orange, gold
100 kΩ: brown, black, yellow, gold
39 kΩ: orange, white, orange, gold

25. **(a)** 27 kΩ ± 10% **(b)** 100 Ω ± 10%
(c) 5.6 MΩ ± 5% **(d)** 6.8 kΩ ± 10%
(e) 33 Ω ± 10% **(f)** 47 kΩ ± 5%

27. **(a)** yellow, violet, silver, gold
(b) red, violet, yellow, gold
(c) green, brown, green, gold

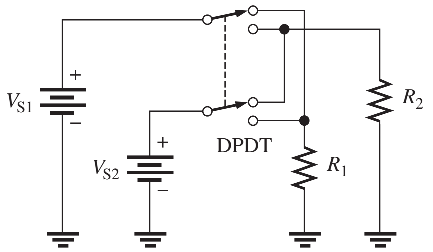
29. **(a)** brown, yellow, violet, red, brown
(b) orange, white, red, gold, brown
(c) white, violet, blue, brown, brown

31. 4.7 kΩ

33. Through lamp 2

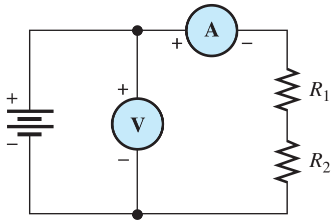
35. Circuit (b)

37. See Figure 76.



▲ FIGURE 76

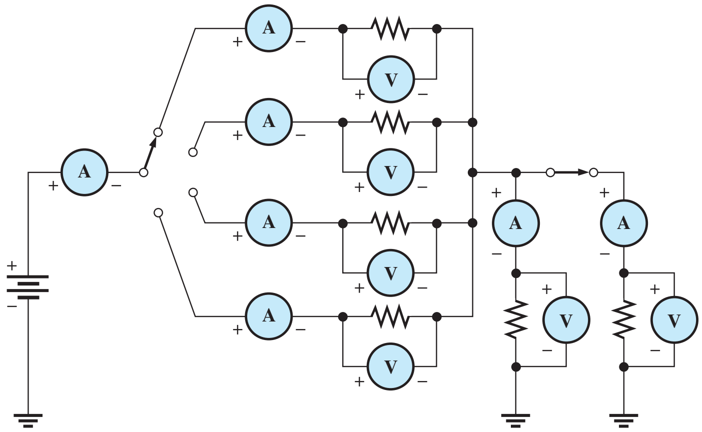
39. See Figure 77.



▲ FIGURE 77

41. Position 1: V1 = 0 V, V2 = V_s
Position 2: V1 = V_s, V2 = 0 V

43. See Figure 78.

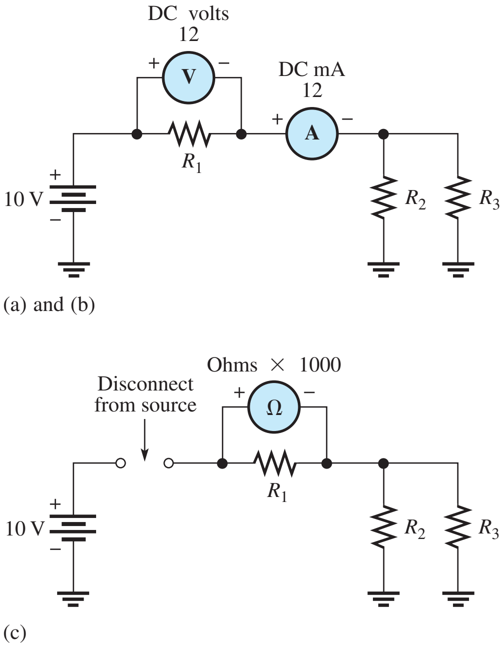


▲ FIGURE 78

45. 250 V

47. **(a)** 20 Ω **(b)** 1.50 MΩ **(c)** 4500 Ω

49. See Figure 79.


DC volts
12
+
−
V
DC mA
12
+
−
A
R_1
+
10 V
−
R_2
R_3
(a) and (b)
Ohms × 1000
Disconnect
from source
+
−
Ω
R_1
+
10 V
−
R_2
R_3
(c)

▲ FIGURE 79

OHM'S LAW

CHAPTER OUTLINE

1 The Relationship of Current, Voltage, and Resistance
2 Current Calculations
3 Voltage Calculations
4 Resistance Calculations
5 Introduction to Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Explain Ohm's law
- Calculate current in a circuit
- Calculate voltage in a circuit
- Calculate resistance in a circuit
- Describe a basic approach to troubleshooting

KEY TERMS

- Ohm's law
- Linear
- Troubleshooting

APPLICATION ACTIVITY PREVIEW

In this application activity, you will see how Ohm's law is used in a practical circuit. The circuit is an array of switch-selectable resistors of various values used to control the speed of a fan. The resistance values will be determined based on fan motor data and by the application of Ohm's law. A test procedure will be developed once the modifications have been made.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

You should already be familiar with the concepts of voltage, current, and resistance. In this chapter, you will learn how voltage, current, and resistance are interrelated. You will also learn how to analyze a simple electric circuit.

Ohm's law is perhaps the single most important tool for the analysis of electric circuits, and you *must* know how to apply it.

In 1826 Georg Simon Ohm found that current, voltage, and resistance are related in a specific and predictable way. Ohm expressed this relationship with a formula that is known today as Ohm's law. In this chapter, you will learn Ohm's law and how to use it in solving circuit problems. A general approach to troubleshooting using an *analysis*, *planning*, and *measurement* (APM) approach is also introduced.

From Chapter 3 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 THE RELATIONSHIP OF CURRENT, VOLTAGE, AND RESISTANCE

Ohm's law describes mathematically how voltage, current, and resistance in a circuit are related. Ohm's law is expressed in three equivalent forms depending on which quantity you need to determine. As you will learn, current and voltage are linearly proportional. However, current and resistance are inversely proportional.

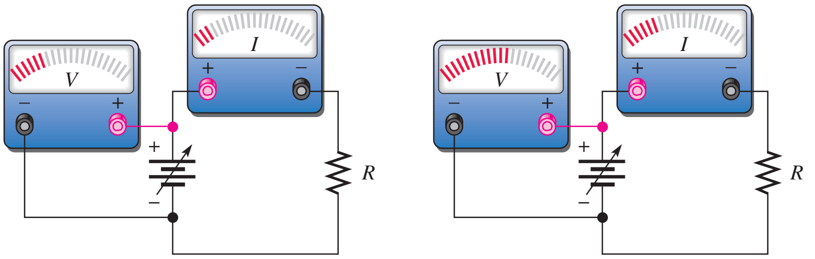
After completing this section, you should be able to

- **Explain Ohm's law**
  - Describe how *V*, *I*, and *R* are related
  - Express *I* as a function of *V* and *R*
  - Express *V* as a function of *I* and *R*
  - Express *R* as a function of *V* and *I*
  - Show graphically that *I* and *V* are directly proportional
  - Show graphically that *I* and *R* are inversely proportional
  - Explain why *I* and *V* are linearly proportional

Ohm determined experimentally that if the voltage across a resistor is increased, the current through the resistor will also increase; and, likewise, if the voltage is decreased, the current will decrease. For example, if the voltage is doubled, the current will double. If the voltage is halved, the current will also be halved. This relationship is illustrated in Figure 1, with relative meter indications of voltage and current.

FIGURE 1

Effect on the current of changing the voltage with the resistance at a constant value.

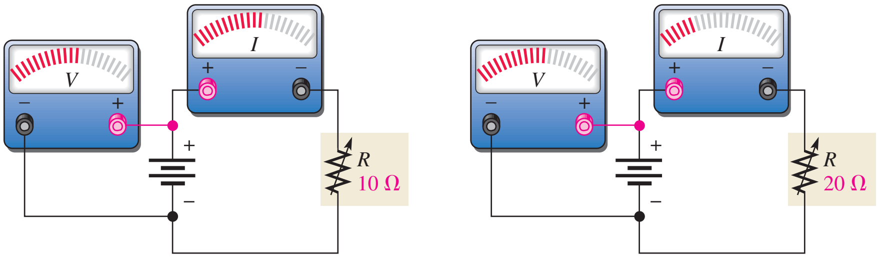


(a) Less *V*, less *I* (b) More *V*, more *I*

Ohm also determined that if the voltage is held constant, less resistance results in more current, and, also, more resistance results in less current. For example, if the resistance is halved, the current doubles. If the resistance is doubled, the current is halved. This concept is illustrated by the meter indications in Figure 2, where the resistance is increased and the voltage is held constant.

FIGURE 2

Effect on the current of changing the resistance with the voltage at a constant value.



(a) Less *R*, more *I* (b) More *R*, less *I*

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Ohm's law states that current is directly proportional to voltage and inversely proportional to resistance. The circuits in Figures 1 and 2 illustrate Ohm's law, which is given in the following formula:

$$I = \frac{V}{R} \qquad \text{Equation 1}$$

where I is current in amperes (A), V is voltage in volts (V), and R is resistance in ohms (Ω). For a constant value of R, if the value of V is increased, the value of I increases; if V is decreased, I decreases. If V is constant and R is increased, I decreases. Similarly, if V is constant and R is decreased, I increases.

Using Equation 1, you can calculate the current if you know the values of voltage and resistance. By manipulating Equation 1, you can obtain expressions for voltage and resistance.

$$V = IR \qquad \text{Equation 2}$$

$$R = \frac{V}{I} \qquad \text{Equation 3}$$

With Equation 2, you can calculate voltage if you know the values of current and resistance. With Equation 3, you can calculate resistance if you know the values of voltage and current.

The three expressions—Equations 1, 2 and 3—are all equivalent. They simply three ways of stating Ohm's law.

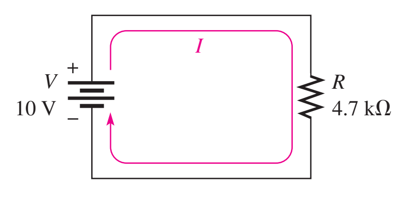
The Linear Relationship of Current and Voltage

In resistive circuits, current and voltage are linearly proportional. **Linear** means that if one of the quantities is increased or decreased by a certain percentage, the other will increase or decrease by the same percentage, assuming that the resistance is constant in value. For example, if the voltage across a resistor is tripled, the current will triple.

EXAMPLE 1

Show that if the voltage in the circuit of Figure 3 is increased to three times its present value, the current will triple in value.

▶ FIGURE 3



Solution With 10 V, the current is

$$I = \frac{V}{R} = \frac{10\text{ V}}{4.7\text{ k}\Omega} = \mathbf{2.13\text{ mA}}$$

If the voltage is increased to 30 V, the current will be

$$I = \frac{V}{R} = \frac{30\text{ V}}{4.7\text{ k}\Omega} = \mathbf{6.38\text{ mA}}$$

The current went from 2.13 mA to 6.38 mA when the voltage was tripled to 30 V.

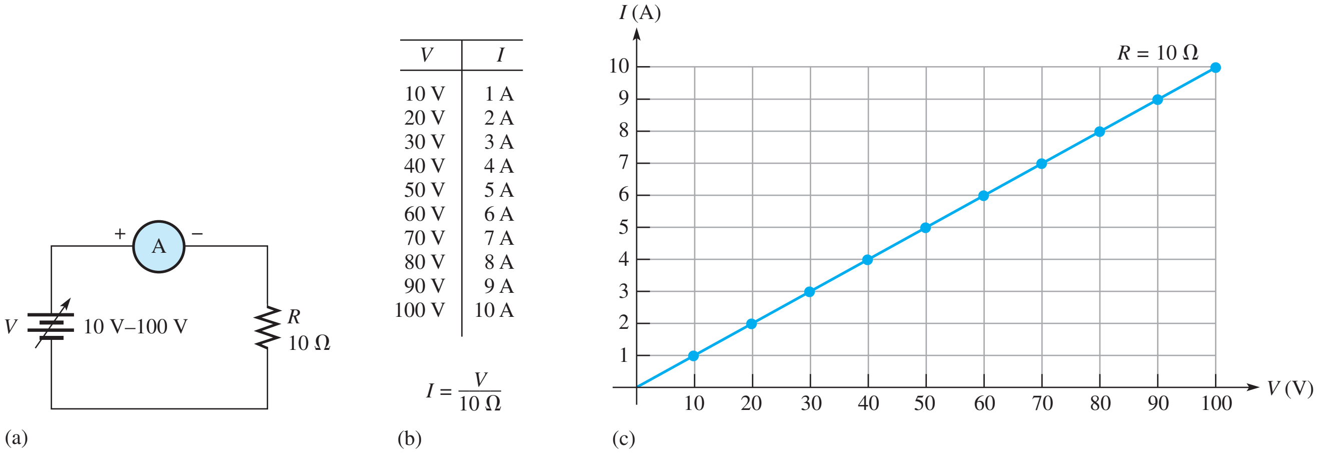
*Related Problem** If the voltage in Figure 3 is quadrupled, will the current also quadruple?

Use Multisim file E03-01 to verify the calculated results in this example and your calculation for the related problem.

---

*Answers are at the end of the chapter.

Let's take a constant value of resistance, for example, 10Ω, and calculate the current for several values of voltage ranging from 10 V to 100 V in the circuit in Figure 4(a). The current values obtained are shown in Figure 4(b). The graph of the *I* values versus the *V* values is shown in Figure 4(c). Note that it is a straight line graph. This graph tells us that a change in voltage results in a linearly proportional change in current. No matter what value *R* is, assuming that *R* is constant, the graph of *I* versus *V* will always be a straight line.



V	*I*
10 V	1 A
20 V	2 A
30 V	3 A
40 V	4 A
50 V	5 A
60 V	6 A
70 V	7 A
80 V	8 A
90 V	9 A
100 V	10 A

$$I = \frac{V}{10\ \Omega}$$

▲ **FIGURE 4**

Graph of current versus voltage for the circuit in part (a).

EXAMPLE 2

Assume that you are measuring the current in a circuit that is operating with 25 V. The ammeter reads 50 mA. Later, you notice that the current has dropped to 40 mA. Assuming that the resistance did not change, you must conclude that the voltage source has changed. How much has the voltage changed, and what is its new value?

Solution The current has dropped from 50 mA to 40 mA, which is a decrease of 20%. Since the voltage is linearly proportional to the current, the voltage has decreased by the same percentage that the current did. Taking 20% of 25 V, you get

$$\text{Change in voltage} = (0.2)(25\text{ V}) = \mathbf{5\ V}$$

Subtract this change from the original voltage to get the new voltage.

$$\text{New voltage} = 25\text{ V} - 5\text{ V} = \mathbf{20\ V}$$

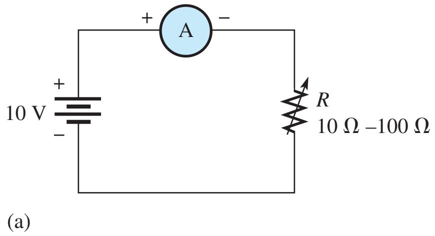
Notice that you did not need the resistance value in order to find the new voltage.

Related Problem If the current drops to 0 A under the same conditions stated in the example, what is the voltage?

The Inverse Relationship of Current and Resistance

As you have seen, current varies inversely with resistance as expressed by Ohm's law, $I = V/R$. When the resistance is reduced, the current goes up; when the resistance is increased, the current goes down. For example, if the source voltage is held constant and the resistance is halved, the current doubles in value; when the resistance is doubled, the current is reduced by half.

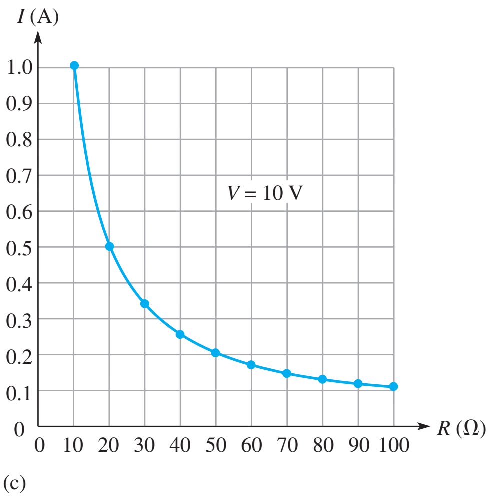
Let's take a constant value of voltage, for example, 10 V, and calculate the current for several values of resistance ranging from 10 Ω to 100 Ω in the circuit in Figure 5(a). The



(a)

R (Ω)	I (A)
10	1.000
20	0.500
30	0.333
40	0.250
50	0.200
60	0.167
70	0.143
80	0.125
90	0.111
100	0.100

(b)



(c)

▲ **FIGURE 5**

Graph of current versus resistance for the circuit in part (a).

values obtained are shown in Figure 5(b). The graph of the I values versus the R values is shown in Figure 5(c).

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Ohm's law defines how three basic quantities are related. What are these quantities?
2. Write the Ohm's law formula for current.
3. Write the Ohm's law formula for voltage.
4. Write the Ohm's law formula for resistance.
5. If the voltage across a fixed-value resistor is tripled, does the current increase or decrease, and by how much?
6. If the voltage across a fixed resistor is cut in half, how much will the current change?
7. There is a fixed voltage across a resistor, and you measure a current of 1 A. If you replace the resistor with one that has twice the resistance value, how much current will you measure?
8. In a circuit the voltage is doubled and the resistance is cut in half. Would the current increase or decrease and if so, by how much?
9. In a circuit, $V = 2$ V and $I = 10$ mA. If V is changed to 1 V, what will I equal?
10. If $I = 3$ A at a certain voltage, what will it be if the voltage is doubled?

2 CURRENT CALCULATIONS

Examples in this section illustrate the Ohm's law formula $I = V/R$.

After completing this section, you should be able to

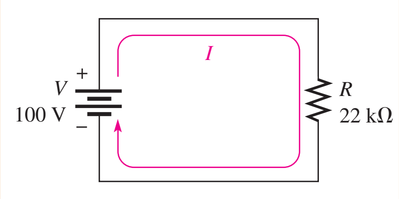
- **Calculate current in a circuit**
    - Use Ohm's law to find current when you know voltage and resistance values
    - Use voltage and resistance values expressed with metric prefixes

The following examples use the formula $I = V/R$. In order to get current in amperes, you must express the value of voltage in volts and the value of resistance in ohms.

EXAMPLE 3

How many amperes of current are in the circuit of Figure 6?

▶ FIGURE 6



Solution Use the formula $I = V/R$, and substitute 100 V for V and 22 kΩ for R.

$$I = \frac{V}{R} = \frac{100\text{ V}}{22\text{ k}\Omega} = \mathbf{4.55\text{ mA}}$$

Related Problem If R is changed to 33 kΩ in Figure 6, what is the current?

Use Multisim file E03-03 to verify the calculated results in this example and your calculation for the related problem.

EXAMPLE 4

If the resistance in Figure 6 is changed to 47 Ω and the voltage to 50 V, what is the new value of current?

Solution Substitute $V = 50$ V and $R = 47\ \Omega$ into the formula $I = V/R$.

$$I = \frac{V}{R} = \frac{50\text{ V}}{47\ \Omega} = \mathbf{1.06\text{ A}}$$

Related Problem If $V = 5$ V and $R = 1000\ \Omega$, what is the current?

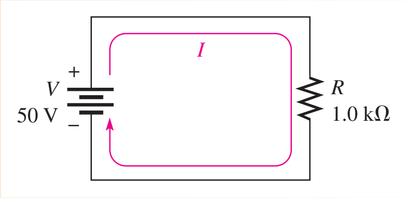
Units with Metric Prefixes

In electronics, resistance values of thousands of ohms or even millions of ohms are common. The metric prefixes *kilo* (k) and *mega* (M) are used to indicate large values. Thus, thousands of ohms are expressed in kilohms (kΩ), and millions of ohms are expressed in megohms (MΩ). The following four examples illustrate how to use kilohms and megohms to calculate current. Volts (V) divided by kilohms (kΩ) results in milliamperes (mA). Volts (V) divided by megohms (MΩ) results in microamperes (μA).

EXAMPLE 5

Calculate the current in Figure 7.

▶ FIGURE 7



Solution Remember that 1.0 kΩ is the same as $1 \times 10^3\ \Omega$. Use the formula $I = V/R$ and substitute 50 V for V and $1 \times 10^3\ \Omega$ for R.

$$I = \frac{V}{R} = \frac{50\text{ V}}{1.0\text{ k}\Omega} = \frac{50\text{ V}}{1 \times 10^3\ \Omega} = 50 \times 10^{-3}\text{ A} = \mathbf{50\ mA}$$

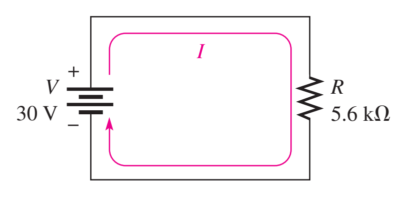
Related Problem Calculate the current in Figure 7 if R is changed to 10 kΩ.

In Example 5, 50×10^{-3} A is expressed as 50 milliamperes (50 mA). This can be used to advantage when you divide volts by kilohms. The current will be in milliamperes, as Example 6 illustrates.

EXAMPLE 6

How many milliamperes are in the circuit of Figure 8?

FIGURE 8



Solution When you divide volts by kilohms, you get current in milliamperes.

$$I = \frac{V}{R} = \frac{30\text{ V}}{5.6\text{ k}\Omega} = \mathbf{5.36\ mA}$$

Related Problem What is the current in milliamperes if R is changed to 2.2 kΩ?

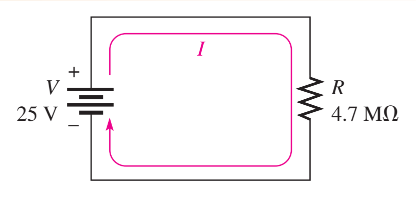
Use Multisim file E03-06 to verify the calculated results in this example and to confirm your calculation for the related problem.

If you apply volts when resistance values are in megohms, the current is in microamperes (μA), as Examples 5 and 6 show.

EXAMPLE 7

Determine the amount of current in the circuit of Figure 9.

FIGURE 9



Solution Recall that 4.7 MΩ equals $4.7 \times 10^6\ \Omega$. Substitute 25 V for V and $4.7 \times 10^6\ \Omega$ for R.

$$I = \frac{V}{R} = \frac{25\text{ V}}{4.7\text{ M}\Omega} = \frac{25\text{ V}}{4.7 \times 10^6\ \Omega} = 5.32 \times 10^{-6}\text{ A} = \mathbf{5.32\ \mu A}$$

Related Problem What is the current if V is increased to 100 V in Figure 6?

EXAMPLE 8

Change the value of R in Figure 9 to 1.8 MΩ. What is the new value of current?

Solution When you divide volts by megohms, you get current in microamperes.

$$I = \frac{V}{R} = \frac{25\text{ V}}{1.8\text{ M}\Omega} = \mathbf{13.9\ \mu A}$$

Related Problem If R is doubled in the circuit of Figure 6, what is the new value of current?

Small voltages, usually less than 50 V are common in semiconductor circuits. Occasionally, however, large voltages are encountered. For example, the high-voltage supply in some television receivers is around 20,000 V (20 kilovolts, or 20 kV), and transmission voltages generated by the power companies may be as high as 345,000 V (345 kV). The following two examples illustrate how to use voltage values in the kilovolt range to calculate current.

EXAMPLE 9

How much current is produced by a voltage of 24 kV across a 12 kΩ resistor?

Solution Since kilovolts are divided by kilohms, the prefixes cancel; therefore, the current is in amperes.

$$I = \frac{V}{R} = \frac{24\text{ kV}}{12\text{ k}\Omega} = \frac{24 \times 10^3\text{ V}}{12 \times 10^3\ \Omega} = \mathbf{2\ A}$$

Related Problem What is the current in mA produced by 1 kV across a 27 kΩ resistor?

EXAMPLE 10

How much current is there through a 100 MΩ resistor when 50 kV are applied?

Solution In this case, divide 50 kV by 100 MΩ to get the current. Substitute 50×10^3 V for 50 kV and $100 \times 10^6\ \Omega$ for 100 MΩ.

$$I = \frac{V}{R} = \frac{50\text{ kV}}{100\text{ M}\Omega} = \frac{50 \times 10^3\text{ V}}{100 \times 10^6\ \Omega} = 0.5 \times 10^{-3}\text{ A} = \mathbf{0.5\ mA}$$

Remember that the power of ten in the denominator is subtracted from the power of ten in the numerator. So 50 was divided by 100, giving 0.5, and 6 was subtracted from 3, giving 10^{-3}.

Related Problem How much current is there through a 6.8 MΩ resistor when 10 kV are applied?

SECTION 2 CHECKUP

In Problems 1–4, calculate I.

1. $V = 10$ V and $R = 5.6\ \Omega$.
2. $V = 100$ V and $R = 560\ \Omega$.
3. $V = 5$ V and $R = 2.2$ kΩ.
4. $V = 15$ V and $R = 4.7$ MΩ.
5. If a 4.7 MΩ resistor has 20 kV across it, how much current is there?
6. How much current will 10 kV across a 2.2 MΩ resistor produce?

3 VOLTAGE CALCULATIONS

Examples in this section illustrate the Ohm's law expression $V = IR$.

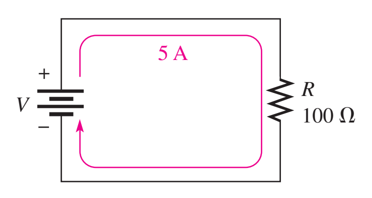
After completing this section, you should be able to

- **Calculate voltage in a circuit**
    - Use Ohm's law to find voltage when you know current and resistance values
    - Use current and resistance values expressed with metric prefixes

The following examples use the formula $V = IR$. To obtain voltage in volts, you must express the value of I in amperes and the value of R in ohms.

EXAMPLE 11 In the circuit of Figure 10, how much voltage is needed to produce 5 A of current?

▶ FIGURE 10



Solution Substitute 5 A for I and 100 Ω for R into the formula $V = IR$.

$$V = IR = (5\text{ A})(100\ \Omega) = \mathbf{500\ V}$$

Thus, 500 V are required to produce 5 A of current through a 100 Ω resistor.

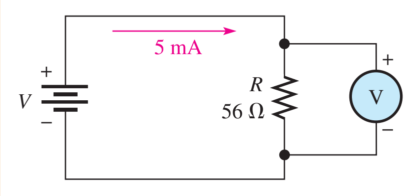
Related Problem In Figure 10, how much voltage is required to produce 12 A of current?

Units with Metric Prefixes

The following two examples illustrate how to use current values in the milliampere (mA) and microampere (μA) ranges to calculate voltage.

EXAMPLE 12 How much voltage will be measured across the resistor in Figure 11?

▶ FIGURE 11



Solution Five milliamperes equals 5×10^{-3} A. Substitute the values for I and R into the formula $V = IR$.

$$V = IR = (5\text{ mA})(56\ \Omega) = (5 \times 10^{-3}\text{ A})(56\ \Omega) = 280 \times 10^{-3}\text{ V} = \mathbf{280\ mV}$$

Related Problem How much voltage is measured across R if $R = 33\ \Omega$ and $I = 1.5$ mA in Figure 11?

Use Multisim file E03-12 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 13

Suppose that a solar cell produces a current of 180 μA through a 100 Ω resistor. How much voltage is across the resistor?

Solution 180 microamperes equals 180×10^{-6} A. Substitute the values for I and R into the formula $V = IR$.

$$V = IR = (180\ \mu\text{A})(100\ \Omega) = (180 \times 10^{-6}\ \text{A})(100\ \Omega) = 18 \times 10^{-3}\ \text{V} = \mathbf{18\ mV}$$

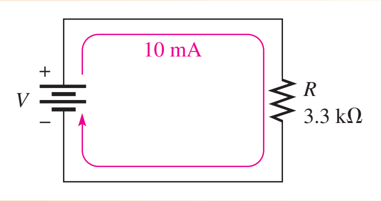
Related Problem If there are 3.2 μA through a 47 Ω resistor, what is the voltage across the resistor?

The following two examples illustrate how to use resistance values in the kilohm (kΩ) and megohm (MΩ) ranges to calculate voltage.

EXAMPLE 14

The circuit in Figure 12 has a current of 10 mA. What is the voltage?

▶ FIGURE 12



Solution Ten milliamperes equals 10×10^{-3} A and 3.3 kΩ equals $3.3 \times 10^{3}\ \Omega$. Substitute these values into the formula $V = IR$.

$$V = IR = (10\ \text{mA})(3.3\ \text{k}\Omega) = (10 \times 10^{-3}\ \text{A})(3.3 \times 10^{3}\ \Omega) = \mathbf{33\ V}$$

Notice that 10^{-3} and 10^{3} cancel. Therefore, milliamperes cancel kilohms when multiplied, and the result is volts.

Related Problem If the current in Figure 12 is 25 mA, what is the voltage?

Use Multisim file E03-14 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 15

If there is a current of 50 μA through a 4.7 MΩ resistor, what is the voltage?

Solution Fifty microamperes equals 50×10^{-6} A and 4.7 MΩ is $4.7 \times 10^{6}\ \Omega$. Substitute these values into the formula $V = IR$.

$$V = IR = (50\ \mu\text{A})(4.7\ \text{M}\Omega) = (50 \times 10^{-6}\ \text{A})(4.7 \times 10^{6}\ \Omega) = \mathbf{235\ V}$$

Notice that 10^{-6} and 10^{6} cancel. Therefore, microamperes cancel megohms when multiplied, and the result is volts.

Related Problem If there are 450 μA through a 3.9 MΩ resistor, what is the voltage?

SECTION 3 CHECKUP

In Problems 1–7, calculate V.

1. $I = 1$ A and $R = 10\ \Omega$.
2. $I = 8$ A and $R = 470\ \Omega$.
3. $I = 3$ mA and $R = 100\ \Omega$.
4. $I = 25\ \mu$A and $R = 56\ \Omega$.
5. $I = 2$ mA and $R = 1.8$ kΩ.
6. $I = 5$ mA and $R = 100$ MΩ.
7. $I = 10\ \mu$A and $R = 2.2$ MΩ.
8. How much voltage is required to produce 100 mA through a 4.7 kΩ resistor?
9. What voltage do you need to cause 3 mA of current in a 3.3 kΩ resistor?
10. A battery produces 2 A through a 6.8 Ω resistive load. What is the battery voltage?

4 RESISTANCE CALCULATIONS

Examples in this section illustrate the Ohm's law expression $R = V/I$.

After completing this section, you should be able to

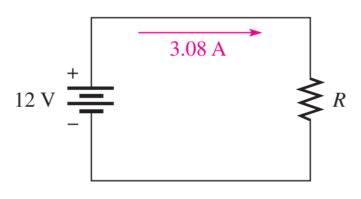
- **Calculate resistance in a circuit**
    - Use Ohm's law to find resistance when you know voltage and current values
    - Use current and voltage values expressed with metric prefixes

The following examples use the formula $R = V/I$. To get resistance in ohms, you must express the value of I in amperes and the value of V in volts.

EXAMPLE 16

In the circuit of Figure 13, how much resistance is needed to draw 3.08 A of current from the battery?

FIGURE 13



Solution Substitute 12 V for V and 3.08 A for I into the formula $R = V/I$.

$$R = \frac{V}{I} = \frac{12\text{ V}}{3.08\text{ A}} = \mathbf{3.90\ \Omega}$$

Related Problem In Figure 13, to what value must R be changed for a current of 5.45 A?

EXAMPLE 17

Use Ohm's law to calculate the resistance of a rear window defroster grid in a certain vehicle. When it is connected to 12.6 V, it draws 15.0 A from the battery. What is the resistance of the defroster grid?

Solution

$$R = \frac{V}{I} = \frac{12.6\text{ V}}{15.0\text{ A}} = \mathbf{0.84\ \Omega}$$

Related Problem If one of the grid wires opens, the current drops to 13.0 A. What is the new resistance?

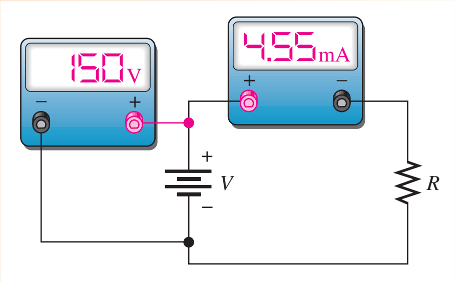
Units with Metric Prefixes

The following two examples illustrate how to use current values in the milliampere (mA) and microampere (μA) ranges to calculate resistance.

EXAMPLE 18

Suppose that the ammeter in Figure 14 indicates 4.55 mA of current and the voltmeter reads 150 V. What is the value of *R*?

▶ FIGURE 14



Solution 4.55 mA equals 4.55×10^{-3} A. Substitute the voltage and current values into the formula $R = V/I$.

$$R = \frac{V}{I} = \frac{150\text{ V}}{4.55\text{ mA}} = \frac{150\text{ V}}{4.55 \times 10^{-3}\text{ A}} = 33 \times 10^{3}\ \Omega = \mathbf{33\ k\Omega}$$

When volts are divided by milliamperes, the resistance is in kilohms.

Related Problem If the ammeter indicates 1.10 mA and the voltmeter reads 75 V, what is the value of *R*?

Use Multisim file E03-18 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 19

Suppose that the value of the resistor in Figure 14 is changed. If the battery voltage is still 150 V and the ammeter reads 68.2 μA, what is the new resistor value?

Solution 68.2 μA equals 68.2×10^{-6} A. Substitute *V* and *I* values into the equation for *R*.

$$R = \frac{V}{I} = \frac{150\text{ V}}{68.2\ \mu\text{A}} = \frac{150\text{ V}}{68.2 \times 10^{-6}\text{ A}} = 2.2 \times 10^{6}\ \Omega = \mathbf{2.2\ M\Omega}$$

Related Problem If the resistor is changed in Figure 14 so that the ammeter reads 48.5 μA, what is the new resistor value? Assume $V = 150$ V.

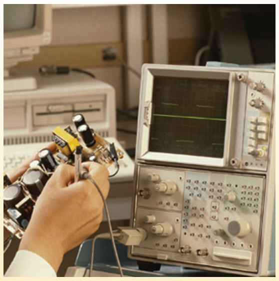
SECTION 4 CHECKUP

In Problems, 1–5, calculate R.

1. $V = 10$ V and $I = 2.13$ A.
2. $V = 270$ V and $I = 10$ A.
3. $V = 20$ kV and $I = 5.13$ A.
4. $V = 15$ V and $I = 2.68$ mA.
5. $V = 5$ V and $I = 2.27$ μA.
6. You have a resistor across which you measure 25 V, and your ammeter indicates 53.2 mA of current. What is the resistor's value in kilohms? In ohms?

5 INTRODUCTION TO TROUBLESHOOTING

Technicians must be able to diagnose and repair malfunctioning circuits and systems. In this section, you learn a general approach to troubleshooting using a simple example.



After completing this section, you should be able to

- **Describe a basic approach to troubleshooting**
  - List three steps in troubleshooting
  - Explain what is meant by half-splitting
  - Discuss and compare the three basic measurements of voltage, current, and resistance

Troubleshooting is the application of logical thinking combined with a thorough knowledge of circuit or system operation to correct a malfunction. The basic approach to troubleshooting consists of three steps: *analysis, planning,* and *measuring*. We will refer to this 3–step approach as APM.

Analysis

The first step in troubleshooting a circuit is to analyze clues or symptoms of the failure. The analysis can begin by determining the answer to certain questions:

1. Has the circuit ever worked?
2. If the circuit once worked, under what conditions did it fail?
3. What are the symptoms of the failure?
4. What are the possible causes of failure?

Planning

The second step in the troubleshooting process, after analyzing the clues, is formulating a logical plan of attack. Much time can be saved by proper planning. A working knowledge of the circuit is a prerequisite to a plan for troubleshooting. If you are not certain how the circuit is supposed to operate, take time to review circuit diagrams (schematics), operating instructions, and other pertinent information. A schematic with proper voltages marked at various test points is particularly useful. Although logical thinking is perhaps the most important tool in troubleshooting, it rarely can solve the problem by itself.

Measuring

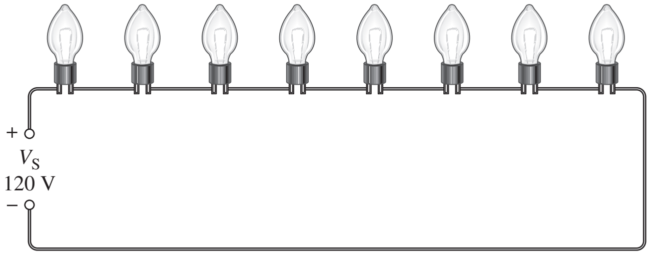
The third step is to narrow the possible failures by making carefully thought out measurements. These measurements usually confirm the direction you are taking in solving the problem, or they may point to a new direction that you should take. Occasionally, you may find a totally unexpected result.

An APM Example

The thought process that is part of the APM approach can be illustrated with a simple example. Suppose you have a string of 8 decorative 12 V bulbs connected in series to a 120 V source V_S, as shown in Figure 15. Assume that this circuit worked properly at one time but stopped working after it was moved to a new location. When plugged in at the new location, the lamps fail to turn on. How do you go about finding the trouble?

FIGURE 15

A string of bulbs connected to a voltage source.



The Analysis Thought Process You may think like this as you proceed to analyze the situation:

- Since the circuit worked before it was moved, the problem could be that there is no voltage at the new location.
- Perhaps the wiring was loose and pulled apart when moved.
- It is possible that a bulb is burned out or loose in its socket.

With this reasoning, you have considered possible causes and failures that may have occurred. The thought process continues:

- The fact that the circuit once worked eliminates the possibility that the original circuit was improperly wired.
- If the fault is due to an open path, it is unlikely that there is more than one break which could be either a bad connection or a burned out bulb.

You have now analyzed the problem and are ready to plan the process of finding the fault in the circuit.

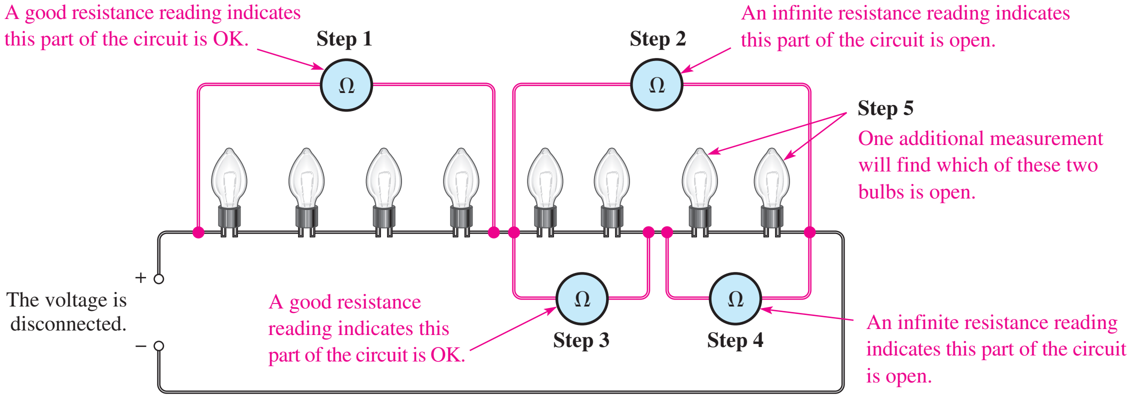
The Planning Thought Process The first part of your plan is to measure for voltage at the new location. If the voltage is present, then the problem is in the string of lights. If voltage is not present, check the circuit breaker at the distribution box in the house. Before resetting breakers, you should think about why a breaker may be tripped. Let's assume that you find the voltage is present. This means that the problem is in the string of lights.

The second part of your plan is to measure either the resistance in the string of lights or to measure voltages across the bulbs. The decision whether to measure resistance or voltage is a toss-up and can be made based on the ease of making the test. Seldom is a troubleshooting plan developed so completely that all possible contingencies are included. You will frequently need to modify the plan as you go along.

The Measurement Process You proceed with the first part of your plan by using a multimeter to check the voltage at the new location. Assume the measurement shows a voltage of 120 V. Now you have eliminated the possibility of no voltage. You know that, since you have voltage across the string and there is no current because no bulb is on, there must be an open in the current path. Either a bulb is burned out, a connection at the lamp socket is broken, or the wire is broken.

Next, you decide to locate the break by measuring resistance with your multimeter. Applying logical thinking, you decide to measure the resistance of each half of the string instead of measuring the resistance of each bulb. By measuring the resistance of half the bulbs at once, you can usually reduce the effort required to find the open. This technique is a type of troubleshooting procedure called **half-splitting**.

Once you have identified the half in which the open occurs, as indicated by an infinite resistance, you use half-splitting again on the faulty half and continue until you narrow the fault down to a faulty bulb or connection. This process is shown in Figure 16, assuming for purposes of illustration that the seventh bulb is burned out.


A good resistance reading indicates this part of the circuit is OK.
Step 1
Ω
Step 2
An infinite resistance reading indicates this part of the circuit is open.
Step 5
One additional measurement will find which of these two bulbs is open.
+
The voltage is disconnected.
−
A good resistance reading indicates this part of the circuit is OK.
Step 3
Step 4
An infinite resistance reading indicates this part of the circuit is open.

▲ FIGURE 16

Illustration of the half-splitting method of troubleshooting. The numbered steps indicate the sequence in which the multimeter is moved from one position to another.

As you can see in the figure, the half-splitting approach in this particular case takes a maximum of five measurements to identify the open bulb. If you had decided to measure each bulb individually and had started at the left, you would have needed seven measurements. Sometimes half-splitting saves steps; sometimes it doesn't. The number of steps required depends on where you make your measurements and in what sequence.

Unfortunately, most troubleshooting is more difficult than this example. However, analysis and planning are essential for effective troubleshooting in any situation. As measurements are made, the plan is often modified; the experienced troubleshooter narrows the search by fitting the symptoms and measurements into a probable cause. In some cases, low-cost equipment is simply discarded when troubleshooting and repair costs are comparable to replacement costs.

Comparison of *V*, *R*, and *I* Measurements

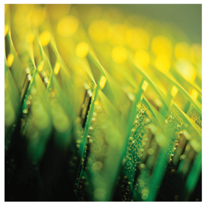
You can measure voltage, current, or resistance in a circuit. To measure voltage, place the voltmeter in parallel across the component; that is, place one lead on each side of the component. This makes voltage measurements the easiest of the three types of measurements.

To measure resistance, connect the ohmmeter across a component; however, the voltage must be first disconnected, and sometimes the component itself must be removed from the circuit. Therefore, resistance measurements are generally more difficult than voltage measurements.

To measure current, place the ammeter in series with the component; that is, the ammeter must be in line with the current path. To do this you must disconnect a component lead or a wire before you connect the ammeter. This usually makes a current measurement the most difficult to perform.

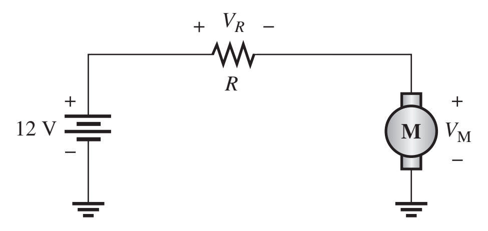
SECTION 5 CHECKUP

1. Name the three steps in the APM approach to troubleshooting.
2. Explain the basic idea of the half-splitting technique.
3. Why are voltages easier to measure than currents in a circuit?



Application Activity

In this application, a resistive circuit is developed for controlling the speed of a small dc motor used to drive a fan for cooling electronic equipment mounted in a cabinet. One way to control the speed of a dc motor is to connect a resistor to the motor, as shown in Figure 17. The resistor reduces the current to the motor and therefore reduces its speed. This method is only recommended for small motors because energy is wasted in the resistance, and the overall efficiency is low.



▲ FIGURE 17

A circuit is required that will allow a selection of four motor speeds. The control circuit is to be mounted in a small box that will be attached to the equipment cabinet to control the fan motor speed, as shown in Figure 18.

The application requires four distinct switchable speeds. Your job is to develop a schematic, determine the values for the required resistors, and prepare a test procedure for the circuit. You will apply Ohm's law in order to complete the assignment. Motor data in graphical form is provided in Figure 19.

Specifications

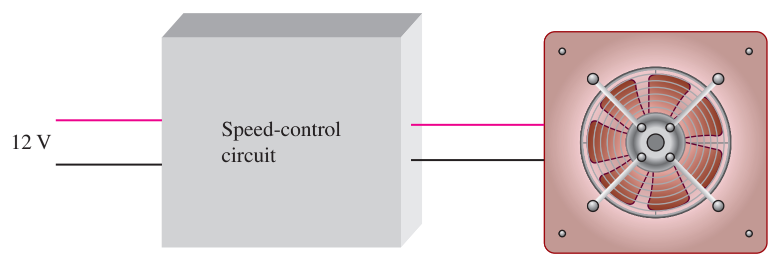
- Each resistor is switch selectable from a rotary switch, so that only one resistor is connected to the motor at a time.
- The four approximate speed settings are 800 rpm, 600 rpm, 400 rpm, and 200 rpm.
- The motor is rated for a maximum of 12 V at 800 rpm.
- The circuit board is to be connected to a 12 V dc source and to the fan motor with wiring that is available inside the cabinet.

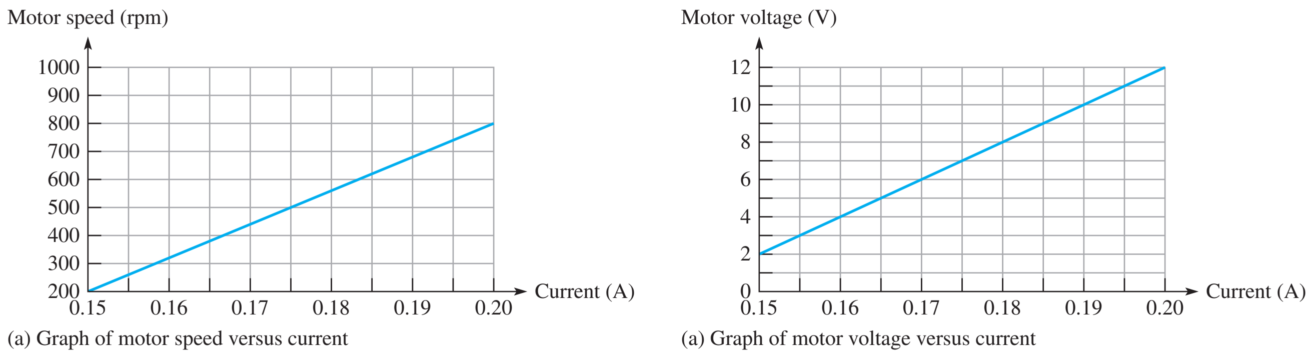
Resistance Calculations

The resistor box has been prepared with speed-value labels and the PC board has been prepared as shown in Figure 20.

1. Use the graphical motor data in Figure 19 and Ohm's law to determine the nearest standard resistor values needed for the project. *Hint:* To find the voltage across the resistor, V_R, subtract the motor voltage, V_M, from 12 V. This is based on Kirchhoff's law.

► FIGURE 18



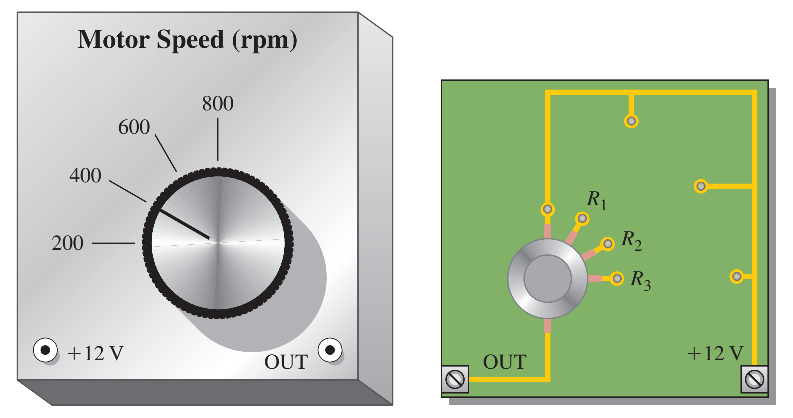


(a) Graph of motor speed versus current

(a) Graph of motor voltage versus current

▲ FIGURE 19

▶ FIGURE 20



(a) Top view of box

(b) Circuit board

The Schematic

2. From the requirements and the board layout, develop a schematic for the circuit.
3. Show the standard resistor values next to each resistor.

Test Procedure

4. After the resistor box is constructed, list steps you would take to ensure that it works properly.
5. List the instruments you would use in testing it.

Troubleshooting

Describe the most likely fault on the circuit board for each of the following problems:

6. When connected between the +12 V terminal and the OUT terminal, an ohmmeter reads infinite resistance for the 600 rpm position.
7. When connected between the +12 V terminal and the OUT terminal, an ohmmeter reads infinite resistance for all positions of the switch.
8. All resistors read 10% higher than the listed value.

Review

9. Explain how Ohm's law was applied to this problem.
10. Describe the relationship between current and motor speed.

SUMMARY

- Voltage and current are linearly proportional.
- Ohm's law gives the relationship of voltage, current, and resistance.
- Current is inversely proportional to resistance.
- A kilohm (kΩ) is one thousand ohms.
- A megohm (MΩ) is one million ohms.

- A microampere (μA) is one-millionth of an ampere.
- A milliampere (mA) is one-thousandth of an ampere.
- Use $I = V/R$ to calculate current.
- Use $V = IR$ to calculate voltage.
- Use $R = V/I$ to calculate resistance.
- APM is a 3-step troubleshooting approach, consisting of analysis, planning, and measuring.
- Half-splitting is a troubleshooting technique that can be used to reduce the number of measurements required to find a problem.

KEY TERMS

Linear Characterized by a straight-line relationship.

Ohm's law A law stating that current is directly proportional to voltage and inversely proportional to resistance.

Troubleshooting A systematic process of isolating, identifying, and correcting a fault in a circuit or system.

FORMULAS

1	$I = \frac{V}{R}$	Form of Ohm's law for calculating current
2	$V = IR$	Form of Ohm's law for calculating voltage
3	$R = \frac{V}{I}$	Form of Ohm's law for calculating resistance

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. If the total resistance of a circuit increases, current decreases.
2. Ohm's law for finding resistance is $R = I/V$.
3. When milliamps and kilohms are multiplied together, the result is volts.
4. If a 10 kΩ resistor is connected to a 10 V source, the current in the resistor will be 1 A.
5. The voltage across a resistor is directly proportional to the current through it.
6. Ohm's law for finding current is $I = V/R$.
7. When microamps and megohms are multiplied, the result is microvolts.
8. The relationship between voltage and current in a resistor is linear.
9. Ohm's law for finding voltage is $V = I/R$.
10. Conductance is the reciprocal of resistance.

SELF-TEST

Answers are at the end of the chapter.

1. Ohm's law states that
   - **(a)** current equals voltage times resistance
   - **(b)** voltage equals current times resistance
   - **(c)** resistance equals current divided by voltage
   - **(d)** voltage equals current squared times resistance
2. When the voltage across a resistor is doubled, the current will
   (a) triple **(b)** halve **(c)** double **(d)** not change
3. When 10 V are applied across a 20 Ω resistor, the current is
   (a) 10 A **(b)** 0.5 A **(c)** 200 A **(d)** 2 A
4. When there are 10 mA of current through 1.0 kΩ resistor, the voltage across the resistor is
   (a) 100 V **(b)** 0.1 V **(c)** 10 kV **(d)** 10 V

5. If 20 V are applied across a resistor and there are 6.06 mA of current, the resistance is

(a) 3.3 kΩ **(b)** 33 kΩ **(c)** 330 kΩ **(d)** 3.03 kΩ

6. A current of 250 μA through a 4.7 kΩ resistor produces a voltage drop of

(a) 53.2 V **(b)** 1.18 mV **(c)** 18.8 V **(d)** 1.18 V

7. A resistance of 2.2 MΩ is connected across a 1 kV source. The resulting current is approximately

(a) 2.2 mA **(b)** 0.455 mA **(c)** 45.5 μA **(d)** 0.455 A

8. How much resistance is required to limit the current from a 10 V battery to 1 mA?

(a) 100 Ω **(b)** 1.0 kΩ **(c)** 10 Ω **(d)** 10 kΩ

9. An electric heater draws 2.5 A from a 110 V source. The resistance of the heating element is

(a) 275 Ω **(b)** 22.7 mΩ **(c)** 44 Ω **(d)** 440 Ω

10. The current through a flashlight bulb is 20 mA and the total battery voltage is 4.5 V. The resistance of the bulb is

(a) 90 Ω **(b)** 225 Ω **(c)** 4.44 Ω **(d)** 45 Ω

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

1. If the current through a fixed resistor goes from 10 mA to 12 mA, the voltage across the resistor

(a) increases **(b)** decreases **(c)** stays the same

2. If the voltage across a fixed resistor goes from 10 V to 7 V, the current through the resistor

(a) increases **(b)** decreases **(c)** stays the same

3. A variable resistor has 5 V across it. If you reduce the resistance, the current through it

(a) increases **(b)** decreases **(c)** stays the same

4. If the voltage across a resistor increases from 5 V to 10 V and the current increases from 1 mA to 2 mA, the resistance

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 14.

5. If the voltmeter reading changes to 175 V, the ammeter reading

(a) increases **(b)** decreases **(c)** stays the same

6. If R is changed to a larger value and the voltmeter reading stays at 150 V, the current

(a) increases **(b)** decreases **(c)** stays the same

7. If the resistor is removed from the circuit leaving an open, the ammeter reading

(a) increases **(b)** decreases **(c)** stays the same

8. If the resistor is removed from the circuit leaving an open, the voltmeter reading

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 24.

9. If the rheostat is adjusted to increase the resistance, the current through the heating element

(a) increases **(b)** decreases **(c)** stays the same

10. If the rheostat is adjusted to increase the resistance, the source voltage

(a) increases **(b)** decreases **(c)** stays the same

11. If the fuse opens, the voltage across the heating element

(a) increases **(b)** decreases **(c)** stays the same

12. If the source voltage increases, the voltage across the heating element

(a) increases **(b)** decreases **(c)** stays the same

13. If the fuse is changed to one with a higher rating, the current through the rheostat

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 26.

14. If the lamp burns out (opens), the current

(a) increases **(b)** decreases **(c)** stays the same

15. If the lamp burns out, the voltage across it

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Relationship of Current, Voltage, and Resistance

1. In a circuit consisting of a voltage source and a resistor, describe what happens to the current when

(a) the voltage is tripled

(b) the voltage is reduced by 75%

(c) the resistance is doubled

(d) the resistance is reduced by 35%

(e) the voltage is doubled and the resistance is cut in half

(f) the voltage is doubled and the resistance is doubled

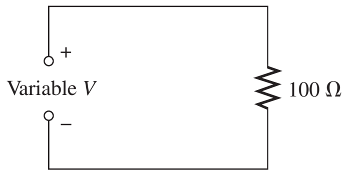
2. State the formula used to find I when the values of V and R are known.

3. State the formula used to find V when the values of I and R are known.

4. State the formula used to find R when the values of V and I are known.

5. A variable voltage source is connected to the circuit of Figure 21. Start at 0 V and increase the voltage in 10 V steps up to 100 V. Determine the current at each voltage point, and plot a graph of V versus I. Is the graph a straight line? What does the graph indicate?

FIGURE 21



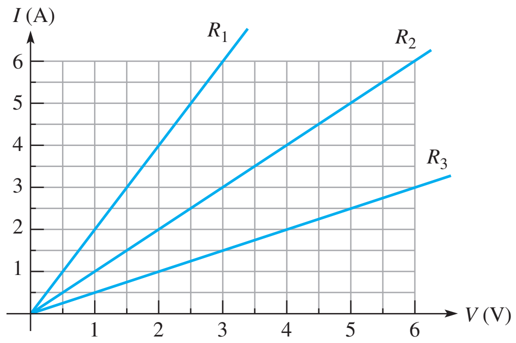
6. In a certain circuit, $I = 5$ mA when $V = 1$ V. Determine the current for each of the following voltages in the same circuit:

(a) $V = 1.5$ V **(b)** $V = 2$ V **(c)** $V = 3$ V

(d) $V = 4$ V **(e)** $V = 10$ V

7. Figure 22 is a graph of current versus voltage for three resistance values. Determine R_1, R_2, and R_3.

FIGURE 22

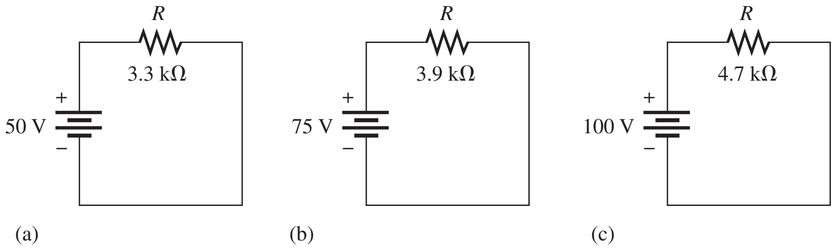


8. Plot the current-voltage relationship for a four-band resistor with the color code gray, red, red, gold.

9. Plot the current-voltage relationship for a five-band resistor with the color code brown, green, gray, brown, red.

10. Which circuit in Figure 23 has the most current? The least current?

▶ **FIGURE 23**



***11.** You are measuring the current in a circuit that is operated on a 10 V battery. The ammeter reads 50 mA. Later, you notice that the current has dropped to 30 mA. Eliminating the possibility of a resistance change, you must conclude that the voltage has changed. How much has the voltage of the battery changed, and what is its new value?

***12.** If you wish to increase the amount of current in a resistor from 100 mA to 150 mA by changing the 20 V source, by how many volts should you change the source? To what new value should you set it?

13. Plot a graph of current versus voltage for voltage values ranging from 10 V to 100 V in 10 V steps for each of the following resistance values:

(a) 1.0 Ω **(b)** 5.0 Ω **(c)** 20 Ω **(d)** 100 Ω

14. Does the graph in Problem 13 indicate a linear relationship between voltage and current? Explain.

SECTION 2 Current Calculations

15. Determine the current in each case:

(a) $V = 5\text{ V}, R = 1.0\ \Omega$
(b) $V = 15\text{ V}, R = 10\ \Omega$
(c) $V = 50\text{ V}, R = 100\ \Omega$
(d) $V = 30\text{ V}, R = 15\text{ k}\Omega$
(e) $V = 250\text{ V}, R = 5.6\text{ M}\Omega$

16. Determine the current in each case:

(a) $V = 9\text{ V}, R = 2.7\text{ k}\Omega$
(b) $V = 5.5\text{ V}, R = 10\text{ k}\Omega$
(c) $V = 40\text{ V}, R = 68\text{ k}\Omega$
(d) $V = 1\text{ kV}, R = 2.2\text{ k}\Omega$
(e) $V = 66\text{ kV}, R = 10\text{ M}\Omega$

17. A 10 Ω resistor is connected across a 12 V battery. What is the current through the resistor?

18. A certain resistor has the following color code: orange, orange, red, gold. Determine the maximum and minimum currents you should expect to measure when a 12 V source is connected across the resistor.

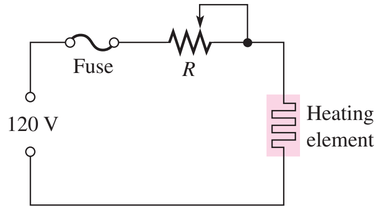
19. A 4-band resistor is connected across the terminals of a 25 V source. Determine the current in the resistor if the color code is yellow, violet, orange, silver.

20. A 5-band resistor is connected across a 12 V source. Determine the current if the color code is orange, violet, yellow, gold, brown.

21. If the voltage in Problem 20 is doubled, will a 0.5 A fuse blow? Explain your answer.

***22.** The potentiometer connected as a rheostat in Figure 24 is used to control the current to a heating element. When the rheostat is adjusted to a value of 8 Ω or less, the heating element can burn out. What is the rated value of the fuse needed to protect the circuit if the voltage across the heating element at the point of maximum current is 100 V and the voltage across the rheostat is the difference between the heating element voltage and the source voltage?

▶ FIGURE 24



23. An indicator light requires a 330 Ω resistor to limit current. The voltage across the current limiting resistor is 3 V. What is the current in the resistor?

SECTION 3 Voltage Calculations

24. A small solar cell is connected to a 27 kΩ resistor. In bright sunlight, the solar cell looks like a current source that can supply 180 μA to the resistor. What is the voltage across the resistor?

25. Calculate the voltage for each value of I and R:

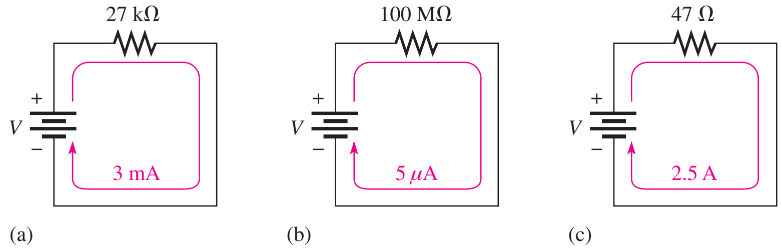
(a) $I = 2\text{ A}, R = 18\ \Omega$
(b) $I = 5\text{ A}, R = 56\ \Omega$
(c) $I = 2.5\text{ A}, R = 680\ \Omega$
(d) $I = 0.6\text{ A}, R = 47\ \Omega$
(e) $I = 0.1\text{ A}, R = 560\ \Omega$

26. Calculate the voltage for each value of I and R:

(a) $I = 1\text{ mA}, R = 10\ \Omega$
(b) $I = 50\text{ mA}, R = 33\ \Omega$
(c) $I = 3\text{ A}, R = 5.6\text{ k}\Omega$
(d) $I = 1.6\text{ mA}, R = 2.2\text{ k}\Omega$
(e) $I = 250\ \mu\text{A}, R = 1.0\text{ k}\Omega$
(f) $I = 500\text{ mA}, R = 1.5\text{ M}\Omega$
(g) $I = 850\ \mu\text{A}, R = 10\text{ M}\Omega$
(h) $I = 75\ \mu\text{A}, R = 47\ \Omega$

27. Three amperes of current are measured through a 27 Ω resistor connected across a voltage source. How much voltage does the source produce?

28. Assign a voltage value to each source in the circuits of Figure 25 to obtain the indicated amounts of current.



▲ FIGURE 25

***29.** A 6 V source is connected to a 100 Ω resistor by two 12 ft lengths of 18 gauge copper wire. The total resistance is the resistance of both wires added to the 100 Ω resistor. Determine the following:

(a) Current
(b) Resistor voltage drop
(c) Voltage drop across each length of wire

SECTION 4 Resistance Calculations

30. Calculate the resistance of a rheostat for each value of V and I:

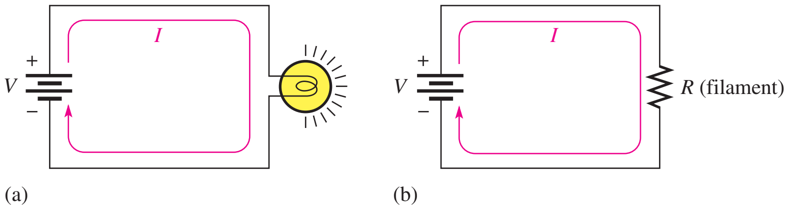
(a) $V = 10\text{ V}, I = 2\text{ A}$
(b) $V = 90\text{ V}, I = 45\text{ A}$
(c) $V = 50\text{ V}, I = 5\text{ A}$
(d) $V = 5.5\text{ V}, I = 10\text{ A}$
(e) $V = 150\text{ V}, I = 0.5\text{ A}$

31. Calculate the resistance of a rheostat for each set of V and I values:

(a) $V = 10\text{ kV}, I = 5\text{ A}$
(b) $V = 7\text{ V}, I = 2\text{ mA}$
(c) $V = 500\text{ V}, I = 250\text{ mA}$
(d) $V = 50\text{ V}, I = 500\ \mu\text{A}$
(e) $V = 1\text{ kV}, I = 1\text{ mA}$

32. Six volts are applied across a resistor. A current of 2 mA is measured. What is the value of the resistor?

33. The filament of a lamp in the circuit of Figure 26(a) has a certain amount of resistance, represented by an equivalent resistance in Figure 26(b). If the lamp operates with 120 V and 0.8 A of current, what is the resistance of its filament when it is on?

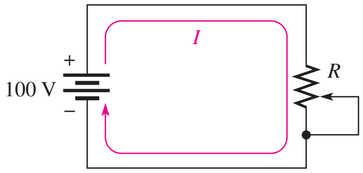

I
+
V
−
I
+
V
−
R (filament)
(a)
(b)

▲ FIGURE 26

34. A certain electrical device has an unknown resistance. You have available a 12 V battery and an ammeter. How would you determine the value of the unknown resistance? Draw the necessary circuit connections.

35. By varying the rheostat (variable resistor) in the circuit of Figure 27, you can change the amount of current. The setting of the rheostat is such that the current is 750 mA. What is the resistance value of this setting? To adjust the current to 1 A, to what resistance value must you set the rheostat? What is the problem with this circuit?

▶ FIGURE 27

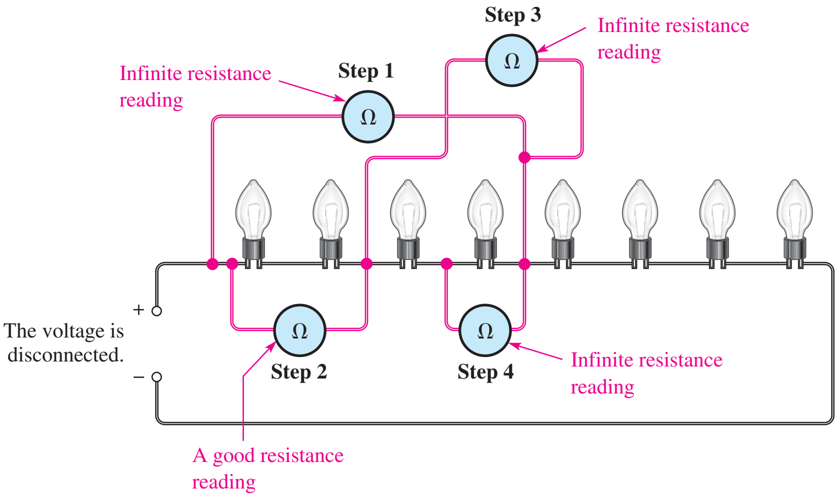


***36.** A 120 V lamp-dimming circuit is controlled by a rheostat and protected from excessive current by a 2 A fuse. To what minimum resistance value can the rheostat be set without blowing the fuse? Assume a lamp resistance of 15 Ω.

37. Repeat Problem 36 for a 110 V circuit and a 1 A fuse.

SECTION 5 Introduction to Troubleshooting

38. In the light circuit of Figure 28, identify the faulty bulb based on the series of ohmmeter readings shown.


Step 3
Infinite resistance reading
Infinite resistance reading
Step 1
Ω
Ω
The voltage is disconnected.
+
−
Ω
Ω
Step 2
Step 4
Infinite resistance reading
A good resistance reading

▲ FIGURE 28

39. Assume you have a 32-light string and one of the bulbs is burned out. Using the half-splitting approach and starting in the left half of the circuit, how many resistance measurements will it take to find the faulty bulb if it is seventeenth from the left?

Multisim Troubleshooting and Analysis

These problems require Multisim.

40. Open file P03-40 and determine which one of the three circuits is not working properly.

41. Open file P03-41 and measure the resistance values of the resistors.

42. Open file P03-42 and determine the values of the current and voltage.

43. Open file P03-43 and determine the value of the source voltage and the resistance.

44. Open file P03-44 and find the problem with the circuit.

ANSWERS

SECTION CHECKUPS

SECTION 1 The Relationship of Current, Voltage, and Resistance

1. Current, voltage, and resistance
2. $I = V/R$
3. $V = IR$
4. $R = V/I$
5. When voltage is tripled, current increases by three times.
6. When voltage is halved, current reduces to one-half of original value.
7. 0.5 A
8. The current would increase by four times if the voltage doubles and the resistance is halved.
9. $I = 5\text{ mA}$
10. $I = 6\text{ A}$

SECTION 2 Current Calculations

1. $I = 10\text{ V}/5.6\ \Omega = 1.79\text{ A}$
2. $I = 100\text{ V}/560\ \Omega = 179\text{ mA}$
3. $I = 5\text{ V}/2.2\text{ k}\Omega = 2.27\text{ mA}$
4. $I = 15\text{ V}/4.7\text{ M}\Omega = 3.19\ \mu\text{A}$
5. $I = 20\text{ kV}/4.7\text{ M}\Omega = 4.26\text{ mA}$
6. $I = 10\text{ kV}/2.2\text{ k}\Omega = 4.55\text{ A}$

SECTION 3 Voltage Calculations

1. $V = (1\text{ A})(10\ \Omega) = 10\text{ V}$
2. $V = (8\text{ A})(470\ \Omega) = 3.76\text{ kV}$
3. $V = (3\text{ mA})(100\ \Omega) = 300\text{ mV}$
4. $V = (25\ \mu\text{A})(56\ \Omega) = 1.4\text{ mV}$
5. $V = (2\text{ mA})(1.8\text{ k}\Omega) = 3.6\text{ V}$
6. $V = (5\text{ mA})(100\text{ M}\Omega) = 500\text{ kV}$
7. $V = (10\ \mu\text{A})(2.2\text{ M}\Omega) = 22\text{ V}$
8. $V = (100\text{ mA})(4.7\text{ k}\Omega) = 470\text{ V}$
9. $V = (3\text{ mA})(3.3\text{ k}\Omega) = 9.9\text{ V}$
10. $V = (2\text{ A})(6.8\ \Omega) = 13.6\text{ V}$

SECTION 4 Resistance Calculations

1. $R = 10\text{ V}/2.13\text{ A} = 4.7\ \Omega$

2. $R = 270\text{ V}/10\text{ A} = 27\ \Omega$

3. $R = 20\text{ kV}/5.13\text{ A} = 3.9\text{ k}\Omega$

4. $R = 15\text{ V}/2.68\text{ mA} = 5.6\text{ k}\Omega$

5. $R = 5\text{ V}/2.27\ \mu\text{A} = 2.2\text{ M}\Omega$

6. $R = 25\text{ V}/53.2\text{ mA} = 0.47\text{ k}\Omega = 470\ \Omega$

SECTION 5 Introduction to Troubleshooting

1. Analysis, planning, and measurement

2. Half-splitting identifies the fault by successively isolating half of the remaining circuit.

3. Voltage is measured across a component; current is measured in series with the component.

RELATED PROBLEMS FOR EXAMPLES

1 Yes
2 0 V
3 3.03 mA
4 0.005 A
5 0.005 A
6 13.6 mA
7 21.3 μA
8 2.66 μA
9 37.0 mA
10 1.47 mA
11 1200 V
12 49.5 mV
13 0.150 mV
14 82.5 V
15 1755 V
16 2.20 Ω
17 0.97 Ω
18 68.2 kΩ
19 3.30 MΩ

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** F **5.** T **6.** T **7.** F **8.** T
9. F **10.** T

SELF-TEST

1. (b) **2.** (c) **3.** (b) **4.** (d) **5.** (a) **6.** (d) **7.** (b) **8.** (d)
9. (c) **10.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (a) **4.** (c) **5.** (a) **6.** (b) **7.** (b) **8.** (c)
9. (b) **10.** (c) **11.** (b) **12.** (a) **13.** (c) **14.** (b) **15.** (c)

ANSWERS TO ODD-NUMBERED PROBLEMS

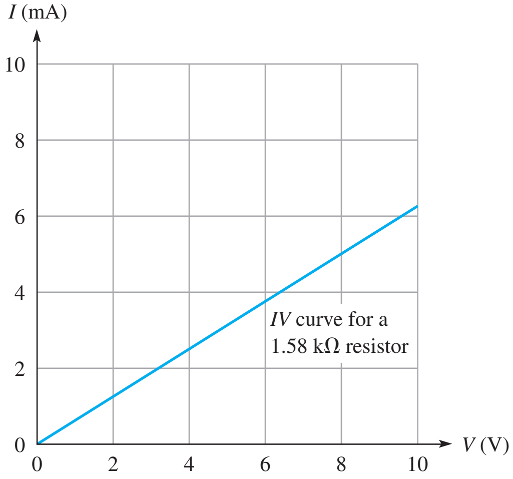
1. **(a)** Current triples. **(b)** Current is reduced 75%.
(c) Current is halved. **(d)** Current increases 54%.
(e) Current quadruples. **(f)** Current is unchanged.

3. $V = IR$

5. The graph is a straight line, indicating a linear relationship between V and I.

7. $R_1 = 0.5\ \Omega, R_2 = 1.0\ \Omega, R_3 = 2\ \Omega$

9. See Figure 29.



▲ FIGURE 29

11. The voltage decreased by 4 V (from 10 V to 6 V).

13. See Figure 30.

15. **(a)** 5 A **(b)** 1.5 A **(c)** 500 mA
(d) 2 mA **(e)** 44.6 μA

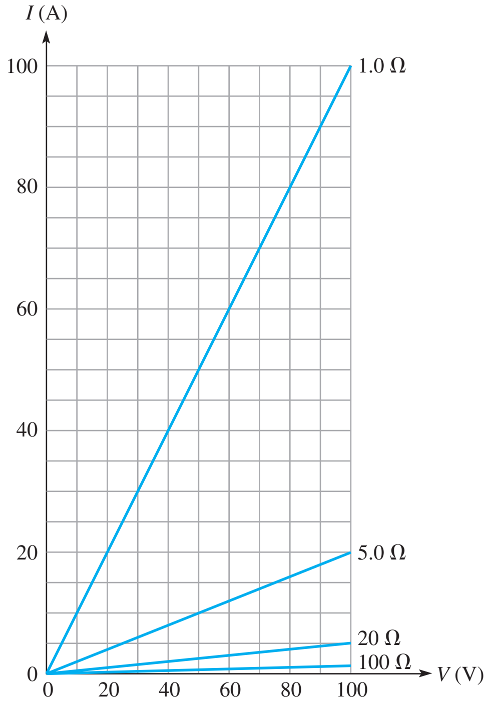
17. 1.2 A

19. 532 μA

21. Yes. The current is now 0.642 A, which exceeds the rating of the fuse.

23. 9.1 mA

25. **(a)** 36 V **(b)** 280 V **(c)** 1700 V
(d) 28.2 V **(e)** 56 V



▲ FIGURE 30

27. 81 V

29. **(a)** 59.9 mA **(b)** 5.99 V **(c)** 4.61 mV

31. **(a)** 2 kΩ **(b)** 3.5 kΩ **(c)** 2 kΩ
(d) 100 kΩ **(e)** 1.0 MΩ

33. 150 Ω

35. 133 Ω; 100 Ω; the source can be shorted if the rheostat is set to 0 Ω.

37. 95 Ω

39. Five

41. $R_A = 560\ \text{k}\Omega; R_B = 2.2\ \text{M}\Omega;$
$R_C = 1.8\ \text{k}\Omega; R_D = 33\ \Omega$

43. $V = 18\ \text{V}; I = 5.455\ \text{mA};$
$R = 3.3\ \text{k}\Omega$

ENERGY AND POWER

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ENERGY AND POWER

CHAPTER OUTLINE

1 Energy and Power
2 Power in an Electric Circuit
3 Resistor Power Ratings
4 Energy Conversion and Voltage Drop in Resistance
5 Power Supplies and Batteries
Application Activity

CHAPTER OBJECTIVES

- Define *energy* and *power*
- Calculate power in a circuit
- Properly select resistors based on power consideration
- Explain energy conversion and voltage drop
- Discuss the characteristics of power supplies and batteries

KEY TERMS

- Energy
- Power
- Joule (J)
- Watt (W)
- Kilowatt-hour
- Watt's law
- Voltage drop
- Efficiency
- Ampere-hour rating

APPLICATION ACTIVITY PREVIEW

In this application activity you will see how the theory learned in this chapter is applicable to a resistance substitution box. The resistance box is to be used in testing circuits in which there will be a maximum of 5 V across all the resistors. You will determine the power rating of each resistor and develop a test procedure for the circuit as well as a cost estimate and parts list.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

You know the relationship of current, voltage, and resistance as stated by Ohm's law. The existence of these three quantities in an electric circuit results in the fourth basic quantity known as power. A specific relationship exists between power and I, V, and R.

Energy is the ability to do work, and power is the rate at which energy is used. Current carries electrical energy through a circuit. As the free electrons pass through the resistance of the circuit, they give up their energy when they collide with atoms in the resistive material. The electrical energy given up by the electrons is converted into heat energy. The rate at which the electrical energy is used is the power in the circuit.

1 ENERGY AND POWER

When there is current through a resistance, electrical energy is converted to heat or other form of energy, such as light. A common example of this is a light bulb that becomes too hot to touch. The current through the filament that produces light also produces unwanted heat because the filament has resistance. Electrical components must be able to dissipate a certain amount of energy in a given period of time.

After completing this section, you should be able to

- **Define *energy* and *power***
  - Express power in terms of energy
  - State the unit of power
  - State the common units of energy
  - Perform energy and power calculations

Energy is the ability to do work, and power is the rate at which energy is used.

Power (P) is a certain amount of energy (W) used in a certain length of time (t), expressed as follows:

$$P = \frac{W}{t}$$

Equation 1

where P is power in watts (W), W is energy in joules (J), and t is time in seconds (s). Note that an italic W is used to represent energy in the form of work and a nonitalic W is used for watts, the unit of power. The **joule (J)** is the SI unit of energy.

Energy in joules divided by time in seconds gives power in watts. For example, if 50 J of energy are used in 2 s, the power is 50 J/2 s = 25 W. By definition,

One watt (W) is the amount of power when one joule of energy is used in one second.

Thus, the number of joules used in one second is always equal to the number of watts. For example, if 75 J are used in 1 s, the power is $P = W/t = 75\text{ J}/1\text{ s} = 75\text{ W}$.

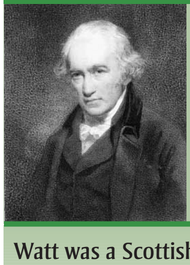
Amounts of power much less than one watt are common in certain areas of electronics. As with small current and voltage values, metric prefixes are used to designate small amounts of power. Thus, milliwatts (mW), microwatts (μW), and even picowatts (pW) are commonly found in some applications.

In the electrical utilities field, kilowatts (kW) and megawatts (MW) are common units. Radio and television stations also use large amounts of power to transmit signals. Electric motors are commonly rated in horsepower (hp) where 1 hp = 746 W.

Since power is the rate at which energy is used, as expressed in Equation 1, power utilized over a period of time represents energy consumption. If you multiply power in watts and time in seconds, you have energy in joules, symbolized by W.

$$W = Pt$$

HISTORY NOTE



James Watt
1736–1819

Watt was a Scottish inventor and was well known for his improvements to the steam engine that made it practical for industrial use. Watt patented several inventions, including the rotary engine. The unit of power is named in his honor. (Photo credit: Library of Congress.)

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

EXAMPLE 1

An amount of energy equal to 100 J is used in 5 s. What is the power in watts?

Solution

$$P = \frac{\text{energy}}{\text{time}} = \frac{W}{t} = \frac{100\text{ J}}{5\text{ s}} = \mathbf{20\text{ W}}$$

*Related Problem** If 100 W of power occurs for 30 s, how much energy, in joules, is used?

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*Answers are at the end of the chapter.

EXAMPLE 2

Express the following values of electrical power using appropriate metric prefixes:

(a) 0.045 W **(b)** 0.000012 W **(c)** 3500 W **(d)** 10,000,000 W

Solution

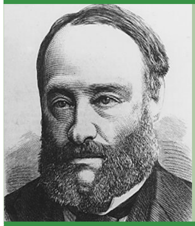
(a) 0.045 W = **45 mW** **(b)** 0.000012 W = **12 μW**

(c) 3500 W = **3.5 kW** **(d)** 10,000,000 W = **10 MW**

Related Problem Express the following amounts of power in watts without metric prefixes:

(a) 1 mW **(b)** 1800 μW **(c)** 1000 mW **(d)** 1 μW

HISTORY NOTE



James Prescott Joule 1818–1889

Joule, a British physicist, is known for his research in electricity and thermodynamics. He formulated the relationship that states that the amount of heat energy produced by an electrical current in a conductor is proportional to the conductor's resistance and the time. The unit of energy is named in his honor. (Photo credit: Library of Congress.)

The Kilowatt-hour (kWh) Unit of Energy

The joule has been defined as a unit of energy. However, there is another way to express energy. Since power is expressed in watts and time in seconds, units of energy called the watt-second (Ws), watt-hour (Wh), and kilowatt-hour (kWh) can be used.

When you pay your electric bill, you are charged on the basis of the amount of energy you use, not the power. Because power companies deal in huge amounts of energy, the most practical unit is the kilowatt-hour. You use a **kilowatt-hour** of energy when you use one thousand watts of power for one hour. For example, a 100 W light bulb burning for 10 h uses 1 kWh of energy.

$$W = Pt = (100\text{ W})(10\text{ h}) = 1000\text{ Wh} = 1\text{ kWh}$$

EXAMPLE 3

Determine the number of kilowatt-hours (kWh) for each of the following energy consumptions:

(a) 1400 W for 1 h **(b)** 2500 W for 2 h **(c)** 100,000 W for 5 h

Solution

(a) 1400 W = 1.4 kW

$$W = Pt = (1.4\text{ kW})(1\text{ h}) = \mathbf{1.4\text{ kWh}}$$

(b) 2500 W = 2.5 kW

$$W = (2.5\text{ kW})(2\text{ h}) = \mathbf{5\text{ kWh}}$$

(c) 100,000 W = 100 kW

$$W = (100\text{ kW})(5\text{ h}) = \mathbf{500\text{ kWh}}$$

Related Problem How many kilowatt-hours are used by a 250 W bulb burning for 8 h?

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Define *power*.
2. Write the formula for power in terms of energy and time.
3. Define *watt*.
4. Express each of the following values of power in the most appropriate units:
   (a) 68,000 W **(b)** 0.005 W **(c)** 0.000025 W
5. If you use 100 W of power for 10 h, how much energy (in kWh) have you used?
6. Convert 2000 Wh to kilowatt-hours.
7. Convert 360,000 Ws to kilowatt-hours.

2 POWER IN AN ELECTRIC CIRCUIT

The generation of heat, which occurs when electrical energy is converted to heat energy, in an electric circuit is often an unwanted by-product of current through the resistance in the circuit. In some cases, however, the generation of heat is the primary purpose of a circuit as, for example, in an electric resistive heater. In any case, you must frequently deal with power in electrical and electronic circuits.

After completing this section, you should be able to

- **Calculate power in a circuit**
  - Determine power when you know I and R values
  - Determine power when you know V and I values
  - Determine power when you know V and R values

When there is current through resistance, the collisions of the electrons produce heat as a result of the conversion of electrical energy, as indicated in Figure 1. The amount of power dissipated in an electric circuit is dependent on the amount of resistance and on the amount of current, expressed as follows:

$$P = I^2R$$

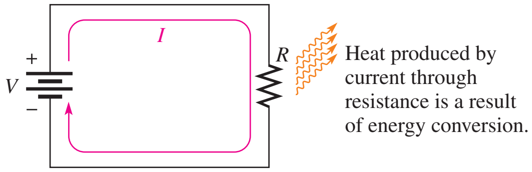
Equation 2

where P is power in watts (W), I is current in amperes (A), and R is resistance in ohms (Ω). You can get an equivalent expression for power in terms of voltage and current by substituting V for IR (I^2 is $I \times I$).

$$P = I^2R = (I \times I)R = I(IR) = (IR)I$$

$$P = VI$$

Equation 3



I
+
V
−
R
Heat produced by current through resistance is a result of energy conversion.

▲ FIGURE 1

Power dissipation in an electric circuit results in heat energy given off by the resistance.

where P is in watts when V is in volts and I is in amperes. You can obtain another equivalent expression by substituting V/R for I (Ohm's law).

$$P = VI = V\left(\frac{V}{R}\right)$$

Equation 4

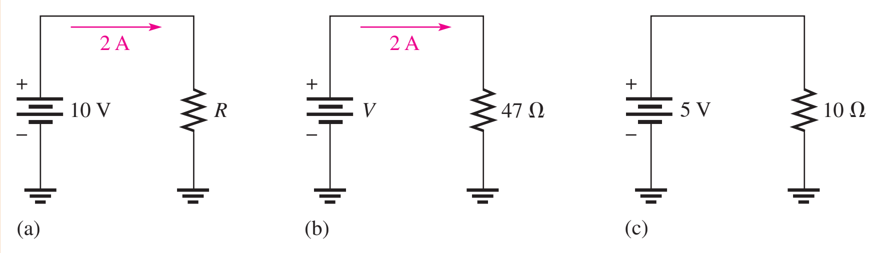
$$P = \frac{V^2}{R}$$

The relationships between power and current, voltage, and resistance expressed in the preceding formulas are known as **Watt's law**. In each case, I must be in amps, V in volts, and R in ohms. To calculate the power in a resistance, you can use any one of the three power formulas, depending on what information you have. For example, assume that you know the values of current and voltage. In this case calculate the power with the formula $P = VI$. If you know I and R, use the formula $P = I^2R$. If you know V and R, use the formula $P = V^2/R$.

EXAMPLE 4

Calculate the power in each of the three circuits of Figure 2.

▶ FIGURE 2



Solution In circuit (a), you know V and I. Therefore, use Equation 3.

$$P = VI = (10\text{ V})(2\text{ A}) = \mathbf{20\text{ W}}$$

In circuit (b), you know I and R. Therefore, use Equation 2.

$$P = I^2R = (2\text{ A})^2(47\ \Omega) = \mathbf{188\text{ W}}$$

In circuit (c), you know V and R. Therefore, use Equation 4.

$$P = \frac{V^2}{R} = \frac{(5\text{ V})^2}{10\ \Omega} = \mathbf{2.5\text{ W}}$$

Related Problem Determine P in each circuit of Figure 2 for the following changes:

Circuit (a): I doubled and V remains the same

Circuit (b): R doubled and I remains the same

Circuit (c): V halved and R remains the same

EXAMPLE 5

A 100 W light bulb operates on 120 V. How much current does it require?

Solution Use the formula $P = VI$ and solve for I by first transposing the terms to get I on the left side in the equation.

$$VI = P$$

Rearranging,

$$I = \frac{P}{V}$$

Substituting 100 W for P and 120 V for V yields

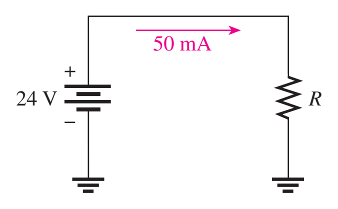
$$I = \frac{P}{V} = \frac{100\ \text{W}}{120\ \text{V}} = 0.833\ \text{A} = \mathbf{833\ mA}$$

Related Problem A light bulb draws 545 mA from a 120 V source. What is the power dissipated?

SECTION 2 CHECKUP

1. If there are 10 V across a resistor and a current of 3 A through it, what is the power dissipated?
2. How much power does the source in Figure 3 generate? What is the power in the resistor? Are the two values the same? Why?

▶ FIGURE 3



3. If there is a current of 5 A through a 56 Ω resistor, what is the power dissipated?
4. How much power is dissipated by 20 mA through a 4.7 kΩ resistor?
5. Five volts are applied to a 10 Ω resistor. What is the power dissipated?
6. How much power does a 2.2 kΩ resistor with 8 V across it dissipate?
7. What is the resistance of a 75 W bulb that takes 0.5 A?

3 RESISTOR POWER RATINGS

As you know, a resistor gives off heat when there is current through it. The limit to the amount of heat that a resistor can give off is specified by its power rating.

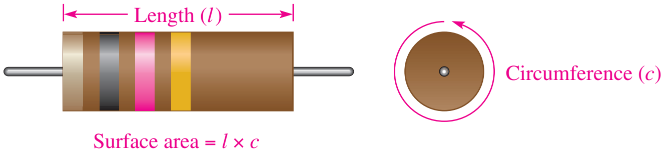
After completing this section, you should be able to

- **Properly select resistors based on power consideration**
  - Define *power rating*
  - Explain how physical characteristics of resistors determine their power rating
  - Check for resistor failure with an ohmmeter

The **power rating** is the maximum amount of power that a resistor can dissipate without being damaged by excessive heat buildup. The power rating is not related to the ohmic value (resistance) but rather is determined mainly by the physical composition, size, and shape of the resistor. All else being equal, the larger the surface area of a resistor, the more power it can dissipate. *The surface area of a cylindrically shaped resistor is equal to the length (l) times the circumference (c),* as indicated in Figure 4. The area of the ends is not included.

▶ **FIGURE 4**

The power rating of a resistor is directly related to its surface area.


Length (l)
Circumference (c)
Surface area = $l \times c$

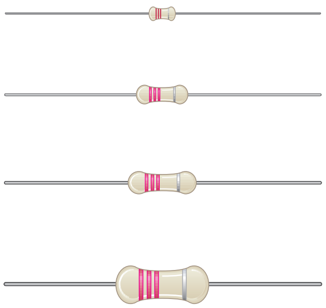
TECH NOTE

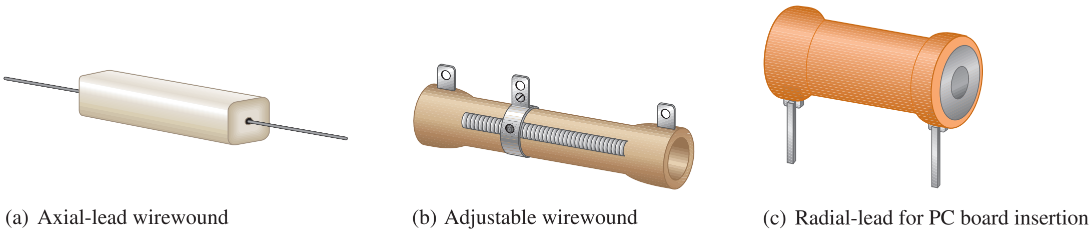
Sometimes an overheated resistor is due to another failure in the circuit. After replacing a heat-damaged resistor and before restoring power, check for visual faults that may cause excessive current, such as a short between two conductors.

Metal-film resistors are available in standard power ratings from ⅛ W to 1 W, as shown in Figure 5. Available power ratings for other types of resistors vary. For example, wirewound resistors have ratings up to 225 W or greater. Figure 6 shows some of these resistors.

▶ **FIGURE 5**

Relative sizes of metal-film resistors with standard power ratings of ⅛ W, ¼ W, ½ W, and 1 W.




(a) Axial-lead wirewound
(b) Adjustable wirewound
(c) Radial-lead for PC board insertion

▲ **FIGURE 6**

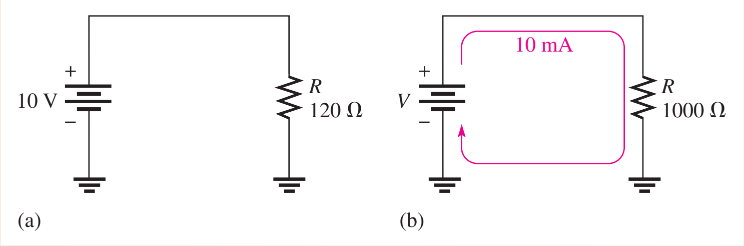
Typical resistors with high power ratings.

When a resistor is used in a circuit, its power rating must be greater than the maximum power that it will have to handle. For example, if a resistor is to dissipate 0.75 W in a circuit application, its rating should be at least the next higher standard value which is 1 W. *A rating larger than the actual power should be used when possible as a safety margin.*

EXAMPLE 6

Choose an adequate power rating for each of the metal-film resistors in Figure 7 (⅛ W, ¼ W, ½ W, or 1 W).

▶ **FIGURE 7**


+
10 V
–
R
120 Ω
(a)
+
V
–
10 mA
R
1000 Ω
(b)

Solution In Figure 7(a), the actual power is

$$P = \frac{V^2}{R} = \frac{(10\text{ V})^2}{120\ \Omega} = \frac{100\text{ V}^2}{120\ \Omega} = 0.833\text{ W}$$

Select a resistor with a power rating higher than the actual power. In this case, a **1 W resistor** should be used.

In Figure 7(b), the actual power is

$$P = I^2R = (10\text{ mA})^2(1000\ \Omega) = (10 \times 10^{-3}\text{ A})^2(1000\ \Omega) = 0.1\text{ W}$$

At least a **⅛ W (0.125 W) resistor** should be used in this case.

Related Problem A certain resistor is required to dissipate 0.25 W. What standard rating should be used?

When the power in a resistor is greater than its rating, the resistor will become excessively hot. As a result, the resistor may burn open or its resistance value may change.

A resistor that has been damaged because of overheating can often be detected by the charred or altered appearance of its surface. If there is no visual evidence, a resistor that is suspected of being damaged can be checked with an ohmmeter for an open or incorrect resistance value. Recall that one or both leads of a resistor should be removed from a circuit to measure resistance.

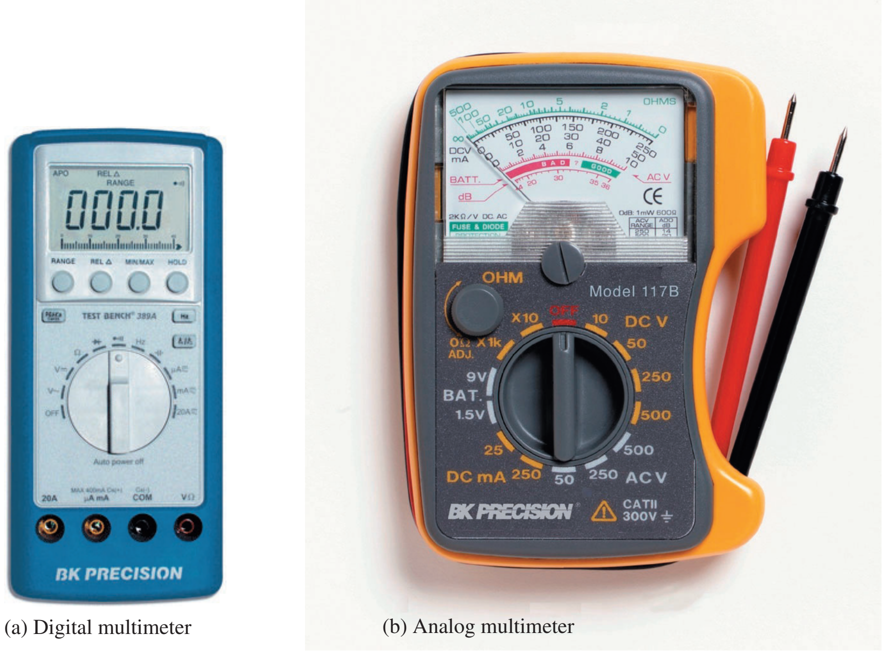
SAFETY NOTE

Some resistors can become very hot in normal operation. To avoid a burn, do not touch a circuit component while the power is connected to the circuit. After power has been turned off, allow time for the components to cool down.

Checking a Resistor with an Ohmmeter

A typical digital multimeter and an analog multimeter are shown in Figures 8(a) and 8(b), respectively. For the digital meter in Figure 8(a), you use the round function switch to select ohms (Ω). You do not have to manually select a range because this particular meter is autoranging and you have a direct digital readout of the resistance value. The large round switch on the analog meter is called a *range switch*. Notice the resistance (OHMS) settings on both meters.

For the analog meter in part (b), each setting indicates the amount by which the ohms scale (top scale) on the meter is to be multiplied. For example, if the pointer is at 50 on the



(a) Digital multimeter

(b) Analog multimeter

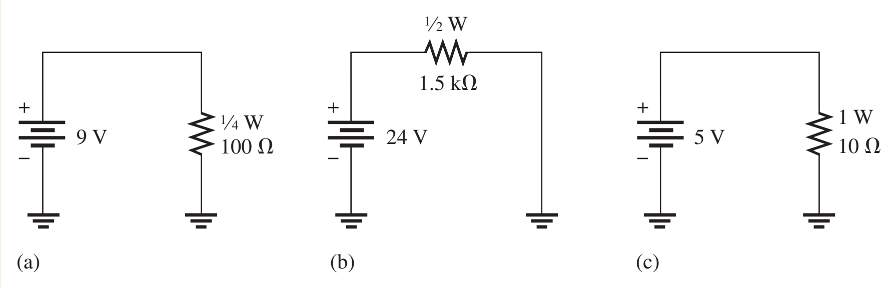
FIGURE 8

Typical portable multimeters. Reproduced with permission. Courtesy of B+K Precision.

ohms scale and the range switch is set at ×10, the resistance being measured is $50 \times 10\ \Omega = 500\ \Omega$. *If the resistor is open, the pointer will stay at full left scale (∞ means infinite) regardless of the range switch setting.*

EXAMPLE 7

Determine whether the resistor in each circuit of Figure 9 has possibly been damaged by overheating.



▲ **FIGURE 9**

Solution In the circuit in Figure 9(a),

$$P = \frac{V^2}{R} = \frac{(9\text{ V})^2}{100\ \Omega} = 0.810\text{ W} = 810\text{ mW}$$

The rating of the resistor is ¼ W (0.25 W), which is insufficient to handle the power. The resistor has been overheated and may be burned out, making it an open.

In the circuit of Figure 9(b),

$$P = \frac{V^2}{R} = \frac{(24\text{ V})^2}{1.5\text{ k}\Omega} = 0.384\text{ W} = 384\text{ mW}$$

The rating of the resistor is ½ W (0.5 W), which is sufficient to handle the power.

In the circuit of Figure 9(c),

$$P = \frac{V^2}{R} = \frac{(5\text{ V})^2}{10\ \Omega} = 2.5\text{ W}$$

The rating of the resistor is 1 W, which is insufficient to handle the power. The resistor has been overheated and may be burned out, making it an open.

Related Problem A 0.25 W, 1.0 kΩ resistor is connected across a 12 V battery. Is the power rating adequate?

SECTION 3 CHECKUP

1. Name two important values associated with a resistor.
2. How does the physical size of a resistor determine the amount of power that it can handle?
3. List the standard power ratings of metal-film resistors.
4. A resistor must handle 0.3 W. What minimum power rating of a metal-film resistor should be used to dissipate the energy properly?
5. What is the maximum voltage that can be applied to a ¼ W 100 Ω resistor if the power rating is not to be exceeded?

4 Energy Conversion and Voltage Drop in Resistance

As you have learned, when there is current through a resistance, electrical energy is converted to heat energy. This heat is caused by collisions of the free electrons within the atomic structure of the resistive material. When a collision occurs, heat is given off; and the electron gives up some of its acquired energy as it moves through the material.

After completing this section, you should be able to

- **Explain energy conversion and voltage drop**
  - Discuss the cause of energy conversion in a circuit
  - Define *voltage drop*
  - Explain the relationship between energy conversion and voltage drop

Figure 10 illustrates charge in the form of electrons flowing from the negative terminal of a battery, through a circuit, and back to the positive terminal. As they emerge from the negative terminal, the electrons are at their highest energy level. The electrons flow through each of the resistors that are connected together to form a current path (this type of connection is called series). As the electrons flow through each resistor, some of their energy is given up in the form of heat. Therefore, the electrons have more energy when they enter a resistor than when they exit the resistor, as illustrated in the figure by the decrease in the intensity of the red color. When they have traveled through the circuit back to the positive terminal of the battery, the electrons are at their lowest energy level.

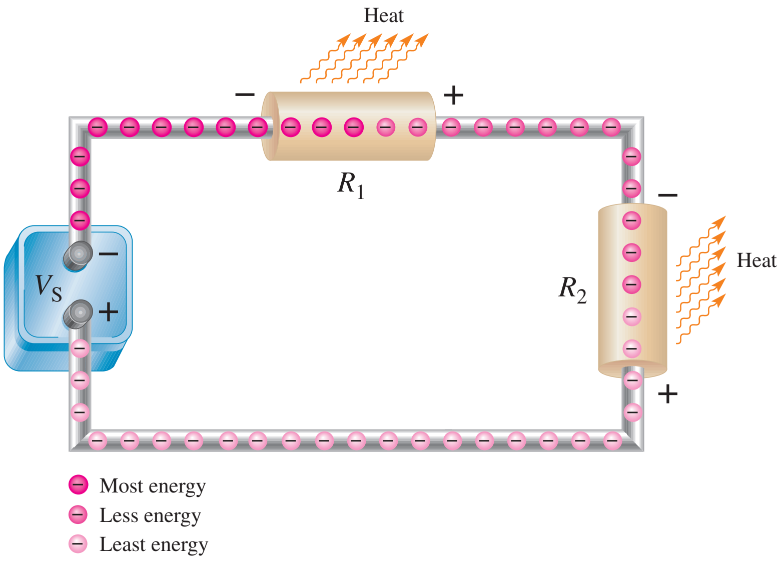


FIGURE 10

A loss of energy by electrons (charge) as they flow through a resistance creates a voltage drop because voltage equals energy divided by charge.

Recall that voltage equals energy per charge ($V = W/Q$) and charge is a property of electrons. Based on the voltage of the battery, a certain amount of energy is imparted to all of the electrons that flow out of the negative terminal. The same number of electrons flow at each point throughout the circuit, but their energy decreases as they move through the resistance of the circuit.

In Figure 10, the voltage at the left end of R_1 is equal to W_{enter}/Q, and the voltage at the right end of R_1 is equal to W_{exit}/Q. The same number of electrons that enter R_1 also exit R_1, so Q is constant. However, the energy W_{exit} is less than W_{enter}, so the voltage at the right end of R_1 is less than the voltage at the left end. This decrease in voltage across the resistor due to a loss of energy is called a **voltage drop**. The voltage at the right end of R_1 is less negative (or more positive) than the voltage at the left end. The voltage drop is indicated by $-$ and $+$ signs (the $+$ implies a less negative or more positive voltage). For conventional current direction, a voltage drop is defined to be from positive ($+$) to negative ($-$), which is the opposite of that for electron flow.

The electrons have lost some energy in R_1 and now they enter R_2 with a reduced energy level. As they flow through R_2, they lose more energy, resulting in another voltage drop across R_2.

SECTION 4 CHECKUP

1. What is the basic reason for energy conversion in a resistor?
2. What is a voltage drop?
3. What is the polarity of a voltage drop in relation to conventional current direction?

5 Power Supplies and Batteries

Power supplies and batteries are types of voltage sources. A *power supply* is generally defined as an electronic device that converts ac (alternating current) from the utility lines to a dc (direct current) voltage that virtually all electronic circuits and some transducers require. Batteries are also capable of supplying dc; in fact, many systems, such as laptop computers, can run from a power supply or internal battery. In this section, both types of voltage sources are described.

After completing this section, you should be able to

- **Discuss characteristics of power supplies and batteries**
    - Describe controls on typical laboratory power supplies
    - Determine the efficiency of a power supply given the input and output power
    - Define ampere-hour rating of batteries

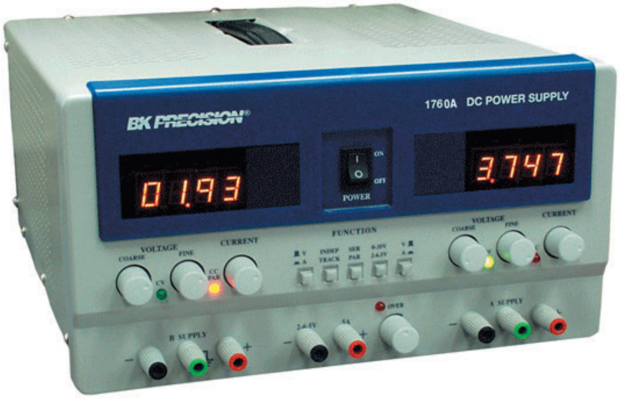
Utilities universally have adopted ac for transmitting electricity from the generating station to the user because it can be readily transformed to high voltages for transmission and low voltages for the end user. High voltages are much more efficient and cost-effective to transmit over long distances. In the United States, the standard voltage supplied to outlets is approximately 120 V or 240 V at 60 Hz, but in Europe and other countries, the outlet voltage is 240 V at 50 Hz.

Virtually all electronic systems require stable dc for the integrated circuits and other devices to work properly. Power supplies fulfill this function by converting ac to stable dc and are usually built into the product. Many electronic systems have a recessed and protected switch that allows the internal power supply to be set for either the 120 V standard or for the 240 V standard. That switch must be set correctly, or serious damage can occur to the equipment.

In the laboratory, circuits are developed and tested. The purpose of a laboratory power supply is to provide the required stable dc to the circuit under test. The test circuit can be anything from a simple resistive network to a complex amplifier or logic circuit. To meet the requirement for a constant voltage, with almost no noise or ripple, laboratory power supplies are **regulated**, meaning the output is constantly sensed and automatically adjusted if it tries to change because of a change in the line voltage or the load.

Many circuits require multiple voltages, as well as the ability to set the voltage to a precise value or change it a small amount for testing. For this reason, laboratory power supplies usually have two or three outputs that are independent of each other and can be controlled separately. Output metering is normally part of a good laboratory power supply, in order to set and monitor the output voltage or current. Control may include fine and coarse controls or digital inputs to set precise voltages.

Figure 11 shows a triple output bench supply such as the type used in many electronic laboratories. The model shown has two 0–30 V independent supplies and a 6.5 V high current supply (commonly referred to a logic supply). Voltages can be precisely set using coarse and fine controls. The 0–30 V supplies have floating outputs, meaning they are not referenced to ground. This allows the user to set them up as a positive or negative supply or even connect them to another external supply. Another feature of this supply is that it can be set up as a current source, with a maximum voltage set for constant current applications.



◀ **FIGURE 11**

A triple output power supply. (Courtesy B+K Precision)

As in many power supplies, there are three output jacks for each of the 0–30 V supplies. The output is taken between the red (more positive) and black terminals. The green jack is referenced to the chassis, which is earth ground, and can be connected to either the red or black jacks. In addition, current and voltage can be monitored using the built-in digital meters.

The power delivered by a power supply is the product of the absolute voltage and current. For example, if a power supply is providing −15.0 V at 3.0 A, the supplied power is 45 W. For a triple output supply, the total power supplied by all three supplies is the sum of the power from each one individually.

EXAMPLE 8

What is the total power delivered by a triple output power supply if the output voltage and current are as follows?

Source 1: 18 V at 2.0 A

Source 2: −18 V at 1.5 A

Source 3: 5.0 V at 1.0 A

Solution Power delivered from each supply is the product of voltage and current (ignoring the sign).

Source 1: $P_1 = V_1 I_1 = (18\text{ V})(2.0\text{ A}) = 36\text{ W}$

Source 2: $P_2 = V_2 I_2 = (18\text{ V})(1.5\text{ A}) = 27\text{ W}$

Source 3: $P_3 = V_3 I_3 = (5.0\text{ V})(1.0\text{ A}) = 5.0\text{ W}$

The total power is

$$P_T = P_1 + P_2 + P_3 = 36\text{ W} + 27\text{ W} + 5.0\text{ W} = \mathbf{68\ W}$$

Related Problem How will the total power delivered change if the current from Source 1 increases to 2.5 A?

Power Supply Efficiency

An important characteristic of electronic power supplies is efficiency. **Efficiency** is the ratio of the output power delivered to a load to the input power to a circuit.

Equation 5

$$\text{Efficiency} = \frac{P_{\text{OUT}}}{P_{\text{IN}}}$$

Efficiency is often expressed as a percentage. For example, if the input power is 100 W and the output power is 50 W, the efficiency is (50 W/100 W) × 100% = 50%.

All electronic power supplies require that power be put into them. For example, an electronic power supply generally uses the ac power from a wall outlet as its input. Its output is usually a regulated dc voltage. The output power is *always* less than the input power because some of the total power must be used internally to operate the power supply circuitry. This internal power dissipation is normally called the *power loss.* The output power is the input power minus the power loss.

Equation 6

$$P_{\text{OUT}} = P_{\text{IN}} - P_{\text{LOSS}}$$

High efficiency means that very little power is dissipated in the power supply and there is a higher proportion of output power for a given input power.

EXAMPLE 9

A certain electronic power supply requires 25 W of input power. It can produce an output power of 20 W. What is its efficiency, and what is the power loss?

Solution

$$\text{Efficiency} = \frac{P_{\text{OUT}}}{P_{\text{IN}}} = \frac{20\text{ W}}{25\text{ W}} = \mathbf{0.8}$$

Expressed as a percentage,

$$\text{Efficiency} = \left(\frac{20\text{ W}}{25\text{ W}}\right)100\% = 80\%$$

The power loss is

$$P_{\text{LOSS}} = P_{\text{IN}} - P_{\text{OUT}} = 25\text{ W} - 20\text{ W} = \mathbf{5\text{ W}}$$

Related Problem A power supply has an efficiency of 92%. If P_{IN} is 50 W, what is P_{OUT}?

Ampere-Hour Ratings of Batteries

Batteries convert stored chemical energy to electrical energy. They are widely used to power small systems, such as laptop computers and cell phones, to supply the stable dc required. The batteries used in these small systems are normally rechargeable, meaning that the chemical reaction can be reversed from an external source. The capacity for any battery is measured in ampere-hours (Ah). For a rechargeable battery, the Ah rating is the capacity before it needs to be recharged. The **ampere-hour rating** determines the length of time a battery can deliver a certain amount of current at the rated voltage.

A rating of one ampere-hour means that a battery can deliver an average of one ampere of current to a load for one hour at the rated voltage output. This same battery can deliver an average of two amperes for one-half hour. The more current the battery is required to deliver, the shorter the life of the battery. In practice, a battery usually is rated for a specified current level and output voltage. For example, a 12 V automobile battery may be rated for 70 Ah at 3.5 A. This means that it can produce an average of 3.5 A for 20 h at the rated voltage.

EXAMPLE 10 For how many hours can a battery deliver 2 A if it is rated at 70 Ah?

Solution The ampere-hour rating is the current times the number of hours (x).

$$70\text{ Ah} = (2\text{ A})(x\text{ h})$$

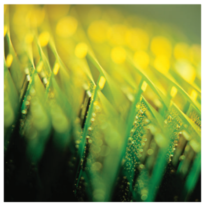
Solving for the number of hours, x, yields

$$x = \frac{70\text{ Ah}}{2\text{ A}} = \mathbf{35\ h}$$

Related Problem A certain battery delivers 10 A for 6 h. What is its Ah rating?

SECTION 5 CHECKUP

1. When a loading device draws an increased amount of current from a power supply, does this change represent a greater or a smaller load on the supply?
2. A power supply produces an output voltage of 10 V. If the supply provides 0.5 A to a load, what is the power to a load?
3. If a battery has an ampere-hour rating of 100 Ah, how long can it provide 5 A to a load?
4. If the battery in Question 3 is a 12 V device, what is its power to a load for the specified value of current?
5. An electronic power supply used in the lab operates with an input power of 1 W. It can provide an output power of 750 mW. What is its efficiency? Determine the power loss.
6. What is the purpose of the green terminal on a laboratory power supply?



Application Activity

In this application, you will work on a resistor substitution box used in testing circuits with up to 5.0 V. The required resistors range from 10 Ω to 4.7 kΩ. Your job is to determine wattage rating for the required resistors, prepare a parts list, determine the cost of the parts, draw a schematic, and prepare a test procedure for the circuit. You will apply Watt's law in order to complete the assignment.

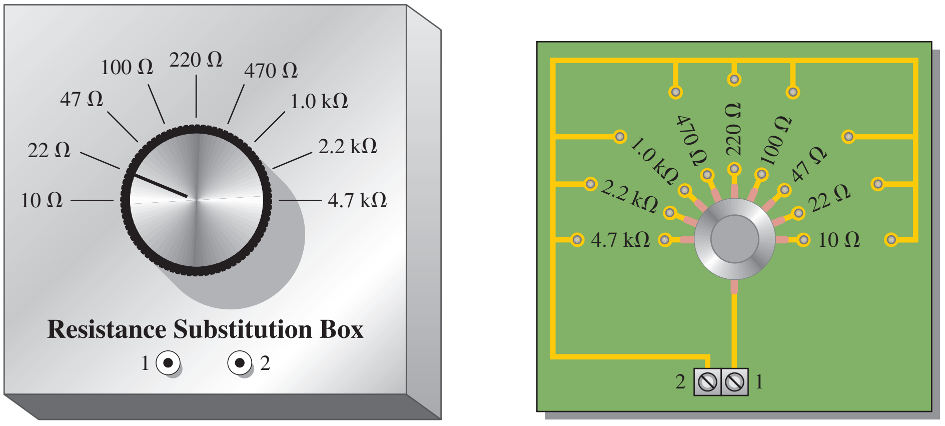
Specifications

- Each resistor is switch-selectable from a rotary switch, so that only one resistor is connected across the output terminals at a time.
- Resistors range in value from 10 Ω to 4.7 kΩ. Each of the sizes required is about twice the size of the previous resistor. In order to use standard values, the following sizes have been selected: 10 Ω, 22 Ω, 47 Ω, 100 Ω, 220 Ω, 470 Ω, and 1.0 kΩ, 2.2 kΩ, and 4.7 kΩ. Resistors are to be ±5%, with a minimum of ¼ W rating (more as needed). Small resistors (½ W or less) are carbon composition; resistors larger than ½ W are metal oxide.
- The maximum voltage that will be applied to the resistor box is 5 V.
- The box is to have two binding posts to connect to the resistors.

Power Ratings

The resistor box has been prepared with values silk-screened onto the enclosure box, and the reverse side of the PC board has been prepared as shown in Figure 12.

1. Use Watt's law and the specified resistor values to determine the power rating of the resistors needed for the project. Table 1 indicates the cost in small quantities for the various power ratings of the resistors.



(a) Top view of enclosure box

(b) Bottom view of PC board

▲ FIGURE 12

▶ TABLE 1

COMPONENT	COST PER ITEM
¼ W resistor (carbon comp.)	$0.08
½ W resistor (carbon comp.)	$0.09
1 W resistor (metal oxide)	$0.09
2 W resistor (metal oxide)	$0.10
5 W resistor (metal oxide)	$0.33
1 pole, 9 position rotary switch	$10.30
Knob	$3.30
Enclosure (4″ × 4″ × 2″ Al)	$8.46
Screw terminal (dual)	$0.20
Binding posts	$1.78
PC board (etched with pattern)	$1.78
Miscellaneous standoffs, etc.	$0.50

Materials List and Estimate of the Total Cost of the Project

2. Based on the specific resistors required, prepare a complete materials list showing quantities and cost.
3. Estimate the total cost of the project, not counting labor.

The Schematic

4. From the requirements and the board layout, develop a schematic for the circuit.
5. Show the resistor values including the wattage rating next to each resistor.

Test Procedure

6. After the resistor substitution box is constructed, list steps you would take to ensure that it works properly.
7. List the instruments you would use in testing it.

Troubleshooting

Describe the most likely fault for each of the following problems and how you would check to verify the problem:

8. An ohmmeter reads infinite resistance between terminals 1 and 2 for the 10 Ω position.
9. An ohmmeter reads infinite resistance between terminals 1 and 2 for all positions of the switch.
10. All resistors read 10% higher than the listed value.

Review

11. Explain how Watt's law was applied to this problem.
12. Could the resistors you specified be used in circuits with a 7 V output? Explain.

SUMMARY

- The power rating in watts of a resistor determines the maximum power that it can handle safely.
- Resistors with a larger physical size can dissipate more power in the form of heat than smaller ones.
- A resistor should have a power rating higher than the maximum power that it is expected to handle in the circuit.
- Power rating is not related to resistance value.
- A resistor normally opens when it overheats and fails.
- Energy is the ability to do work and is equal to power multiplied by time.
- The kilowatt-hour is a unit of energy.
- One kilowatt-hour equals one thousand watts used for one hour or any other combination of watts and hours that has a product of one.
- A power supply is an energy source used to operate electrical and electronic devices.
- A battery is one type of power supply that converts chemical energy into electrical energy.
- An electronic power supply converts commercial energy (ac from the power company) to regulated dc at various voltage levels.
- The output power of a supply is the output voltage times the load current.
- A load is a device that draws current from the power supply.
- The capacity of a battery is measured in ampere-hours (Ah).
- One ampere-hour equals one ampere used for one hour, or any other combination of amperes and hours that has a product of one.
- A circuit with a high efficiency has a smaller percentage power loss than one with a lower efficiency.

KEY TERMS

Ampere-hour (Ah) rating A capacity rating for batteries determined by multiplying the current (A) times the length of time (h) a battery can deliver that current to a load.

Efficiency The ratio of the output power delivered to a load to the input power to a circuit, usually expressed as a percentage.

Energy The ability to do work.

Joule (J) The SI unit of energy.

Kilowatt-hour (kWh) A large unit of energy used mainly by utility companies.

Power The rate of energy usage. The unit is the watt.

Voltage drop The decrease in voltage across a resistor due to a loss of energy.

Watt (W) The unit of power. One watt is the power when 1 J of energy is used in 1 s.

Watt's law A law that states the relationships of power to current, voltage, and resistance.

FORMULAS

1	$P = \frac{W}{t}$	Power equals energy divided by time.
2	$P = I^2R$	Power equals current squared times resistance.
3	$P = VI$	Power equals voltage times current.
4	$P = \frac{V^2}{R}$	Power equals voltage squared divided by resistance.
5	$\textbf{Efficiency} = \frac{P_{OUT}}{P_{IN}}$	Power supply efficiency
6	$P_{OUT} = P_{IN} - P_{LOSS}$	Output power is input power less power loss.

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The kilowatt-hour is a unit of power.
2. One watt is equal to one joule per second.
3. The power rating of a resistor should always be less than the required power dissipation in the circuit.
4. The amount of heat that a resistor can dissipate is proportional to its surface area.
5. If the voltage across a resistor doubles, the power also doubles.
6. Watt's law states that power equals voltage times current.
7. If the current through a resistor doubles, the power increases by four.
8. Within limits, a regulated power supply can automatically keep the output voltage constant even if the load changes.
9. A power supply that has a negative output voltage absorbs power from the load.
10. When analyzing a circuit problem, you should consider the conditions under which it failed.

SELF-TEST

Answers are at the end of the chapter.

1. Power can be defined as
   (a) energy **(b)** heat
   (c) the rate at which energy is used **(d)** the time required to use energy
2. Two hundred joules of energy are consumed in 10 s. The power is
   (a) 2000 W **(b)** 10 W **(c)** 20 W **(d)** 2 W
3. If it takes 300 ms to use 10,000 J of energy, the power is
   (a) 33.3 kW **(b)** 33.3 W **(c)** 33.3 mW
4. In 50 kW, there are
   (a) 500 W **(b)** 5000 W **(c)** 0.5 MW **(d)** 50,000 W
5. In 0.045 W, there are
   (a) 45 kW **(b)** 45 mW **(c)** 4,500 μW **(d)** 0.00045 MW
6. For 10 V and 50 mA, the power is
   (a) 500 mW **(b)** 0.5 W **(c)** 500,000 μW **(d)** answers (a), (b), and (c)
7. When the current through a 10 kΩ resistor is 10 mA, the power is
   (a) 1 W **(b)** 10 W **(c)** 100 mW **(d)** 1000 μW
8. A 2.2 kΩ resistor dissipates 0.5 W. The current is
   (a) 15.1 mA **(b)** 0.227 mA **(c)** 1.1 mA **(d)** 4.4 mA
9. A 330 Ω resistor dissipates 2 W. The voltage is
   (a) 2.57 V **(b)** 660 V **(c)** 6.6 V **(d)** 25.7 V
10. If you used 500 W of power for 24 h, you have used
    (a) 0.5 kWh **(b)** 2400 kWh **(c)** 12,000 kWh **(d)** 12 kWh
11. How many watt-hours represent 75 W used for 10 h?
    (a) 75 Wh **(b)** 750 Wh **(c)** 0.75 Wh **(d)** 7500 Wh
12. A 100 Ω resistor must carry a maximum current of 35 mA. Its rating should be at least
    (a) 35 W **(b)** 35 mW **(c)** 123 mW **(d)** 3500 mW
13. The power rating of a resistor that is to handle up to 1.1 W should be
    (a) 0.25 W **(b)** 1 W **(c)** 2 W **(d)** 5 W
14. A 22 Ω half-watt resistor and a 220 Ω half-watt resistor are connected across a 10 V source. Which one(s) will overheat?
    (a) 22 Ω **(b)** 220 Ω **(c)** both **(d)** neither
15. When the needle of an analog ohmmeter indicates infinity, the resistor being measured is
    (a) overheated **(b)** shorted **(c)** open **(d)** reversed

16. A 12 V battery is connected to a 600 Ω load. Under these conditions, it is rated at 50 Ah. How long can it supply current to the load?

(a) 2500 h **(b)** 50 h **(c)** 25 h **(d)** 4.16 h

17. A given power supply is capable of providing 8 A for 2.5 h. Its ampere-hour rating is

(a) 2.5 Ah **(b)** 20 Ah **(c)** 8 Ah

18. A power supply produces a 0.5 W output with an input of 0.6 W. Its percentage of efficiency is

(a) 50% **(b)** 60% **(c)** 83.3% **(d)** 45%

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

1. If the current through a fixed resistor goes from 10 mA to 12 mA, the power in the resistor

(a) increases **(b)** decreases **(c)** stays the same

2. If the voltage across a fixed resistor goes from 10 V to 7 V, the power in the resistor

(a) increases **(b)** decreases **(c)** stays the same

3. A variable resistor has 5 V across it. If you reduce the resistance, the power in the resistor

(a) increases **(b)** decreases **(c)** stays the same

4. If the voltage across a resistor increases from 5 V to 10 V and the current increases from 1 mA to 2 mA, the power

(a) increases **(b)** decreases **(c)** stays the same

5. If the resistance of a load connected to a battery is increased, the amount of time the battery can supply current

(a) increases **(b)** decreases **(c)** stays the same

6. If the amount of time that a battery supplies current to a load is decreased, its ampere-hour rating

(a) increases **(b)** decreases **(c)** stays the same

7. If the current that a battery supplies to a load is increased, the battery life

(a) increases **(b)** decreases **(c)** stays the same

8. If there is no load connected to a battery, its ampere-hour rating

(a) increases **(b)** decreases **(c)** stays the same

9. If the output voltage of a power supply increases, the power to a constant load

(a) increases **(b)** decreases **(c)** stays the same

10. For a constant power supply output voltage, if the current to a load decreases, the load power

(a) increases **(b)** decreases **(c)** stays the same

11. For a constant power supply output voltage, if the resistance of a load increases, the power in the load

(a) increases **(b)** decreases **(c)** stays the same

12. If the load is removed leaving the power supply terminals open, ideally the power supply output voltage

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 Energy and Power

1. Prove that the unit for power (the watt) is equivalent to one volt × one amp.

2. Show that there are 3.6×10^6 joules in a kilowatt-hour.

3. What is the power when energy is consumed at the rate of 350 J/s?

4. How many watts are used when 7500 J of energy are consumed in 5 h?

5. How many watts does 1000 J in 50 ms equal?

6. Convert the following to kilowatts:

(a) 1000 W **(b)** 3750 W **(c)** 160 W **(d)** 50,000 W

7. Convert the following to megawatts:

(a) 1,000,000 W **(b)** 3×10^6 W **(c)** 15×10^7 W **(d)** 8700 kW

8. Convert the following to milliwatts:

(a) 1 W **(b)** 0.4 W **(c)** 0.002 W **(d)** 0.0125 W

9. Convert the following to microwatts:

(a) 2 W **(b)** 0.0005 W **(c)** 0.25 mW **(d)** 0.00667 mW

10. Convert the following to watts:

(a) 1.5 kW **(b)** 0.5 MW **(c)** 350 mW **(d)** 9000 μW

11. A particular electronic device uses 100 mW of power. If it runs for 24 h, how many joules of energy does it consume?

***12.** If a 300 W bulb is allowed to burn continuously for 30 days, how many kilowatt-hours of energy does it consume?

***13.** At the end of a 31 day period, your utility bill shows that you have used 1500 kWh. What is your average daily power usage?

14. Convert 5×10^6 watt-minutes to kWh.

15. Convert 6700 watt-seconds to kWh.

16. For how many seconds must there be 5 A of current through a 47 Ω resistor in order to consume 25 J?

SECTION 2 Power in an Electric Circuit

17. If a 75 V source is supplying 2 A to a load, what is the resistance value of the load?

18. If a resistor has 5.5 V across it and 3 mA through it, what is the power?

19. An electric heater works on 120 V and draws 3 A of current. How much power does it use?

20. What is the power when there are 500 mA of current through a 4.7 kΩ resistor?

21. Calculate the power dissipated by a 10 kΩ resistor carrying 100 μA.

22. If there are 60 V across a 680 Ω resistor, what is the power?

23. A 56 Ω resistor is connected across the terminals of a 1.5 V battery. What is the power dissipation in the resistor?

24. If a resistor is to carry 2 A of current and handle 100 W of power, how many ohms must it be? Assume that the voltage can be adjusted to any required value.

25. A 12 V source is connected across a 10 Ω resistor.

- **(a)** How much energy is used in two minutes?
- **(b)** If the resistor is disconnected after one minute, is the power during the first minute greater than, less than, or equal to the power during a two minute interval?

SECTION 3 Resistor Power Ratings

26. Refer to the Application Activity at the end of this chapter and determine the required power rating of the resistors in the motor speed-control circuit.

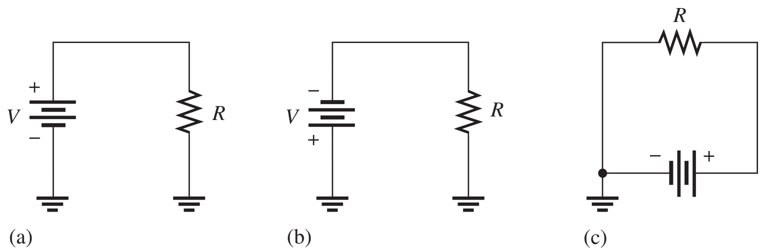
27. The maximum voltage is 1 V and the maximum current is 1 A in a given resistor. Should a 1 W or a 2 W resistor be used? Why?

28. A 6.8 kΩ resistor has burned out in a circuit. You must replace it with another resistor with the same resistance value. If the resistor carries 10 mA, what should its power rating be? Assume that you have available resistors in all the standard power ratings.

29. A certain type of power resistor comes in the following ratings: 3 W, 5 W, 8 W, 12 W, 20 W. Your particular application requires a resistor that can handle approximately 8 W. Which rating would you use for a minimum safety margin of 20% above the rated value? Why?

SECTION 4 Energy Conversion and Voltage Drop in Resistance

30. For each circuit in Figure 13, assign the proper polarity for the voltage drop across the resistor.



▲ FIGURE 13

SECTION 5 Power Supplies and Batteries

31. A 50 Ω load uses 1 W of power. What is the output voltage of the power supply?

32. Assume that an alkaline D-cell battery can maintain an average voltage of 1.25 V for 90 hours in a 10 Ω load before becoming unusable. What average power is delivered to the load during the life of the battery?

33. What is the total energy in joules that is delivered during the 90 hours for the battery in Problem 32?

34. A battery can provide an average of 1.5 A of current for 24 h. What is its ampere-hour rating?

35. How much average current can be drawn from an 80 Ah battery for 10 h?

36. If a battery is rated at 650 mAh, how much average current will it provide for 48 h?

37. If the input power is 500 mW and the output power is 400 mW, how much power is lost? What is the efficiency of this power supply?

38. To operate at 85% efficiency, how much output power must a source produce if the input power is 5 W?

***39.** A certain power supply provides a continuous 2 W to a load. It is operating at 60% efficiency. In a 24 h period, how many kilowatt-hours does the power supply use?

Multisim Troubleshooting and Analysis

These problems require Multisim.

40. Open file P04-40 and determine the current, voltage, and resistance. Using the measured values, calculate the power.

41. Open file P04-41 and determine the current, voltage, and resistance. Calculate the power from these values.

42. Open file P04-42. Measure the current in the lamp and determine if the value agrees with that determined using the power and voltage rating of the lamp.

ANSWERS

SECTION CHECKUPS

SECTION 1 Energy and Power

1. Power is the rate at which energy is used.
2. $P = W/t$
3. Watt is the unit of power. One watt is the power when 1 J of energy is used in 1 s.
4. **(a)** 68,000 W = 68 kW **(b)** 0.005 W = 5 mW **(c)** 0.000025 W = 25 μW
5. $W = (0.1\text{ kW})(10\text{ h}) = 1\text{ kWh}$
6. 2000 Wh = 2 kWh
7. 360,000 Ws = 0.1 kWh

SECTION 2 Power in an Electric Circuit

1. $P = (10\text{ V})(3\text{ A}) = 30\text{ W}$
2. $P = (24\text{ V})(50\text{ mA}) = 1.2\text{ W}$; 1.2 W; the values are the same because all energy generated by the source is dissipated by the resistance.
3. $P = (5\text{ A})^2(56\ \Omega) = 1400\text{ W}$
4. $P = (20\text{ mA})^2(4.7\text{ k}\Omega) = 1.88\text{ W}$
5. $P = (5\text{ V})^2/10\ \Omega = 2.5\text{ W}$
6. $P = (8\text{ V})^2/2.2\text{ k}\Omega = 29.1\text{ mW}$
7. $R = 75\text{ W}/(0.5\text{ A})^2 = 300\ \Omega$

SECTION 3 Resistor Power Ratings

1. Resistors have resistance and a power rating.
2. A larger surface area of a resistor dissipates more power.
3. 0.125 W, 0.25 W, 0.5 W, 1 W
4. A 0.5 W rating should be used for 0.3 W.
5. 5 V

SECTION 4 Energy Conversion and Voltage Drop in Resistance

1. Energy conversion in a resistor is caused by collisions of free electrons with the atoms in the material.
2. Voltage drop is a decrease in voltage across a resistor due to a loss of energy.
3. Voltage drop is positve to negative in the direction of conventional current.

SECTION 5 Power Supplies and Batteries

1. More current means a greater load.
2. $P = (10\text{ V})(0.5\text{ A}) = 5\text{ W}$
3. $t = 100\text{ Ah}/5\text{ A} = 20\text{ h}$
4. $P = (12\text{ V})(5\text{ A}) = 60\text{ W}$
5. $\text{Eff} = (0.75\text{ W}/1\text{ W})100\% = 75\%$; $P_{\text{LOSS}} = 1000\text{ mW} - 750\text{ mW} = 250\text{ mW}$
6. To connect to the chassis ground

RELATED PROBLEMS FOR EXAMPLES

1 3000 J
2 **(a)** 0.001 W **(b)** 0.0018 W **(c)** 1 W **(d)** 0.000001 W
3 2 kWh
4 **(a)** 40 W **(b)** 376 W **(c)** 625 mW
5 65.4 W
6 0.5 W
7 Yes
8 77 W
9 46 W
10 60 Ah

TRUE/FALSE QUIZ

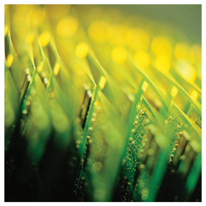
1. F **2.** T **3.** F **4.** T **5.** F **6.** T **7.** T **8.** T
9. F **10.** T

SELF-TEST

1. (c) **2.** (c) **3.** (a) **4.** (d) **5.** (b) **6.** (d) **7.** (a) **8.** (a)
9. (d) **10.** (d) **11.** (b) **12.** (c) **13.** (c) **14.** (a) **15.** (c) **16.** (a)
17. (b) **18.** (c)

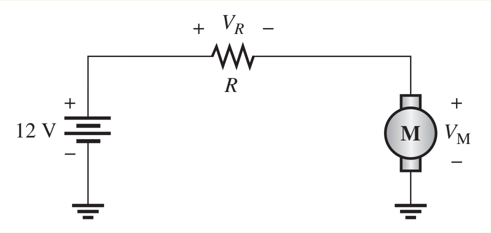
CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (a) **6.** (c)
7. (b) **8.** (c) **9.** (a) **10.** (b) **11.** (b) **12.** (c)



Application Activity

In this application, a resistive circuit is developed for controlling the speed of a small dc motor used to drive a fan for cooling electronic equipment mounted in a cabinet. One way to control the speed of a dc motor is to connect a resistor to the motor, as shown in Figure 14. The resistor reduces the current to the motor and therefore reduces its speed. This method is only recommended for small motors because energy is wasted in the resistance, and the overall efficiency is low.



▲ FIGURE 14

A circuit is required that will allow a selection of four motor speeds. The control circuit is to be mounted in a small box that will be attached to the equipment cabinet to control the fan motor speed, as shown in Figure 15.

The application requires four distinct switchable speeds. Your job is to develop a schematic, determine the values for the required resistors, and prepare a test procedure for the circuit. You will apply Ohm's law in order to complete the assignment. Motor data in graphical form is provided in Figure 16.

Specifications

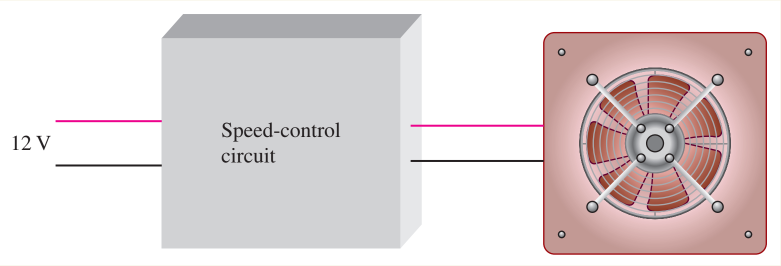
- Each resistor is switch selectable from a rotary switch, so that only one resistor is connected to the motor at a time.
- The four approximate speed settings are 800 rpm, 600 rpm, 400 rpm, and 200 rpm.
- The motor is rated for a maximum of 12 V at 800 rpm.
- The circuit board is to be connected to a 12 V dc source and to the fan motor with wiring that is available inside the cabinet.

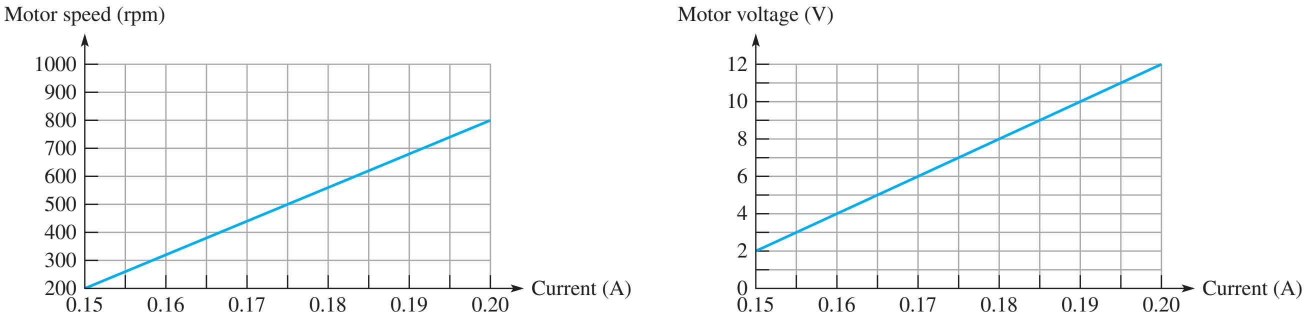
Resistance Calculations

The resistor box has been prepared with speed-value labels and the PC board has been prepared as shown in Figure 17.

1. Use the graphical motor data in Figure 16 and Ohm's law to determine the nearest standard resistor values needed for the project. *Hint:* To find the voltage across the resistor, V_R, subtract the motor voltage, V_M, from 12 V. This is based on Kirchhoff's law.

► FIGURE 15



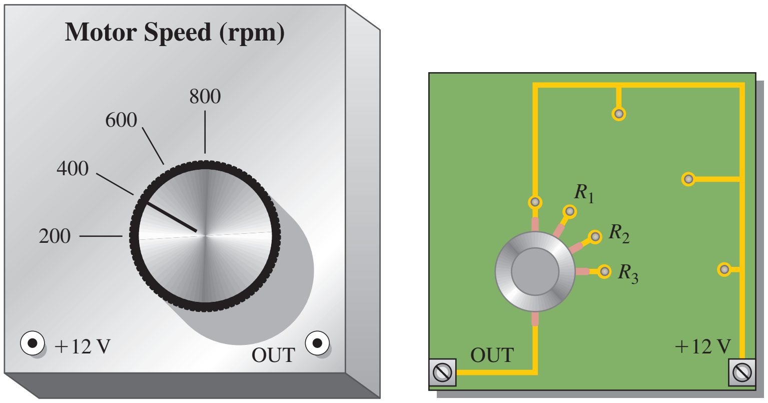


(a) Graph of motor speed versus current

(a) Graph of motor voltage versus current

▲ FIGURE 16

▶ FIGURE 17



(a) Top view of box

(b) Circuit board

The Schematic

2. From the requirements and the board layout, develop a schematic for the circuit.
3. Show the standard resistor values next to each resistor.

Test Procedure

4. After the resistor box is constructed, list steps you would take to ensure that it works properly.
5. List the instruments you would use in testing it.

Troubleshooting

Describe the most likely fault on the circuit board for each of the following problems:

6. When connected between the +12 V terminal and the OUT terminal, an ohmmeter reads infinite resistance for the 600 rpm position.
7. When connected between the +12 V terminal and the OUT terminal, an ohmmeter reads infinite resistance for all positions of the switch.
8. All resistors read 10% higher than the listed value.

Review

9. Explain how Ohm's law was applied to this problem.
10. Describe the relationship between current and motor speed.

ANSWERS TO ODD-NUMBERED PROBLEMS

1. volt = joule/coulomb
amp = coulomb/second
VI = joule/coulomb × coulomb/second = joule/sec = watt

3. 350 W

5. 20 kW

7. **(a)** 1 MW **(b)** 3 MW
(c) 150 MW **(d)** 8.7 MW

9. **(a)** 2,000,000 μW **(b)** 500 μW
(c) 250 μW **(d)** 6.67 μW

11. 8640 J

13. 2.02 kW/day

15. 0.00186 kWh

17. 37.5 Ω

19. 360 W

21. 100 μW

23. 40.2 mW

25. **(a)** 0.480 Wh **(b)** Equal

27. 2 W, to provide a safety margin

29. At least 12 W, to allow a safety margin of 20%

31. 7.07 V

33. 50,544 J

35. 8 A

37. 100 mW, 80%

39. 0.08 kWh

41. $V = 5$ V; $I = 5$ mA;
$R = 1$ kΩ

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SERIES CIRCUITS

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Series Circuits

CHAPTER OUTLINE

1 Resistors in Series
2 Total Series Resistance
3 Current in a Series Circuit
4 Application of Ohm's Law
5 Voltage Sources in Series
6 Kirchhoff's Voltage Law
7 Voltage Dividers
8 Power in Series Circuits
9 Voltage Measurements
10 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Identify a series resistive circuit
- Determine total series resistance
- Determine the current throughout a series circuit
- Apply Ohm's law in series circuits
- Determine the total effect of voltage sources connected in series
- Apply Kirchhoff's voltage law
- Use a series circuit as a voltage divider
- Determine power in a series circuit
- Measure voltage with respect to ground
- Troubleshoot series circuits

KEY TERMS

- Series
- Kirchhoff's voltage law
- Voltage divider
- Reference ground
- Open
- Short

APPLICATION ACTIVITY PREVIEW

In this application activity, you will evaluate a voltage-divider circuit board connected to a 12 V battery to provide a selection of fixed reference voltages for use with an electronic instrument.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, the concepts of Ohm's law and power in resistors are applied to circuits in which resistors are connected in a series arrangement.

Resistive circuits can be of two basic forms: series and parallel. In this chapter, series circuits are studied. You will see how Ohm's law is used in series circuits; and you will learn another important circuit law, Kirchhoff's voltage law. Also, several applications of series circuits, including voltage dividers, are presented.

When resistors are connected in series and a voltage is applied across the series connection, there is only one path for current; therefore, each resistor in series has the same amount of current through it. All of the resistances in series add together to produce a total resistance. The voltage drops across each of the resistors add up to the voltage applied across the entire series connection.

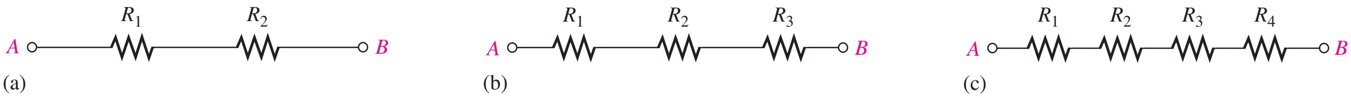
1 RESISTORS IN SERIES

When connected in series, resistors form a "string" in which there is only one path for current.

After completing this section, you should be able to

- **Identify a series resistive circuit**
    - Translate a physical arrangement of resistors into a schematic

The schematic in Figure 1(a) shows two resistors connected in series between point *A* and point *B*. Part (b) shows three resistors in series, and part (c) shows four in series. Of course, there can be any number of resistors in a series circuit.



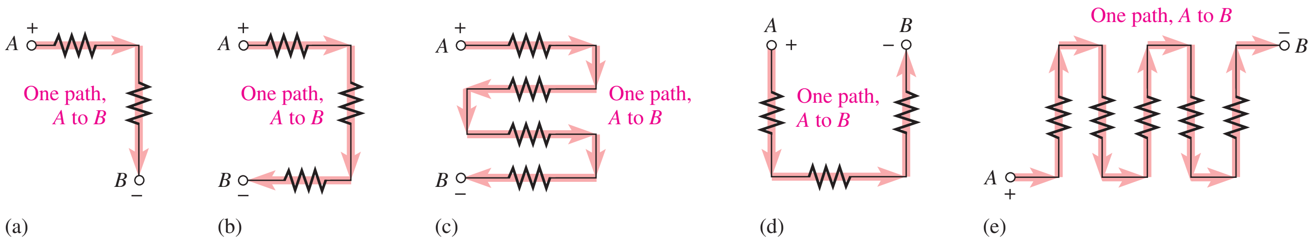
▲ FIGURE 1

Resistors in series.

When a voltage source is connected between point *A* and *B*, the only way for current to get from one point to the other in any of the connections of Figure 1 is to go through each of the resistors. The following statement describes a series circuit:

A series circuit provides only one path for current between two points so that the current is the same through each series resistor.

In an actual circuit diagram, a series circuit may not always be as easy to visually identify as those in Figure 1. For example, Figure 2 shows series resistors drawn in other ways with voltage applied. Remember, if there is only one current path between two points, the resistors between those two points are in series, no matter how they appear in a diagram.



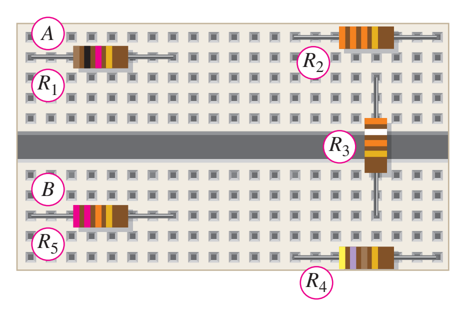
▲ FIGURE 2

Some examples of series circuits. Notice that the current is the same at all points because the current has only one path.

EXAMPLE 1

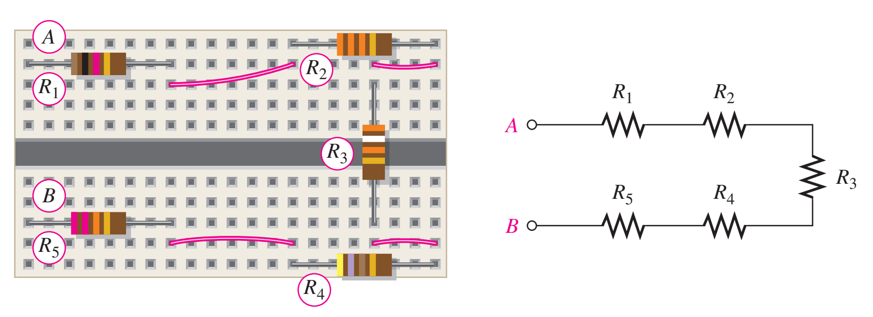
Suppose that there are five resistors positioned on a protoboard as shown in Figure 3. Wire them together in series so that, starting from the positive (+) terminal, R_1 is first, R_2 is second, R_3 is third, and so on. Draw a schematic showing this connection.

FIGURE 3


A
R1
R2
R3
B
R5
R4

Solution The wires are connected as shown in Figure 4(a), which is the assembly diagram. The schematic is shown in Figure 4(b). Note that the schematic does not necessarily show the actual physical arrangement of the resistors as does the assembly diagram. The schematic shows how components are connected electrically; the assembly diagram shows how components are arranged and interconnected physically.

FIGURE 4


A
R1
R2
R3
B
R5
R4
A
B
R1
R2
R3
R5
R4

(a) Assembly diagram (b) Schematic

*Related Problem** **(a)** Show how you would rewire the protoboard in Figure 4(a) so that all the odd-numbered resistors come first followed by the even-numbered ones. **(b)** Determine the resistance value of each resistor.

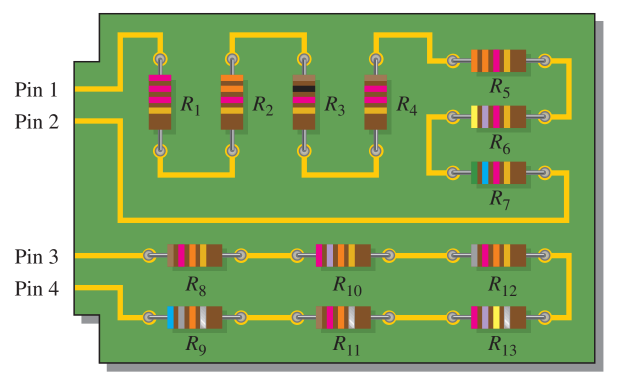
---

*Answers are at the end of the chapter.

EXAMPLE 2

Describe how the resistors on the printed circuit (PC) board in Figure 5 are related electrically. Determine the resistance value of each resistor.

FIGURE 5


Pin 1
Pin 2
Pin 3
Pin 4
R1
R2
R3
R4
R5
R6
R7
R8
R10
R12
R9
R11
R13

Solution Resistors R_1 through R_7 are in series with each other. This series combination is connected between pins 1 and 2 on the PC board.

Resistors R_8 through R_{13} are in series with each other. This series combination is connected between pins 3 and 4 on the PC board.

The values of the resistors are $R_1 = 2.2\text{ k}\Omega$, $R_2 = 3.3\text{ k}\Omega$, $R_3 = 1.0\text{ k}\Omega$, $R_4 = 1.2\text{ k}\Omega$, $R_5 = 3.3\text{ k}\Omega$, $R_6 = 4.7\text{ k}\Omega$, $R_7 = 5.6\text{ k}\Omega$, $R_8 = 12\text{ k}\Omega$, $R_9 = 68\text{ k}\Omega$, $R_{10} = 27\text{ k}\Omega$, $R_{11} = 12\text{ k}\Omega$, $R_{12} = 82\text{ k}\Omega$, and $R_{13} = 270\text{ k}\Omega$.

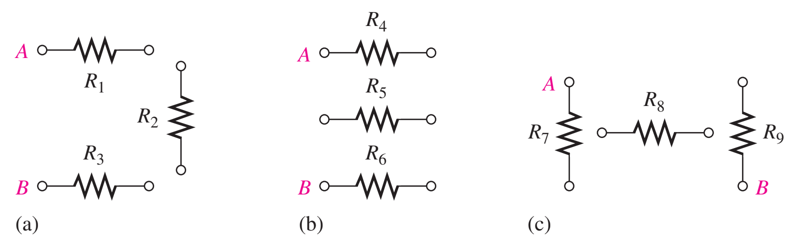
Related Problem How is the circuit changed when pin 2 and pin 3 in Figure 5 are connected?

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. How are the resistors connected in a series circuit?
2. How can you identify a series circuit?
3. Complete the schematics for the circuits in each part of Figure 6 by connecting each group of resistors in series in numerical order from terminal *A* to terminal *B*.
4. Connect each group of series resistors in Figure 6 in series with each other.

▶ FIGURE 6



2 TOTAL SERIES RESISTANCE

The total resistance of a series circuit is equal to the sum of the resistances of each individual series resistor.

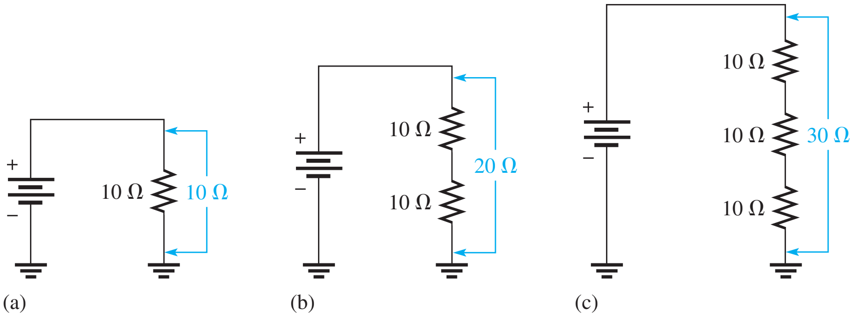
After completing this section, you should be able to

- **Determine total series resistance**
    - Explain why resistance values add when resistors are connected in series
    - Apply the series resistance formula

Series Resistor Values Add

When resistors are connected in series, the resistor values add because each resistor offers opposition to the current in direct proportion to its resistance. A greater number of resistors connected in series creates more opposition to current. More opposition to current implies a higher value of resistance. Thus, every time a resistor is added in series, the total resistance increases.

Figure 7 illustrates how series resistances add to increase the total resistance. Part (a) has a single 10 Ω resistor. Part (b) shows another 10 Ω resistor connected in series with the first one, making a total resistance of 20 Ω. If a third 10 Ω resistor is connected in series with the first two, as shown in part (c), the total resistance becomes 30 Ω.



▲ **FIGURE 7**

Total resistance increases with each additional series resistor.

Series Resistance Formula

For any number of individual resistors connected in series, the total resistance is the sum of each of the individual values.

Equation 1

$$R_T = R_1 + R_2 + R_3 + \cdots + R_n$$

where R_T is the total resistance and R_n is the last resistor in the series string (n can be any positive integer equal to the number of resistors in series). For example, if there are four resistors in series ($n = 4$), the total resistance formula is

$$R_T = R_1 + R_2 + R_3 + R_4$$

If there are six resistors in series ($n = 6$), the total resistance formula is

$$R_T = R_1 + R_2 + R_3 + R_4 + R_5 + R_6$$

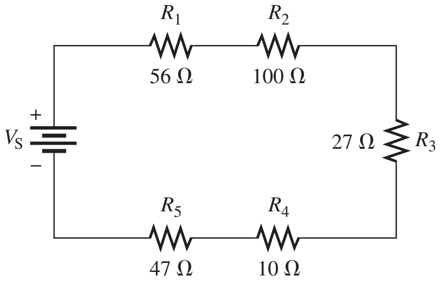
To illustrate the calculation of total series resistance, let's determine R_T in the circuit of Figure 8, where V_S is the source voltage. The circuit has five resistors in series. To get the total resistance, simply add the values.

$$R_T = 56\ \Omega + 100\ \Omega + 27\ \Omega + 10\ \Omega + 47\ \Omega = 240\ \Omega$$

Note in Figure 8 that the order in which the resistances are added does not matter. You can physically change the positions of the resistors in the circuit without affecting the total resistance or the current.

▶ **FIGURE 8**

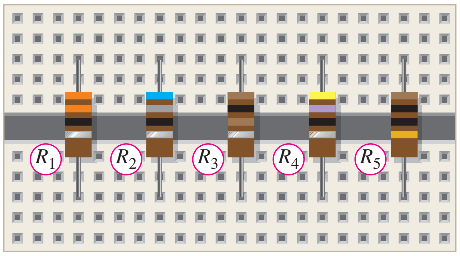
Example of five resistors in series.



EXAMPLE 3

Connect the resistors in Figure 9 in series, and determine the total resistance, R_T, from the color codes.

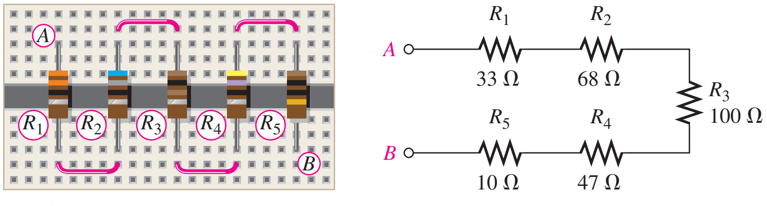
▶ **FIGURE 9**



R_1
R_2
R_3
R_4
R_5

Solution The resistors are connected as shown in Figure 10. Find the total resistance by adding all the values.

$$R_T = R_1 + R_2 + R_3 + R_4 + R_5 = 33\ \Omega + 68\ \Omega + 100\ \Omega + 47\ \Omega + 10\ \Omega = \mathbf{258\ \Omega}$$



A
B
R_1
R_2
R_3
R_4
R_5
A
B
R_1
33 Ω
R_2
68 Ω
R_3
100 Ω
R_5
10 Ω
R_4
47 Ω

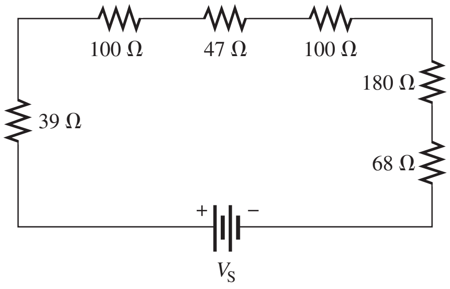
(a) Circuit assembly (b) Schematic

▲ **FIGURE 10**

Related Problem Determine the total resistance in Figure 10(a) if the positions of R_2 and R_4 are interchanged.

EXAMPLE 4 What is the total resistance (R_T) in the circuit of Figure 11?

▶ **FIGURE 11**



100 Ω
47 Ω
100 Ω
180 Ω
39 Ω
68 Ω
+
−
V_S

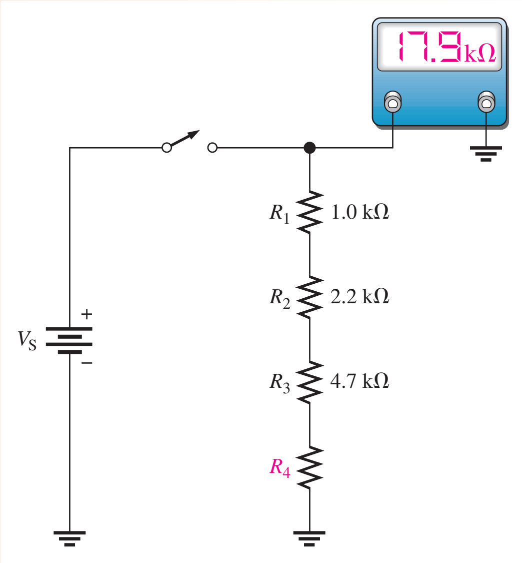
Solution Sum all the values.

$$R_T = 39\ \Omega + 100\ \Omega + 47\ \Omega + 100\ \Omega + 180\ \Omega + 68\ \Omega = \mathbf{534\ \Omega}$$

Related Problem What is the total resistance for the following series resistors: 1.0 kΩ, 2.2 kΩ, 3.3 kΩ, and 5.6 kΩ?

EXAMPLE 5 Determine the value of R_4 in the circuit of Figure 12.

▶ FIGURE 12



Solution From the ohmmeter reading, $R_T = 17.9\text{ k}\Omega$.

$$R_T = R_1 + R_2 + R_3 + R_4$$

Solving for R_4 yields

$$R_4 = R_T - (R_1 + R_2 + R_3) = 17.9\text{ k}\Omega - (1.0\text{ k}\Omega + 2.2\text{ k}\Omega + 4.7\text{ k}\Omega) = \mathbf{10\text{ k}\Omega}$$

Related Problem Determine the value of R_4 in Figure 12 if the ohmmeter reading is 14.7 kΩ.

Equal-Value Series Resistors

When a circuit has more than one resistor of the same value in series, there is a shortcut method to obtain the total resistance: Simply multiply the resistance value by the number of equal-value resistors that are in series. This method is essentially the same as adding the values. For example, five 100 Ω resistors in series have an R_T of $5(100\ \Omega) = 500\ \Omega$. In general, the formula is expressed as

Equation 2

$$R_T = nR$$

where n is the number of equal-value resistors and R is the resistance value.

EXAMPLE 6 Find the R_T of eight 22 Ω resistors in series.

Solution Find R_T by adding the values.

$$R_T = 22\ \Omega + 22\ \Omega + 22\ \Omega + 22\ \Omega + 22\ \Omega + 22\ \Omega + 22\ \Omega + 22\ \Omega = \mathbf{176\ \Omega}$$

However, it is much easier to multiply to get the same result.

$$R_T = 8(22\ \Omega) = \mathbf{176\ \Omega}$$

Related Problem Find R_T for three 1.0 kΩ resistors and two 720 Ω resistors in series.

SECTION 2 CHECKUP

1. The following resistors (one each) are in series: 1.0 Ω, 2.2 Ω, 3.3 Ω, and 4.7 Ω. What is the total resistance?
2. The following resistors are in series: one 100 Ω, two 56 Ω, four 12 Ω, and one 330 Ω. What is the total resistance?
3. Suppose that you have one resistor each of the following values: 1.0 kΩ, 2.7 kΩ, 5.6 kΩ, and 560 kΩ. To get a total resistance of approximately 13.8 kΩ, you need one more resistor. What should its value be?
4. What is the R_T for twelve 56 Ω resistors in series?
5. What is the R_T for twenty 5.6 kΩ resistors and thirty 8.2 kΩ resistors in series?

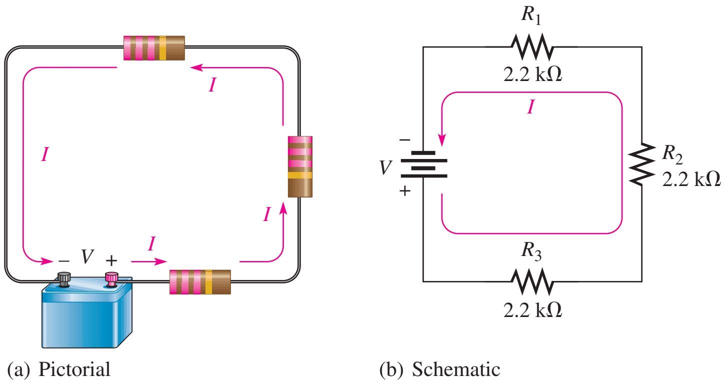
3 CURRENT IN A SERIES CIRCUIT

The current is the same through all points in a series circuit. The current through each resistor in a series circuit is the same as the current through all the other resistors that are in series with it.

After completing this section, you should be able to

- **Determine the current throughout a series circuit**
  - Show that the current is the same at all points in a series circuit

Figure 13 shows three resistors connected in series to a dc voltage source. *At any point in this circuit, the current into that point must equal the current out of that point,* as illustrated by the current directional arrows. Notice also that the current out of each resistor must equal the current into each resistor because there is no place where part of the current can branch off and go somewhere else. Therefore, the current in each section of the circuit is the same as the current in all other sections. It has only one path going from the positive (+) side of the source to the negative (−) side.

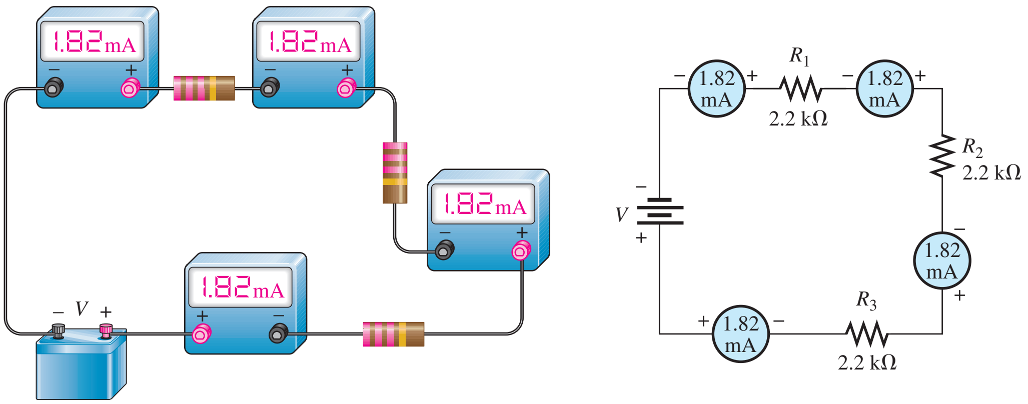


(a) Pictorial (b) Schematic

▲ FIGURE 13

Current into any point in a series circuit is the same as the current out of that point.

Let's assume that the battery in Figure 13 supplies 1.82 mA of current to the series resistance. There are 1.82 mA of current out of the battery's positive terminal. When ammeters are connected at several points in the circuit, as shown in Figure 14, each meter reads 1.82 mA.


1.82 mA
1.82 mA
1.82 mA
1.82 mA
–
+
V
R1
R2
R3
2.2 kΩ
2.2 kΩ
2.2 kΩ

(a) Pictorial

(b) Schematic

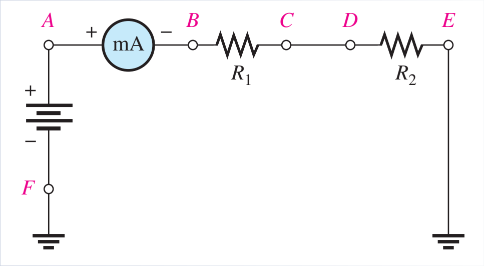
FIGURE 14

Current is the same at all points in a series circuit.

SECTION 3 CHECKUP

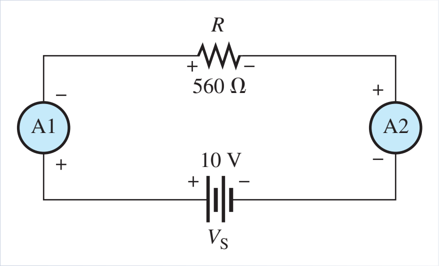
1. In a circuit with a 100 Ω resistor and a 47 Ω resistor in series, there are 20 mA of current through the 100 Ω resistor. How much current is through the 47 Ω resistor?
2. A milliammeter is connected between points *A* and *B* in Figure 15. It measures 50 mA. If you move the meter and connect it between points *C* and *D*, how much current will it indicate? Between *E* and *F*?

FIGURE 15


A
B
C
D
E
F
mA
+
–
R1
R2

3. In Figure 16, how much current does ammeter 1 indicate? How much current does ammeter 2 indicate?
4. Describe current in a series circuit?

FIGURE 16


R
560 Ω
A1
A2
10 V
VS
+
–

4 APPLICATION OF OHM'S LAW

The basic concepts of series circuits and Ohm's law can be applied to series circuit analysis.

After completing this section, you should be able to

- **Apply Ohm's law in series circuits**
    - Find the current in a series circuit
    - Find the voltage across each resistor in series

The following are key points to remember when you analyze series circuits:

1. Current through any of the series resistors is the same as the total current.
2. If you know the total applied voltage and the total resistance, you can determine the total current by Ohm's law.

$$I_T = \frac{V_T}{R_T}$$

3. If you know the voltage drop across one of the series resistors (R_x), you can determine the total current by Ohm's law.

$$I_T = \frac{V_x}{R_x}$$

4. If you know the total current, you can find the voltage drop across any of the series resistors by Ohm's law.

$$V_x = I_T R_x$$

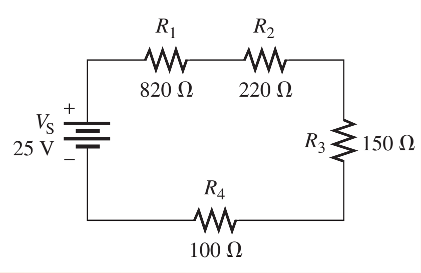
5. The polarity of a voltage drop across a resistor is positive at the end of the resistor that is closest to the positive terminal of the voltage source.
6. The current through a resistor is defined to be in a direction from the positive end of the resistor to the negative end.
7. An open in a series circuit prevents current; and, therefore, there is zero voltage drop across each series resistor. The total voltage appears across the points between which there is an open.

Now let's look at several examples that use Ohm's law for series circuit analysis.

EXAMPLE 7

Find the current in the circuit of Figure 17.

FIGURE 17



This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Solution The current is determined by the source voltage V_S and the total resistance R_T. First, calculate the total resistance.

$$R_T = R_1 + R_2 + R_3 + R_4 = 820\ \Omega + 220\ \Omega + 150\ \Omega + 100\ \Omega = 1.29\ \text{k}\Omega$$

Next, use Ohm's law to calculate the current.

$$I = \frac{V_S}{R_T} = \frac{25\ \text{V}}{1.29\ \text{k}\Omega} = 0.0194\ \text{A} = \mathbf{19.4\ mA}$$

where V_S is the total voltage and I is the total current. Remember, the same current exists at all points in the circuit. Thus, each resistor has 19.4 mA through it.

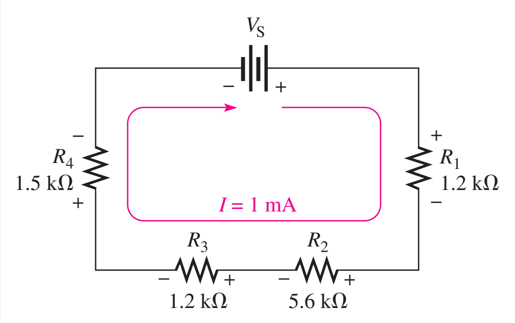
Related Problem What is the current in the circuit of Figure 17 if R_4 is changed to 200 Ω?

Use Multisim file E05–07 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 8

The current in the circuit of Figure 18 is 1 mA. For this amount of current, what must the source voltage V_S be?

▶ **FIGURE 18**



Solution In order to calculate V_S, first determine R_T.

$$R_T = R_1 + R_2 + R_3 + R_4 = 1.2\ \text{k}\Omega + 5.6\ \text{k}\Omega + 1.2\ \text{k}\Omega + 1.5\ \text{k}\Omega = 9.5\ \text{k}\Omega$$

Next, use Ohm's law to determine V_S.

$$V_S = IR_T = (1\ \text{mA})(9.5\ \text{k}\Omega) = \mathbf{9.5\ V}$$

Related Problem Calculate V_S if the 5.6 kΩ resistor is changed to 3.9 kΩ with the current the same.

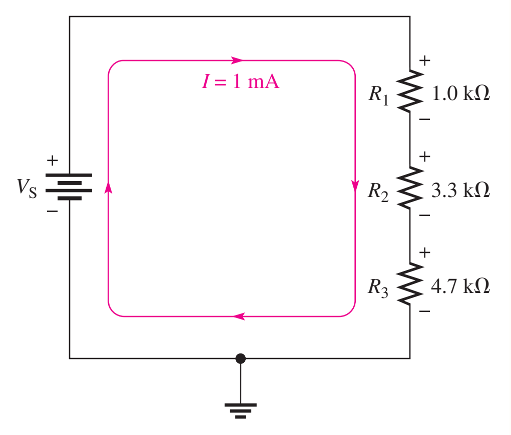
Use Multisim file E05–08 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 9

Calculate the voltage across each resistor in Figure 19, and find the value of V_S. To what maximum value can V_S be raised if the current is to be limited to 5 mA?

Solution By Ohm's law, the voltage across each resistor is equal to its resistance multiplied by the current through it. Use the Ohm's law formula $V = IR$ to determine the voltage

▶ **FIGURE 19**



across each of the resistors. Keep in mind that there is the same current through each series resistor. The voltage across R_1 (designated V_1) is

$$V_1 = IR_1 = (1\text{ mA})(1.0\text{ k}\Omega) = \mathbf{1\ V}$$

The voltage across R_2 is

$$V_2 = IR_2 = (1\text{ mA})(3.3\text{ k}\Omega) = \mathbf{3.3\ V}$$

The voltage across R_3 is

$$V_3 = IR_3 = (1\text{ mA})(4.7\text{ k}\Omega) = \mathbf{4.7\ V}$$

To find the value of V_S, first determine R_T.

$$R_T = 1.0\text{ k}\Omega + 3.3\text{ k}\Omega + 4.7\text{ k}\Omega = 9\text{ k}\Omega$$

The source voltage V_S is equal to the current times the total resistance.

$$V_S = IR_T = (1\text{ mA})(9\text{ k}\Omega) = \mathbf{9\ V}$$

Notice that if you add the voltage drops of the resistors, they total 9 V, which is the same as the source voltage.

V_S can be increased to a value where $I = 5$ mA. Calculate the maximum value of V_S as follows:

$$V_{S(max)} = IR_T = (5\text{ mA})(9\text{ k}\Omega) = \mathbf{45\ V}$$

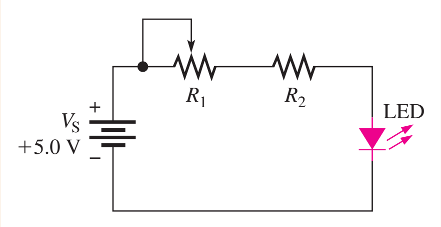
Related Problem Repeat the calculations for V_1, V_2, V_3, V_S, and $V_{S(max)}$ if $R_3 = 2.2\text{ k}\Omega$ and I is maintained at 1 mA.

Use Multisim files E05-09A, E05-09B, and E05-09C to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 10

Series connections often involve using resistance to limit current to some level. For example, it is necessary to limit current to a light-emitting diode (LED) to prevent the LED from burning out. The circuit in Figure 20 shows a basic application, where a red LED is used as an indicator as part of a more complicated circuit. The rheostat is included to dim the LED depending on ambient conditions. We will focus on these two current-limiting resistors.

FIGURE 20



A red LED will always have a voltage across it of about +1.7 V when it is on and working within its normal operating range. The remaining voltage from the power supply will be across the two series resistors. Together, the rheostat and the fixed resistor will have a total of 3.3 V across them.

Assume you want current in the LED to range from a minimum of 2.5 mA (dim) to a maximum of 10 mA (bright). What values of R_1 and R_2 would you choose to accomplish this?

Solution Start with the brightest condition when the resistance of the rheostat is adjusted for 0 Ω. In this case, there will be no voltage across R_1 and 3.3 V will be across R_2. Because it is a series circuit, the same current is in R_2 as the LED. Therefore,

$$R_2 = \frac{V}{I} = \frac{3.3\text{ V}}{10\text{ mA}} = \mathbf{330\ \Omega}$$

Now determine the total resistance required to limit the current to 2.5 mA. The total resistance is $R_T = R_1 + R_2$, and the voltage drop across R_T is 3.3 V. From Ohm's law,

$$R_T = \frac{V}{I} = \frac{3.3\text{ V}}{2.5\text{ mA}} = \mathbf{1.32\ k\Omega}$$

To find R_1, subtract the value of R_2 from the total resistance.

$$R_1 = R_T - R_2 = 1.32\text{ k}\Omega - 330\ \Omega = 990\ \Omega$$

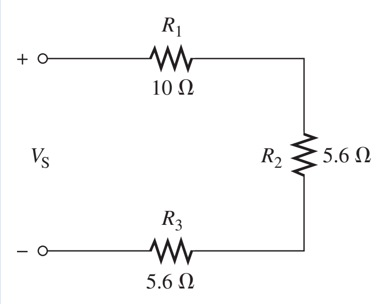
Choose a **1.0 kΩ** rheostat as the nearest standard value.

Related Problem What is the value of R_2 if the highest current is 12 mA?

SECTION 4 CHECKUP

1. A 6 V battery is connected across three 100 Ω resistors in series. What is the current through each resistor?
2. How much voltage is required to produce 50 mA through the circuit of Figure 21?

FIGURE 21



3. How much voltage is dropped across each resistor in Figure 21 when the current is 50 mA?
4. There are four equal-value resistors connected in series with a 5 V source. A current of 4.63 mA is measured. What is the value of each resistor?
5. What value should a series current-limiting resistor be to limit the current in a red LED to 10 mA if the source is 3 V and the LED drops 1.7 V?

5 VOLTAGE SOURCES IN SERIES

Recall that an ideal voltage source is an energy source that provides a constant voltage to a load. Batteries and electronic power supplies are practical examples of dc voltage sources.

After completing this section, you should be able to

- **Determine the total effect of voltage sources connected in series**
  - Determine the total voltage of series sources with the same polarities
  - Determine the total voltage of series sources with opposite polarities

When two or more voltage sources are in series, the total voltage is equal to the algebraic sum of the individual source voltages. The algebraic sum means that the polarities of the sources must be included when the sources are combined in series. Sources with opposite polarities have voltages with opposite signs.

$$V_{S(tot)} = V_{S1} + V_{S2} + \cdots + V_{Sn}$$

When the voltage sources are all in the same direction in terms of their polarities, as in Figure 22(a), all of the voltages have the same sign when added; there is a total of 4.5 V from terminal *A* to terminal *B* with *A* more positive than *B*.

$$V_{AB} = 1.5\text{ V} + 1.5\text{ V} + 1.5\text{ V} = +4.5\text{ V}$$

The voltage has a double subscript, *AB,* to indicate that it is the voltage at point *A* with respect to point *B*.

In Figure 22(b), the middle voltage source is opposite to the other two; so its voltage has an opposite sign when added to the others. For this case the total voltage from *A* to *B* is

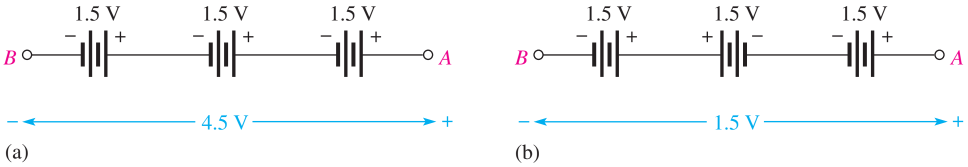
$$V_{AB} = +1.5\text{ V} - 1.5\text{ V} + 1.5\text{ V} = +1.5\text{ V}$$

Terminal *A* is 1.5 V more positive than terminal *B*.

A familiar example of voltage sources in series is the flashlight. When you put two 1.5 V batteries in your flashlight, they are connected in series, giving a total of 3 V. When connecting batteries or other voltage sources in series to increase the total voltage, always connect from the positive (+) terminal of one to the negative (−) terminal of another. Such a connection is illustrated in Figure 23.

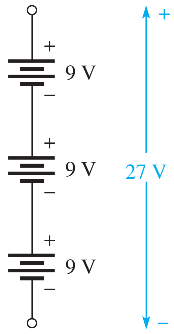
TECH NOTE

When replacing batteries in a portable electronic device, it is best to use all of the same type of battery and not mix old batteries with new batteries. In particular, do not mix alkaline with nonalkaline batteries. Incorrect use of batteries can cause hydrogen gas to build inside the battery and cause the casing to rupture. Worse, the mixing of hydrogen gas with oxygen can be an explosion hazard.



▲ FIGURE 22

Voltage sources in series add algebraically. If a source is reversed, it subtracts from the total voltage as shown in part (b). This is not a normal configuration for batteries.



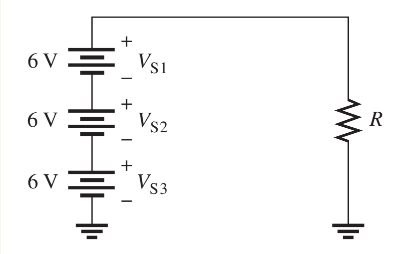
▲ FIGURE 23

Connection of three 9 V batteries to obtain 27 V.

EXAMPLE 11

What is the total source voltage ($V_{S(tot)}$) in Figure 24?

▶ FIGURE 24

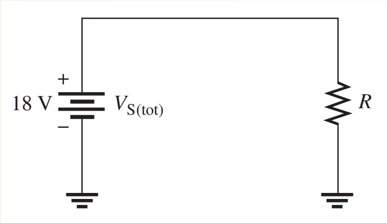


Solution The polarity of each source is the same (the sources are connected in the same direction in the circuit). Add the three voltages to get the total.

$$V_{S(tot)} = V_{S1} + V_{S2} + V_{S3} = 6\text{ V} + 6\text{ V} + 6\text{ V} = \mathbf{18\text{ V}}$$

The three individual sources can be replaced by a single equivalent source of 18 V with its polarity as shown in Figure 25.

▶ FIGURE 25



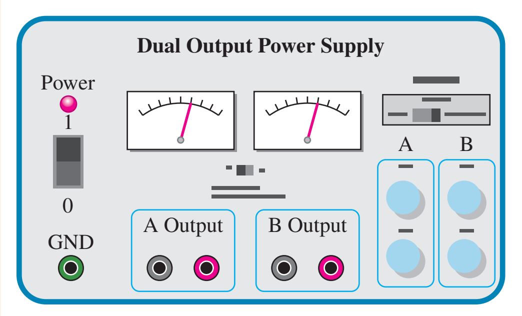
Related Problem If V_{S3} in Figure 24 is accidentally installed backwards, what is the total source voltage?

Use Multisim file E05-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 12

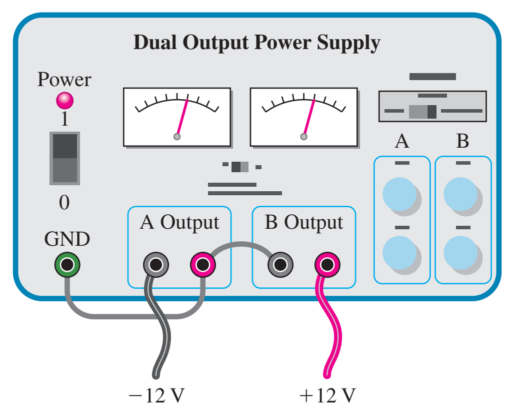
Many circuits use positive and negative supply voltages. A dual-power supply will normally have two independent outputs such as those shown in Figure 26. Show how to connect the two 12 V outputs from the power supply so that there is both a positive and a negative output.

▶ FIGURE 26



Solution See Figure 27. The positive output of one supply is connected to the negative output of the second supply. The ground terminal is connected to this same point, forcing the *A* output to be negative and the *B* output to be positive.

▶ FIGURE 27



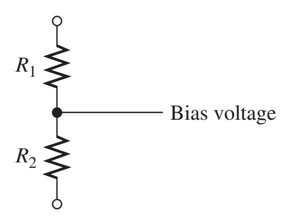
Related Problem Draw the schematic of the setup in Figure 27.

Open Multisim file E05-12. Verify that the connection for the dual power supply will result in negative and positive voltages on the A and B outputs.

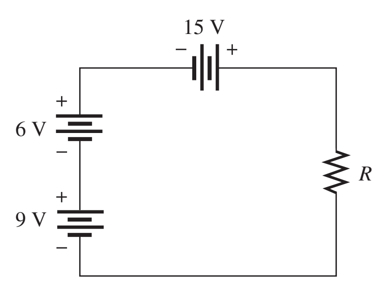
SECTION 5 CHECKUP

1. Four 1.5 V flashlight batteries are connected in series plus to minus. What is the total voltage of all four cells?
2. How many 12 V batteries must be connected in series to produce 60 V? Draw a schematic that shows the battery connections.
3. The resistive circuit in Figure 28 is used to bias a transistor amplifier. Show how to connect two 15 V power supplies in order to get 30 V across the two resistors.
4. Determine the total source voltage for the circuit of Figure 29.
5. One of four 1.5 V batteries was accidentally installed in the wrong direction in a flashlight. What is the voltage across the bulb?

▶ FIGURE 28



▶ FIGURE 29



6 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law is a fundamental circuit law that states that the algebraic sum of all the voltages around a single closed path is zero or, in other words, the sum of the voltage drops equals the total source voltage.

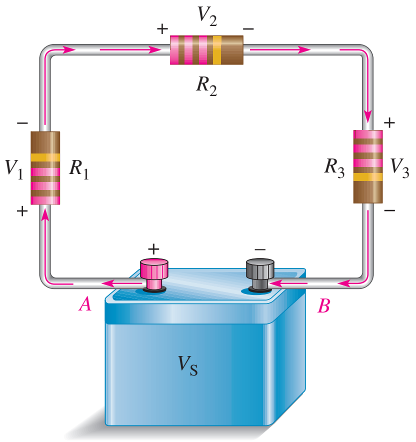
After completing this section, you should be able to

- **Apply Kirchhoff's voltage law**
  - State Kirchhoff's voltage law
  - Determine the source voltage by adding the voltage drops
  - Determine an unknown voltage drop

In an electric circuit, the voltages across the resistors (voltage drops) *always* have polarities opposite to the source voltage polarity. For example, in Figure 30, follow a clockwise loop around the circuit. Note that the source polarity is minus-to-plus and each voltage drop is plus-to-minus. The voltage drops across resistors are designated as V_1, V_2, and so on.

FIGURE 30

Illustration of voltage polarities in a closed-loop circuit.



In Figure 30, the current is out of the positive side of the source and through the resistors as the arrows indicate. The current is into the positive side of each resistor and out the negative side. The drop in energy level across a resistor creates a potential difference, or voltage drop, with a plus-to-minus polarity in the direction of the current.

The voltage from point *A* to point *B* in the circuit of Figure 30 is the source voltage, V_S. Also, the voltage from *A* to *B* is the sum of the series resistor voltage drops. Therefore, the source voltage is equal to the sum of the three voltage drops, as stated by **Kirchhoff's voltage law**.

The sum of all the voltage drops around a single closed path in a circuit is equal to the total source voltage in that loop.

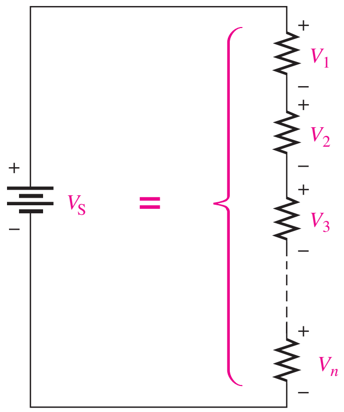
Kirchhoff's voltage law applied to a series circuit is illustrated in Figure 31. For this case, Kirchhoff's voltage law can be expressed by Equation 3.

Equation 3

$$V_S = V_1 + V_2 + V_3 + \cdots + V_n$$

where the subscript n represents the number of voltage drops.

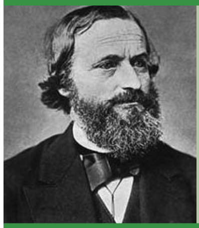
If all the voltage drops around a closed path are added and then this total is subtracted from the source voltage, the result is zero. This result occurs because the sum of the voltage drops always equals the source voltage.



◀ **FIGURE 31**

Sum of *n* voltage drops equals the source voltage.

HISTORY NOTE



Gustav Robert Kirchhoff (1824–1887)

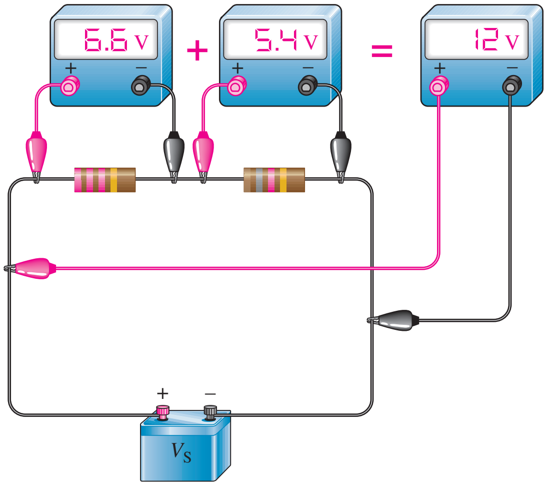
Kirchhoff was a German physicist who contributed to the fundamental understanding of electrical circuits, spectroscopy, and the emission of black-body radiation by heated objects. Concepts in both circuit theory and thermal emission are named Kirchhoff's laws in his honor. Kirchhoff formulated his circuit laws, which are now commonly used in electrical engineering and technology, in 1845 while still a student. He completed this study as a seminar exercise; it later became his doctoral dissertation. (Photo credit: Photographische Gesellschaft, Berlin, courtesy AIP Emilio Segre Visual Archives, W. F. Meggers Collection, Brittle Books Collection, Harvard University Collection.)

The algebraic sum of all the voltages (both source and drops) around a single closed path is zero.

Therefore, another way of expressing Kirchhoff's voltage law in equation form is

$$V_S - V_1 - V_2 - V_3 - \cdots - V_n = 0$$

You can verify Kirchhoff's voltage law by connecting a circuit and measuring each resistor voltage and the source voltage as illustrated in Figure 32. When the resistor voltages are added together, their sum will equal the source voltage. Any number of resistors can be added.



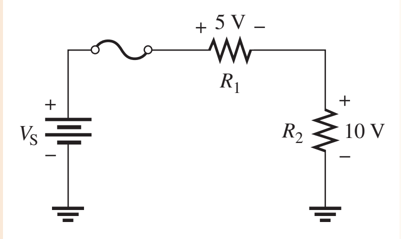
◀ **FIGURE 32**

Illustration of an experimental verification of Kirchhoff's voltage law.

EXAMPLE 13

Determine the source voltage V_S in Figure 33 where the two voltage drops are given. There is no voltage drop across the fuse.

▶ **FIGURE 33**



Solution By Kirchhoff's voltage law, (Eq. 3), the source voltage (applied voltage) must equal the sum of the voltage drops. Adding the voltage drops gives the value of the source voltage.

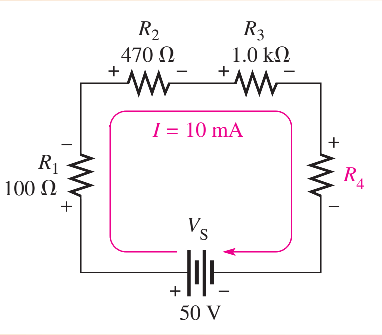
$$V_S = 5\text{ V} + 10\text{ V} = \mathbf{15\text{ V}}$$

Related Problem If V_S is increased to 30 V, determine the two voltage drops. What is the voltage across each component (including the fuse) if the fuse is blown?

Open Multisim files E05-13A and E05-13B to verify the calculated results in the example and to confirm your calculations for the related problem.

EXAMPLE 14 Find the value of R_4 in Figure 34.

▶ **FIGURE 34**



Solution In this problem you will use both Ohm's law and Kirchhoff's voltage law.

First, use Ohm's law to find the voltage drop across each of the known resistors.

$$\begin{aligned} V_1 &= IR_1 = (10\text{ mA})(100\ \Omega) = 1.0\text{ V} \\ V_2 &= IR_2 = (10\text{ mA})(470\ \Omega) = 4.7\text{ V} \\ V_3 &= IR_3 = (10\text{ mA})(1.0\text{ k}\Omega) = 10\text{ V} \end{aligned}$$

Next, use Kirchhoff's voltage law to find V_4, the voltage drop across the unknown resistor.

$$\begin{aligned} V_S - V_1 - V_2 - V_3 - V_4 &= 0\text{ V} \\ 50\text{ V} - 1.0\text{ V} - 4.7\text{ V} - 10\text{ V} - V_4 &= 0\text{ V} \\ 34.3\text{ V} - V_4 &= 0\text{ V} \\ V_4 &= 34.3\text{ V} \end{aligned}$$

Now that you know V_4, use Ohm's law to calculate R_4.

$$R_4 = \frac{V_4}{I} = \frac{34.3\text{ V}}{10\text{ mA}} = \mathbf{3.43\text{ k}\Omega}$$

R_4 is most likely a 3.3 kΩ resistor because 3.43 kΩ is within a standard tolerance range (±5%) of 3.3 kΩ.

Related Problem Determine R_4 in Figure 34 for $V_S = 25\text{ V}$ and $I = 10\text{ mA}$.

Use Multisim file E05-14 to verify the calculated results in the example and to confirm your calculation for the related problem.

So far, you have seen how Kirchhoff's voltage law can be applied to a series circuit with a voltage source, but it can also be applied to other types of circuits. For example, there are cases where there is no source voltage in a given closed loop. Even so, Kirchhoff's voltage law still applies. This leads to a more-general form of Equation 3.

$$V_1 + V_2 + V_3 + \cdots + V_n = 0$$

Equation 4

If a voltage source is present, it is simply treated as one of the terms in Equation 4.

In applying Equation 4, an algebraic sign is given to each voltage in the path depending on whether it is voltage rise (+) or a voltage drop (−). A voltage across a resistor can appear as either a rise or a drop depending on the path direction chosen, so you will need to assign algebraic signs to voltage rises and voltage drops in a consistent manner.

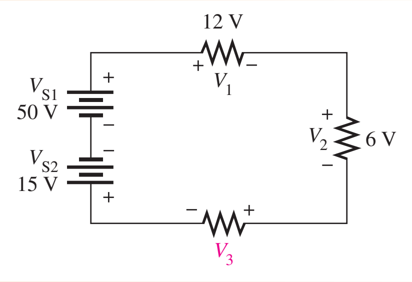
Sometimes you will see an equation such as Equation 4 expressed in shorthand notation using the Greek letter sigma (Σ) to denote summation, as follows:

$$\sum_{i=1}^{n} V_i = 0$$

This mathematical expression is equivalent to Equation 4 and means that you add the voltages from the first ($i = 1$) to the last ($i = n$).

EXAMPLE 15 Determine the unknown voltage drop, V_3, in Figure 35.

▶ FIGURE 35



Solution By Kirchhoff's voltage law (Eq. 4), the algebraic sum of all the voltages around the circuit is zero. The value of each voltage drop except V_3 is known. Substitute these values into the equation.

$$V_{S1} + V_{S2} + V_3 + V_2 + V_1 = 0$$
$$50\text{ V} - 15\text{ V} - V_3 - 6\text{ V} - 12\text{ V} = 0$$

Next, combine the known values, transpose 17 V to the right side of the equation, and cancel the minus signs.

$$17\text{ V} - V_3 = 0\text{ V}$$
$$-V_3 = -17\text{ V}$$
$$V_3 = \mathbf{17\ V}$$

The voltage drop across R_3 is 17 V, and its polarity is as shown in Figure 35.

Related Problem Determine V_3 if the polarity of V_{S2} is reversed in Figure 35.

SECTION 6 CHECKUP

1. State Kirchhoff's voltage law in two ways.
2. A 50 V source is connected to a series resistive circuit. What is the sum of the voltage drops in this circuit?
3. Two equal-value resistors are connected in series across a 10 V battery. What is the voltage drop across each resistor?
4. In a series circuit with a 25 V source, there are three resistors. One voltage drop is 5 V, and the other is 10 V. What is the value of the third voltage drop?
5. The individual voltage drops in a series string are as follows: 1 V, 3 V, 5 V, 8 V, and 7 V. What is the total voltage applied across the series string?

7 VOLTAGE DIVIDERS

A series circuit acts as a voltage divider. The voltage divider is an important application of series circuits.

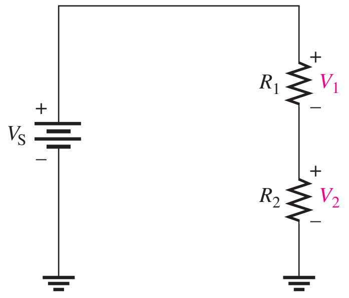
After completing this section, you should be able to

- **Use a series circuit as a voltage divider**
  - Apply the voltage-divider formula
  - Use a potentiometer as an adjustable voltage divider
  - Describe some voltage-divider applications

A circuit consisting of a series string of resistors connected to a voltage source acts as a **voltage divider**. Figure 36 shows a circuit with two resistors in series, although there can be any number. There are two voltage drops across the resistors: one across R_1 and one

FIGURE 36

Two-resistor voltage divider.

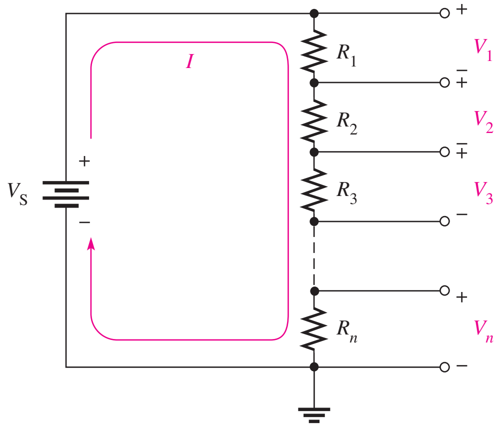


across R_2. These voltage drops are V_1 and V_2, respectively, as indicated in the schematic. Since each resistor has the same current, the voltage drops are proportional to the resistance values. For example, if the value of R_2 is twice that of R_1, then the value of V_2 is twice that of V_1.

The total voltage drop around a single closed path divides among the series resistors in amounts directly proportional to the resistance values. For example, in Figure 36, if V_S is 10 V, R_1 is 50 Ω, and R_2 is 100 Ω, then V_1 is one-third the total voltage, or 3.33 V, because R_1 is one-third the total resistance of 150 Ω. Likewise, V_2 is two-thirds V_S, or 6.67 V.

Voltage-Divider Formula

With a few calculations, you can develop a formula for determining how the voltages divide among series resistors. Assume a circuit with n resistors in series as shown in Figure 37, where n can be any number.



◀ **FIGURE 37**

Generalized voltage divider with *n* resistors.

Let V_x represent the voltage drop across any one of the resistors and R_x represent the number of a particular resistor or combination of resistors. By Ohm's law, you can express the voltage drop across R_x as follows:

$$V_x = IR_x$$

The current through the circuit is equal to the source voltage divided by the total resistance ($I = V_S/R_T$). In the circuit of Figure 37, the total resistance is $R_1 + R_2 + R_3 + \cdots + R_n$. By substitution of V_S/R_T for I in the expression for V_x,

$$V_x = \left(\frac{V_S}{R_T}\right)R_x$$

Rearranging the terms you get

$$V_x = \left(\frac{R_x}{R_T}\right)V_S$$

Equation 5

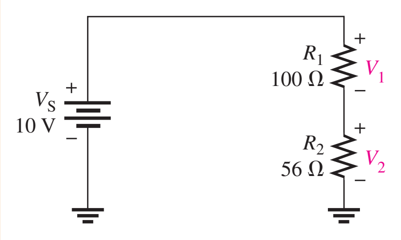
Equation 5 is the general voltage-divider formula, which can be stated as follows:

> **The voltage drop across any resistor or combination of resistors in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.**

EXAMPLE 16

Determine V_1 (the voltage across R_1) and V_2 (the voltage across R_2) in the voltage divider in Figure 38.

▶ FIGURE 38



Solution To determine V_1, use the voltage-divider formula, $V_x = (R_x/R_T)V_S$, where $x = 1$. The total resistance is

$$R_T = R_1 + R_2 = 100\ \Omega + 56\ \Omega = 156\ \Omega$$

R_1 is 100 Ω and V_S is 10 V. Substitute these values into the voltage-divider formula.

$$V_1 = \left(\frac{R_1}{R_T}\right)V_S = \left(\frac{100\ \Omega}{156\ \Omega}\right)10\ \text{V} = \mathbf{6.41\ V}$$

There are two ways to find the value of V_2: Kirchhoff's voltage law or the voltage-divider formula. If you use Kirchhoff's voltage law ($V_S = V_1 + V_2$), substitute the values for V_S and V_1 as follows:

$$V_2 = V_S - V_1 = 10\ \text{V} - 6.41\ \text{V} = \mathbf{3.59\ V}$$

To determine V_2, use the voltage-divider formula where $x = 2$.

$$V_2 = \left(\frac{R_2}{R_T}\right)V_S = \left(\frac{56\ \Omega}{156\ \Omega}\right)10\ \text{V} = \mathbf{3.59\ V}$$

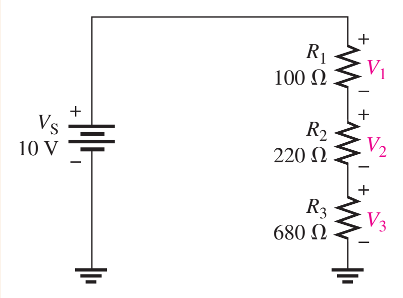
Related Problem Find the voltages across R_1 and R_2 in Figure 38 if R_2 is changed to 180 Ω.

Use Multisim file E05-16 to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 17

Calculate the voltage drop across each resistor in the voltage divider of Figure 39.

▶ FIGURE 39



Solution Look at the circuit for a moment and consider the following: The total resistance is 1000 Ω. Ten percent of the total voltage is across R_1 because it is 10% of the total resistance (100 Ω is 10% of 1000 Ω). Likewise, 22% of the total voltage is dropped across R_2 because it is 22% of the total resistance (220 Ω is 22% of 1000 Ω). Finally, R_3 drops 68% of the total voltage because 680 Ω is 68% of 1000 Ω.

Because of the convenient values in this problem, it is easy to figure the voltages mentally. ($V_1 = 0.10 \times 10\text{ V} = 1\text{ V}$, $V_2 = 0.22 \times 10\text{ V} = 2.2\text{ V}$, and $V_3 = 0.68 \times 10\text{ V} = 6.8\text{ V}$). Such is usually not the case, but sometimes a little thinking will produce a result more efficiently and eliminate some calculating. This is also a good way to estimate what your results should be so that you will recognize an unreasonable answer as a result of a calculation error.

Although you have already reasoned through this problem, the calculations will verify your results.

$$V_1 = \left(\frac{R_1}{R_T}\right)V_S = \left(\frac{100\ \Omega}{1000\ \Omega}\right)10\text{ V} = \mathbf{1\ V}$$

$$V_2 = \left(\frac{R_2}{R_T}\right)V_S = \left(\frac{220\ \Omega}{1000\ \Omega}\right)10\text{ V} = \mathbf{2.2\ V}$$

$$V_3 = \left(\frac{R_3}{R_T}\right)V_S = \left(\frac{680\ \Omega}{1000\ \Omega}\right)10\text{ V} = \mathbf{6.8\ V}$$

Notice that the sum of the voltage drops is equal to the source voltage, in accordance with Kirchhoff's voltage law. This check is a good way to verify your results.

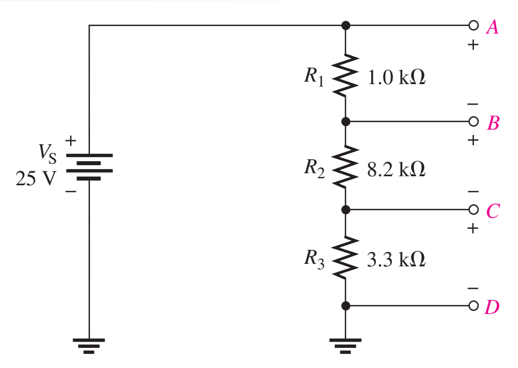
Related Problem If R_1 and R_2 in Figure 39 are changed to 680 Ω, what are the voltage drops?

Use Multisim file E05-17 to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 18 Determine the voltages between the following points in the voltage divider of Figure 40:

(a) *A* to *B* **(b)** *A* to *C* **(c)** *B* to *C* **(d)** *B* to *D* **(e)** *C* to *D*

▶ **FIGURE 40**



Solution First, determine R_T.

$$R_T = R_1 + R_2 + R_3 = 1.0\text{ k}\Omega + 8.2\text{ k}\Omega + 3.3\text{ k}\Omega = 12.5\text{ k}\Omega$$

Next, apply the voltage-divider formula to obtain each required voltage.

(a) The voltage A to B is the voltage drop across R_1.

$$V_{AB} = \left(\frac{R_1}{R_T}\right)V_S = \left(\frac{1.0\text{ k}\Omega}{12.5\text{ k}\Omega}\right)25\text{ V} = \mathbf{2\text{ V}}$$

(b) The voltage from A to C is the combined voltage drop across both R_1 and R_2. In this case, R_x in the general formula given in Equation 5 is $R_1 + R_2$.

$$V_{AC} = \left(\frac{R_1 + R_2}{R_T}\right)V_S = \left(\frac{9.2\text{ k}\Omega}{12.5\text{ k}\Omega}\right)25\text{ V} = \mathbf{18.4\text{ V}}$$

(c) The voltage from B to C is the voltage drop across R_2.

$$V_{BC} = \left(\frac{R_2}{R_T}\right)V_S = \left(\frac{8.2\text{ k}\Omega}{12.5\text{ k}\Omega}\right)25\text{ V} = \mathbf{16.4\text{ V}}$$

(d) The voltage from B to D is the combined voltage drop across both R_2 and R_3. In this case, R_x in the general formula is $R_2 + R_3$.

$$V_{BD} = \left(\frac{R_2 + R_3}{R_T}\right)V_S = \left(\frac{11.5\text{ k}\Omega}{12.5\text{ k}\Omega}\right)25\text{ V} = \mathbf{23\text{ V}}$$

(e) Finally, the voltage from C to D is the voltage drop across R_3.

$$V_{CD} = \left(\frac{R_3}{R_T}\right)V_S = \left(\frac{3.3\text{ k}\Omega}{12.5\text{ k}\Omega}\right)25\text{ V} = \mathbf{6.6\text{ V}}$$

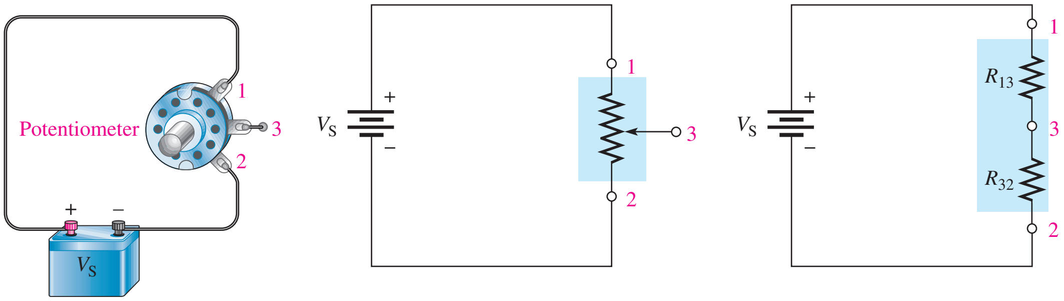
If you connect this voltage divider, you can verify each of the calculated voltages by connecting a voltmeter between the appropriate points in each case.

Related Problem Determine each of the previously calculated voltages if V_S is doubled.

Use Multisim file E05-18 to verify the calculated results in this example and to confirm your calculations for the related problem.

A Potentiometer as an Adjustable Voltage Divider

A potentiometer is a variable resistor with three terminals. A linear potentiometer connected to a voltage source is shown in Figure 41(a) with the schematic shown in part (b). Notice that the two end terminals are labeled 1 and 2. The adjustable terminal or wiper is labeled 3. The potentiometer functions as a voltage divider,



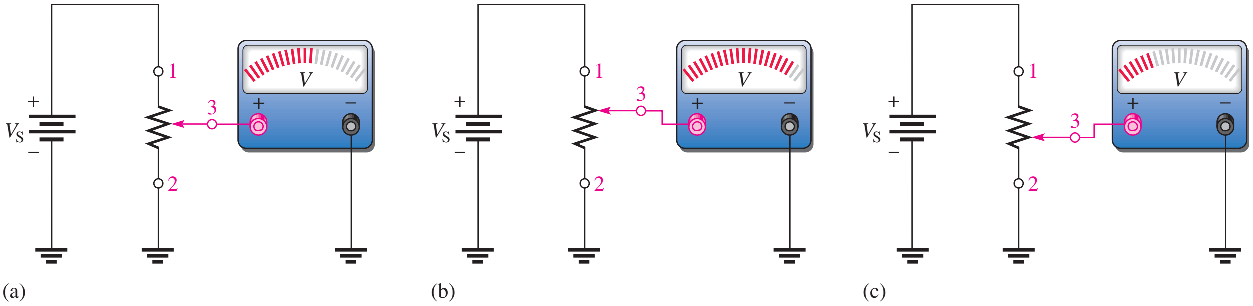
(a) Pictorial (b) Schematic (c) Equivalent schematic

▲ **FIGURE 41**

The potentiometer used as a voltage divider.

which can be illustrated by separating the total resistance into two parts, as shown in Figure 41(c). The resistance between terminal 1 and terminal 3 (R_{13}) is one part, and the resistance between terminal 3 and terminal 2 (R_{32}) is the other part. So this potentiometer is equivalent to a two-resistor voltage divider that can be manually adjusted.

Figure 42 shows what happens when the wiper contact (3) is moved. In part (a), the wiper is exactly centered, making the two resistances equal. If you measure the voltage across terminals 3 to 2 as indicated by the voltmeter symbol, you have one-half of the total source voltage. When the wiper is moved up, as in part (b), the resistance between terminals 3 and 2 increases, and the voltage across it increases proportionally. When the wiper is moved down, as in part (c), the resistance between terminals 3 and 2 decreases, and the voltage decreases proportionally.

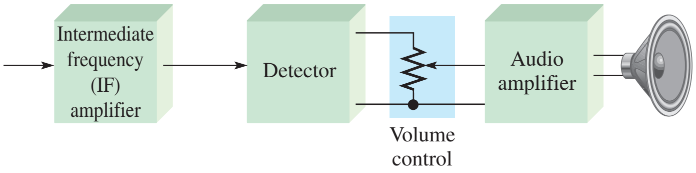


▲ FIGURE 42

Adjusting the voltage divider.

Applications

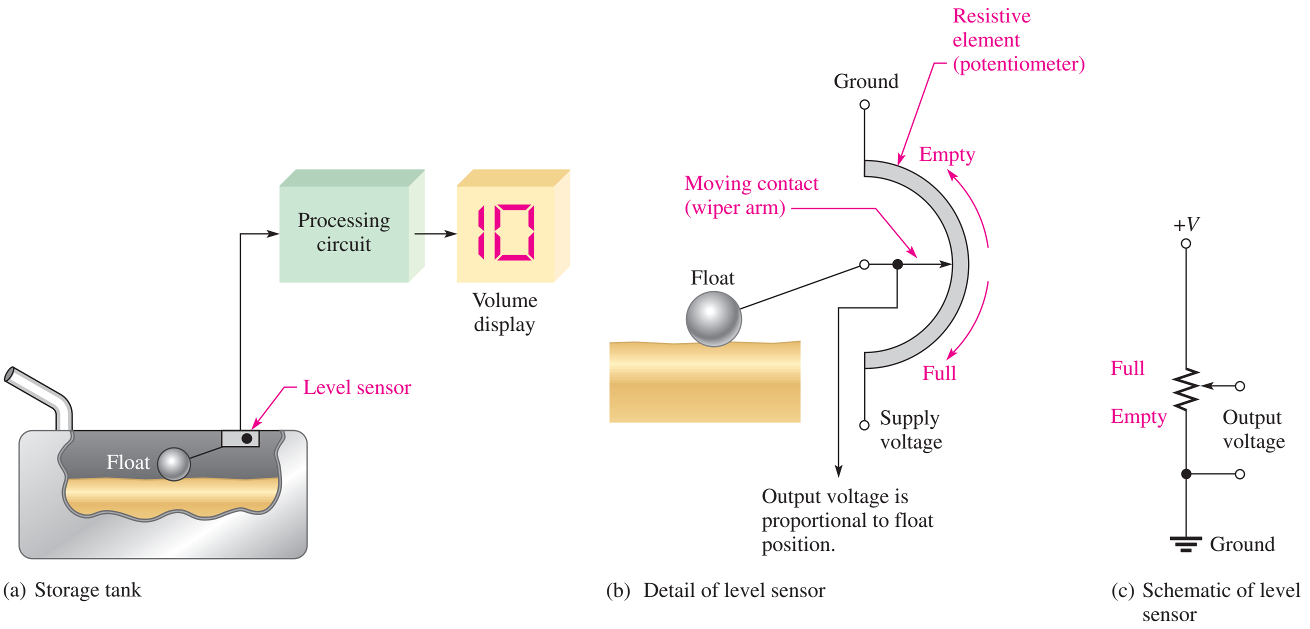
The volume control of some radio receivers is a common application of a potentiometer used as a voltage divider. Since the loudness of the sound is dependent on the amount of voltage associated with the audio signal, you can increase or decrease the volume by adjusting the potentiometer, that is, by turning the knob of the volume control on the set. The block diagram in Figure 43 shows how a potentiometer can be used for volume control in a typical receiver.



▲ FIGURE 43

A variable voltage divider used for volume control in a radio receiver.

Figure 44 illustrates another application in which a potentiometer voltage divider can be used as a level sensor in a liquid storage tank. As shown in part (a), the float moves up as the tank is filled and moves down as the tank empties. The float is mechanically linked to the wiper arm of a potentiometer, as shown in part (b). The output voltage varies proportionally with the position of the wiper arm. As the liquid in the tank decreases, the sensor output voltage also decreases. The output voltage goes to the indicator circuitry, which controls a digital readout to show the amount of liquid in the tank. The schematic of this system is shown in part (c).


Processing circuit
10
Volume display
Level sensor
Float
Resistive element (potentiometer)
Ground
Empty
Moving contact (wiper arm)
Float
Full
Supply voltage
Output voltage is proportional to float position.
+V
Full
Empty
Output voltage
Ground

(a) Storage tank (b) Detail of level sensor (c) Schematic of level sensor

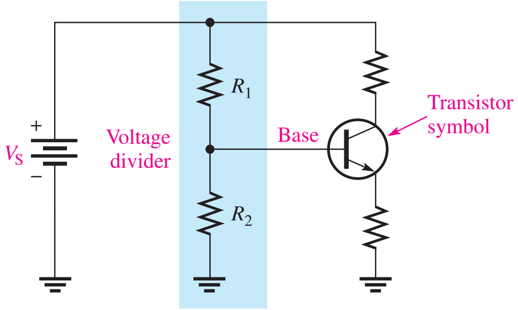
▲ **FIGURE 44**

A potentiometer voltage divider used in a level sensor.

Still another application for voltage dividers is in setting the dc operating voltage (bias) in transistor amplifiers. Figure 45 shows a voltage divider used for this purpose. You will study transistor amplifiers and biasing in a later course, so it is important that you understand the basics of voltage dividers at this point.

▶ **FIGURE 45**

The voltage divider used as a bias circuit for a transistor amplifier, where the voltage at the base terminal of the transistor is determined by the voltage divider as $V_{base} = (R_2/(R_1 + R_2))V_S$.

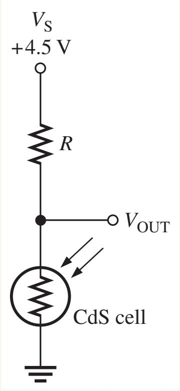

+
V_S
−
Voltage divider
R_1
R_2
Base
Transistor symbol

A voltage divider is useful for converting a resistance sensor output to a voltage. Resistance sensors include thermistors, CdS cells, and strain gauges. To convert a change in resistance to an output voltage, the resistance sensor can replace one of the resistors in a voltage divider.

EXAMPLE 19

Assume you have a CdS cell configured as shown in Figure 46 that uses three AA batteries for a voltage source (4.5 V). At dusk, the resistance of the cell rises from a low resistance to above 90 kΩ. The cell is used to trigger a logic circuit that will turn on lights if V_{OUT} is greater than approximately 1.5 V. What value of R will produce an output voltage of 1.5 V when the cell resistance is 90 kΩ?

▶ FIGURE 46



Solution Notice that the threshold voltage (1.5 V) is one-third of the supply voltage. You know that 90 kΩ represents one-third of the total resistance at this point. Therefore, the total resistance is

$$R_T = 3(90\text{ k}\Omega) = 270\text{ k}\Omega$$

The resistance value needed to produce $V_{OUT} = 1.5$ V is

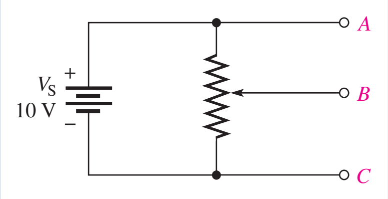
$$R = R_T - 90\text{ k}\Omega = 270\text{ k}\Omega - 90\text{ k}\Omega = \mathbf{180\text{ k}\Omega}$$

Related Problem Starting with Equation 5, prove that the required resistance needed to produce an output voltage of 1.5 V is 180 kΩ when the cell resistance is 90 kΩ.

SECTION 7 CHECKUP

1. What is a voltage divider?
2. How many resistors can there be in a series voltage-divider circuit?
3. Write the general formula for voltage dividers.
4. If two series resistors of equal value are connected across a 10 V source, how much voltage is there across each resistor?
5. A 47 kΩ resistor and an 82 kΩ resistor are connected as a voltage divider. The source voltage is 10 V. Draw the circuit, and determine the voltage across each of the resistors.
6. The circuit of Figure 47 is an adjustable voltage divider. If the potentiometer is linear, where would you set the wiper in order to get 5 V from *A* to *B* and 5 V from *B* to *C*?

▶ FIGURE 47



8 POWER IN SERIES CIRCUITS

The power dissipated by each individual resistor in a series circuit contributes to the total power in the circuit. The individual powers are additive.

After completing this section, you should be able to

- **Determine power in a series circuit**
    - Apply one of the power formulas

The total amount of power in a series resistive circuit is equal to the sum of the powers in each resistor in series.

Equation 6

$$P_T = P_1 + P_2 + P_3 + \cdots + P_n$$

where P_T is the total power and P_n is the power in the last resistor in series.

Power formulas are applicable to series circuits. Since there is the same current through each resistor in series, the following formulas are used to calculate the total power:

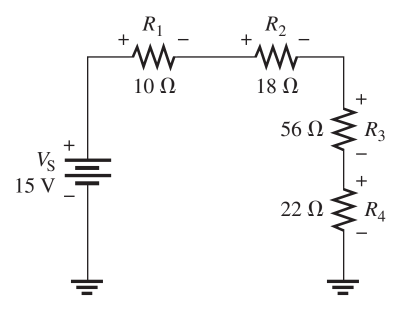
$$P_T = V_S I$$
$$P_T = I^2 R_T$$
$$P_T = \frac{V_S^2}{R_T}$$

where I is the current through the circuit, V_S is the total source voltage across the series connection, and R_T is the total resistance.

EXAMPLE 20

Determine the total amount of power in the series circuit in Figure 48.

▶ FIGURE 48



Solution The source voltage is 15 V. The total resistance is

$$R_T = R_1 + R_2 + R_3 + R_4 = 10\ \Omega + 18\ \Omega + 56\ \Omega + 22\ \Omega = 106\ \Omega$$

The easiest formula to use is $P_T = V_S^2/R_T$ since you know both V_S and R_T.

$$P_T = \frac{V_S^2}{R_T} = \frac{(15\text{ V})^2}{106\ \Omega} = \frac{225\text{ V}^2}{106\ \Omega} = 2.12\text{ W}$$

If you determine the power in each resistor separately and all of these powers are added, you obtain the same result. First, find the current.

$$I = \frac{V_S}{R_T} = \frac{15\text{ V}}{106\ \Omega} = 142\text{ mA}$$

Next, calculate the power for each resistor using $P = I^2R$.

$$P_1 = I^2R_1 = (142\text{ mA})^2(10\ \Omega) = 200\text{ mW}$$
$$P_2 = I^2R_2 = (142\text{ mA})^2(18\ \Omega) = 360\text{ mW}$$
$$P_3 = I^2R_3 = (142\text{ mA})^2(56\ \Omega) = 1.12\text{ W}$$
$$P_4 = I^2R_4 = (142\text{ mA})^2(22\ \Omega) = 441\text{ mW}$$

Then, add these powers to get the total power.

$$P_T = P_1 + P_2 + P_3 + P_4 = 200\text{ mW} + 360\text{ mW} + 1.12\text{ W} + 441\text{ mW} = \mathbf{2.12\text{ W}}$$

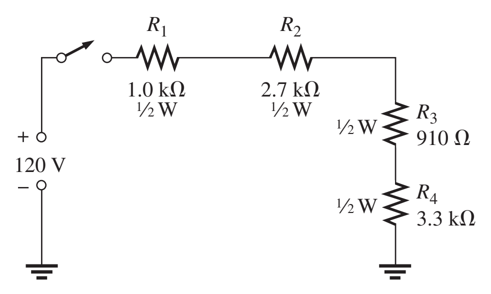
This result compares to the total power as determined previously by the formula $P_T = V_S^2/R_T$.

Related Problem What is the power in the circuit of Figure 48 if V_S is increased to 30 V?

The amount of power in a resistor is important because the power rating of the resistors must be high enough to handle the expected power in the circuit. The following example illustrates practical considerations relating to power in a series circuit.

EXAMPLE 21 Determine if the indicated power rating (½ W) of each resistor in Figure 49 is sufficient to handle the actual power when the switch is closed. If a rating is not adequate, specify the required minimum rating.

▶ FIGURE 49



Solution First, determine the total resistance.

$$R_T = R_1 + R_2 + R_3 + R_4 = 1.0\text{ k}\Omega + 2.7\text{ k}\Omega + 910\ \Omega + 3.3\text{ k}\Omega = 7.91\text{ k}\Omega$$

Next, calculate the current.

$$I = \frac{V_S}{R_T} = \frac{120\text{ V}}{7.91\text{ k}\Omega} = 15\text{ mA}$$

Then calculate the power in each resistor.

$$P_1 = I^2R_1 = (15\text{ mA})^2(1.0\text{ k}\Omega) = \mathbf{225\text{ mW}}$$
$$P_2 = I^2R_2 = (15\text{ mA})^2(2.7\text{ k}\Omega) = \mathbf{608\text{ mW}}$$
$$P_3 = I^2R_3 = (15\text{ mA})^2(910\ \Omega) = \mathbf{205\text{ mW}}$$
$$P_4 = I^2R_4 = (15\text{ mA})^2(3.3\text{ k}\Omega) = \mathbf{743\text{ mW}}$$

R_2 and R_4 do not have a rating sufficient to handle the actual power, which exceeds ½ W in each of these two resistors, and they may burn out if the switch is closed. These resistors should be replaced by 1 W resistors.

Related Problem Determine the minimum power rating required for each resistor in Figure 49 if the source voltage is increased to 240 V.

SECTION 8 CHECKUP

1. If you know the power in each resistor in a series circuit, how can you find the total power?
2. The resistors in a series circuit dissipate the following powers: 2 W, 5 W, 1 W, and 8 W. What is the total power in the circuit?
3. A circuit has a 100 Ω, a 330 Ω, and a 680 Ω resistor in series. There is a current of 1 A through the circuit. What is the total power?

9 VOLTAGE MEASUREMENTS

The concept of *reference ground* is designated as the 0 V reference point for a circuit. Voltage is always measured with respect to another point in the circuit. In this section, ground is discussed in more detail.

After completing this section, you should be able to

- **Measure voltage with respect to ground**
  - Define the term *reference ground*
  - Explain the use of single and double subscripts for indicating voltages
  - Identify ground in a circuit

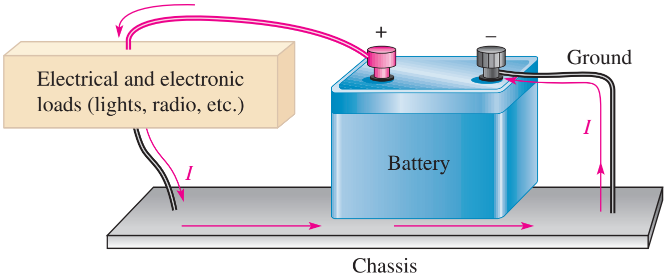
The term *ground* has its origin in telephone systems in which one of the conductors was the earth itself. The term was also used in early radio receiving antennas (called *aerials*) where one part of the system was connected to a metal pipe driven into the earth. Today, *ground* can mean different things and is not necessarily at the same potential as the earth. In electronic systems, **reference ground** (or *common*) refers to a conductor that is the comparison point for voltage measurements in a circuit. Frequently, it is the conductor that carries the power supply return current. Most electronic circuit boards have a larger conducting surface area for ground. For many multilayer circuit boards the ground surface area is a separate internal layer, which is referred to as a *ground plane*.

In electrical wiring, the reference ground is usually the same as the earth potential because neutral and earth ground are connected at the entrance point to a building. In this case, reference ground and earth ground are at the same potential.

The concept of *reference ground* is also used in automotive electrical systems. In most automotive systems, the chassis of the automobile is the ground reference. In nearly all modern automobiles, the negative post of the battery is connected to the chassis with a solid low-resistance connection. This makes the chassis of the vehicle serve as the return path for all of the electrical circuits in the vehicle, as illustrated in the simplified drawing in Figure 50. In some vintage cars, the positive terminal was connected to the chassis in an arrangement called *positive ground*. In both cases, the chassis represents the reference ground point.

SAFETY NOTE

When removing a battery from an automobile, remove the ground wire first. If a tool should then accidentally come in contact with the chassis and the positive terminal, a spark will be avoided; there is no current because there is no return path. When installing a battery, always install the ground connection last.

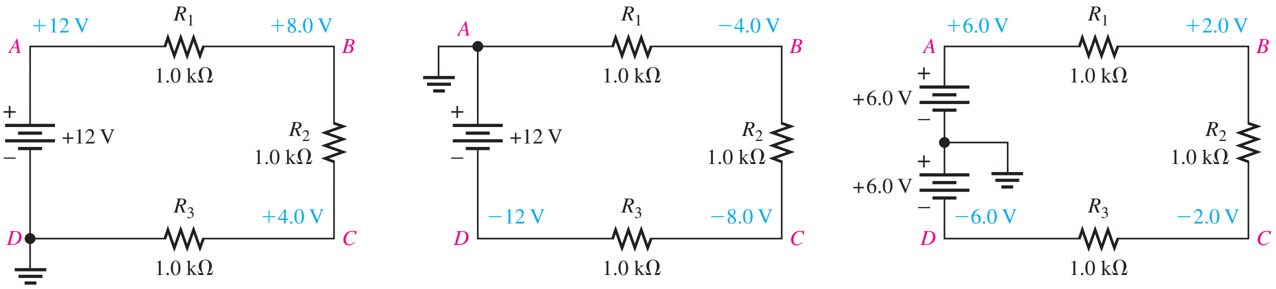


◀ **FIGURE 50**

The chassis serves as a return current path for the electrical circuits in a vehicle.

Measuring Voltages with Respect to Ground

When voltages are measured with respect to ground, they are indicated with a single letter subscript. For example, V_A is voltage at point *A* with respect to ground. The circuits in Figure 51 consist of three 1.0 kΩ series resistors and four lettered reference points. Reference ground represents a potential of 0 V with respect to all other points in the circuit as illustrated. In part (a), the reference point is *D* and all voltages are positive with respect to *D*. In part (b), the reference point is *A* in the circuit, and all other points have a negative voltage.



(a) Positive voltages with respect to ground

(b) Negative voltages with respect to ground

(c) Positive and negative voltages with respect to ground

▲ **FIGURE 51**

The ground point does not affect the current in the circuit or the voltage drops across the resistors.

Many circuits use both positive and negative voltages and as mentioned, the return path for these supplies is designated as reference ground. Figure 51(c) shows the same circuit but with two 6 V sources replacing the 12 V source. In this case, the reference point is arbitrarily designated between the two voltage sources. There is exactly the same current in all three circuits, but now voltages are referenced to the new ground point. As you can see from these examples, the reference ground point is arbitrary and does not change the current.

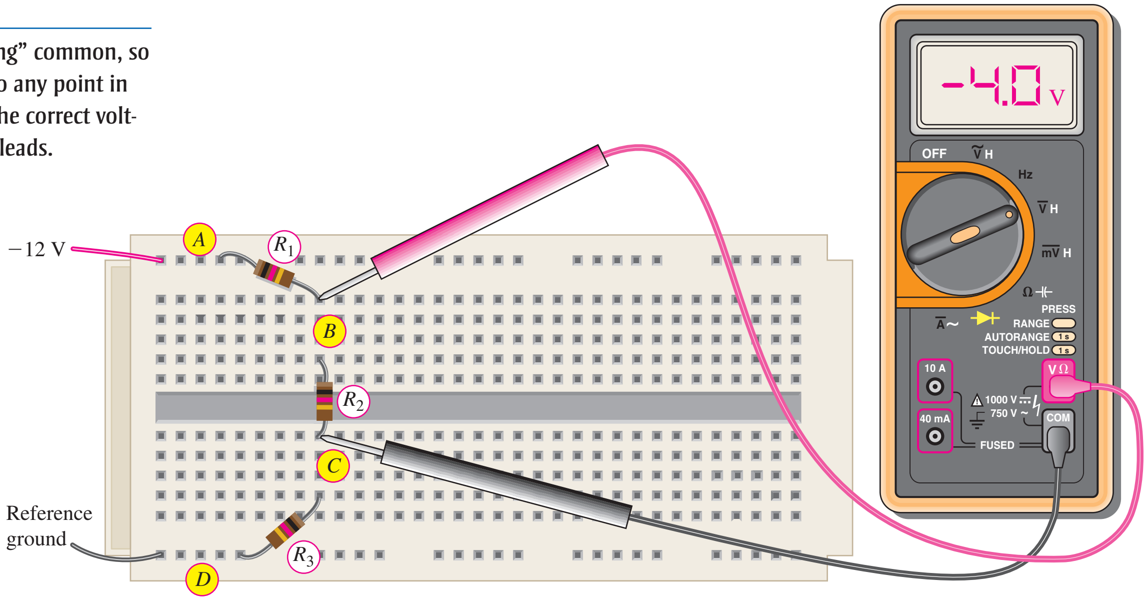
Not all voltages are measured with respect to ground. If you wish to specify the voltage drop across an ungrounded resistor, you can name the resistor in the subscript or use two subscripts. When two subscripts are used, the voltage represents the difference between the points. For example, V_{BC} means $V_B - V_C$. In Figure 51, V_{BC} is the same in all three circuits (+4.0 V) as you can confirm by performing the subtraction in each case. Another way of designating V_{BC} is to write simply V_{R2}.

There is one more convention commonly used to express voltages using subscripts. Power supply voltages are usually given with a double-letter subscript. The reference point is ground or common. For example, a voltage written as V_{CC} is a positive power supply voltage with respect to ground. Other common power supply voltages are V_{SS} (positive), V_{EE} (negative), and V_{SS} (negative).

To measure voltages with a digital meter, the meter leads can be connected across any two points and the meter will indicate the voltage, either positive or negative. The meter reference jack is labeled "COM" (normally black). This is common only to the meter, not to the

▶ **FIGURE 52**

A DMM has a “floating” common, so it can be connected to any point in the circuit and read the correct voltage between the two leads.

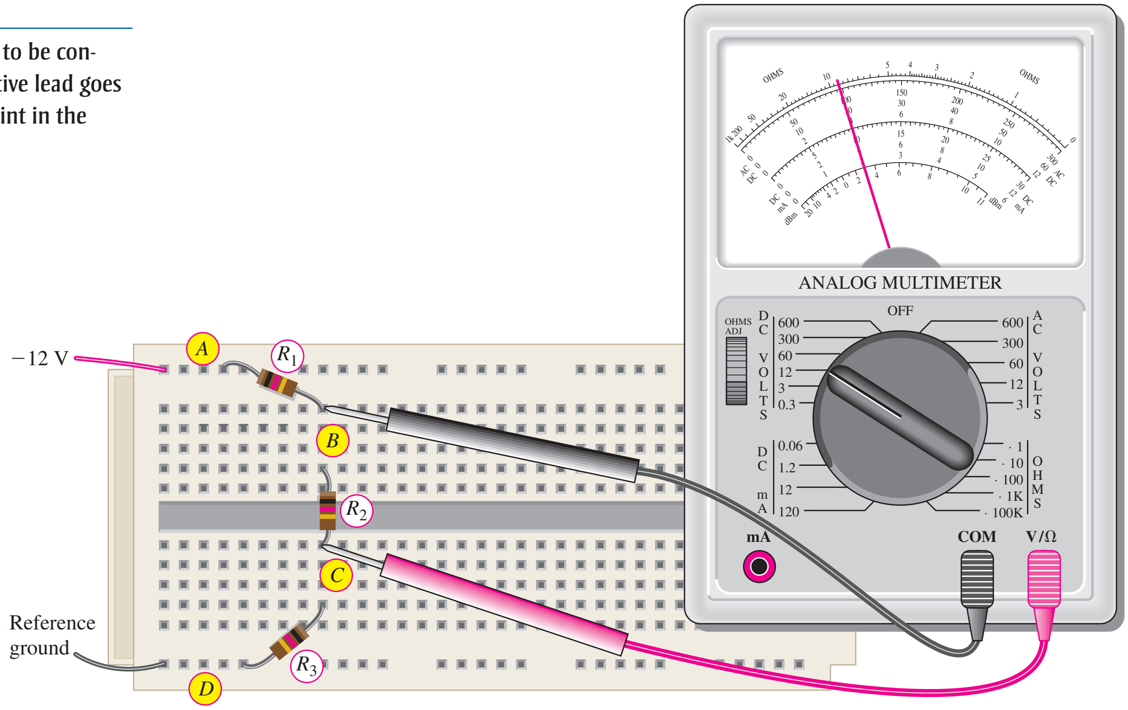

-4.0 V
OFF
Ṽ H
Hz
V H
mV H
Ω
A~
PRESS
RANGE
AUTORANGE
TOUCH/HOLD
10 A
40 mA
VΩ
1000 V
750 V ~
COM
FUSED
−12 V
A
R1
B
R2
C
Reference ground
R3
D

circuit. Figure 52 shows measuring a floating voltage using a DMM to measure the voltage across the ungrounded resistor R_2. The circuit is the same one given in Figure 51(b), which has a negative power supply. It is shown as it might be constructed in a lab. Notice that the meter indicates a negative voltage, which means that the meter’s COM lead is the more positive lead. If you wanted to measure voltages with respect to the circuit’s reference ground, you would connect the COM on the meter to the circuit’s reference ground. The voltage with respect to the reference ground will be indicated.

If you are using an analog meter to make circuit measurements, you must connect the meter so that its common lead is connected to the most negative point in the circuit; otherwise, the meter movement will try to move backward. Figure 53 shows an analog meter connected to the same circuit as before; notice that the meter leads are reversed. To measure

▶ **FIGURE 53**

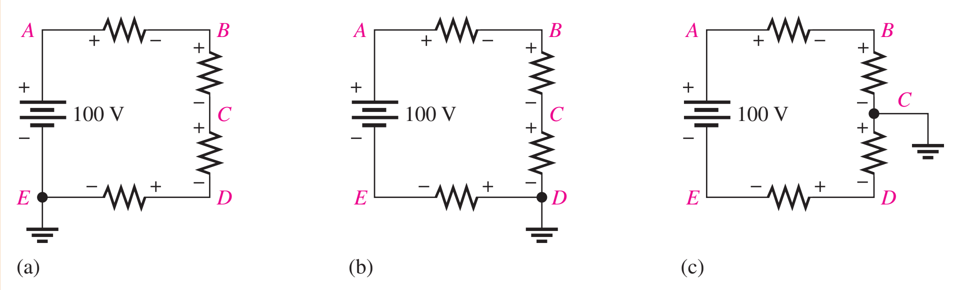
An analog meter needs to be connected so that the positive lead goes to the more positive point in the circuit.


OHMS
ANALOG MULTIMETER
OFF
OHMS ADJ
DC VOLTS
600
300
60
12
3
0.3
DC mA
0.06
1.2
12
120
AC VOLTS
600
300
60
12
3
OHMS
1
10
100
1K
100K
mA
COM
V/Ω
−12 V
A
R1
B
R2
C
Reference ground
R3
D

voltage across R_2, the leads must be connected so that the meter will deflect in the positive direction. Notice that the positive lead on the meter is connected to the more positive point in the circuit, which in this case is the circuit's ground. The user appends a minus sign to the reading when recording the reading.

EXAMPLE 22

Determine the voltages with respect to ground of each of the indicated points in each circuit of Figure 54. Assume that 25 V are dropped across each resistor.



(a) (b) (c)

▲ **FIGURE 54**

Solution In circuit (a), the voltage polarities are as shown. Point *E* is ground. Single-letter subscripts denote voltage at a point with respect to ground. The voltages with respect to ground are as follows:

$$V_E = \mathbf{0\ V}, \quad V_D = \mathbf{+25\ V}, \quad V_C = \mathbf{+50\ V}, \quad V_B = \mathbf{+75\ V}, \quad V_A = \mathbf{+100\ V}$$

In circuit (b), the voltage polarities are as shown. Point *D* is ground. The voltages with respect to ground are as follows:

$$V_E = \mathbf{-25\ V}, \quad V_D = \mathbf{0\ V}, \quad V_C = \mathbf{+25\ V}, \quad V_B = \mathbf{+50\ V}, \quad V_A = \mathbf{+75\ V}$$

In circuit (c), the voltage polarities are as shown. Point *C* is ground. The voltages with respect to ground are as follows:

$$V_E = \mathbf{-50\ V}, \quad V_D = \mathbf{-25\ V}, \quad V_C = \mathbf{0\ V}, \quad V_B = \mathbf{+25\ V}, \quad V_A = \mathbf{+50\ V}$$

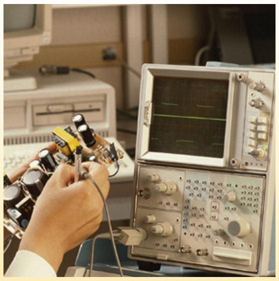
Related Problem If the ground is at point *A* in the circuit in Figure 54, what are the voltages at each of the points with respect to ground?

Use Multisim file E05-22 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 9 CHECKUP

1. What is the reference point in a circuit called?
2. Voltages in a circuit are generally referenced with respect to ground. (T or F)
3. The housing or chassis is often used as reference ground. (T or F)
4. If V_{AB} in a circuit is $+5.0$ V, what is V_{BA}?

10 TROUBLESHOOTING



Open resistors or contacts and one point shorted to another are common problems in all circuits including series circuits.

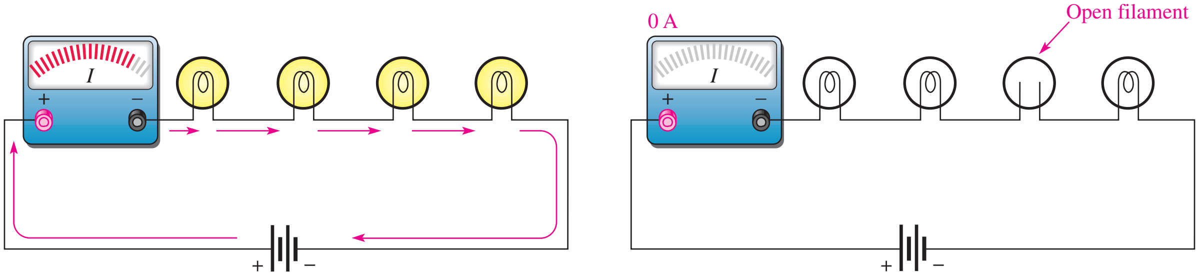
After completing this section, you should be able to

- **Troubleshoot series circuits**
  - Check for an open circuit
  - Check for a short circuit
  - Identify primary causes of opens and shorts

Open Circuit

The most common failure in a series circuit is an **open**. For example, when a resistor or a lamp burns out, it causes a break in the current path and creates an open circuit, as illustrated in Figure 55.

An open in a series circuit prevents current.



(a) A complete series circuit has current.

(b) An open series circuit has no current.

▲ FIGURE 55

Current ceases when an open occurs.

TECH NOTE

When measuring a resistance, make sure that you do not touch the meter leads or the resistor leads. If you hold both ends of a high-value resistor in your fingers along with the meter probes, the measurement will be inaccurate because your body resistance can affect the measured value. When body resistance is placed in parallel with a high-value resistor, the measured value will be less than the actual value of the resistor.

Troubleshooting an Open You should already be familiar with the analysis, planning, and measurement (APM) approach to troubleshooting, as well as the half-splitting method. Now, these principles will be applied using voltage measurements instead of resistance measurements. As you know, voltage measurements are generally the easiest to make because you do not have to disconnect anything.

As a beginning step, prior to analysis, it is a good idea to make a visual check of the faulty circuit. Occasionally, you can find a charred resistor, a broken lamp filament, a loose wire, or a loose connection this way. However, it is possible (and probably more common) for a resistor or other component to open without showing visible signs of damage. When a visual check reveals nothing, then proceed with the APM approach.

When an open occurs in a series circuit, all of the source voltage appears across the open. The reason for this is that the open condition prevents current through the series circuit. With no current, there can be no voltage drop across any of the other resistors (or other component). Since $IR = (0\text{ A})R = 0\text{ V}$, the voltage on each end of a good resistor is the same. Therefore, the voltage applied across a series string also appears across the open component because there are no other voltage drops in the circuit, as illustrated in

Figure 56. The source voltage will appear across the open resistor in accordance with Kirchhoff's voltage law as follows:

$$
\begin{aligned}
V_S &= V_1 + V_2 + V_3 + V_4 + V_5 + V_6 \\
V_4 &= V_S - V_1 - V_2 - V_3 - V_5 - V_6 \\
&= 10\text{ V} - 0\text{ V} - 0\text{ V} - 0\text{ V} - 0\text{ V} - 0\text{ V} \\
V_4 &= V_S = 10\text{ V}
\end{aligned}
$$

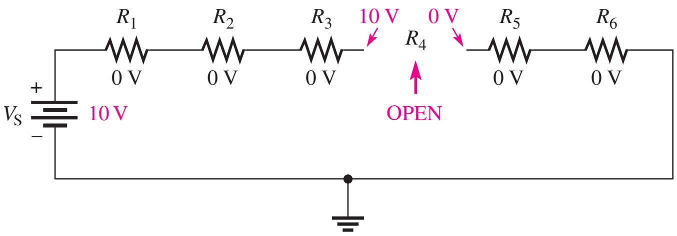


FIGURE 56

The source voltage appears across the open series resistor.

Example of Half-Splitting Using Voltage Measurements Suppose a circuit has four resistors in series. You have determined by *analyzing* the symptoms (there is voltage but no current) that one of the resistors is open, and you are *planning* to find the open resistor using a voltmeter for *measuring* by the half-splitting method. A sequence of measurements for this particular case is illustrated in Figure 57.

Step 1: Measure across R_1 and R_2 (the left half of the circuit). A 0 V reading indicates that neither of these resistors is open.

Step 2: Move the meter to measure across R_3 and R_4; the reading is 10 V. This indicates there is an open in the right half of the circuit, so either R_3 or R_4 is the faulty resistor (assume no bad connections).

Step 3: Move the meter to measure across R_3. A measurement of 10 V across R_3 identifies it as the open resistor. If you had measured across R_4, it would have indicated 0 V. This would have also identified R_3 as the faulty component because it would have been the only one left that could have 10 V across it.

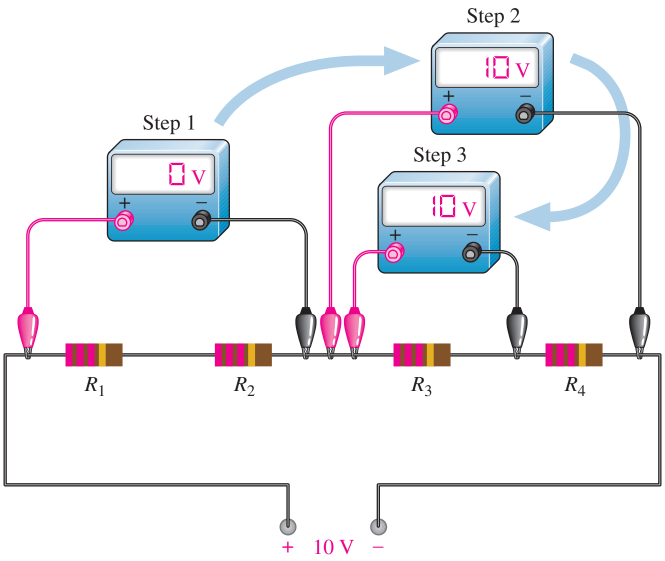


FIGURE 57

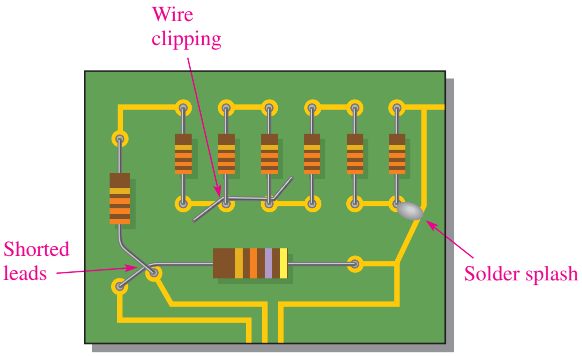
Troubleshooting a series circuit for an open using half-splitting.

Short Circuit

Sometimes an unwanted short circuit occurs when two conductors touch or a foreign object such as solder or a wire clipping accidentally connects two sections of a circuit together. This situation is particularly common in circuits with a high component density. Several potential causes of short circuits are illustrated on the PC board in Figure 58.

▶ **FIGURE 58**

Examples of shorts on a PC board.

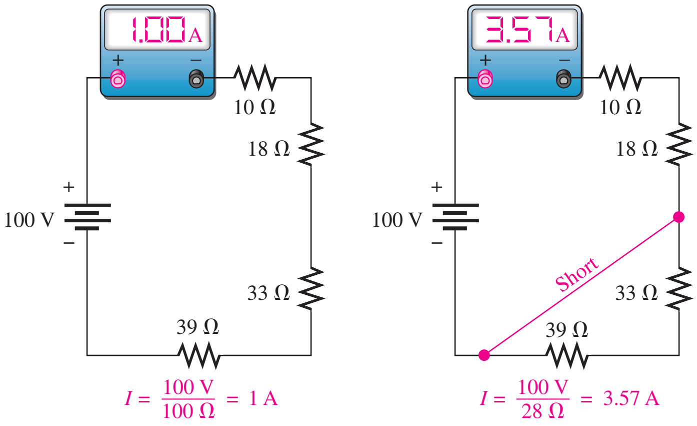


When there is a **short**, a portion of the series resistance is bypassed (all of the current goes through the short), thus reducing the total resistance as illustrated in Figure 59. Notice that the current increases as a result of the short.

A short in a series circuit causes more current than normal.

▶ **FIGURE 59**

Example of the effect of a short in a series circuit.



(a) Before short

(b) After short

Troubleshooting a Short A short is very difficult to troubleshoot. As in any troubleshooting situation, it is a good idea to make a visual check of the faulty circuit. In the case of a short in the circuit, a wire clipping, solder splash, or touching leads is often found to be the culprit. In terms of component failure, shorts are less common than opens in many types of components. Furthermore, a short in one part of a circuit can cause overheating in another part due to the higher current caused by the short. As a result two failures, an open and a short, may occur together.

When a short occurs in a series circuit, there is essentially no voltage across the shorted part. A short has zero or near zero resistance, although shorts with significant resistance values can occur from time to time. These are called *resistive shorts.* For purposes of illustration, zero resistance is assumed for all shorts.

In order to troubleshoot a short, measure the voltage across each resistor until you get a reading of 0 V. This is the straightforward approach and does not use half-splitting. In order to apply the half-splitting method, you must know the correct voltage at each point in the circuit and compare it to measured voltages. Example 23 illustrates using half-splitting to find a short.

EXAMPLE 23

Assume you have determined that there is a short in a circuit with four series resistors because the current is higher than it should be. You know that the voltage at each point in the circuit should be as shown in Figure 60 if the circuit is working properly. The voltages are shown relative to the negative terminal of the source. Find the location of the short.

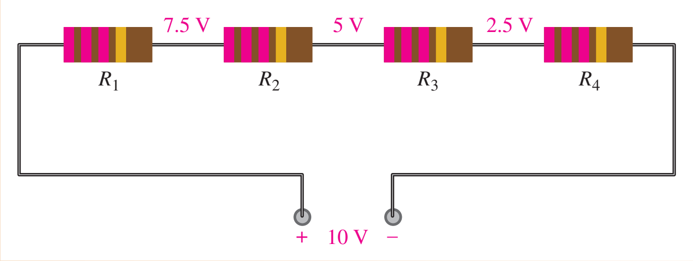


FIGURE 60 Series circuit (without a short) with correct voltages indicated.

Solution Use the half-splitting method to troubleshoot the short.

Step 1: Measure across R_1 and R_2. The meter shows a reading of 6.67 V, which is higher than the normal voltage (it should be 5 V). Look for a voltage that is lower than normal because a short will make the voltage less across that part of the circuit.

Step 2: Move the meter and measure across R_3 and R_4; the reading of 3.33 V is incorrect and lower than normal (it should be 5 V). This shows that the short is in the right half of the circuit and that either R_3 or R_4 is shorted.

Step 3: Again move the meter and measure across R_3. A reading of 3.3 V across R_3 tells you that R_4 is shorted because it must have 0 V across it. Figure 61 illustrates this troubleshooting technique.

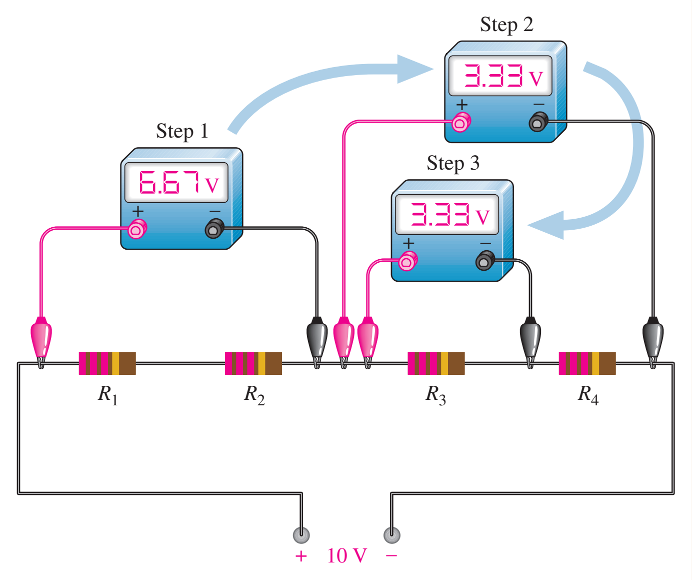
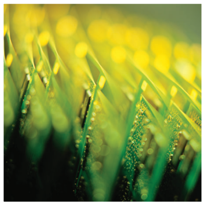


FIGURE 61 Troubleshooting a series circuit for a short using half-splitting.

Related Problem Assume that R_1 is shorted in Figure 61. What would the Step 1 measurement be?

SECTION 10 CHECKUP

1. Define *short*.
2. Define *open*.
3. What happens when a series circuit opens?
4. Name two general ways in which an open circuit can occur in practice. What may cause a short circuit to occur?
5. When a resistor fails, it will normally fail open. (T or F)
6. The total voltage across a string of series resistors is 24 V. If one of the resistors is open, how much voltage is there across it? How much is there across each of the good resistors?

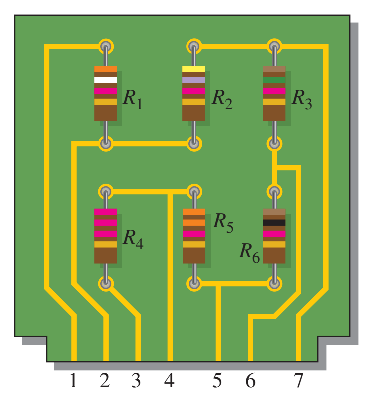


Application Activity

For this application activity, you have a voltage-divider circuit board to evaluate and modify if necessary. You will use it to obtain five different voltage levels from a 12 V battery that has a 6.5 Ah rating. The voltage divider provides positive reference voltages to an electronic circuit in an analog-to-digital converter. Your job is to check the circuit to see if it provides the following voltages within a tolerance of ±5% with respect to the negative side of the battery: 10.4 V, 8.0 V, 7.3 V, 6.0 V, and 2.7 V. If the existing circuit does not provide the specified voltages, you will modify it so that it does. Also, you must make sure that the power ratings of the resistors are adequate for the application and determine how long the battery will last with the voltage divider connected to it.

The Schematic of the Circuit

1. Use Figure 62 to determine the resistor values and draw the schematic of the voltage-divider circuit. All the resistors on the board are $\frac{1}{4}$ W.



▲ FIGURE 62

The Voltages

2. Determine the voltage at each pin on the circuit board with respect to the negative side of the battery when the positive side of the 12 V battery is connected to pin 3 and the negative side is connected to pin 1. Compare the existing voltages to the following specifications:

   Pin 1: negative terminal of 12 V battery

   Pin 2: 2.7 V ± 5%

   Pin 3: positive terminal of 12 V battery

   Pin 4: 10.4 V ± 5%

   Pin 5: 8.0 V ± 5%

   Pin 6: 7.3 V ± 5%

   Pin 7: 6.0 V ± 5%

3. If the output voltages of the existing circuit are not the same as those stated in the specifications, make the necessary changes in the circuit to meet the specifications. Draw a schematic of the modified circuit showing resistor values and adequate power ratings.

The Battery

4. Find the total current drawn from the 12 V battery when the voltage-divider circuit is connected and determine how many days the 6.5 Ah battery will last.

A Test Procedure

5. Determine how you would test the voltage-divider circuit board and what instruments you would use. Then detail your test procedure in a step-by-step format.

Troubleshooting

6. Determine the most likely fault for each of the following cases. Voltages are with respect to the negative battery terminal (pin 1 on the circuit board).
    - No voltage at any of the pins on the circuit board
    - 12 V at pins 3 and 4. All other pins have 0 V.
    - 12 V at all pins except 0 V at pin 1
    - 12 V at pin 6 and 0 V at pin 7
    - 3.3 V at pin 2

Review

7. What is the total power dissipated by the voltage-divider circuit in Figure 62 with a 12 V battery?
8. What are the output voltages from the voltage divider if the positive terminal of a 6 V battery is connected to pin 3 and the negative terminal to pin 1?
9. When the voltage-divider board is connected to the electronic circuit to which it is providing positive reference voltages, which pin on the board should be connected to the ground of the electronic circuit?

Multisim Analysis

Using Multisim connect the circuit based on the schematic from Activity 1 and verify the output voltages specified in Activity 2. Insert faults determined in Activity 6 and verify the resulting voltage measurements.

SUMMARY

- The current is the same at all points in a series circuit.
- The total series resistance is the sum of all resistors in the series circuit.
- The total resistance between any two points in a series circuit is equal to the sum of all resistors connected in series between those two points.
- If all of the resistors in a series circuit are of equal value, the total resistance is the number of resistors multiplied by the resistance value of one resistor.
- Voltage sources in series add algebraically.
- Kirchhoff's voltage law: The sum of all the voltage drops around a single closed path in a circuit is equal to the total source voltage in that loop.
- Kirchhoff's voltage law: The algebraic sum of all the voltages (both source and drops) around a single closed path is zero.
- The voltage drops in a circuit are always opposite in polarity to the total source voltage.
- Conventional current is defined to be out of the positive side of a source and into the negative side.
- Conventional current is defined to be into the positive side of each resistor and out of the more negative (less positive) side.
- A voltage drop results from a decrease in energy level across a resistor.
- A voltage divider is a series arrangement of resistors connected to a voltage source.
- A voltage divider is so named because the voltage drop across any resistor in the series circuit is divided down from the total voltage by an amount proportional to that resistance value in relation to the total resistance.
- A potentiometer can be used as an adjustable voltage divider.
- The total power in a resistive circuit is the sum of all the individual powers of the resistors making up the series circuit.
- Ground (common) is zero volts with respect to all points referenced to it in the circuit.
- *Negative ground* is the term used when the negative side of the source is grounded.
- *Positive ground* is the term used when the positive side of the source is grounded.
- The voltage across an open component always equals the source voltage.
- The voltage across a shorted component is always 0 V.

KEY TERMS

Kirchhoff's voltage law A law stating that (1) the sum of the voltage drops around a single closed path equals the source voltage in that loop or (2) the algebraic sum of all the voltages around any closed path in a circuit is zero.

Open A circuit condition in which the current path is broken.

Reference ground A method of grounding whereby the metal chassis that houses the assembly or a large conductive area on a printed circuit board is used as the common or reference point.

Series In an electric circuit, a relationship of components in which the components are connected such that they provide a single current path between two points.

Short A circuit condition in which there is a zero or abnormally low resistance path between two points; usually an inadvertent condition.

Voltage divider A circuit consisting of series resistors across which one or more output voltages are taken.

FORMULAS

1	$R_T = R_1 + R_2 + R_3 + \cdots + R_n$	Total resistance of *n* resistors in series
2	$R_T = nR$	Total resistance of *n* equal-value resistors in series
3	$V_S = V_1 + V_2 + V_3 + \cdots + V_n$	Kirchhoff's voltage law
4	$V_1 + V_2 + V_3 + \cdots + V_n = 0$	Kirchhoff's voltage law stated another way
5	$V_x = \left(\frac{R_x}{R_T}\right)V_S$	Voltage-divider formula
6	$P_T = P_1 + P_2 + P_3 + \cdots + P_n$	Total power

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. A series circuit can have more than one path for current.
2. The total resistance of a series circuit can be less than the largest resistor in that circuit.
3. If two series resistors are different sizes, the larger resistor will have the larger current.
4. If two series resistors are different sizes, the larger resistor will have the larger voltage.
5. If three equal resistors are used in a voltage divider, the voltage across each one will be one-third of the source voltage.
6. There is no valid electrical reason for installing flashlight batteries so that they are not all in the same direction.
7. Kirchhoff's voltage law is valid only if a loop contains a voltage source.
8. The voltage-divider equation can be written as $V_x = (R_x/R_T)V_S$.
9. The power dissipated by the resistors in a series circuit is the same as the power supplied by the source.
10. If point *A* in a circuit has a voltage of +10 V and point *B* has a voltage of −2 V, then V_{AB} is +8 V.

SELF-TEST

Answers are at the end of the chapter.

1. Five equal-value resistors are connected in series and there is a current of 2 mA into the first resistor. The amount of current out of the second resistor is

   (a) equal to 2 mA **(b)** less than 2 mA **(c)** greater than 2 mA

2. To measure the current out of the third resistor in a circuit consisting of four series resistors, an ammeter can be placed

   (a) between the third and fourth resistors **(b)** between the second and third resistors
   (c) at the positive terminal of the source **(d)** at any point in the circuit

3. When a third resistor is connected in series with two series resistors, the total resistance

(a) remains the same **(b)** increases **(c)** decreases **(d)** increases by one-third

4. When one of four series resistors is removed from a circuit and the circuit reconnected, the current

(a) decreases by the amount of current through the removed resistor

(b) decreases by one-fourth

(c) quadruples

(d) increases

5. A series circuit consists of three resistors with values of 100 Ω, 220 Ω, and 330 Ω. The total resistance is

(a) less than 100 Ω **(b)** the average of the values **(c)** 550 Ω **(d)** 650 Ω

6. A 9 V battery is connected across a series combination of 68 Ω, 33 Ω, 100 Ω, and 47 Ω resistors. The amount of current is

(a) 36.3 mA **(b)** 27.6 A **(c)** 22.3 mA **(d)** 363 mA

7. While putting four 1.5 V batteries in a flashlight, you accidentally put one of them in backward. The voltage across the bulb will be

(a) 6 V **(b)** 3 V **(c)** 4.5 V **(d)** 0 V

8. If you measure all the voltage drops and the source voltage in a series circuit and add them together, taking into consideration the polarities, you will get a result equal to

(a) the source voltage **(b)** the total of the voltage drops

(c) zero **(d)** the total of the source voltage and the voltage drops

9. There are six resistors in a given series circuit and each resistor has 5 V dropped across it. The source voltage is

(a) 5 V **(b)** 30 V

(c) dependent on the resistor values **(d)** dependent on the current

10. A series circuit consists of a 4.7 kΩ, a 5.6 kΩ, and a 10 kΩ resistor. The resistor that has the most voltage across it is

(a) the 4.7 kΩ **(b)** the 5.6 kΩ

(c) the 10 kΩ **(d)** impossible to determine from the given information

11. Which of the following series combinations dissipates the most power when connected across a 100 V source?

(a) One 100 Ω resistor **(b)** Two 100 Ω resistors

(c) Three 100 Ω resistors **(d)** Four 100 Ω resistors

12. The total power in a certain circuit is 1 W. Each of the five equal-value series resistors making up the circuit dissipates

(a) 1 W **(b)** 5 W **(c)** 0.5 W **(d)** 0.2 W

13. When you connect an ammeter in a series-resistive circuit and turn on the source voltage, the meter reads zero. You should check for

(a) a broken wire **(b)** a shorted resistor **(c)** an open resistor **(d)** answers (a) and (c)

14. While checking out a series-resistive circuit, you find that the current is higher than it should be. You should look for

(a) an open circuit **(b)** a short **(c)** a low resistor value **(d)** answers (b) and (c)

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 68.

1. With a 10 V voltage source connected between points *A* and *B*, when the switches are thrown from position 1 to position 2, the total current from the source

(a) increases **(b)** decreases **(c)** stays the same

2. For the conditions described in Question 1, the current through R_3

(a) increases **(b)** decreases **(c)** stays the same

3. When the switches are in position 1 and a short develops across R_3, the current through R_2

(a) increases **(b)** decreases **(c)** stays the same

4. When the switches are in position 2 and a short develops across R_3, the current through R_5

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 69.

5. If the current shown by one of the milliammeters increases, the current shown by the other two

(a) increases **(b)** decreases **(c)** stays the same

6. If the source voltage decreases, the current indicated by each milliammeter

(a) increases **(b)** decreases **(c)** stays the same

7. If the current through R_1 increases as a result of R_1 being replaced by a different resistor, the current indicated by each milliammeter

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 73.

8. If the switch is thrown from position *A* to position *B*, the ammeter reading

(a) increases **(b)** decreases **(c)** stays the same

9. If the switch is thrown from position *B* to position *C,* the voltage across R_4

(a) increases **(b)** decreases **(c)** stays the same

10. If the switch is thrown from position *C* to position *D,* the current through R_3

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 80(b).

11. If R_1 is changed to 1.2 kΩ, the voltage from *A* to *B*

(a) increases **(b)** decreases **(c)** stays the same

12. If R_2 and R_3 are interchanged, the voltage from *A* to *B*

(a) increases **(b)** decreases **(c)** stays the same

13. If the source voltage increases from 8 V to 10 V, the voltage from *A* to *B*

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 87.

14. If the 9 V source is reduced to 5 V, the current in the circuit

(a) increases **(b)** decreases **(c)** stays the same

15. If the 9 V source is reversed, the voltage at point *B* with respect to ground

(a) increases **(b)** decreases **(c)** stays the same

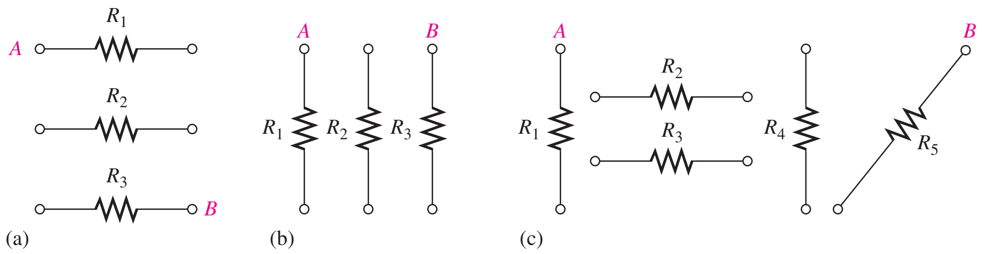
PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 Resistors in Series

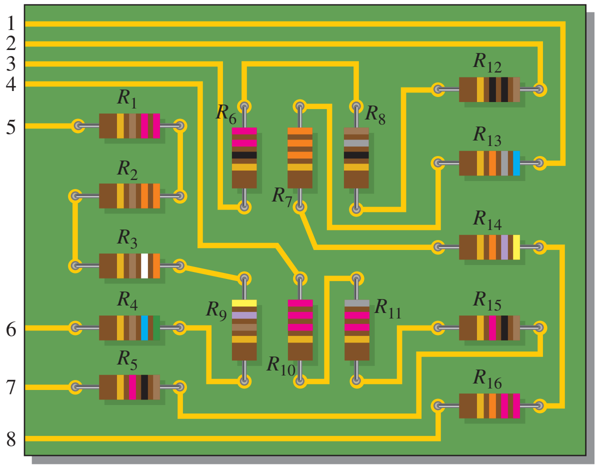
1. Connect each set of resistors in Figure 63 in series between points *A* and *B*.

▶ FIGURE 63



2. Determine the groupings of resistors in Figure 64 that are series connections. Show how to interconnect the pins to put all the resistors in series.

▶ FIGURE 64

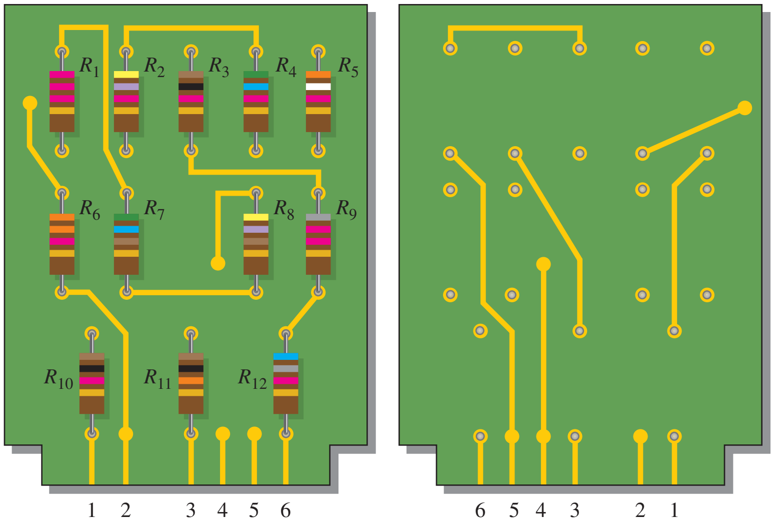

1
2
3
4
5
6
7
8
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
R_{10}
R_{11}
R_{12}
R_{13}
R_{14}
R_{15}
R_{16}

3. Determine the nominal resistance between pins 1and 8 in the circuit board in Figure 64.

4. Determine the nominal resistance between pins 2 and 3 in the circuit board in Figure 64.

5. On the double-sided PC board in Figure 65, identify each group of series resistors. Note that many of the interconnections feed through the board from the top side to the bottom side.

▶ FIGURE 65


R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
R_{10}
R_{11}
R_{12}
1
2
3
4
5
6
6
5
4
3
2
1

Top

Bottom

SECTION 2 Total Series Resistance

6. The following resistors (one each) are connected in a series circuit: 1.0 Ω, 2.2 Ω, 5.6 Ω, 12 Ω, and 22 Ω. Determine the total resistance.

7. Find the total resistance of each of the following groups of series resistors:

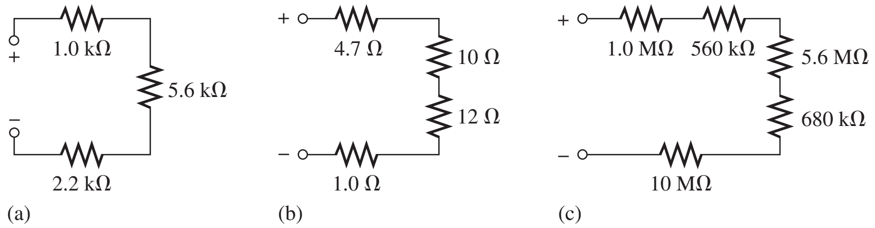
(a) 560 Ω and 1000 Ω

(b) 47 Ω and 56 Ω

(c) 1.5 kΩ, 2.2 kΩ, and 10 kΩ

(d) 1.0 MΩ, 470 kΩ, 1.0 kΩ, 2.2 MΩ

8. Calculate R_T for each circuit of Figure 66.



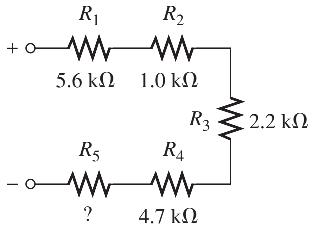
▲ FIGURE 66

9. What is the total resistance of twelve 5.6 kΩ resistors in series?

10. Six 56 Ω resistors, eight 100 Ω resistors, and two 22 Ω resistors are all connected in series. What is the total resistance?

11. If the total resistance in Figure 67 is 17.4 kΩ, what is the value of R_5?

▶ FIGURE 67

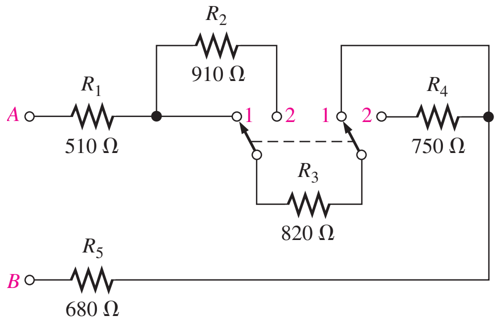


***12.** You have the following resistor values available to you in the lab in unlimited quantities: 10 Ω, 100 Ω, 470 Ω, 560 Ω, 680 Ω, 1.0 kΩ, 2.2 kΩ, and 5.6 kΩ. All of the other standard values are out of stock. A project that you are working on requires an 18 kΩ resistance. What combinations of the available values would you use in series to achieve this total resistance?

13. Find the total resistance in Figure 66 if all three circuits are connected in series.

14. What is the total resistance from *A* to *B* for each switch position in Figure 68?

▶ FIGURE 68



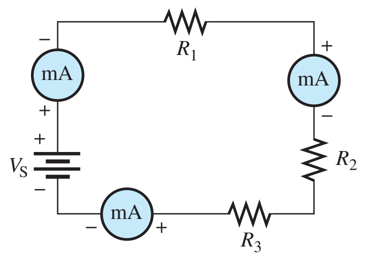
SECTION 3 Current in a Series Circuit

15. What is the current through each resistor in a series circuit if the total voltage is 12 V and the total resistance is 120 Ω?

16. The current from the source in Figure 69 is 5 mA. How much current does each milliammeter in the circuit indicate?

17. Show how to connect a voltage source and an ammeter to the PC board in Figure 64 to measure the current in R_1. Which other resistor currents are measured by this setup?

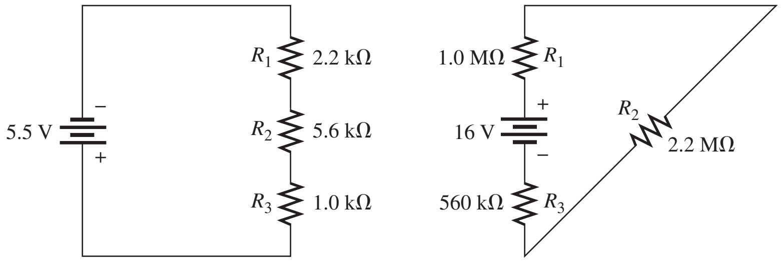
FIGURE 69


mA
mA
mA
R_1
R_2
R_3
V_S
+
−

*18. Using 1.5 V batteries, a switch, and three lamps, devise a circuit to apply 4.5 V across either one lamp, two lamps in series, or three lamps in series with a single-control switch. Draw the schematic.

SECTION 4 Application of Ohm's Law

19. What is the current in each circuit of Figure 70?


5.5 V
R_1 2.2 kΩ
R_2 5.6 kΩ
R_3 1.0 kΩ
1.0 MΩ R_1
16 V
R_2 2.2 MΩ
560 kΩ R_3
+
−

(a)

(b)

FIGURE 70

20. Determine the voltage drop across each resistor in Figure 70.

21. Three 470 Ω resistors are connected in series with a 48 V source.

- **(a)** What is the current is in the circuit?
- **(b)** What is the voltage across each resistor?
- **(c)** What is the minimum power rating of the resistors?

22. Four equal-value resistors are in series with a 5 V battery, and 2.23 mA are measured. What is the value of each resistor?

23. What is the value of each resistor in Figure 71?

24. Determine V_{R1}, R_2, and R_3 in Figure 72.

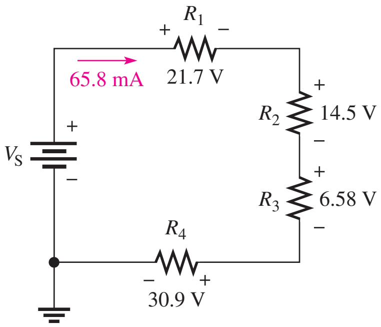

R_1
21.7 V
65.8 mA
R_2 14.5 V
R_3 6.58 V
R_4
30.9 V
V_S
+
−

FIGURE 71

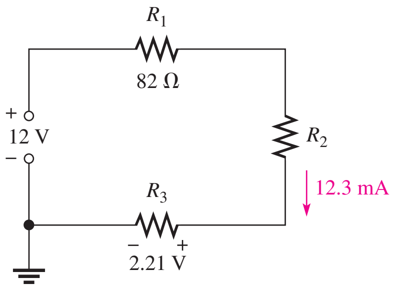

R_1
82 Ω
12 V
R_2
12.3 mA
R_3
2.21 V
+
−

FIGURE 72

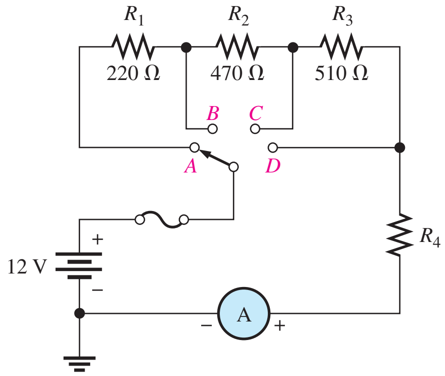
25. For the circuit in Figure 73 the meter reads 7.84 mA when the switch is in position *A*.

(a) What is the resistance of R_4?

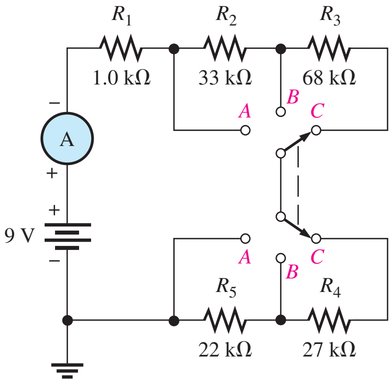
(b) What should be the meter reading for switch positions *B*, *C*, and *D*?

(c) Will a ¼ A fuse blow in any position of the switch?

26. Determine the current measured by the meter in Figure 74 for each position of the ganged switch.



▲ FIGURE 73



▲ FIGURE 74

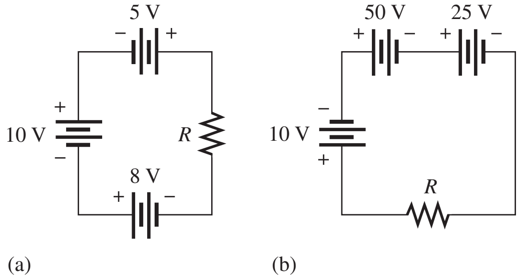
SECTION 5 Voltage Sources in Series

27. *Series aiding* is a term sometimes used to describe voltage sources of the same polarity in series. If a 5 V and a 9 V source are connected in this manner, what is the total voltage?

28. The term *series opposing* means that sources are in series with opposite polarities. If a 12 V and a 3 V battery are series opposing, what is the total voltage?

29. Determine the total source voltage in each circuit of Figure 75.

▶ FIGURE 75



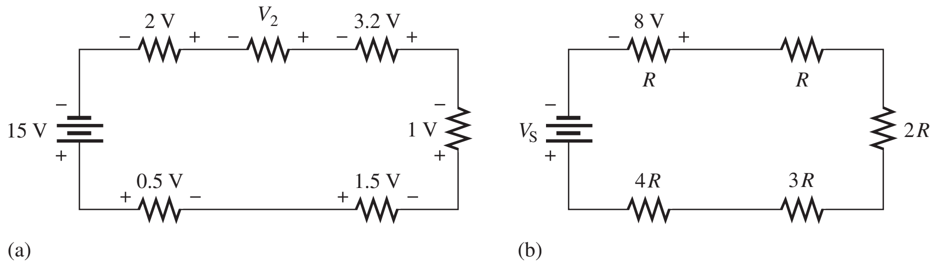
SECTION 6 Kirchhoff's Voltage Law

30. The following voltage drops are measured across three resistors in series: 5.5 V, 8.2 V, and 12.3 V. What is the value of the source voltage to which these resistors are connected?

31. Five resistors are in series with a 20 V source. The voltage drops across four of the resistors are 1.5 V, 5.5 V, 3 V, and 6 V. How much voltage is dropped across the fifth resistor?

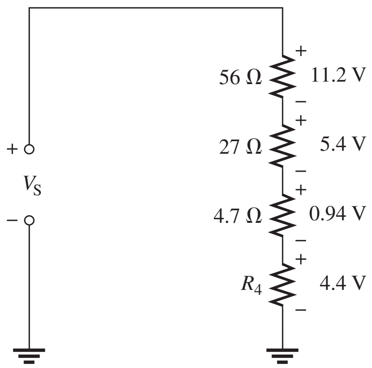
32. Determine the unspecified voltage drop(s) in each circuit of Figure 76. Show how to connect a voltmeter to measure each unknown voltage drop.

▶ FIGURE 76

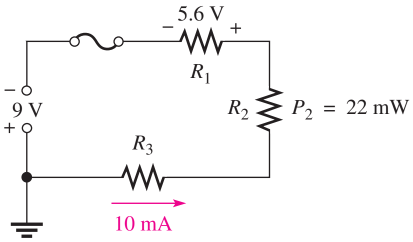


33. In the circuit of Figure 77, determine the resistance of R_4.

34. Find R_1, R_2, and R_3 in Figure 78.


56 Ω
11.2 V
27 Ω
5.4 V
4.7 Ω
0.94 V
R_4
4.4 V
V_S

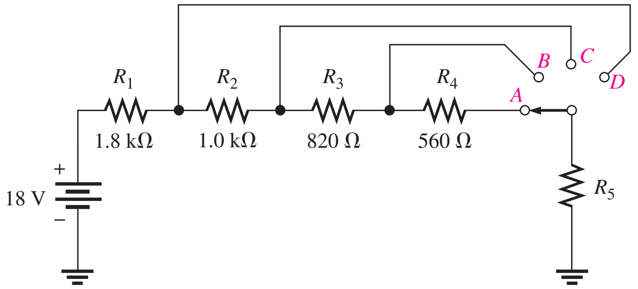
▲ FIGURE 77


5.6 V
R_1
R_2
P_2 = 22 mW
9 V
R_3
10 mA

▲ FIGURE 78

35. Determine the voltage across R_5 for each position of the switch in Figure 79. The current in each position is as follows: *A*, 3.35 mA; *B*, 3.73 mA; *C*, 4.50 mA; *D*, 6.00 mA.

36. Using the result of Problem 35, determine the voltage across each resistor in Figure 79 for each switch position.

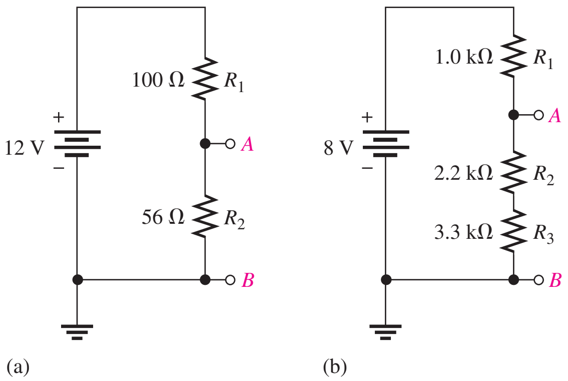

R_1
1.8 kΩ
R_2
1.0 kΩ
R_3
820 Ω
R_4
560 Ω
A
B
C
D
R_5
18 V

▲ FIGURE 79

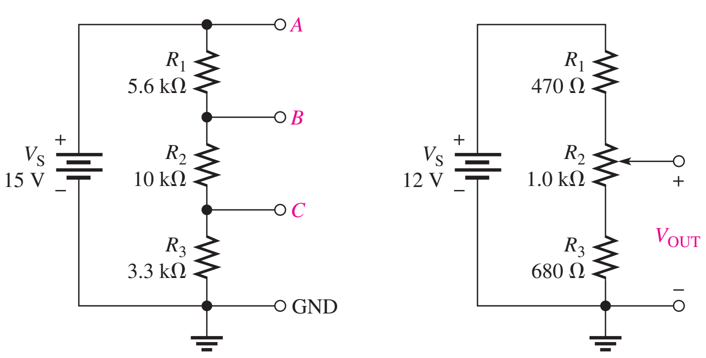
SECTION 7 Voltage Dividers

***37.** The total resistance of a circuit is 560 Ω. What percentage of the total voltage appears across a 27 Ω resistor that makes up part of the total series resistance?

38. Determine the voltage between points *A* and *B* in each voltage divider of Figure 80.


100 Ω
R_1
12 V
A
56 Ω
R_2
B
(a)
1.0 kΩ
R_1
A
8 V
2.2 kΩ
R_2
3.3 kΩ
R_3
B
(b)

▲ FIGURE 80



(a) (b)

▲ **FIGURE 81**

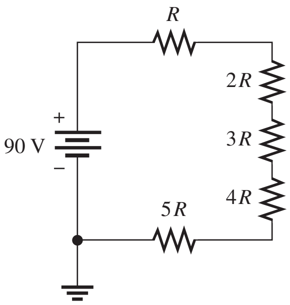
39. Determine the voltage with respect to ground for output A, B, and C in Figure 81(a).

40. Determine the minimum and maximum voltage from the voltage divider in Figure 81(b).

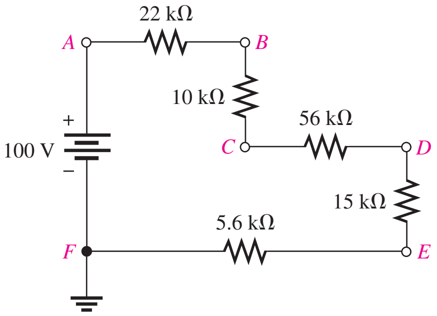
***41.** What is the voltage across each resistor in Figure 82? R is the lowest-value resistor, and all others are multiples of that value as indicated.

42. Determine the voltage at each point in Figure 83 with respect to the negative side of the battery.

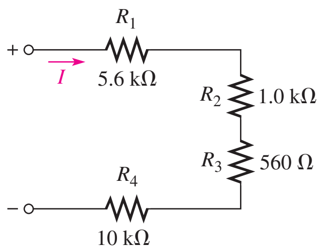
43. If there are 10 V across R_1 in Figure 84, what is the voltage across each of the other resistors?



▲ **FIGURE 82**



▲ **FIGURE 83**



▲ **FIGURE 84**

***44.** With the table of standard resistor values given in Appendix A, design a voltage divider to provide the following approximate voltages with respect to ground using a 30 V source: 8.18 V, 14.7 V, and 24.6 V. The current drain on the source must be limited to no more than 1 mA. The number of resistors, their values, and their wattage ratings must be specified. A schematic showing the circuit arrangement and resistor placement must be provided.

***45.** Design a variable voltage divider to provide an output voltage adjustable from a minimum of 10 V to a maximum of 100 V within ±1% using a 1 to 120 V source. The maximum voltage must occur at the maximum resistance setting of the potentiometer, and the minimum voltage must occur at the minimum resistance (zero) setting. The current is to be 10 mA.

SECTION 8 Power in Series Circuits

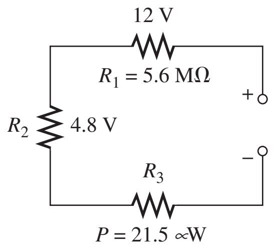
46. Five series resistors each handle 50 mW. What is the total power?

47. What is the total power in the circuit in Figure 84? Use the results of Problem 43.

48. The following ¼ W resistors are in series: 1.2 kΩ, 2.2 kΩ, 3.9 kΩ, and 5.6 kΩ. What is the maximum voltage that can be applied across the series resistors without exceeding a power rating? Which resistor will burn out first if excessive voltage is applied?

49. Find R_T in Figure 85.

FIGURE 85


12 V
R_1 = 5.6 MΩ
+
R_2
4.8 V
–
R_3
P = 21.5 ∝W

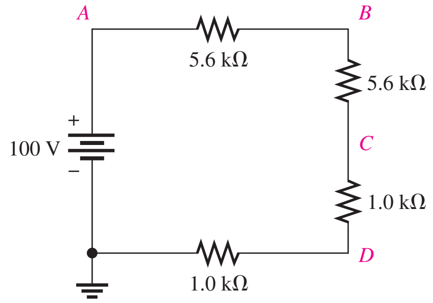
50. A certain series circuit consists of a ⅛ W resistor, a ¼ W resistor and a ½ W resistor. The total resistance is 2400 Ω. If each of the resistors is operating in the circuit at its maximum power dissipation, determine the following:

(a) I **(b)** V_T **(c)** The value of each resistor

SECTION 9 Voltage Measurements

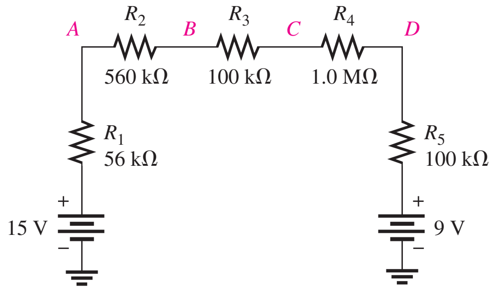
51. Determine the voltage at each point with respect to ground in Figure 86.

FIGURE 86


A
B
5.6 kΩ
5.6 kΩ
+
100 V
–
C
1.0 kΩ
D
1.0 kΩ

52. In Figure 87, how would you determine the voltage across R_2 by measuring, without connecting a meter directly across the resistor?

FIGURE 87


A
R_2
B
R_3
C
R_4
D
560 kΩ
100 kΩ
1.0 MΩ
R_1
56 kΩ
R_5
100 kΩ
+
15 V
–
+
9 V
–

53. Determine the voltage at each point with respect to ground in Figure 87.

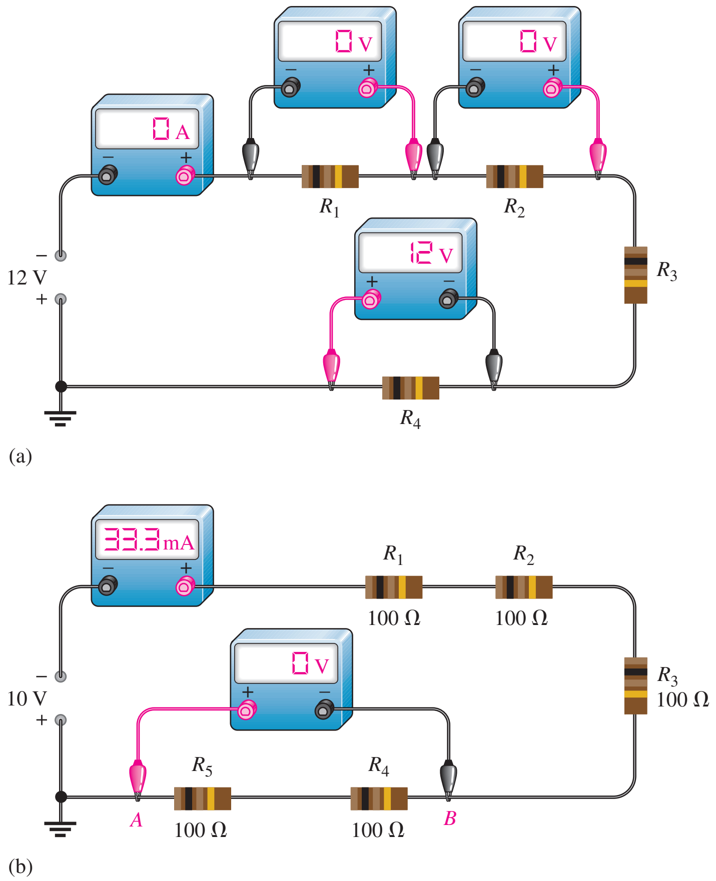
54. In Figure 87, what is V_{AC}?

55. In Figure 87, what is V_{CA}?

SECTION 10 Troubleshooting

56. A string of five series resistors is connected across a 12 V battery. Zero volts is measured across all of the resistors except R_2. What is wrong with the circuit? What voltage will be measured across R_2?

FIGURE 88


0 A
0 V
0 V
12 V
R1
R2
R3
R4
−
12 V
+
(a)
33.3 mA
R1
R2
100 Ω
100 Ω
R3
100 Ω
0 V
−
10 V
+
R5
R4
A
100 Ω
100 Ω
B
(b)

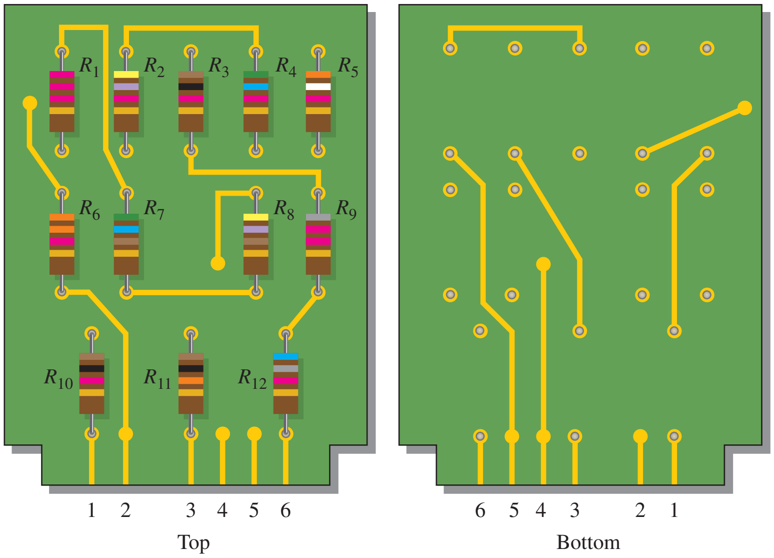
57. By observing the meters in Figure 88, determine the types of failures in the circuits and which components have failed.

58. What current would you measure in Figure 88(b) if only R_2 were shorted?

***59.** Table 1 shows the results of resistance measurements on the PC board in Figure 89. Are these results correct? If not, identify the possible problems.

TABLE 1

BETWEEN PINS	RESISTANCE
1 and 2	∞
1 and 3	∞
1 and 4	4.23 kΩ
1 and 5	∞
1 and 6	∞
2 and 3	23.6 kΩ
2 and 4	∞
2 and 5	∞
2 and 6	∞
3 and 4	∞
3 and 5	∞
3 and 6	∞
4 and 5	∞
4 and 6	∞
5 and 6	19.9 kΩ



▲ FIGURE 89

***60.** You measure 15 kΩ between pins 5 and 6 on the PC board in Figure 89. Does this indicate a problem? If so, identify it.

***61.** In checking out the PC board in Figure 89, you measure 17.83 kΩ between pins 1 and 2. Also, you measure 13.6 kΩ between pins 2 and 4. Does this indicate a problem on the PC board? If so, identify the fault.

***62.** The three groups of series resistors on the PC board in Figure 89 are connected in series with each other to form a single series circuit by connecting pin 2 to pin 4 and pin 3 to pin 5. A voltage source is connected across pins 1 and 6 and an ammeter is placed in series. As you increase the source voltage, you observe the corresponding increase in current. Suddenly, the current drops to zero and you smell smoke. All resistors are ½ W.

(a) What has happened?

(b) Specifically, what must you do to fix the problem?

(c) At what voltage did the failure occur?

Multisim Troubleshooting and Analysis

These problems require Multisim.

63. Open file P05-63 and measure the total series resistance.

64. Open file P05-64 and determine by measurement if there is an open resistor and, if so, which one.

65. Open file P05-65 and determine the unspecified resistance value.

66. Open file P05-66 and determine the unspecified source voltage.

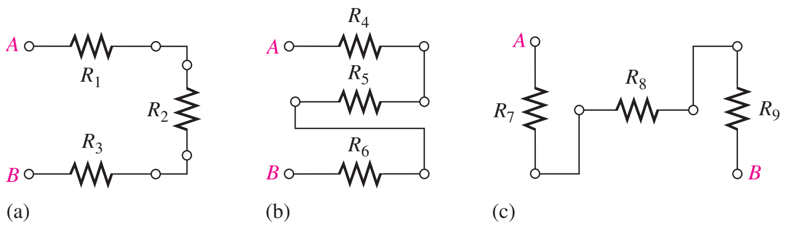
67. Open file P05-67 and find the shorted resistor if there is one.

ANSWERS

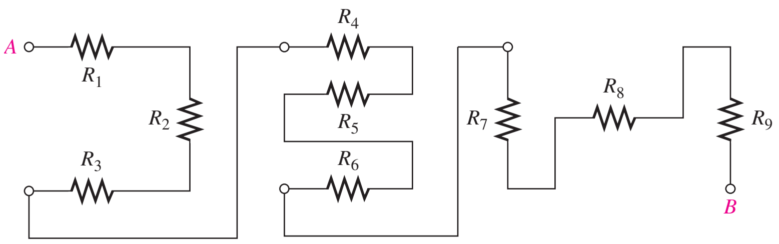
SECTION CHECKUPS

SECTION 1 Resistors in Series

1. Series resistors are connected end-to-end in a "string" with each lead of a given resistor connected to a different resistor.
2. There is a single current path in a series circuit.



▲ FIGURE 90



▲ FIGURE 91

3. See Figure 90.

4. See Figure 91.

SECTION 2 Total Series Resistance

1. $R_T = 1.0\ \Omega + 2.2\ \Omega + 3.3\ \Omega + 4.7\ \Omega = 11.2\ \Omega$

2. $R_T = 100\ \Omega + 2(56\ \Omega) + 4(12\ \Omega) + 330\ \Omega = 590\ \Omega$

3. $R_5 = 13.8\ \text{k}\Omega - (1.0\ \text{k}\Omega + 2.7\ \text{k}\Omega + 5.6\ \text{k}\Omega + 560\ \Omega) = 3.94\ \text{k}\Omega$

4. $R_T = 12(56\ \Omega) = 672\ \Omega$

5. $R_T = 20(5.6\ \text{k}\Omega) + 30(8.2\ \text{k}\Omega) = 358\ \text{k}\Omega$

SECTION 3 Current in a Series Circuit

1. $I = 20\ \text{mA}$

2. The milliammeter measures 50 mA between *C* and *D* and 50 mA between *E* and *F*.

3. $I = 10\ \text{V}/560\ \Omega = 17.9\ \text{mA}$; 17.9 mA

4. In a series circuit, current is the same at all points.

SECTION 4 Application of Ohm's Law

1. $I = 6\ \text{V}/300\ \Omega = 20\ \text{mA}$

2. $V_S = (50\ \text{mA})(21.2\ \Omega) = 1.06\ \text{V}$

3. $V_1 = (50\ \text{mA})(10\ \Omega) = 0.5\ \text{V}$; $V_2 = (50\ \text{mA})(5.6\ \Omega) = 0.28\ \text{V}$;
$V_3 = (50\ \text{mA})(5.6\ \Omega) = 0.28\ \text{V}$

4. $R = \frac{1}{4}\,(5\ \text{V}/4.63\ \text{mA}) = 270\ \Omega$

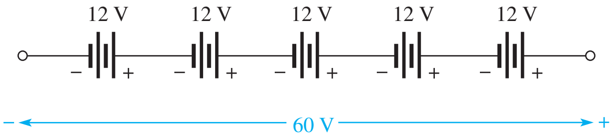
5. $R = 130\ \Omega$

SECTION 5 Voltage Sources in Series

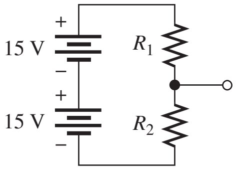
1. $V_T = 4(1.5\ \text{V}) = 6.0\ \text{V}$

2. 60 V/12 V = 5; see Figure 92.

▶ FIGURE 92



▶ FIGURE 93



3. See Figure 93.
4. $V_{S(tot)} = 9\text{ V} + 6\text{ V} + 15\text{ V} = 30\text{ V}$
5. 3.0 V

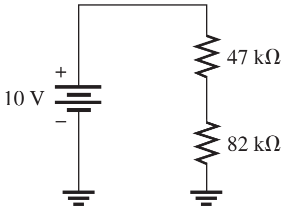
SECTION 6 Kirchhoff's Voltage Law

1. **(a)** Kirchhoff's law states the algebraic sum of the voltages around a closed path is zero;
   (b) Kirchhoff's law states the sum of the voltage drops equals the total source voltage.
2. $V_T = V_S = 50\text{ V}$
3. $V_1 = V_2 = 5\text{ V}$
4. $V_3 = 25\text{ V} - 10\text{ V} - 5\text{ V} = 10\text{ V}$
5. $V_S = 1\text{ V} + 3\text{ V} + 5\text{ V} + 8\text{ V} + 7\text{ V} = 24\text{ V}$

SECTION 7 Voltage Dividers

1. A voltage divider is a circuit with two or more series resistors in which the voltage taken across any resistor or combination of resistors is proportional to the value of that resistance.
2. Two or more resistors form a voltage divider.
3. $V_x = (R_x/R_T)V_S$
4. $V_R = 10\text{ V}/2 = 5\text{ V}$
5. $V_{47} = (47\text{ k}\Omega/129\text{ k}\Omega)10\text{ V} = 3.64\text{ V}$; $V_{82} = (82\text{ k}\Omega/129\text{ k}\Omega)10\text{ V} = 6.36\text{ V}$; see Figure 94.
6. Set the wiper at the midpoint.

▶ FIGURE 94



SECTION 8 Power in Series Circuits

1. Add the power in each resistor to get total power.
2. $P_T = 2\text{ W} + 5\text{ W} + 1\text{ W} + 8\text{ W} = 16\text{ W}$
3. $P_T = (1\text{ A})^2(1110\ \Omega) = 1110\text{ W}$

SECTION 9 Voltage Measurements

1. The reference point in a circuit is called ground or common.
2. True
3. True
4. $V_{BA} = -5.0\text{ V}$

SECTION 10 Troubleshooting

1. A short is a zero resistance path that bypasses a portion of a circuit.
2. An open is a break in the current path.

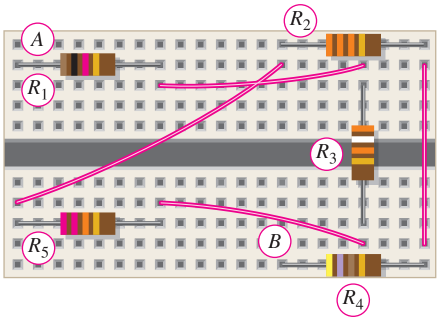
3. When a circuit opens, current ceases.
4. An open can be created by a switch or by a component failure. A short can be created by a switch or, unintentionally, by a wire clipping or solder splash.
5. True, a resistor normally fails open.
6. 24 V across the open *R*; 0 V across the other *R*s

RELATED PROBLEMS FOR EXAMPLES

1 **(a)** See Figure 95.

(b) $R_1 = 1.0\,\text{k}\Omega, R_2 = 33\,\text{k}\Omega, R_3 = 39\,\text{k}\Omega, R_4 = 470\,\Omega, R_5 = 22\,\text{k}\Omega$

▶ FIGURE 95



2 All resistors on the board are in series.

3 258 Ω

4 12.1 kΩ

5 6.8 kΩ

6 4440 Ω

7 18.0 mA

8 7.8 V

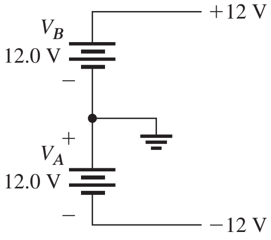
9 $V_1 = 1\text{ V}, V_2 = 3.3\text{ V}, V_3 = 2.2\text{ V}; V_S = 6.5\text{ V}; V_{S(\text{max})} = 32.5\text{ V}$

10 $R_2 = 275\ \Omega$

11 6 V

12 See Figure 96.

▶ FIGURE 96



13 10 V and 20 V; $V_{\text{fuse}} = V_S = 30\text{ V}; V_{R1} = V_{R2} = 0\text{ V}$

14 930 Ω

15 47 V

16 $V_1 = 3.57\text{ V}; V_2 = 6.43\text{ V}$

17 $V_1 = V_2 = V_3 = 3.33\text{ V}$

18 $V_{AB} = 4\text{ V}; V_{AC} = 36.8\text{ V}; V_{BC} = 32.8\text{ V}; V_{BD} = 46\text{ V}; V_{CD} = 13.2\text{ V}$

19
$$V_x = \left(\frac{R_x}{R_T}\right)V_S$$
$$\frac{V_x}{V_S} = \frac{R_x}{R_T} = \frac{R_x}{R + R_x}$$
$$\frac{1.5\text{ V}}{4.5\text{ V}} = \frac{90\text{ k}\Omega}{R + 90\text{ k}\Omega}$$
$$1.5\text{ V}(R + 90\text{ k}\Omega) = (4.5\text{ V})(90\text{ k}\Omega)$$
$$1.5R = 270\ \Omega$$
$$R = 180\ \Omega$$

20 8.49 W

21 $P_1 = 0.92$ W (1 W); $P_2 = 2.49$ W (5 W); $P_3 = 0.838$ W (1 W); $P_4 = 3.04$ W (5 W)

22 $V_A = 0$ V; $V_B = -25$ V; $V_C = -50$ V; $V_D = -75$ V; $V_E = -100$ V

23 3.33 V

TRUE/FALSE QUIZ

1. F **2.** F **3.** F **4.** T **5.** T
6. T **7.** F **8.** T **9.** T **10.** F

SELF-TEST

1. (a) **2.** (d) **3.** (b) **4.** (d) **5.** (d) **6.** (a) **7.** (b) **8.** (c)
9. (b) **10.** (c) **11.** (a) **12.** (d) **13.** (d) **14.** (d)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (b) **3.** (c) **4.** (a) **5.** (a) **6.** (b)
7. (a) **8.** (a) **9.** (a) **10.** (b) **11.** (b) **12.** (c)
13. (a) **14.** (a) **15.** (b)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. See Figure 97.

3. 170 kΩ

5. R_1, R_7, R_8, and R_{10} are in series.

R_2, R_4, R_6, and R_{11} are in series.

R_3, R_5, R_9, and R_{12} are in series.

7. **(a)** 1560 Ω **(b)** 103 Ω

(c) 13.7 kΩ **(d)** 3.671 MΩ

9. 67.2 kΩ

11. 3.9 kΩ

13. 17.8 MΩ

15. $I = 100$ mA

17. See Figure 98. The current through R_2, R_3, R_4, and R_9 is also measured by this set-up.

19. **(a)** 625 μA **(b)** 4.26 μA

21. **(a)** 34 mA **(b)** 16 V **(c)** 0.543 W

23. $R_1 = 330\ \Omega$, $R_2 = 220\ \Omega$, $R_3 = 100\ \Omega$, $R_4 = 470\ \Omega$

25. **(a)** 331 Ω

(b) Position *B:* 9.15 mA

Position *C:* 14.3 mA

Position *D:* 36.3 mA

(c) No

27. 14 V

29. **(a)** 23 V **(b)** 85 V

31. 4 V

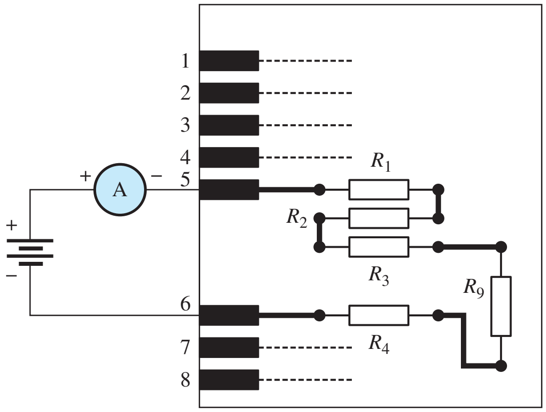
33. 22 Ω

35. Position *A:* 4.0 V

Position *B:* 4.5 V

Position *C:* 5.4 V

Position *D:* 7.2 V



▲ **FIGURE 98**

37. 4.82%

39. *A* output $= 15$ V

B output $= 10.6$ V

C output $= 2.62$ V

41. $V_R = 6$ V, $V_{2R} = 12$ V, $V_{3R} = 18$ V,

$V_{4R} = 24$ V, $V_{5R} = 30$ V

43. $V_2 = 1.79$ V, $V_3 = 1$ V, $V_4 = 17.9$ V

45. See Figure 99.

47. 54.9 mW

49. 12.5 MΩ

51. $V_A = 100$ V, $V_B = 57.7$ V, $V_C = 15.2$ V, $V_D = 7.58$ V

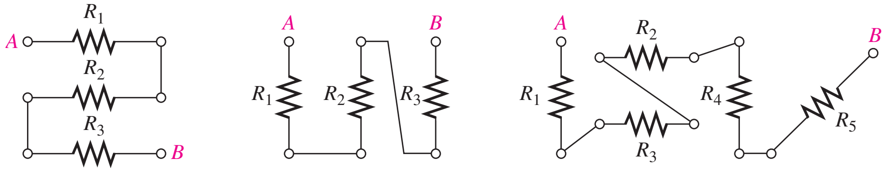
53. $V_A = 14.82$ V, $V_B = 12.97$ V, $V_C = 12.64$ V, $V_D = 9.34$ V

55. -2.18 V

57. **(a)** R_4 is open. **(b)** Short from *A* to *B*

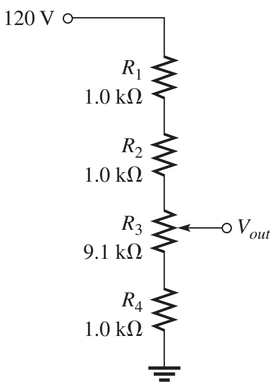
59. Table 1 is correct.

▶ **FIGURE 97**



(a) (b) (c)

FIGURE 99


120 V
R_1
1.0 kΩ
R_2
1.0 kΩ
R_3
9.1 kΩ
V_{out}
R_4
1.0 kΩ

61. Yes. There is a short between pin 4 and the upper side of R_{11}.

63. $R_T = 7.481\text{ k}\Omega$

65. $R_3 = 22\ \Omega$

67. R_1 shorted

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PARALLEL CIRCUITS

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PARALLEL CIRCUITS

CHAPTER OUTLINE

1 Resistors in Parallel
2 Voltage in a Parallel Circuit
3 Kirchhoff's Current Law
4 Total Parallel Resistance
5 Application of Ohm's Law
6 Current Sources in Parallel
7 Current Dividers
8 Power in Parallel Circuits
9 Parallel Circuit Applications
10 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Identify a parallel resistive circuit
- Determine the voltage across each branch in a parallel circuit
- Apply Kirchhoff's current law
- Determine total parallel resistance
- Apply Ohm's law in a parallel circuit
- Determine the total effect of current sources in parallel
- Use a parallel circuit as a current divider
- Determine power in a parallel circuit
- Describe some basic applications of parallel circuits
- Troubleshoot parallel circuits

KEY TERMS

- Branch
- Parallel
- Kirchhoff's current law
- Node
- Current divider

APPLICATION ACTIVITY PREVIEW

In this application activity, a panel-mounted power supply will be modified by adding a milliammeter to indicate current to a load. Expansion of the meter for multiple current ranges using parallel (shunt) resistors will be demonstrated. The problem with very low-value resistors when a switch is used to select the current ranges will be introduced and the effect of switch contact resistance will be demonstrated. A way of eliminating the contact resistance problem will be presented. Finally, the ammeter circuit will be installed in the power supply. The knowledge of parallel circuits and of basic ammeters that you will acquire in this chapter plus your understanding of Ohm's law and current dividers will be put to good use.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, you will see how Ohm's law is used in parallel circuits; and you will learn Kirchhoff's current law. Also, several applications of parallel circuits, including automotive lighting, residential wiring, control circuit, and the internal wiring of analog ammeters are presented. You will learn how to determine total parallel resistance and how to troubleshoot for open resistors.

When resistors are connected in parallel and a voltage is applied across the parallel circuit, each resistor provides a separate path for current. The total resistance of a parallel circuit is reduced as more resistors are connected in parallel. The voltage across each of the parallel resistors is equal to the voltage applied across the entire parallel circuit.

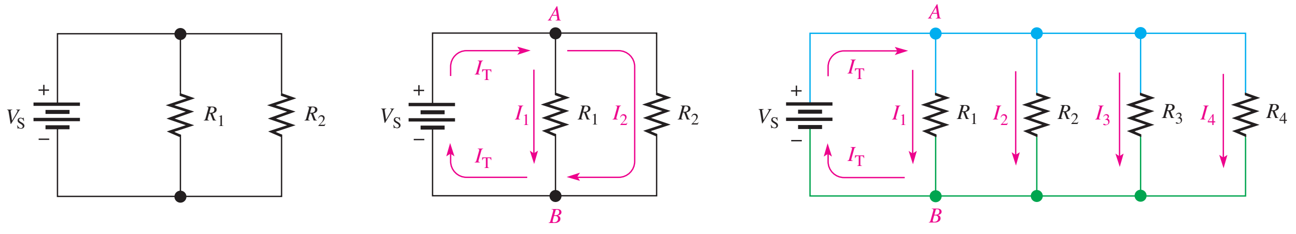
1 RESISTORS IN PARALLEL

When two or more resistors are individually connected between two separate points (nodes) in a circuit, they are in parallel with each other. A parallel circuit provides more than one path for current.

After completing this section, you should be able to

- **Identify a parallel resistive circuit**
    - Translate a physical arrangement of parallel resistors into a schematic

Each current path is called a **branch**, and a **parallel** circuit is one that has more than one branch. Two resistors connected in parallel are shown in Figure 1(a). As shown in part (b), the current out of the source (I_T) divides when it gets to point *A*. I_1 goes through R_1 and I_2 goes through R_2. If additional resistors are connected in parallel with the first two, more current paths are provided between point *A* and point *B*, as shown in Figure 1(c). All points along the top shown in blue are electrically the same as point *A*, and all points along the bottom shown in green are electrically the same as point *B*.



(a) (b) (c)

▲ FIGURE 1

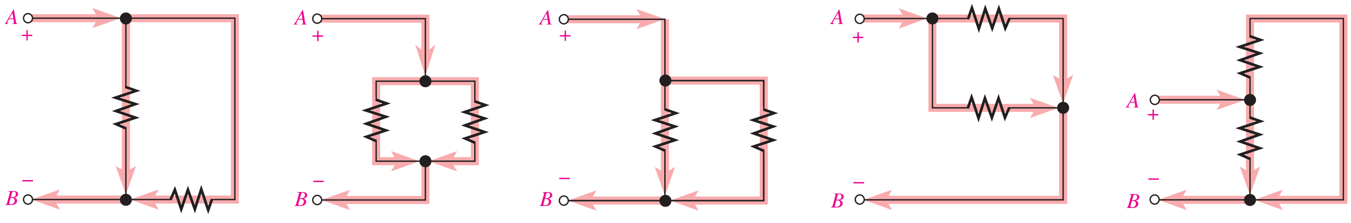
Resistors in parallel.

In Figure 1, it is obvious that the resistors are connected in parallel. Often, in actual circuit diagrams, the parallel relationship is not as clear. It is important that you learn to recognize parallel circuits regardless of how they may be drawn.

A rule for identifying parallel circuits is as follows:

If there is more than one current path (branch) between two separate points and if the voltage between those two points also appears across each of the branches, then there is a parallel circuit between those two points.

Figure 2 shows parallel resistors drawn in different ways between two separate points labeled *A* and *B*. Notice that in each case, the current has two paths going from *A* to *B*, and



(a) (b) (c) (d) (e)

▲ FIGURE 2

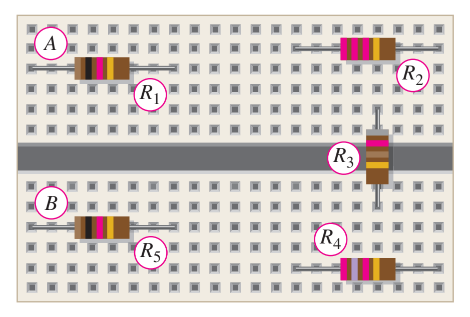
Examples of circuits with two parallel paths.

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

the voltage across each branch is the same. Although these examples show only two parallel paths, there can be any number of resistors in parallel.

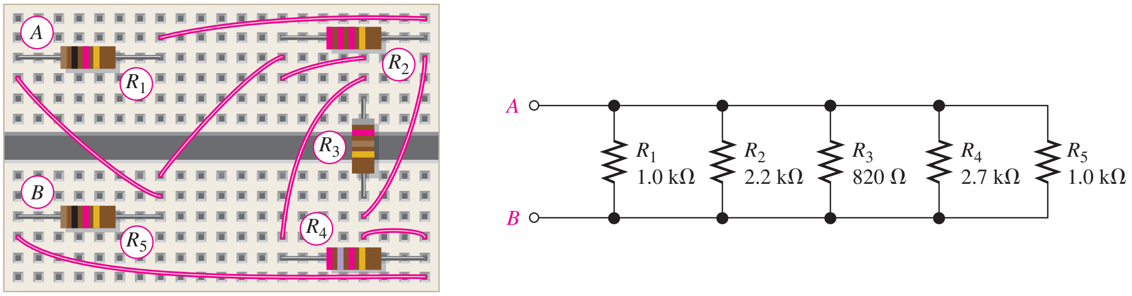
EXAMPLE 1

Five resistors are positioned on a protoboard as shown in Figure 3. Show the wiring required to connect all of the resistors in parallel between *A* and *B*. Draw a schematic and label each of the resistors with its value.



▲ FIGURE 3

Solution Wires are connected as shown in the assembly diagram of Figure 4(a). The schematic is shown in Figure 4(b). Again, note that the schematic does not necessarily have to show the actual physical arrangement of the resistors. The schematic shows how components are connected electrically.



(a) Assembly wiring diagram

(b) Schematic

▲ FIGURE 4

*Related Problem** How would the circuit have to be rewired if R_2 is removed?

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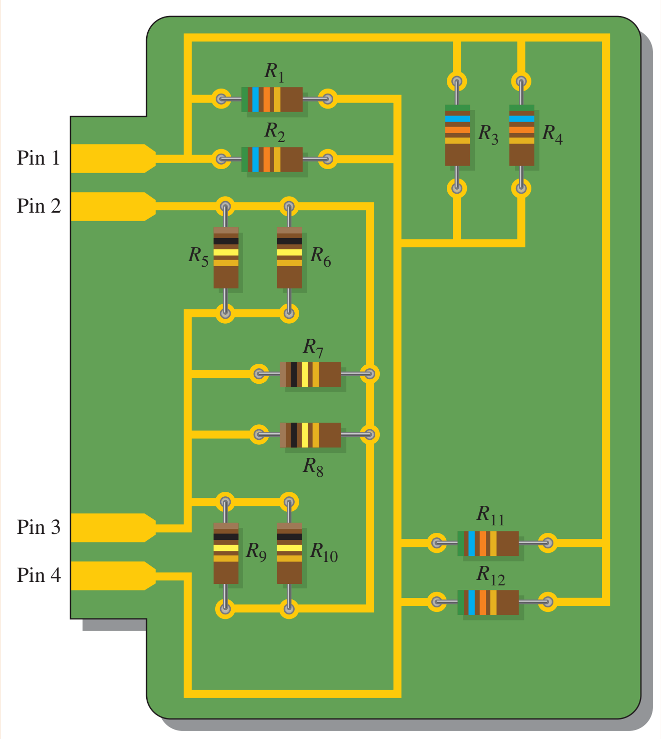
*Answers are at the end of the chapter.

EXAMPLE 2 Determine the parallel groupings in Figure 5 and the value of each resistor.



▲ FIGURE 5

Solution Resistors R_1 through R_4 and R_{11} and R_{12} are all in parallel. This parallel combination is connected to pins 1 and 4. Each resistor in this group is 56 kΩ.

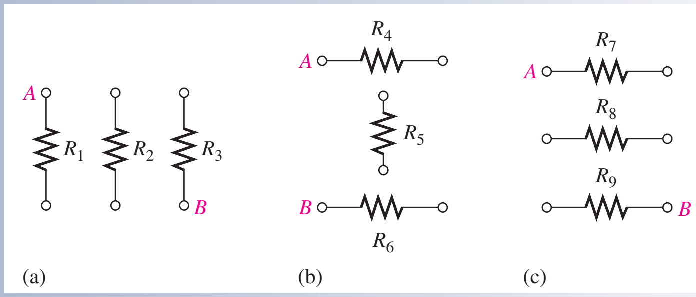
Resistors R_5 through R_{10} are all in parallel. This combination is connected to pins 2 and 3. Each resistor in this group is 100 kΩ.

Related Problem How would you connect all of the resistors in Figure 5 in parallel?

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. How are the resistors connected in a parallel circuit?
2. How do you identify a parallel circuit?
3. Complete the schematics for the circuits in each part of Figure 6 by connecting the resistors in parallel between points *A* and *B*.

4. Connect each group of parallel resistors in Figure 6 in parallel with each other.



▲ FIGURE 6

2 VOLTAGE IN A PARALLEL CIRCUIT

The voltage across any given branch of a parallel circuit is equal to the voltage across each of the other branches in parallel. As you know, each current path in a parallel circuit is called a *branch*.

After completing this section, you should be able to

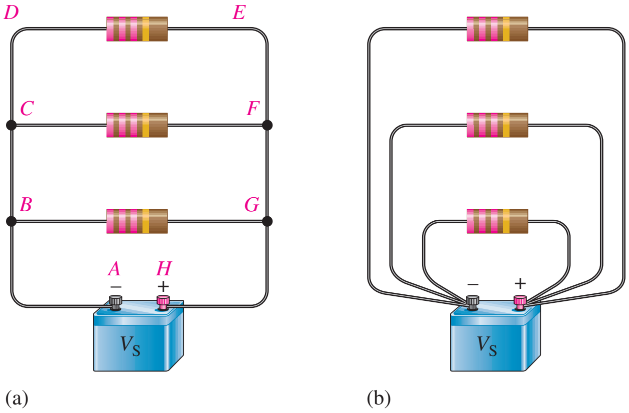
- **Determine the voltage across each branch in a parallel circuit**
  - Explain why the voltage is the same across all parallel resistors

To illustrate voltage in a parallel circuit, let's examine Figure 7(a). Points *A*, *B*, *C*, and *D* along the left side of the parallel circuit are electrically the same point because the voltage is the same along this line. You can think of all of these points as being connected by a single wire to the negative terminal of the battery. The points *E*, *F*, *G*, and *H* along the right side of the circuit are all at a voltage equal to that of the positive terminal of the source. Thus, voltage across each parallel resistor is the same, and each is equal to the source voltage. Note that the parallel circuit in Figure 7 resembles a ladder.

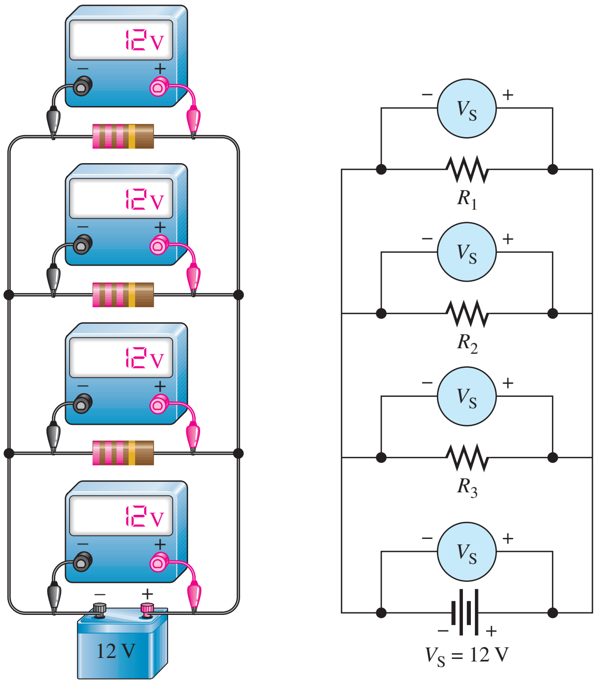
Figure 7(b) is the same circuit as in part (a), drawn in a slightly different way. Here the left side of each resistor is connected to a single point, which is the negative battery terminal. The right side of each resistor is connected to a single point, which is the positive battery terminal. The resistors are still all in parallel across the source.

▶ FIGURE 7

Voltage across parallel branches is the same.



In Figure 8, a 12 V battery is connected across three parallel resistors. When the voltage is measured across the battery and then across each of the resistors, the readings are the same. As you can see, the same voltage appears across each branch in a parallel circuit.



(a) Pictorial (b) Schematic

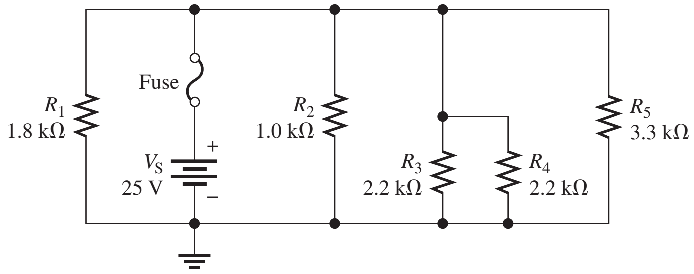
FIGURE 8

The same voltage appears across each resistor in parallel.

EXAMPLE 3

Determine the voltage across each resistor in Figure 9.

FIGURE 9



Solution The five resistors are in parallel, so the voltage across each one is equal to the applied source voltage. There is no voltage across the fuse. The voltage across the resistors is

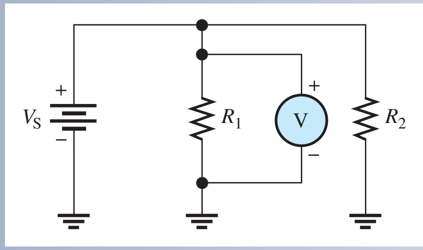
$$V_1 = V_2 = V_3 = V_4 = V_5 = V_S = \mathbf{25\ V}$$

Related Problem If R_4 is removed from the circuit, what is the voltage across R_3?

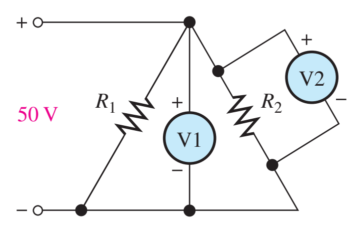
Use Multisim file E06-03 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 2 CHECKUP

1. A 10 Ω and a 22 Ω resistor are connected in parallel with a 5 V source. What is the voltage across each of the resistors?
2. A voltmeter is connected across R_1 in Figure 10. It measures 118 V. If you move the meter and connect it across R_2, how much voltage will it indicate? What is the source voltage?
3. In Figure 11, how much voltage does voltmeter 1 indicate? Voltmeter 2?
4. How are voltages across each branch of a parallel circuit related?



▲ FIGURE 10



▲ FIGURE 11

3 Kirchhoff's Current Law

Kirchhoff's voltage law deals with voltages in a single closed path. Kirchhoff's current law applies to currents in multiple paths.

After completing this section, you should be able to

- **Apply Kirchhoff's current law**
    - State Kirchhoff's current law
    - Define *node*
    - Determine the total current by adding the branch currents
    - Determine an unknown branch current

Kirchhoff's current law, often abbreviated KCL, can be stated as follows:

The sum of the currents into a node (total current in) is equal to the sum of the currents out of that node (total current out).

A **node** is any point or junction in a circuit where two or more components are connected. In a parallel circuit, a node or junction is a point where the parallel branches come together. For example, in the circuit of Figure 12, point *A* is one node and point *B* is another. Let's start at the positive terminal of the source and follow the current. The total current I_T from the source is *into* node *A*. At this point, the current splits up among the three branches as indicated. Each of the three branch currents (I_1, I_2, and I_3) is *out of* node *A*. Kirchhoff's current law says that the total current into node *A* is equal to the total current out of node *A*; that is,

$$I_T = I_1 + I_2 + I_3$$

Now, following the currents in Figure 12 through the three branches, you see that they come back together at node *B*. Currents I_1, I_2, and I_3 are into node *B*, and I_T is

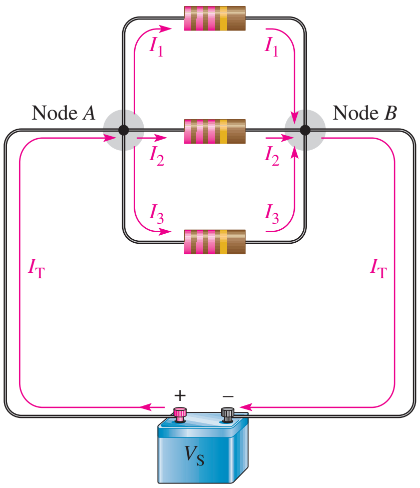


FIGURE 12

Kirchhoff's current law: The current into a node equals the current out of that node.

TECH NOTE

Kirchhoff's current law can be applied to anything that acts as a load for an electrical system. For example, the power connections to a circuit board will have the same current supplied to the board as leaving the board. Likewise, a building has the same current on the incoming "hot" line as the neutral. Normally, the supply current and return current are always the same.

out of node *B*. Kirchhoff's current law formula at node *B* is therefore the same as at node *A*.

$$I_T = I_1 + I_2 + I_3$$

Figure 13 illustrates the general case of Kirchhoff's current law and can be written as a mathematical relationship:

$$I_{IN(1)} + I_{IN(2)} + I_{IN(3)} + \cdots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + I_{OUT(3)} + \cdots + I_{OUT(m)}$$

Equation 1

Moving the terms on the right side to the left side and changing the sign results in the following equivalent equation:

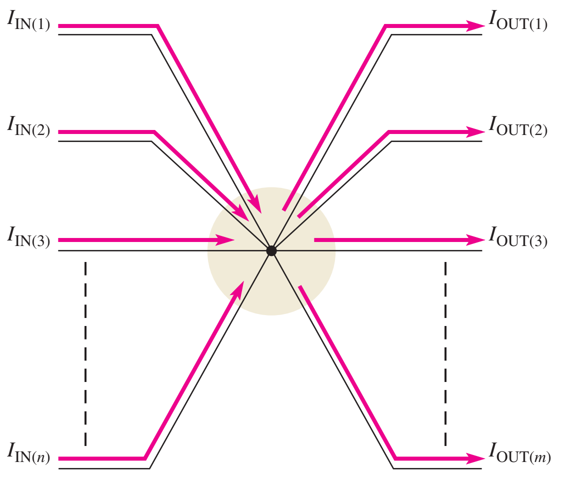
$$I_{IN(1)} + I_{IN(2)} + I_{IN(3)} + \cdots + I_{IN(n)} - I_{OUT(1)} - I_{OUT(2)} - I_{OUT(3)} - \cdots - I_{OUT(m)} = 0$$

This equation shows that all current into and out of the junction sums to zero and can be stated as

The algebraic sum of all of the currents entering and leaving a node is equal to zero.

An equivalent way of writing Kirchhoff's current law can be expressed using mathematical summation shorthand.

$$\sum_{i=1}^{n} I_i = 0$$



$$I_{IN(1)} + I_{IN(2)} + I_{IN(3)} + \cdot\ \cdot\ \cdot + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + I_{OUT(3)} + \cdot\ \cdot\ \cdot\ I_{OUT(m)}$$

FIGURE 13

Generalized circuit node illustrating Kirchhoff's current law.

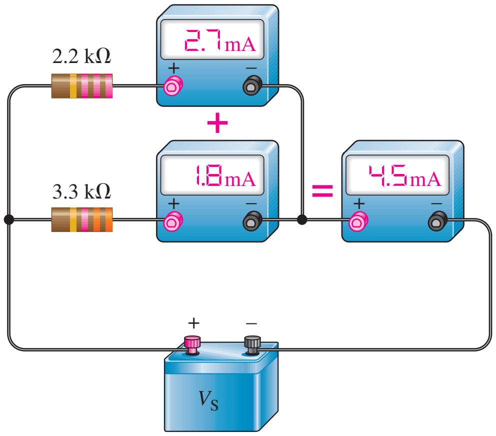
To express Kirchhoff's current law with this method, all currents are assigned a sequential subscript (1, 2, 3, and so on) no matter if the current is into or out of the node. Currents into the node are positive and currents leaving the node are negative.

You can verify Kirchhoff's current law by connecting a circuit and measuring each branch current and the total current from the source, as illustrated in Figure 14. When the branch currents are added together, their sum will equal the total current. This rule applies for any number of branches.

The following four examples illustrate use of Kirchhoff's current law.

▶ FIGURE 14

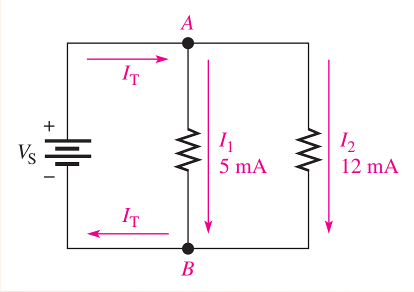
An illustration of Kirchhoff's current law.



EXAMPLE 4

The branch currents are shown in the circuit of Figure 15. Determine the total current entering node A and the total current leaving node B.

▶ FIGURE 15



Solution The total current out of node A is the sum of the two branch currents. So the total current into node A is

$$I_T = I_1 + I_2 = 5\text{ mA} + 12\text{ mA} = \mathbf{17\text{ mA}}$$

The total current entering node B is the sum of the two branch currents. So the total current out of node B is

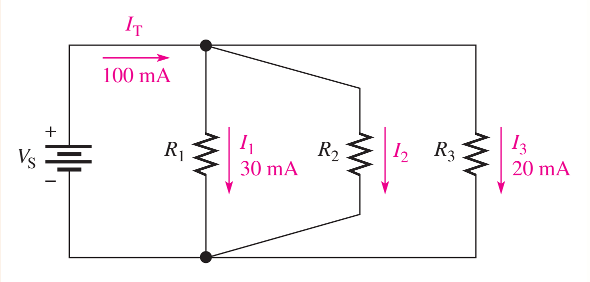
$$I_T = I_1 + I_2 = 5\text{ mA} + 12\text{ mA} = \mathbf{17\text{ mA}}$$

Related Problem If a third branch is added to the circuit in Figure 15 and its current is 3 mA, what is the total current into node A and out of node B?

EXAMPLE 5

Determine the current I_2 through R_2 in Figure 16.

▶ FIGURE 16



Solution The total current into the junction of the three branches is $I_T = I_1 + I_2 + I_3$. From Figure 16, you know the total current and the branch currents through R_1 and R_3. Solve for I_2 as follows:

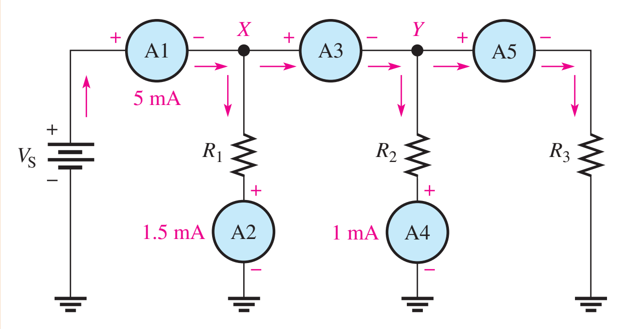
$$I_2 = I_T - I_1 - I_3 = 100\text{ mA} - 30\text{ mA} - 20\text{ mA} = \mathbf{50\text{ mA}}$$

Related Problem Determine I_T and I_2 if a fourth branch is added to the circuit in Figure 16 and it has 12 mA through it.

EXAMPLE 6

Use Kirchhoff's current law to find the current measured by ammeters A3 and A5 in Figure 17.

▶ FIGURE 17



Solution The total current into node X is 5 mA. Two currents are out of node X: 1.5 mA through resistor R_1 and the current through A3. Kirchhoff's current law applied at node X gives

$$5\text{ mA} - 1.5\text{ mA} - I_{A3} = 0$$

Solving for I_{A3} yields

$$I_{A3} = 5\text{ mA} - 1.5\text{ mA} = \mathbf{3.5\text{ mA}}$$

The total current into node Y is $I_{A3} = 3.5$ mA. Two currents are out of node Y: 1 mA through resistor R_2 and the current through A5 and R_3. Kirchhoff's current law applied at node Y gives

$$3.5\text{ mA} - 1\text{ mA} - I_{A5} = 0$$

Solving for I_{A5} yields

$$I_{A5} = 3.5\text{ mA} - 1\text{ mA} = \mathbf{2.5\text{ mA}}$$

Related Problem How much current will an ammeter measure when it is placed in the circuit right below R_3 in Figure 17? Below the negative battery terminal?

EXAMPLE 7

Using Kirchhoff's current law, find the current in each of two taillights given that the total current from the battery is 8.0 A and together the headlights draw 5.6 A.

Solution

$$I_{\text{BAT}} - I_{\text{T(HEAD)}} - I_{\text{T(TAIL)}} = 0$$

Rearranging and changing the signs,

$$I_{\text{T(TAIL)}} = I_{\text{BAT}} - I_{\text{T(HEAD)}} = 8.0\text{ A} - 5.6\text{ A} = 2.4\text{ A}$$

Because the taillights are identical, each draws 2.4 A/2 = **1.2 A**

Related Problem

When the brake lights are applied, the total current from the battery increases to +9.0 A. How much current does each brake light require?

SECTION 3 CHECKUP

1. State Kirchhoff's current law in two ways.
2. There is a total current of 2.5 mA into a node and the current out of the node divides into three parallel branches. What is the sum of all three branch currents?
3. 100 mA and 300 mA are into a node. What is the amount of current out of the node?
4. Each of two taillights on a trailer draws 1 A and the two brake lights each draw 1 A. What is the current in the ground lead to the trailer when all lights are on?
5. An underground pump has 10 A of current in the hot line. What should be the current on the neutral line?

4 TOTAL PARALLEL RESISTANCE

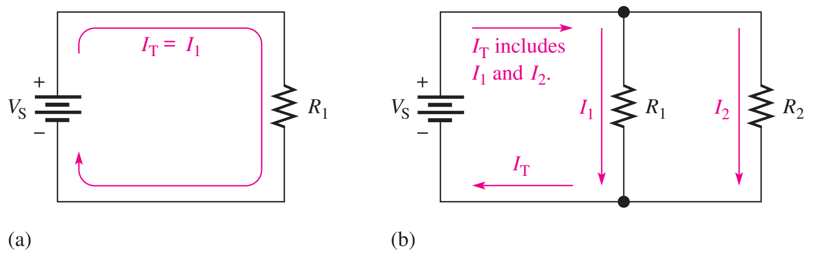
When resistors are connected in parallel, the total resistance of the circuit decreases. The total resistance of a parallel circuit is always less than the value of the smallest resistor. For example, if a 10 Ω resistor and a 100 Ω resistor are connected in parallel, the total resistance is less than 10 Ω.

After completing this section, you should be able to

- **Determine total parallel resistance**
    - Explain why resistance decreases as resistors are connected in parallel
    - Apply the parallel-resistance formula

As you know, when resistors are connected in parallel, the current has more than one path. The number of current paths is equal to the number of parallel branches.

In Figure 18(a), there is only one current path because it is a series circuit. There is a certain amount of current, I_1, through R_1. If resistor R_2 is connected in parallel with R_1, as shown in Figure 18(b), there is an additional amount of current, I_2, through R_2. The total current from the source has increased with the addition of the parallel resistor. Assuming that the source voltage is constant, an increase in the total current from the source means that the total resistance has decreased, in accordance with Ohm's law. Additional resistors connected in parallel will further reduce the resistance and increase the total current.



(a) (b)

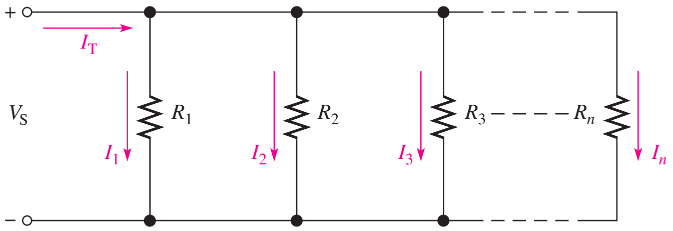
▲ **FIGURE 18**

Addition of resistors in parallel reduces total resistance and increases total current.

Formula for Total Parallel Resistance

The circuit in Figure 19 shows a general case of n resistors in parallel (n can be any integer greater than 1). From Kirchhoff's current law, the equation for current is

$$I_T = I_1 + I_2 + I_3 + \cdots + I_n$$



▲ **FIGURE 19**

Circuit with n resistors in parallel.

Since V_S is the voltage across each of the parallel resistors, by Ohm's law, $I_1 = V_S/R_1$, $I_2 = V_S/R_2$, and so on. By substitution into the equation for current,

$$\frac{V_S}{R_T} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} + \cdots + \frac{V_S}{R_n}$$

The term V_S can be factored out of the right side of the equation and canceled with V_S on the left side, leaving only the resistance terms.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Recall that the reciprocal of resistance ($1/R$) is called *conductance,* which is symbolized by G. The unit of conductance is the siemens (S). The equation for $1/R_T$ can be expressed in terms of conductance as

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

Solve for R_T by taking the reciprocal of (that is, by inverting) both sides of the equation for $1/R_T$.

Equation 2

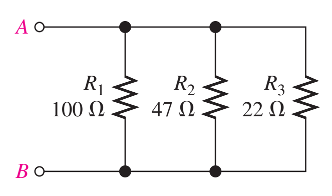
$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_n}\right)}$$

Equation 2 shows that to find the total parallel resistance, add all the $1/R$ (or conductance, G) terms and then take the reciprocal of the sum.

$$R_T = \frac{1}{G_T}$$

EXAMPLE 8 Calculate the total parallel resistance between points *A* and *B* of the circuit in Figure 20.

▶ **FIGURE 20**



Solution To calculate the total parallel resistance when you know the individual resistances, first find the conductance, which is the reciprocal of the resistance, of each of the three resistors.

$$G_1 = \frac{1}{R_1} = \frac{1}{100\ \Omega} = 10\text{ mS}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{47\ \Omega} = 21.3\text{ mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{22\ \Omega} = 45.5\text{ mS}$$

Next, calculate R_T by adding G_1, G_2, and G_3 and taking the reciprocal of the sum.

$$R_T = \frac{1}{G_T} = \frac{1}{10\text{ mS} + 21.3\text{ mS} + 45.5\text{ mS}} = \frac{1}{76.8\text{ mS}} = \mathbf{13.0\ \Omega}$$

For a quick accuracy check, notice that the value of R_T (13.0 Ω) is smaller than the smallest value in parallel, which is R_3 (22 Ω), as it should be.

Related Problem If a 33 Ω resistor is connected in parallel in Figure 20, what is the new value of R_T?

Calculator Tip

The parallel-resistance formula is easily solved on a calculator using Equation 2. The general procedure is to enter the value of R_1 and then take its reciprocal by pressing the x^{-1} key. (The reciprocal is a secondary function on some calculators.) Next press the + key; then enter the value of R_2 and take its reciprocal using the x^{-1} key and press the + key. Repeat this procedure until all of the resistor values have been entered; then press ENTER. The final step is to press the x^{-1} key and the ENTER key to get R_T. The total parallel resistance is now on the display. The display format may vary, depending on the particular calculator. For example, the steps required for a typical calculator solution of Example 7 are as follows:

1. Enter 100. Display shows 100.
2. Press x^{-1} (or 2nd then x^{-1}). Display shows 100^{-1}.
3. Press +. Display shows 100^{-1} + .
4. Enter 47. Display shows $100^{-1} + 47$.
5. Press x^{-1} (or 2nd then x^{-1}). Display shows $100^{-1} + 47^{-1}$.
6. Press +. Display shows $100^{-1} + 47^{-1}$ + .
7. Enter 22. Display shows $100^{-1} + 47^{-1} + 22$.

8. Press x^{-1} (or 2nd then x^{-1}). Display shows $100^{-1} + 47^{-1} + 22^{-1}$.
9. Press ENTER. Display shows a result of 76.7311411992E^{-3}.
10. Press x^{-1} (or 2nd then x^{-1}) and then ENTER. Display shows a result of 13.0325182758E0.

The number displayed in Step 10 is the total resistance in ohms. Round it to 13.0 Ω.

The Case of Two Resistors in Parallel

Equation 2 is a general formula for finding the total resistance for any number of resistors in parallel. The combination of two resistors in parallel occurs commonly in practice. Also, any number of resistors in parallel can be broken down into pairs as an alternate way to find the R_T. Based on Equation 2, the formula for the total resistance of two resistors in parallel is

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)}$$

Combining the terms in the denominator yields

$$R_T = \frac{1}{\left(\frac{R_1 + R_2}{R_1R_2}\right)}$$

which can be rewritten as follows:

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

Equation 3

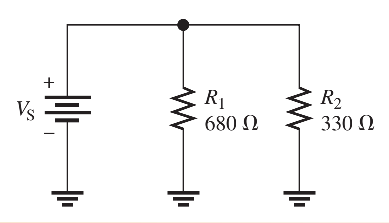
Equation 3 states

The total resistance for two resistors in parallel is equal to the product of the two resistors divided by the sum of the two resistors.

This equation is sometimes referred to as the "product over the sum" formula.

EXAMPLE 9

Calculate the total resistance connected to the voltage source of the circuit in Figure 21.



▲ FIGURE 21

Solution Use Equation 3.

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(680\ \Omega)(330\ \Omega)}{680\ \Omega + 330\ \Omega} = \frac{224{,}400\ \Omega^2}{1010\ \Omega} = \mathbf{222\ \Omega}$$

Related Problem Determine R_T if a 220 Ω replaces R_1 in Figure 21.

The Case of Equal-Value Resistors in Parallel

Another special case of parallel circuits is the parallel connection of several resistors each having the same resistance value. There is a shortcut method of calculating R_T when this case occurs.

If several resistors in parallel have the same resistance, they can be assigned the same symbol R. For example, $R_1 = R_2 = R_3 = \cdots = R_n = R$. Starting with Equation 2, you can develop a special formula for finding R_T.

$$R_T = \frac{1}{\left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \cdots + \left(\frac{1}{R}\right)}$$

Notice that in the denominator, the same term, $1/R$, is added n times (n is the number of equal-value resistors in parallel). Therefore, the formula can be written as

$$R_T = \frac{1}{n/R}$$

or

Equation 4

$$R_T = \frac{R}{n}$$

Equation 4 says that when any number of resistors (n), all having the same resistance (R), are connected in parallel, R_T is equal to the resistance divided by the number of resistors in parallel.

EXAMPLE 10

Four 8 Ω speakers are connected in parallel to the output of an amplifier. What is the total resistance across the output of the amplifier?

Solution There are four 8 Ω resistors in parallel. Use Equation 4 as follows:

$$R_T = \frac{R}{n} = \frac{8\ \Omega}{4} = \mathbf{2\ \Omega}$$

Related Problem If two of the speakers are removed, what is the resistance across the output?

Determining an Unknown Parallel Resistor

Sometimes you need to determine the values of resistors that are to be combined to produce a desired total resistance. For example, you use two parallel resistors to obtain a known total resistance. If you know or arbitrarily choose one resistor value, then you can calculate the second resistor value using Equation 3 for two parallel resistors. The formula for determining the value of an unknown resistor R_x is developed as follows:

$$\frac{1}{R_T} = \frac{1}{R_A} + \frac{1}{R_x}$$

$$\frac{1}{R_x} = \frac{1}{R_T} - \frac{1}{R_A}$$

$$\frac{1}{R_x} = \frac{R_A - R_T}{R_A R_T}$$

Equation 5

$$R_x = \frac{R_A R_T}{R_A - R_T}$$

where R_x is the unknown resistor and R_A is the known or selected value.

EXAMPLE 11

Suppose that you wish to obtain a resistance as close to 150 Ω as possible by combining two resistors in parallel. There is a 330 Ω resistor available. What other value do you need?

Solution $R_T = 150\ \Omega$ and $R_A = 330\ \Omega$. Therefore,

$$R_x = \frac{R_A R_T}{R_A - R_T} = \frac{(330\ \Omega)(150\ \Omega)}{330\ \Omega - 150\ \Omega} = 275\ \Omega$$

The closest standard value is **270 Ω**.

Related Problem If you need to obtain a total resistance of 130 Ω, what value can you add in parallel to the parallel combination of 330 Ω and 270 Ω? First find the value of 330 Ω and 270 Ω in parallel and treat that value as a single resistor.

Notation for Parallel Resistors

Sometimes, for convenience, parallel resistors are designated by two parallel vertical marks. For example, R_1 in parallel with R_2 can be written as $R_1 \| R_2$. Also, when several resistors are in parallel with each other, this notation can be used. For example,

$$R_1 \| R_2 \| R_3 \| R_4 \| R_5$$

indicates that R_1 through R_5 are all in parallel.

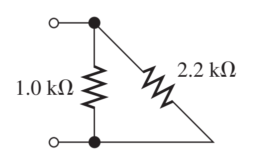
This notation is also used with resistance values. For example,

$$10\ \text{k}\Omega \| 5\ \text{k}\Omega$$

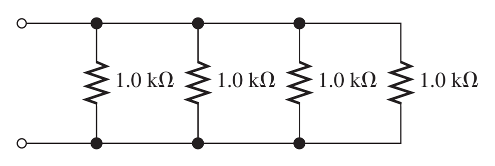
means that a 10 kΩ resistor is in parallel with a 5 kΩ resistor.

SECTION 4 CHECKUP

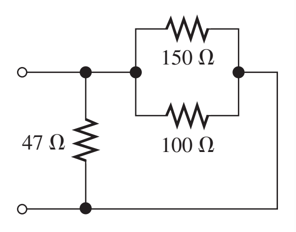
1. Does the total resistance increase or decrease as more resistors are connected in parallel?
2. The total parallel resistance is always less than what value?
3. Write the general formula for R_T with any number of resistors in parallel.
4. Write the special formula for two resistors in parallel.
5. Write the special formula for any number of equal-value resistors in parallel.
6. Calculate R_T for Figure 22.
7. Determine R_T for Figure 23.
8. Find R_T for Figure 24.



▲ FIGURE 22



▲ FIGURE 23



▲ FIGURE 24

5 APPLICATION OF OHM'S LAW

Ohm's law can be applied to parallel circuit analysis.

After completing this section, you should be able to

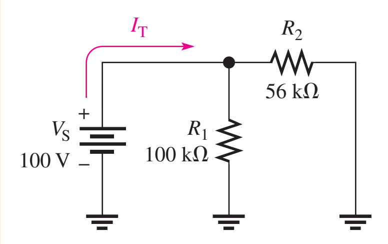
- **Apply Ohm's law in a parallel circuit**
    - Find the total current in a parallel circuit
    - Find each branch current in a parallel circuit
    - Find the voltage across a parallel circuit
    - Find the resistance of a parallel circuit

The following examples illustrate how to apply Ohm's law to determine the total current, branch currents, voltage, and resistance in parallel circuits.

EXAMPLE 12

Find the total current produced by the battery in Figure 25.

FIGURE 25



Solution The battery "sees" a total parallel resistance that determines the amount of current that it generates. First, calculate R_T.

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(100\text{ k}\Omega)(56\text{ k}\Omega)}{100\text{ k}\Omega + 56\text{ k}\Omega} = \frac{5600\text{ k}\Omega^2}{156\text{ k}\Omega} = 35.9\text{ k}\Omega$$

The battery voltage is 100 V. Use Ohm's law to find I_T.

$$I_T = \frac{V_S}{R_T} = \frac{100\text{ V}}{35.9\text{ k}\Omega} = \mathbf{2.79\text{ mA}}$$

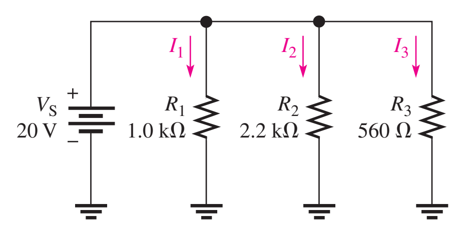
Related Problem What is I_T in Figure 25 if R_2 is changed to 120 kΩ? What is the current through R_1?

Use Multisim file E06-12 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 13

Determine the current through each resistor in the parallel circuit of Figure 26.

FIGURE 26



Solution The voltage across each resistor (branch) is equal to the source voltage. That is, the voltage across R_1 is 20 V, the voltage across R_2 is 20 V, and the voltage across R_3 is 20 V. Determine the current through each resistor as follows:

$$I_1 = \frac{V_S}{R_1} = \frac{20\text{ V}}{1.0\text{ k}\Omega} = \mathbf{20\text{ mA}}$$

$$I_2 = \frac{V_S}{R_2} = \frac{20\text{ V}}{2.2\text{ k}\Omega} = \mathbf{9.09\text{ mA}}$$

$$I_3 = \frac{V_S}{R_3} = \frac{20\text{ V}}{560\ \Omega} = \mathbf{35.7\text{ mA}}$$

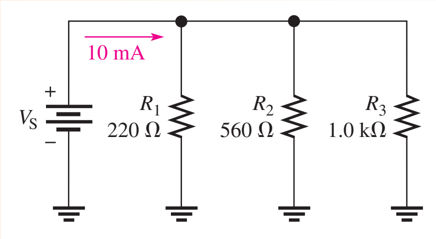
Related Problem If an additional resistor of 910 Ω is connected in parallel to the circuit in Figure 26, determine all of the branch currents.

Use Multisim file E06-13 to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 14

Find the voltage V_S across the parallel circuit in Figure 27.

▶ **FIGURE 27**



Solution The total current into the parallel circuit is 10 mA. If you know the total resistance, then you can apply Ohm's law to get the voltage. The total resistance is

$$R_T = \frac{1}{G_1 + G_2 + G_3}$$

$$= \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$= \frac{1}{\left(\frac{1}{220\ \Omega}\right) + \left(\frac{1}{560\ \Omega}\right) + \left(\frac{1}{1.0\text{ k}\Omega}\right)}$$

$$= \frac{1}{4.55\text{ mS} + 1.79\text{ mS} + 1\text{ mS}} = \frac{1}{7.34\text{ mS}} = 136\ \Omega$$

Therefore, the source voltage is

$$V_S = I_T R_T = (10\text{ mA})(136\ \Omega) = \mathbf{1.36\text{ V}}$$

Related Problem Find the voltage if R_3 is decreased to 680 Ω in Figure 27 and I_T is 10 mA.

Use Multisim file E06-14 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 15

Sometimes a direct measurement of resistance is not practical. For example, tungsten filament bulbs get hot after turning them on, causing the resistance to increase. An ohmmeter can only measure the cold resistance. Assume you want to know the equivalent hot resistance of the two headlight bulbs and the two taillight bulbs in a car. The two headlights normally operate on 12.6 V and draw 2.8 A each when they are on.

(a) What is the total equivalent hot resistance when two headlights are on?

(b) Assume the total current for headlights and taillights is 8.0 A when all four bulbs are on. What is the equivalent resistance of each taillight?

Solution

(a) Use Ohm's law to calculate the equivalent resistance of one headlight.

$$R_{\text{HEAD}} = \frac{V}{I} = \frac{12.6\ \text{V}}{2.8\ \text{A}} = 4.5\ \Omega$$

Because the two bulbs are in parallel and have equal resistances,

$$R_{\text{T(HEAD)}} = \frac{R_{\text{HEAD}}}{n} = \frac{4.5\ \Omega}{2} = \mathbf{2.25\ \Omega}$$

(b) Use Ohm's law to find the total resistance with two taillights and two headlights on.

$$R_{\text{T(HEAD+TAIL)}} = \frac{12.6\ \text{V}}{8.0\ \text{A}} = 1.58\ \Omega$$

Apply the formula for parallel resistors to find the resistance of the two taillights together.

$$\frac{1}{R_{\text{T(HEAD + TAIL)}}} = \frac{1}{R_{\text{T(HEAD)}}} + \frac{1}{R_{\text{T(TAIL)}}}$$

$$\frac{1}{R_{\text{T(TAIL)}}} = \frac{1}{R_{\text{T(HEAD + TAIL)}}} - \frac{1}{R_{\text{T(HEAD)}}} = \frac{1}{1.58\ \Omega} - \frac{1}{2.25\ \Omega}$$

$$R_{\text{T(TAIL)}} = 5.25\ \Omega$$

The two taillights are in parallel, so each light has a resistance of

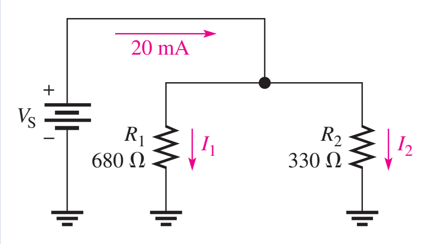
$$R_{\text{(TAIL)}} = nR_{\text{T(TAIL)}} = 2(5.25\ \Omega) = \mathbf{10.5\ \Omega}$$

Related Problem What is the total equivalent resistance of two headlights that each draw 3.15 A?

SECTION 5 CHECKUP

1. A 10 V battery is connected across three 680 Ω resistors that are in parallel. What is the total current from the battery?
2. How much voltage is required to produce 20 mA of current through the circuit of Figure 28?

▶ FIGURE 28



3. How much current is there through each resistor of Figure 28?
4. There are four equal-value resistors in parallel with a 12 V source, and 5.85 mA of current from the source. What is the value of each resistor?
5. A 1.0 kΩ and a 2.2 kΩ resistor are connected in parallel. There is a total of 100 mA through the parallel combination. How much voltage is dropped across the resistors?

6 CURRENT SOURCES IN PARALLEL

A **current source** is a type of energy source that provides a constant current to a load even if the resistance of that load changes. A transistor can be used as a current source; therefore, current sources are important in electronic circuits. Although the study of transistors is beyond the scope of this chapter, you should understand how current sources act in parallel.

After completing this section, you should be able to

- **Determine the total effect of current sources in parallel**
  - Determine the total current from parallel sources having the same direction
  - Determine the total current from parallel sources having opposite directions

In general, the total current produced by current sources in parallel is equal to the algebraic sum of the individual current sources. The algebraic sum means that you must consider the direction of current when you combine the sources in parallel. For example, in Figure 29(a), the three current sources in parallel provide current in the same direction (into node *A*). So the total current into node *A* is

$$I_T = 1\text{ A} + 2\text{ A} + 2\text{ A} = 5\text{ A}$$

In Figure 29(b), the 1 A source provides current in a direction opposite to the other two. The total current into node *A* in this case is

$$I_T = 2\text{ A} + 2\text{ A} - 1\text{ A} = 3\text{ A}$$

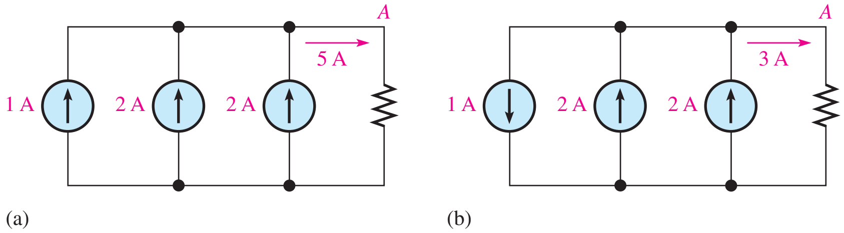
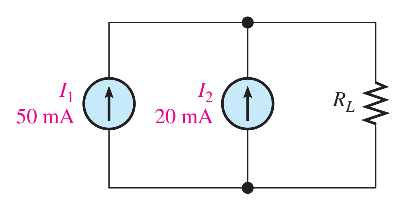


FIGURE 29

EXAMPLE 16

Determine the current through R_L in Figure 30.

FIGURE 30



Solution The two current sources are in the same direction; so the current through R_L is

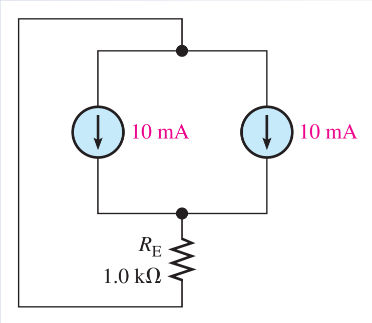
$$I_{R_L} = I_1 + I_2 = 50\text{ mA} + 20\text{ mA} = \mathbf{70\text{ mA}}$$

Related Problem Determine the current through R_L if the direction of I_2 is reversed.

SECTION 6 CHECKUP

1. Four 0.5 A current sources are connected in parallel in the same direction. What current will be produced through a load resistor?
2. How many 100 mA current sources must be connected in parallel to produce a total current output of 300 mA? Draw a schematic showing the sources connected.
3. In a certain transistor amplifier circuit, the transistor can be represented by a 10 mA current source, as shown in Figure 31. In a certain transistor amplifier, two transistors act in parallel. How much current is there through the resistor R_E?

▶ FIGURE 31



7 CURRENT DIVIDERS

A parallel circuit acts as a current divider because the current entering the junction of parallel branches "divides" up into several individual branch currents.

After completing this section, you should be able to

- **Use a parallel circuit as a current divider**
  - Apply the current-divider formula
  - Determine an unknown branch current

In a parallel circuit, the total current into the junction of the parallel branches divides among the branches. Thus, a parallel circuit acts as a **current divider**. This current-divider principle is illustrated in Figure 32 for a two-branch parallel circuit in which part of the total current I_T goes through R_1 and part through R_2.

Since the same voltage is across each of the resistors in parallel, the branch currents are inversely proportional to the values of the resistors. For example, if the value of R_2 is twice that of R_1, then the value of I_2 is one-half that of I_1. In other words,

> **The total current divides among parallel resistors into currents with values inversely proportional to the resistance values.**

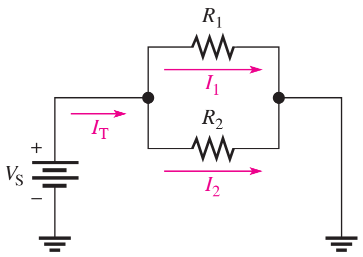


FIGURE 32

Total current divides between the two branches.

The branches with higher resistance have less current, and the branches with lower resistance have more current, in accordance with Ohm's law. If all the branches have the same resistance, the branch currents are all equal.

Figure 33 shows specific values to demonstrate how the currents divide according to the branch resistances. Notice that in this case the resistance of the upper branch is one-tenth the resistance of the lower branch, but the upper branch current is ten times the lower branch current.

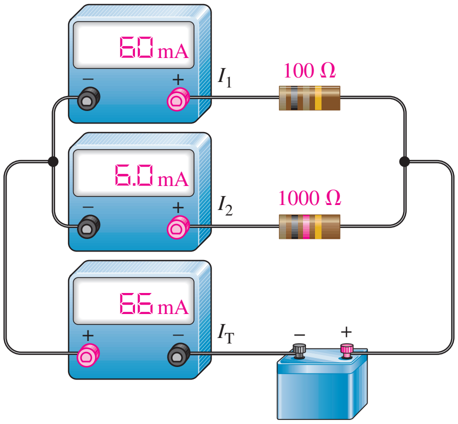


FIGURE 33

The branch with the lower resistance has more current, and the branch with the higher resistance has less current.

Current-Divider Formula

You can develop a formula for determining how currents divide among any number of parallel resistors as shown in Figure 34, where *n* is the total number of resistors.

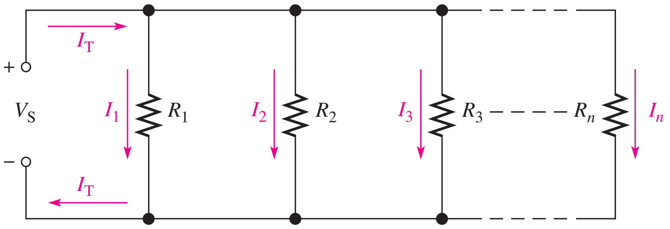


FIGURE 34

A parallel circuit with *n* branches.

The current through any one of the parallel resistors is I_x, where x represents the number of a particular resistor (1, 2, 3, and so on). By Ohm's law, you can express the current through any one of the resistors in Figure 34 as follows:

$$I_x = \frac{V_S}{R_x}$$

The source voltage, V_S, appears across each of the parallel resistors, and R_x represents any one of the parallel resistors. The total source voltage, V_S, is equal to the total current times the total parallel resistance.

$$V_S = I_T R_T$$

Substituting $I_T R_T$ for V_S in the expression for I_x results in

$$I_x = \frac{I_T R_T}{R_x}$$

Rearranging terms yields

Equation 6

$$I_x = \left(\frac{R_T}{R_x}\right) I_T$$

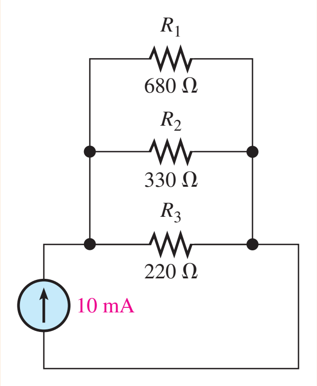
where $x = 1, 2, 3$, etc. Equation 6 is the general current-divider formula and applies to a parallel circuit with any number of branches.

> **The current (I_x) through any branch equals the total parallel resistance (R_T) divided by the resistance (R_x) of that branch, and then multiplied by the total current (I_T) into the junction of parallel branches.**

EXAMPLE 17

Determine the current through each resistor in the circuit of Figure 35.

▶ FIGURE 35



Solution First calculate the total parallel resistance.

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)} = \frac{1}{\left(\frac{1}{680\ \Omega}\right) + \left(\frac{1}{330\ \Omega}\right) + \left(\frac{1}{220\ \Omega}\right)} = 111\ \Omega$$

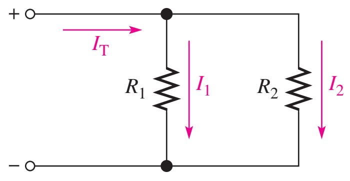
The total current is 10 mA. Use Equation 6 to calculate each branch current.

$$I_1 = \left(\frac{R_T}{R_1}\right) I_T = \left(\frac{111\ \Omega}{680\ \Omega}\right) 10\text{ mA} = \mathbf{1.63\ mA}$$

$$I_2 = \left(\frac{R_T}{R_2}\right) I_T = \left(\frac{111\ \Omega}{330\ \Omega}\right) 10\text{ mA} = \mathbf{3.36\ mA}$$

$$I_3 = \left(\frac{R_T}{R_3}\right) I_T = \left(\frac{111\ \Omega}{220\ \Omega}\right) 10\text{ mA} = \mathbf{5.05\ mA}$$

Related Problem Determine the current through each resistor in Figure 35 if R_3 is removed.



▲ FIGURE 36

Current-Divider Formulas for Two Branches Two parallel resistors are common in practical circuits, as shown in Figure 36. As you know from Equation 3,

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

Using the general current-divider formula in Equation 6, the formulas for I_1 and I_2 can be written as follows:

$$I_1 = \left(\frac{R_T}{R_1}\right)I_T \quad \text{and} \quad I_2 = \left(\frac{R_T}{R_2}\right)I_T$$

Substituting $R_1R_2/(R_1 + R_2)$ for R_T and canceling terms result in

$$I_1 = \frac{\left(\frac{\cancel{R_1}R_2}{R_1 + R_2}\right)}{\cancel{R_1}}I_T \quad \text{and} \quad I_2 = \frac{\left(\frac{R_1\cancel{R_2}}{R_1 + R_2}\right)}{\cancel{R_2}}I_T$$

Therefore, the current-divider formulas for the special case of two branches are

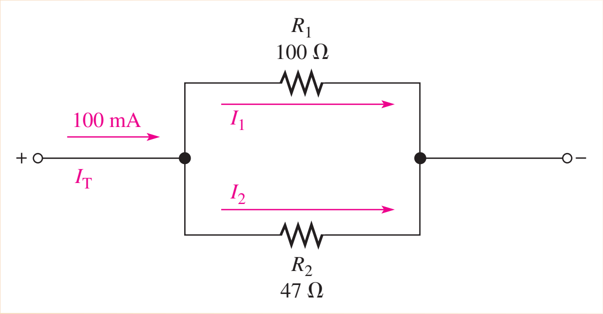
$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T \qquad \text{Equation 7}$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T \qquad \text{Equation 8}$$

Note that in Equations 7 and 8, the current in one of the branches is equal to the opposite branch resistance divided by the sum of the two resistors, all multiplied by the total current. In all applications of the current-divider equations, you must know the total current into the parallel branches.

EXAMPLE 18

Find I_1 and I_2 in Figure 37.



► FIGURE 37

Solution Use Equation 7 to determine I_1.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right)100\text{ mA} = \mathbf{32.0\ mA}$$

Use Equation 8 to determine I_2.

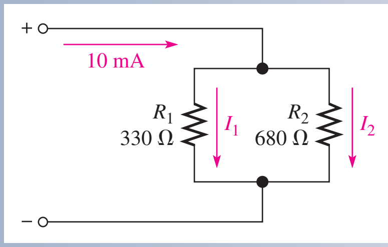
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T = \left(\frac{100\ \Omega}{147\ \Omega}\right)100\text{ mA} = \mathbf{68.0\ mA}$$

Related Problem If $R_1 = 56\ \Omega$, and $R_2 = 82\ \Omega$ in Figure 37 and I_T stays the same, what will each branch current be?

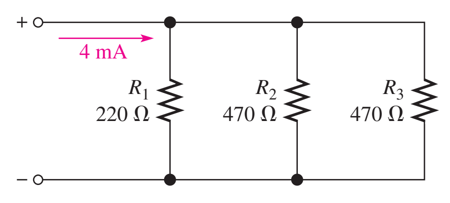
Calculate the source voltage for Figure 37. Open Multisim file E06-18 and add the voltage source to the circuit. Verify the calculated results.

SECTION 7 CHECKUP

1. Write the general current-divider formula.
2. Write the two special formulas for calculating each branch current for a two-branch circuit.
3. A circuit has the following resistors in parallel with a voltage source: 220 kΩ, 100 kΩ, 82 kΩ, 47 kΩ, and 22 kΩ. Which resistor has the most current through it? The least current?
4. Find I_1 and I_2 in the circuit of Figure 38.
5. Determine the current through R_3 in Figure 39.



▲ FIGURE 38



▲ FIGURE 39

8 POWER IN PARALLEL CIRCUITS

Total power in a parallel circuit is found by adding up the powers of all the individual resistors, the same as for series circuits.

After completing this section, you should be able to

- **Determine power in a parallel circuit**

Equation 9 states the formula for finding total power in a concise way for any number of resistors in parallel.

Equation 9

$$P_T = P_1 + P_2 + P_3 + \cdots + P_n$$

where P_T is the total power and P_n is the power in the last resistor in parallel. As you can see, the powers are additive, just as in a series circuit.

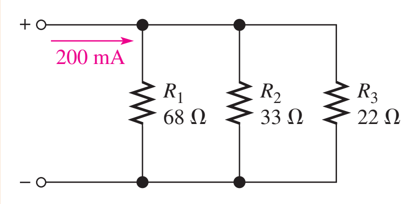
Power formulas are directly applicable to parallel circuits. The following formulas are used to calculate the total power P_T:

$$P_T = VI_T$$
$$P_T = I_T^2R_T$$
$$P_T = \frac{V^2}{R_T}$$

where V is the voltage across the parallel circuit, I_T is the total current into the parallel circuit, and R_T is the total resistance of the parallel circuit. Examples 19 and 20 show how total power can be calculated in a parallel circuit.

EXAMPLE 19

Determine the total amount of power in the parallel circuit in Figure 40.



▲ FIGURE 40

Solution The total current is 200 mA. The total resistance is

$$R_T = \frac{1}{\left(\frac{1}{68\ \Omega}\right) + \left(\frac{1}{33\ \Omega}\right) + \left(\frac{1}{22\ \Omega}\right)} = 11.1\ \Omega$$

The easiest power formula to use is $P_T = I_T^2 R_T$ because you know both I_T and R_T.

$$P_T = I_T^2 R_T = (200\text{ mA})^2(11.1\ \Omega) = \mathbf{444\ mW}$$

Let's demonstrate that if you determine the power in each resistor and if you add all of these values together, you will get the same result. First, find the voltage across each branch of the circuit.

$$V = I_T R_T = (200\text{ mA})(11.1\ \Omega) = 2.22\text{ V}$$

Remember that the voltage across all branches is the same.

Next, use $P = V^2/R$ to calculate the power for each resistor.

$$P_1 = \frac{(2.22\text{ V})^2}{68\ \Omega} = 72.5\text{ mW}$$

$$P_2 = \frac{(2.22\text{ V})^2}{33\ \Omega} = 149\text{ mW}$$

$$P_3 = \frac{(2.22\text{ V})^2}{22\ \Omega} = 224\text{ mW}$$

Add these powers to get the total power.

$$P_T = 72.5\text{ mW} + 149\text{ mW} + 224\text{ mW} = 446\text{ mW}$$

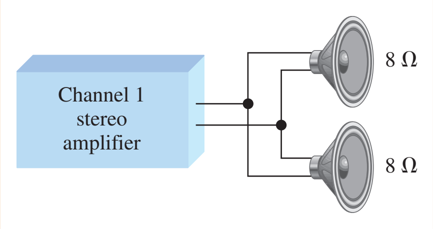
This calculation shows that the sum of the individual powers is equal (approximately) to the total power as determined by one of the power formulas. Rounding to three significant figures accounts for the difference.

Related Problem Find the total power in Figure 40 if the voltage is doubled.

EXAMPLE 20

The amplifier in one channel of a stereo system as shown in Figure 41 drives two speakers. If the maximum voltage* to the speakers is 15 V, how much power must the amplifier be able to deliver to the speakers?

▶ FIGURE 41



Solution The speakers are connected in parallel to the amplifier output, so the voltage across each is the same. The maximum power to each speaker is

$$P_{\text{max}} = \frac{V_{\text{max}}^2}{R} = \frac{(15\text{ V})^2}{8\ \Omega} = 28.1\text{ W}$$

The total power that the amplifier must be capable of delivering to the speaker system is twice the power in an individual speaker because the total power is the sum of the individual powers.

$$P_{\text{T(max)}} = P_{\text{(max)}} + P_{\text{(max)}} = 2P_{\text{(max)}} = 2(28.1\text{ W}) = \mathbf{56.2\text{ W}}$$

Related Problem If the amplifier can produce a maximum of 18 V, what is the maximum total power to the speakers?

---

*Voltage is ac in this case; but power is determined the same for ac voltage as for dc voltage, as you will see later.

SECTION 8 CHECKUP

1. If you know the power in each resistor in a parallel circuit, how can you find the total power?
2. The resistors in a parallel circuit dissipate the following powers: 238 mW, 512 mW, 109 mW, and 876 mW. What is the total power in the circuit?
3. A circuit has a 1.0 kΩ, a 2.7 kΩ, and a 3.9 kΩ resistor in parallel. There is a total current of 1 A into the parallel circuit. What is the total power?
4. Normally, circuits are protected with a fuse that is at least 120% of the maximum expected current. What fuse rating should be used for an automotive rear window defroster that is rated for 100 W? Assume $V = 12.6$ V.

9 PARALLEL CIRCUIT APPLICATIONS

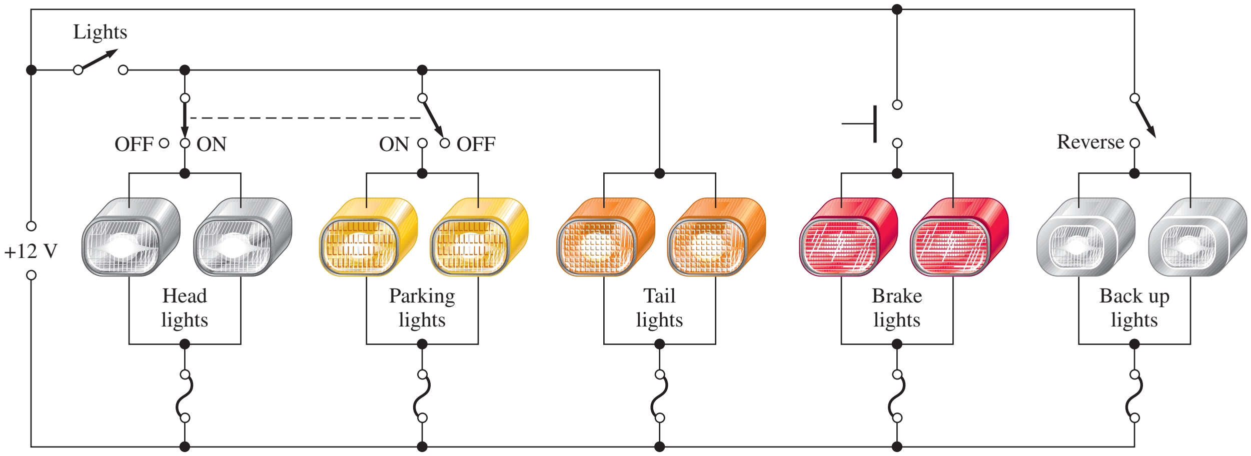
Parallel circuits are found in some form in virtually every electronic system. In many of these applications, the parallel relationship of components may not be obvious until you have covered some advanced topics that you will study later. For now, let's look at some examples of common and familiar applications of parallel circuits.

After completing this section, you should be able to

- **Describe some basic applications of parallel circuits**
  - Discuss the lighting system in automobiles
  - Discuss residential wiring
  - Explain basically how a multiple-range ammeter works

Automotive

One advantage of a parallel circuit over a series circuit is that when one branch opens, the other branches are not affected. For example, Figure 42 shows a simplified diagram of an automobile lighting system. When one headlight on a car goes out, it does not cause the other lights to go out because they are all in parallel.



▲ FIGURE 42

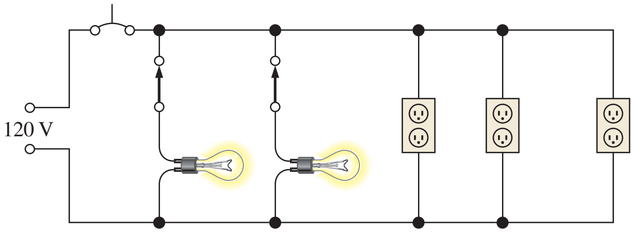
Simplified diagram of the exterior light system of an automobile.

Notice that the brake lights are switched on independently of the headlights and taillights. They come on only when the driver closes the brake light switch by depressing the brake pedal. When the lights switch is closed, both headlights and both taillights are on. When the headlights are on, the parking lights are off and vice versa, as indicated by the dashed line connecting the switches. If any one of the lights burns out (opens), there is still current in each of the other lights. The back-up lights are switched on when the reverse gear is engaged.

Another automotive application of parallel resistances is the rear window defroster. As you know, power is dissipated in resistance in the form of heat. The defroster consists of a group of parallel resistance wires that warm the glass when power is applied. A typical defroster can dissipate over 100 W across the window. Although resistance heating is not as efficient as other forms of heat, in applications like this it is simple and cost effective.

Residential

Another common use of parallel circuits is in residential electrical systems. All the lights and appliances in a home are wired in parallel. Figure 43 shows a typical room wiring arrangement with two switch-controlled lights and three wall outlets in parallel.



▲ FIGURE 43

Example of a parallel circuit in residential wiring.

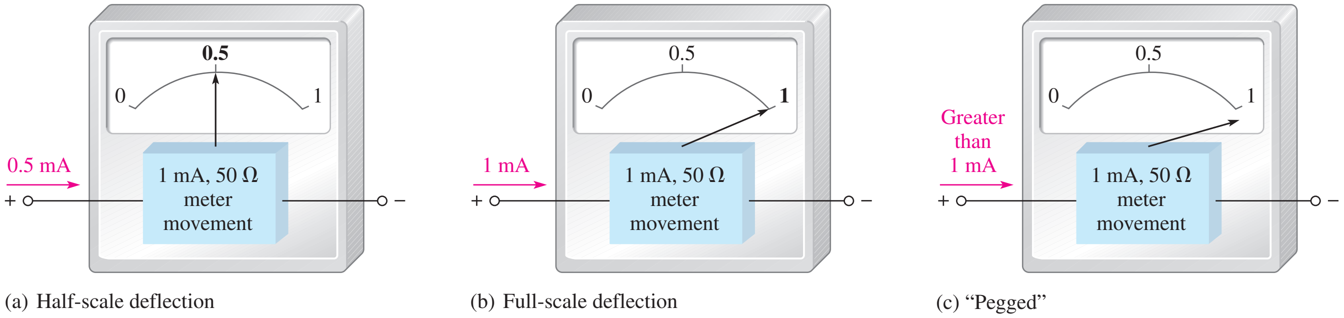
Control Circuits

Many control systems use parallel circuits or equivalent circuits for control and monitoring an industrial process such as a production line. Most complex control applications are implemented on a dedicated computer called a *programmable logic controller* (PLC). A PLC shows an equivalent parallel circuit, which is displayed on a computer screen. Internally, the circuit may exist only as a computer code, written in a computer programming language. However, the displayed circuit is one that could in principle be constructed with hardware. These circuits can be drawn to resemble a ladder—with rungs representing the load (and a source) and the rails representing the two principal nodes. (For example, notice that Figure 43 resembles a ladder on its side, with the lights and wall outlets as loads.) Parallel control circuits use ladder diagrams but add additional control elements (switches, relays, timers, and more). The result of adding the control elements creates a logical diagram, which is referred to as *ladder logic*. Ladder logic is easy to understand, so it is popular for showing control logic in industrial environments, such as factories or food processors. Entire books are written about ladder logic, but a ladder diagram (parallel circuit) is at the core of all ladder logic. The most important aspect is that ladder logic shows the basic functionality of the circuit in an easy-to-read format.

Analog Ammeters

Parallel circuits are used in the analog (needle-type) ammeter or milliammeter. Although analog meters are not as common as they once were, they are still used as panel meters in certain applications and analog multimeters are still available. Parallel circuits are an important part of analog ammeter operation because they allow the selection of various ranges in order to measure many different current values.

The mechanism in an ammeter that causes the pointer to move in proportion to the current is called the *meter movement,* which is based on a magnetic principle that you will learn later. Right now, it is sufficient to know that a given meter movement has a certain resistance and a maximum current. This maximum current, called the *full-scale deflection current,* causes the pointer to go all the way to the end of the scale. For example, a certain meter movement has a 50 Ω resistance and a full-scale deflection current of 1 mA. A meter with this particular movement can measure currents of 1 mA or less as indicated in Figure 44(a) and (b). Currents greater than 1 mA will cause the pointer to "peg" (or stop) slightly past the full scale mark as indicated in part (c), which can damage the meter.

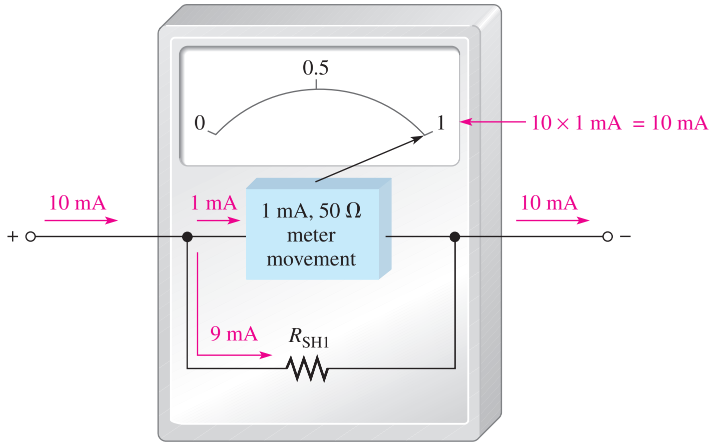

0.5
0
1
0.5 mA
+
−
1 mA, 50 Ω meter movement
0.5
0
1
1 mA
+
−
1 mA, 50 Ω meter movement
0.5
0
1
Greater than 1 mA
+
−
1 mA, 50 Ω meter movement

(a) Half-scale deflection (b) Full-scale deflection (c) "Pegged"

▲ FIGURE 44

A 1 mA analog ammeter.

Figure 45 shows a simple ammeter with a resistor in parallel with the 1 mA meter movement; this resistor is called a *shunt resistor.* Its purpose is to bypass a portion of current around the meter movement to extend the range of current that can be measured. The figure specifically shows 9 mA through the shunt resistor and 1 mA through the meter


0.5
0
1
10 × 1 mA = 10 mA
10 mA
1 mA
1 mA, 50 Ω meter movement
10 mA
+
–
9 mA
R_{SH1}

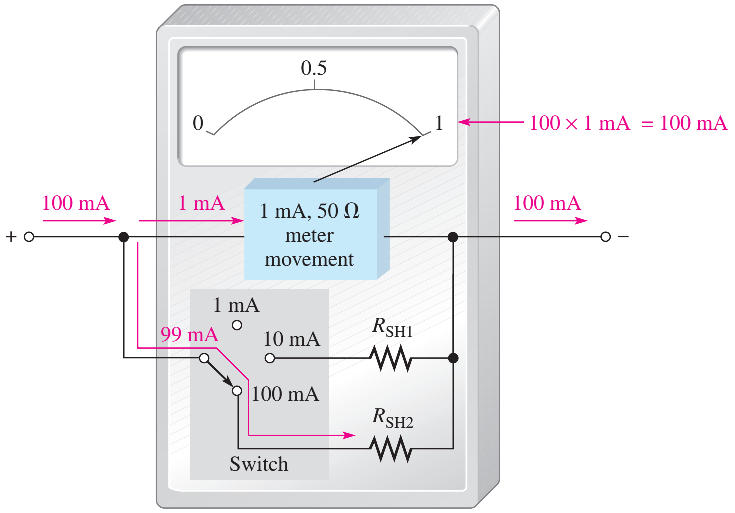
▲ **FIGURE 45**

A 10 mA analog ammeter.

movement. Thus, up to 10 mA can be measured. To find the actual current value, simply multiply the reading on the scale by 10.

A multiple-range ammeter has a range switch that permits the selection of several full-scale current settings. In each switch position, a certain amount of current is bypassed through a parallel resistor as determined by the resistance value. In our example, the current though the movement is never greater than 1 mA.

Figure 46 illustrates a meter with three ranges: 1 mA, 10 mA, and 100 mA. When the range switch is in the 1 mA position, all of the current into the meter goes through the meter movement. In the 10 mA setting, up to 9 mA goes through R_{SH1} and up to 1 mA through the movement. In the 100 mA setting, up to 99 mA goes through R_{SH2}, and the movement can still have only 1 mA for full-scale.


0.5
0
1
100 × 1 mA = 100 mA
100 mA
1 mA
1 mA, 50 Ω meter movement
100 mA
+
–
1 mA
99 mA
10 mA
R_{SH1}
100 mA
R_{SH2}
Switch

▲ **FIGURE 46**

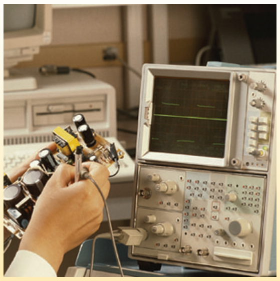
An analog ammeter with three ranges.

The scale reading is interpreted based on the range setting. For example, in Figure 46, if 50 mA of current are being measured, the needle points at the 0.5 mark on the scale; you must multiply 0.5 by 100 to find the current value. In this situation, 0.5 mA is through the movement (half-scale deflection) and 49.5 mA are through R_{SH2}.

SECTION 9 CHECKUP

1. What is the purpose of a shunt resistor in an analog meter movement?
2. Do the shunt resistors have resistance values considerably less than or more than that of the meter movement? Why?

10 TROUBLESHOOTING



Recall that an open circuit is one in which the current path is interrupted and there is no current. In this section we examine what happens when a branch of a parallel circuit opens.

After completing this section, you should be able to

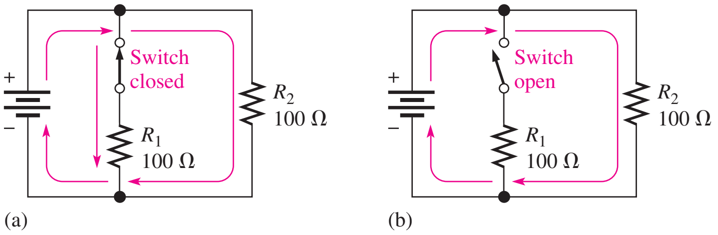
- **Troubleshoot parallel circuits**
  - Check for an open in a circuit

Open Branches

If a switch is connected in a branch of a parallel circuit, as shown in Figure 47, an open or a closed path can be made by the switch. When the switch is closed, as in Figure 47(a), R_1 and R_2 are in parallel. The total resistance is 50 Ω (two 100 Ω resistors in parallel). Current is through both resistors. If the switch is opened, as in Figure 47(b), R_1 is effectively removed from the circuit, and the total resistance is 100 Ω. Current is now only through R_2. In general,

> **When an open occurs in a parallel branch, the total resistance increases, the total current decreases, and the same current continues through each of the remaining parallel paths.**

The decrease in total current equals the amount of current that was previously in the open branch. The other branch currents remain the same.



▲ FIGURE 47

When switch opens, total current decreases and current through R_2 remains unchanged.

Consider the lamp circuit in Figure 48. There are four bulbs in parallel with a 12 V source. In part (a), there is current through each bulb. Now suppose that one of the bulbs burns out, creating an open path as shown in Figure 48(b). This light will go out because there is no current through the open path. Notice, however, that current continues through all the other parallel bulbs, and they continue to glow. The open branch does not change the voltage across the parallel branches; it remains at 12 V, and the current through each branch remains the same.

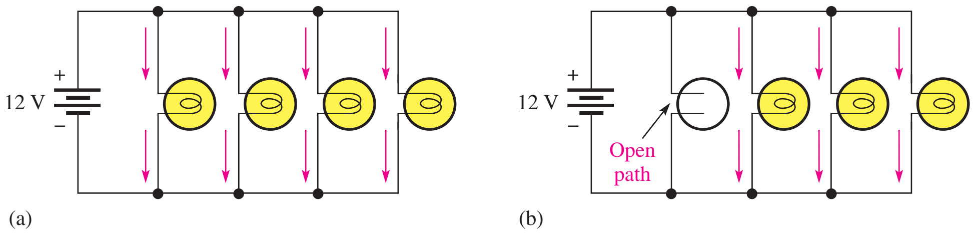


FIGURE 48

When one lamp opens, total current decreases and other branch currents remain unchanged.

You can see that a parallel circuit has an advantage over a series circuit in lighting systems because if one or more of the parallel bulbs burn out, the others will stay on. In a series circuit, when one bulb goes out, all of the others go out also because the current path is completely interrupted.

When a resistor in a parallel circuit opens, the open resistor cannot be located by measurement of the voltage across the branches because the same voltage exists across all the branches. Thus, there is no way to tell which resistor is open by simply measuring voltage. The good resistors will always have the same voltage as the open one, as illustrated in Figure 49 (note that the middle resistor is open).

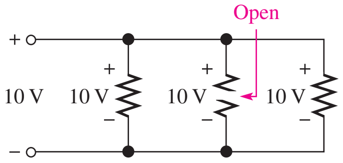


FIGURE 49

Parallel branches (open or not) have the same voltage.

If a visual inspection does not reveal the open resistor, it must be located by current measurements. In practice, measuring current is more difficult than measuring voltage because you must insert the ammeter in series to measure the current. Thus, a wire or a PC board connection must be cut or disconnected, or one end of a component must be lifted off the circuit board, in order to connect the ammeter in series. This procedure, of course, is not required when voltage measurements are made because the meter leads are simply connected across a component.

Finding an Open Branch by Current Measurement

In a parallel circuit with a suspected open branch, the total current can be measured to find the open. *When a parallel resistor opens, the total current, I_T, is always less than its normal value.* Once you know I_T and the voltage across the branches, a few calculations will determine the open resistor when all the resistors are of different resistance values.

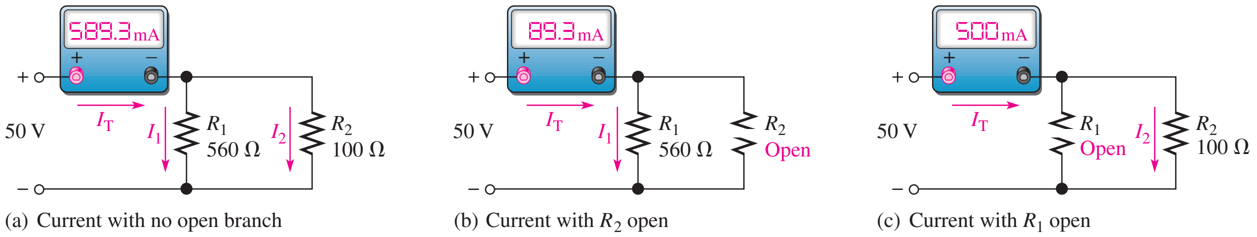
Consider the two-branch circuit in Figure 50(a). If one of the resistors opens, the total current will equal the current in the good resistor. Ohm's law quickly tells you what the current in each resistor should be.

$$I_1 = \frac{50\text{ V}}{560\ \Omega} = 89.3\text{ mA}$$

$$I_2 = \frac{50\text{ V}}{100\ \Omega} = 500\text{ mA}$$

$$I_T = I_1 + I_2 = 589.3\text{ mA}$$

If R_2 is open, the total current is 89.3 mA, as indicated in Figure 50(b). If R_1 is open, the total current is 500 mA, as indicated in Figure 50(c).



(a) Current with no open branch (b) Current with R_2 open (c) Current with R_1 open

FIGURE 50

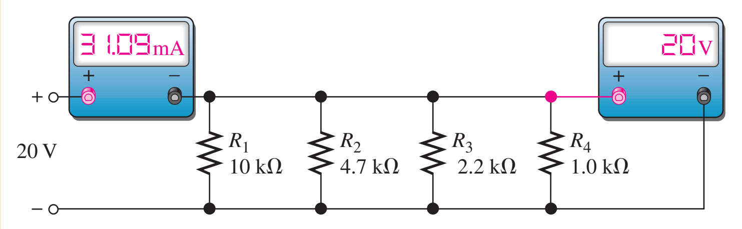
Finding an open path by current measurement.

This procedure can be extended to any number of branches having unequal resistances. If the parallel resistances are all equal, the current in each branch must be checked until a branch is found with no current. This is the open resistor.

EXAMPLE 21

In Figure 51, there is a total current of 31.09 mA, and the voltage across the parallel branches is 20 V. Is there an open resistor, and, if so, which one is it?

▶ **FIGURE 51**



Solution Calculate the current in each branch.

$$I_1 = \frac{V}{R_1} = \frac{20\text{ V}}{10\text{ k}\Omega} = 2\text{ mA}$$

$$I_2 = \frac{V}{R_2} = \frac{20\text{ V}}{4.7\text{ k}\Omega} = 4.26\text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{20\text{ V}}{2.2\text{ k}\Omega} = 9.09\text{ mA}$$

$$I_4 = \frac{V}{R_4} = \frac{20\text{ V}}{1.0\text{ k}\Omega} = 20\text{ mA}$$

The total current should be

$$I_T = I_1 + I_2 + I_3 + I_4 = 2\text{ mA} + 4.26\text{ mA} + 9.09\text{ mA} + 20\text{ mA} = 35.35\text{ mA}$$

The actual measured current is 31.09 mA, as stated, which is 4.26 mA less than normal, indicating that the branch carrying 4.26 mA is open. Thus, ***R_2* must be open**.

Related Problem What is the total current measured in Figure 51 if R_4 and not R_2 is open?

Finding an Open Branch by Resistance Measurement

If the parallel circuit to be checked can be disconnected from its voltage source and from any other circuit to which it may be connected, a measurement of the total resistance can be used to locate an open branch.

Recall that conductance, G, is the reciprocal of resistance ($1/R$) and its unit is the siemens (S). The total conductance of a parallel circuit is the sum of the conductances of all the resistors.

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

To locate an open branch, do the following steps:

1. Calculate what the total conductance should be using the individual resistor values.

$$G_{T(calc)} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

2. Measure the total resistance with an ohmmeter and calculate the total measured conductance.

$$G_{T(meas)} = \frac{1}{R_{T(meas)}}$$

3. Subtract the measured total conductance (Step 2) from the calculated total conductance (Step 1). The result is the conductance of the open branch and the resistance is obtained by taking its reciprocal ($R = 1/G$).

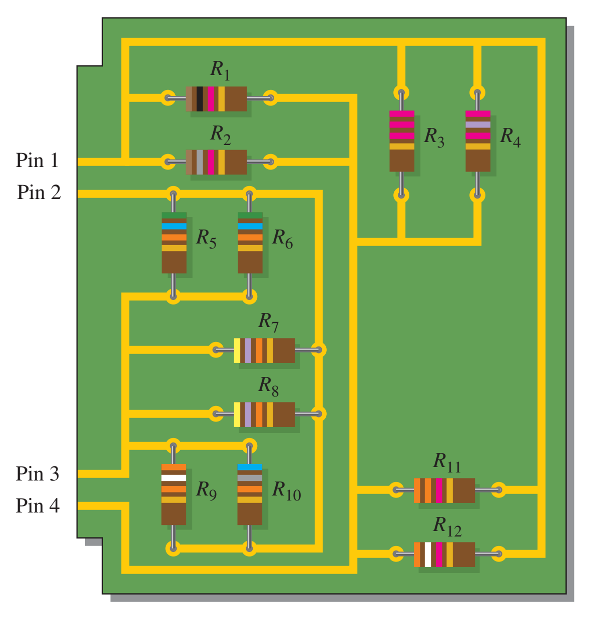
$$R_{\text{open}} = \frac{1}{G_{\text{T(calc)}} - G_{\text{T(meas)}}}$$

Equation 10

EXAMPLE 22

Your ohmmeter measures 402 Ω between pin 1 and pin 4 in Figure 52. Check the PC board between these two pins for open branches.

▶ **FIGURE 52**



Solution There are two separate parallel circuits on the board. The circuit between pin 1 and pin 4 is checked as follows (we will assume one of the resistors is open):

1. Calculate what the total conductance should be using the individual resistor values.

$$G_{\text{T(calc)}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{11}} + \frac{1}{R_{12}}$$

$$= \frac{1}{1.0\,\text{k}\Omega} + \frac{1}{1.8\,\text{k}\Omega} + \frac{1}{2.2\,\text{k}\Omega} + \frac{1}{2.7\,\text{k}\Omega} + \frac{1}{3.3\,\text{k}\Omega} + \frac{1}{3.9\,\text{k}\Omega} = 2.94\text{ mS}$$

2. Measure the total resistance with an ohmmeter and calculate the total measured conductance.

$$G_{\text{T(meas)}} = \frac{1}{402\ \Omega} = 2.49\text{ mS}$$

3. Subtract the measured total conductance (Step 2) from the calculated total conductance (Step 1). The result is the conductance of the open branch and the resistance is obtained by taking its reciprocal.

$$G_{\text{open}} = G_{\text{T(calc)}} - G_{\text{T(meas)}} = 2.94\text{ mS} - 2.49\text{ mS} = 0.45\text{ mS}$$

$$R_{\text{open}} = \frac{1}{G_{\text{open}}} = \frac{1}{0.45\text{ mS}} = 2.2\,\text{k}\Omega$$

Resistor R_3 is open and must be replaced.

Related Problem Your ohmmeter indicates 9.6 kΩ between pin 2 and pin 3 on the PC board in Figure 52. Determine if this is correct and, if not, which resistor is open.

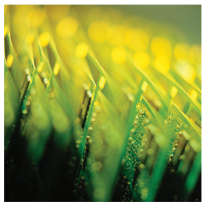
Shorted Branches

When a branch in a parallel circuit shorts, the current increases to an excessive value, causing a fuse or circuit breaker to blow. This results in a difficult troubleshooting problem because it is hard to isolate the shorted branch.

A pulser and a current tracer are tools often used to find shorts in a circuit. They are not restricted to use in digital circuits but can be effective in any type of circuit. The pulser is a pen-shaped tool that applies pulses to a selected point in a circuit, causing pulses of current to flow through the shorted path. The current tracer is also a pen-shaped tool that senses pulses of current. By following the current with the tracer, the current path can be identified.

SECTION 10 CHECKUP

1. If a parallel branch opens, what changes can be detected in the circuit's voltage and the currents, assuming that the parallel circuit is across a constant-voltage source?
2. What happens to the total resistance if one branch opens?
3. If several light bulbs are connected in parallel and one of the bulbs opens (burns out), will the others continue to glow?
4. There is 100 mA of current in each branch of a parallel circuit. If one branch opens, what is the current in each of the remaining branches?
5. A three-branch circuit normally has the following branch currents: 100 mA, 250 mA, and 120 mA. If the total current measures 350 mA, which branch is open?



Application Activity

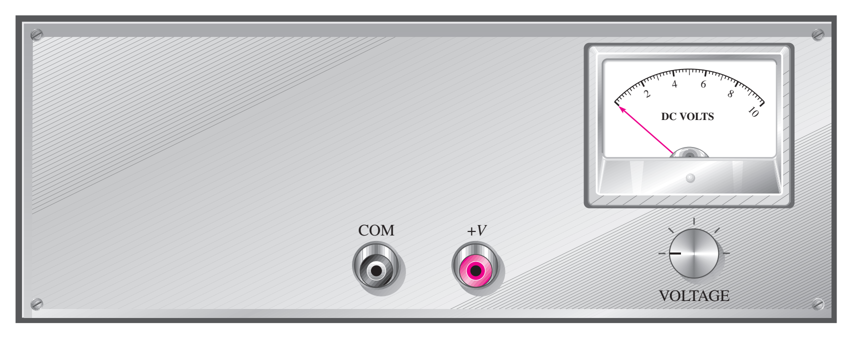
In this application activity, a dc power supply is modified by adding a 3-range ammeter to indicate current to the load. As you have learned, parallel resistances can be used to extend the range of an ammeter. These parallel resistors, called *shunts,* bypass current around the meter movement, allowing for the meter to effectively measure higher currents than the maximum current for which the meter movement is designed.

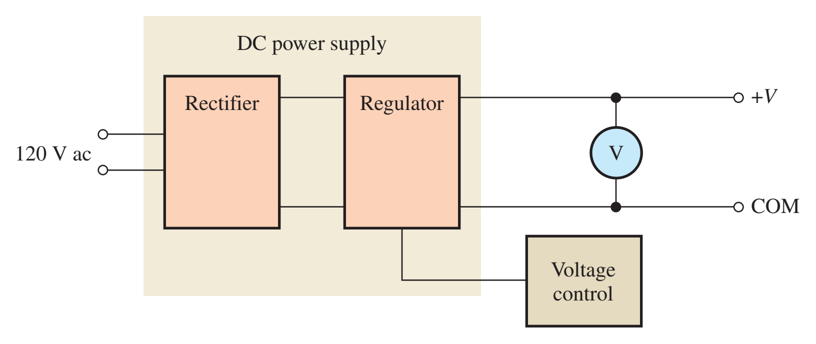
The Power Supply

A rack-mounted power supply is shown in Figure 53. The voltmeter indicates the output voltage, which can be adjusted

▶ FIGURE 53

Front panel view of a rack-mounted power supply.





▲ FIGURE 54

Basic block diagram of the dc power supply.

from 0 V to 10 V using the voltage control. The power supply is capable of providing up to 2 A to a load. A basic block diagram of the power supply is shown in Figure 54. It consists of a rectifier circuit that converts ac voltage from the wall outlet to dc voltage and a regulator circuit that keeps the output voltage at a constant value.

It is required that the power supply be modified by adding an ammeter with three switch-selected current ranges of 25 mA, 250 mA, and 2.5 A. To accomplish this, two shunt resistances are used that can each be switched into a parallel connection with the meter movement. This approach works fine as long as the required values of the shunt resistors are not too small. However, there are problems at very low values of shunt resistance and you will see why next.

The Shunt Circuit

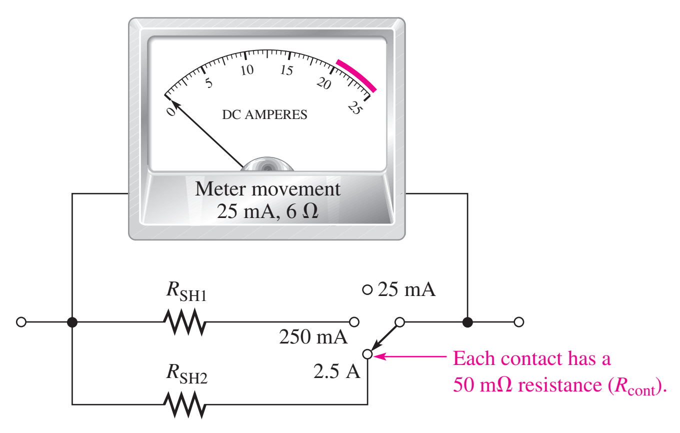
An ammeter is selected that has a full-scale deflection of 25 mA and a resistance of 6 Ω. Two shunt resistors must be added—one for 250 mA and one for 2.5 A full-scale deflections. The internal meter movement provides the 25 mA range. This is shown in Figure 55. The range selection is provided by a 1-pole, 3-position rotary switch with a contact resistance of 50 mΩ. Contact resistance of switches can be from less than 20 mΩ to about 100 mΩ. The contact resistance of a given switch can vary with temperature, current, and usage and, therefore, cannot be relied upon to remain within a reasonable tolerance of the specified value. Also, the switch is a make-before-break type, which means that contact with the previous position is not broken until contact with the new position is made.

The shunt resistance value for the 2.5 A range is determined as follows where the voltage across the meter movement is

$$V_M = I_M R_M = (25\text{ mA})(6\ \Omega) = 150\text{ mV}$$

The current through the shunt resistor for full-scale deflection is

$$I_{SH2} = I_{FULL\ SCALE} - I_M = 2.5\text{ A} - 25\text{ mA} = 2.475\text{ A}$$



▲ FIGURE 55

Ammeter modified to provide three current ranges.

The total shunt resistance is

$$R_{\text{SH2(tot)}} = \frac{V_{\text{M}}}{I_{\text{SH2}}} = \frac{150\ \text{mV}}{2.475\ \text{A}} = 60.6\ \text{m}\Omega$$

Low-ohm precision resistors are generally available in values from 1 mΩ to 10 Ω or greater from various manufacturers.

Notice in Figure 55 that the contact resistance, R_{CONT}, of the switch appears in series with R_{SH2}. Since the total shunt resistance must be 60.6 mΩ, the value of the shunt resistor R_{SH2} is

$$R_{\text{SH2}} = R_{\text{SH2(tot)}} - R_{\text{CONT}} = 60.6\ \text{m}\Omega - 50\ \text{m}\Omega = 10.6\ \text{m}\Omega$$

Although this value, or one close to it, may be available, the problem in this case is that the switch contact resistance is nearly five times that of R_{SH2} and any variation in it would create a significant inaccuracy in the meter. As you can see, this approach is not acceptable for these particular requirements.

Another Approach

A variation of the standard shunt resistance circuit is shown in Figure 56. The shunt resistor, R_{SH}, is connected in parallel for the two higher current range settings and disconnected for the 25 mA setting using a 2-pole, 3-position switch. This circuit avoids dependency on the switch contact resistance by using resistor values that are large enough to make it insignificant. The disadvantages of this meter circuit are that it requires a more complex switch and the voltage drops from input to output are greater than in the previous shunt circuit.

For the 250 mA range, the current through the meter movement for full-scale deflection is 25 mA. The voltage across the meter movement is 150 mV.

$$I_{\text{SH}} = 250\ \text{mA} - 25\ \text{mA} = 225\ \text{mA}$$

$$R_{\text{SH}} = \frac{150\ \text{mV}}{225\ \text{mA}} = 0.67\ \Omega = 670\ \text{m}\Omega$$

This value of R_{SH} is more than thirteen times the expected switch contact resistance of 50 mΩ, thus minimizing the effect of the contact resistance.

For the 2.5 A range, the current through the meter movement for full-scale deflection is still 25 mA. This is also the current through R_1.

$$I_{\text{SH}} = 2.5\ \text{A} - 25\ \text{mA} = 2.475\ \text{A}$$

The voltage across the meter circuit from A to B is

$$V_{AB} = I_{\text{SH}}R_{\text{SH}} = (2.475\ \text{A})(670\ \text{m}\Omega) = 1.66\ \text{V}$$

Applying Kirchhoff's voltage law and Ohm's law to find R_1,

$$V_{R1} + V_{\text{M}} = V_{AB}$$

$$V_{R1} = V_{AB} - V_{M} = 1.66\ \text{V} - 150\ \text{mV} = 1.51\ \text{V}$$

$$R_1 = \frac{V_{R1}}{I_{\text{M}}} = \frac{1.51\ \text{V}}{25\ \text{mA}} = 60.4\ \Omega$$

This value is much greater than the contact resistance of the switch.

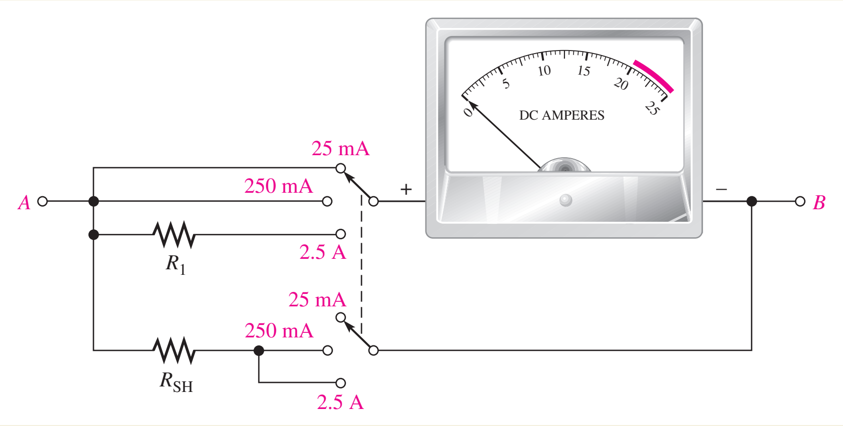
1. Determine the maximum power dissipated by R_{SH} in Figure 56 for each range setting.
2. How much voltage is there from A to B in Figure 56 when the switch is set to the 2.5 A range and the current is 1 A?
3. The meter indicates 250 mA. How much does the voltage across the meter circuit from A to B change when the switch is moved from the 250 mA position to the 2.5 A position?
4. Assume the meter movement has a resistance of 4 Ω instead of 6 Ω. Specify any changes necessary in the circuit of Figure 56.

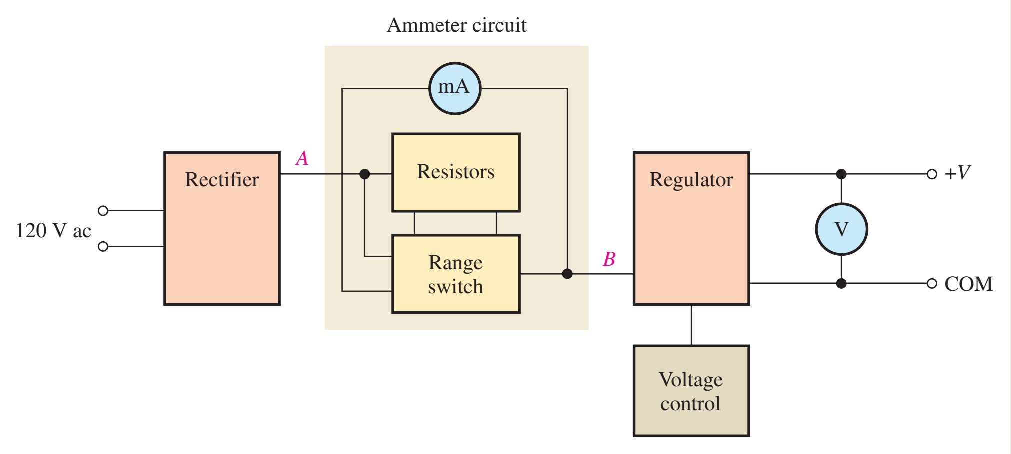
Implementing the Power Supply Modification

Once the proper values are obtained, the resistors are placed on a board which is then mounted in the power supply. The resistors and the range switch are connected to the power supply as shown in Figure 57. The ammeter circuit is connected between the rectifier circuit in the power supply and the regulator circuit in order to reduce the impact of the voltage drop across the meter circuit on the output voltage. The regulator maintains, within certain limits, a constant dc output voltage even though its input voltage coming through the meter circuit may change.

▶ FIGURE 56

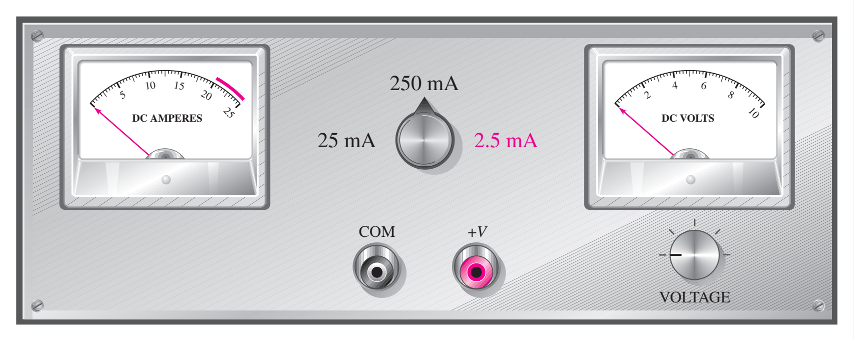
Meter circuit redesigned to eliminate or minimize the effect of switch contact resistance. The switch is a 2-pole, 3-position make-before-break rotary type.




Ammeter circuit
mA
Resistors
Range switch
Rectifier
A
B
120 V ac
Regulator
V
+V
COM
Voltage control

▲ FIGURE 57

Block diagram of dc power supply with 3-range milliammeter.


DC AMPERES
5
10
15
20
25
250 mA
25 mA
2.5 mA
DC VOLTS
2
4
6
8
10
COM
+V
VOLTAGE

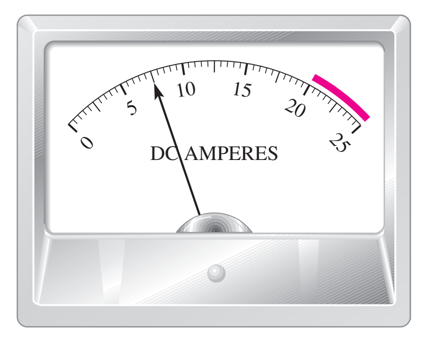
◀ FIGURE 58

The power supply with the addition of the milliammeter and the current range selection switch.

Figure 58 shows the modified power supply front panel with the rotary range switch and milliammeter installed. The red portion of the scale indicates excess current for the 2.5 A range since the power supply has a maximum current of 2 A for safe operation.

Review

5. When the meter is set to the 250 mA range, which resistance has the most current through it?
6. Determine the total resistance from *A* to *B* of the meter circuit in Figure 56 for each of the three current ranges.
7. Explain why the circuit in Figure 56 was used instead of the one in Figure 55.
8. If the pointer is at the 15 and the range switch is set to 250 mA, what is the current?
9. How much current is indicated by the ammeter in Figure 59 for each of the three range switch settings in Figure 56?


0
5
10
15
20
25
DC AMPERES

◀ FIGURE 59

SUMMARY

- Resistors in parallel are connected between two points (nodes).
- A parallel combination has more than one path for current.
- The total parallel resistance is less than the lowest-value resistor.
- The voltages across all branches of a parallel circuit are the same.
- Current sources in parallel add algebraically.
- Kirchhoff's current law: The sum of the currents into a junction (total current in) equals the sum of the currents out of the junction (total current out).
- The algebraic sum of all the currents entering and leaving a junction is equal to zero.
- A parallel circuit is a current divider, so called because the total current entering the junction of parallel branches divides up into each of the branches.
- If all of the branches of a parallel circuit have equal resistance, the currents through all of the branches are equal.
- The total power in a parallel-resistive circuit is the sum of all of the individual powers of the resistors making up the parallel circuit.
- The total power for a parallel circuit can be calculated with the power formulas using values of total current, total resistance, or total voltage.
- If one of the branches of a parallel circuit opens, the total resistance increases, and therefore the total current decreases.
- If a branch of a parallel circuit opens, there is no change in current through the remaining branches.

KEY TERMS

Branch One current path in a parallel circuit.

Current divider A parallel circuit in which the currents divide inversely proportional to the parallel branch resistances.

Kirchhoff's current law A circuit law stating that the total current into a node equals the total current out of the node. Equivalently, the algebraic sum of all the currents entering and leaving a node is zero.

Node A point in a circuit at which two or more components are connected; also known as a *junction.*

Parallel The relationship between two circuit components that exists when they are connected between the same pair of nodes.

FORMULAS

1	$I_{\mathrm{IN}(1)} + I_{\mathrm{IN}(2)} + \cdots + I_{\mathrm{IN}(n)} = I_{\mathrm{OUT}(1)} + I_{\mathrm{OUT}(2)} + \cdots + I_{\mathrm{OUT}(m)}$	Kirchhoff's current law
2	$R_{\mathrm{T}} = \dfrac{1}{\left(\dfrac{1}{R_1}\right) + \left(\dfrac{1}{R_2}\right) + \left(\dfrac{1}{R_3}\right) + \cdots + \left(\dfrac{1}{R_n}\right)}$	Total parallel resistance
3	$R_{\mathrm{T}} = \dfrac{R_1 R_2}{R_1 + R_2}$	Special case for two resistors in parallel
4	$R_{\mathrm{T}} = \dfrac{R}{n}$	Special case for *n* equal-value resistors in parallel
5	$R_x = \dfrac{R_A R_{\mathrm{T}}}{R_A - R_{\mathrm{T}}}$	Unknown parallel resistor

6	$I_x = \left(\dfrac{R_T}{R_x}\right)I_T$	General current-divider formula
7	$I_1 = \left(\dfrac{R_2}{R_1 + R_2}\right)I_T$	Two-branch current-divider formula
8	$I_2 = \left(\dfrac{R_1}{R_1 + R_2}\right)I_T$	Two-branch current-divider formula
9	$P_T = P_1 + P_2 + P_3 + \cdots + P_n$	Total power
10	$R_{open} = \dfrac{1}{G_{T(calc)} - G_{T(meas)}}$	Open branch resistance

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. To find the total conductance of parallel resistors, you can add the conductance of each of the resistors.
2. The total resistance of parallel resistors is always smaller than the smallest resistor.
3. The product-over-sum rule works for any number of parallel resistors.
4. In a parallel circuit, the voltage is larger across a larger resistor and smaller across a smaller resistor.
5. When a new path is added to a parallel circuit, the total resistance goes up.
6. When a new path is added to a parallel circuit, the total current goes up.
7. The total current entering a node is always equal to the total current leaving the node.
8. In the current-divider formula ($I_x = (R_T/R_x)I_T$), the quantity R_T/R_x is a fraction that is never greater than 1.
9. When two resistors are in parallel, the smaller resistor will have the smaller power dissipation.
10. The total power dissipated by parallel resistors can be larger than the power supplied by the source.

SELF-TEST

Answers are at the end of the chapter.

1. In a parallel circuit, each resistor has
   (a) the same current **(b)** the same voltage
   (c) the same power **(d)** all of the above
2. When a 1.2 kΩ resistor and a 100 Ω resistor are connected in parallel, the total resistance is
   (a) greater than 1.2 kΩ
   (b) greater than 100 Ω but less than 1.2 kΩ
   (c) less than 100 Ω but greater than 90 Ω
   (d) less than 90 Ω
3. A 330 Ω resistor, a 270 Ω resistor, and a 68 Ω resistor are all in parallel. The total resistance is approximately
   (a) 668 Ω **(b)** 47 Ω **(c)** 68 Ω **(d)** 22 Ω
4. Eight resistors are in parallel. The two lowest-value resistors are both 1.0 kΩ. The total resistance
   (a) is less than 8 kΩ **(b)** is greater than 1.0 kΩ
   (c) is less than 1.0 kΩ **(d)** is less than 500 Ω
5. When an additional resistor is connected across an existing parallel circuit, the total resistance
   (a) decreases **(b)** increases
   (c) remains the same **(d)** increases by the value of the added resistor
6. If one of the resistors in a parallel circuit is removed, the total resistance
   (a) decreases by the value of the removed resistor **(b)** remains the same
   (c) increases **(d)** doubles

7. One current into a junction is 500 mA and the other current into the same junction is 300 mA. The total current out of the junction is

   (a) 200 mA **(b)** unknown **(c)** 800 mA **(d)** the larger of the two

8. The following resistors are in parallel across a voltage source: 390 Ω, 560 Ω, and 820 Ω. The resistor with the least current is

   (a) 390 Ω **(b)** 560 Ω

   (c) 820 Ω **(d)** impossible to determine without knowing the voltage

9. A sudden decrease in the total current into a parallel circuit may indicate

   (a) a short **(b)** an open resistor

   (c) a drop in source voltage **(d)** either (b) or (c)

10. In a four-branch parallel circuit, there are 10 mA of current in each branch. If one of the branches opens, the current in each of the other three branches is

    (a) 13.3 mA **(b)** 10 mA **(c)** 0 A **(d)** 30 mA

11. In a certain three-branch parallel circuit, R_1 has 10 mA through it, R_2 has 15 mA through it, and R_3 has 20 mA through it. After measuring a total current of 35 mA, you can say that

    (a) R_1 is open **(b)** R_2 is open

    (c) R_3 is open **(d)** the circuit is operating properly

12. If there are a total of 100 mA into a parallel circuit consisting of three branches and two of the branch currents are 40 mA and 20 mA, the third branch current is

    (a) 60 mA **(b)** 20 mA **(c)** 160 mA **(d)** 40 mA

13. A complete short develops across one of five parallel resistors on a PC board. The most likely result is

    (a) the shorted resistor will burn out

    (b) one or more of the other resistors will burn out

    (c) the fuse in the power supply will blow

    (d) the resistance values will be altered

14. The power dissipation in each of four parallel branches is 1 W. The total power dissipation is

    (a) 1 W **(b)** 4 W **(c)** 0.25 W **(d)** 16 W

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 63.

1. If R_1 opens with the switch in the position shown, the voltage at terminal *A* with respect to ground

   (a) increases **(b)** decreases **(c)** stays the same

2. If the switch is thrown from position *A* to position *B*, the total current

   (a) increases **(b)** decreases **(c)** stays the same

3. If R_4 opens with the switch in position *C*, the total current

   (a) increases **(b)** decreases **(c)** stays the same

4. If a short develops between *B* and *C* while the switch is in position *B*, the total current

   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 69(b).

5. If R_2 opens, the current through R_1

   (a) increases **(b)** decreases **(c)** stays the same

6. If R_3 opens, the voltage across it

   (a) increases **(b)** decreases **(c)** stays the same

7. If R_1 opens, the voltage across it
  (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 70.

8. If the resistance of the rheostat R_2 is increased, the current through R_1
  (a) increases **(b)** decreases **(c)** stays the same

9. If the fuse opens, the voltage across the rheostat R_2
  (a) increases **(b)** decreases **(c)** stays the same

10. If the rheostat R_2 develops a short between the wiper and ground, the current through it
  (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 74.

11. If the 2.25 mA source opens while the switch is in position C, the current through R
  (a) increases **(b)** decreases **(c)** stays the same

12. If the 2.25 mA source opens while the switch is in position B, the current through R
  (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 82.

13. If pins 4 and 5 are shorted together, the resistance between pins 3 and 6
  (a) increases **(b)** decreases **(c)** stays the same

14. If the bottom connection of R_1 is shorted to the top connection of R_5, the resistance between pins 1 and 2
  (a) increases **(b)** decreases **(c)** stays the same

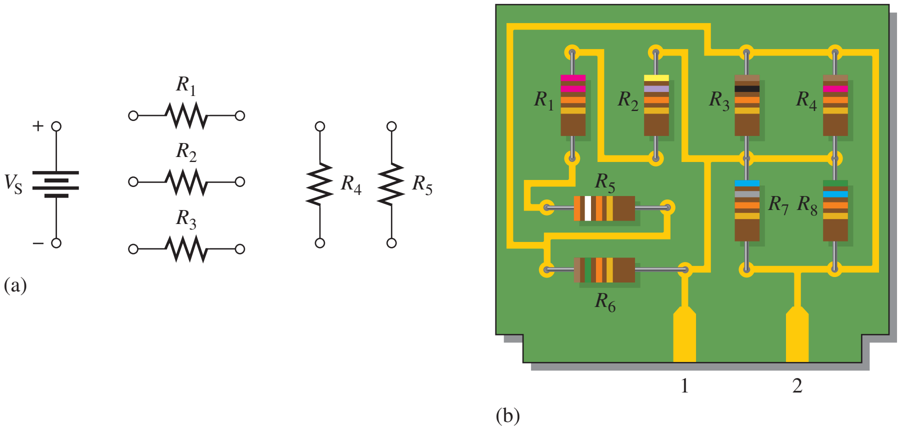
15. If R_7 opens, the resistance between pins 5 and 6
  (a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

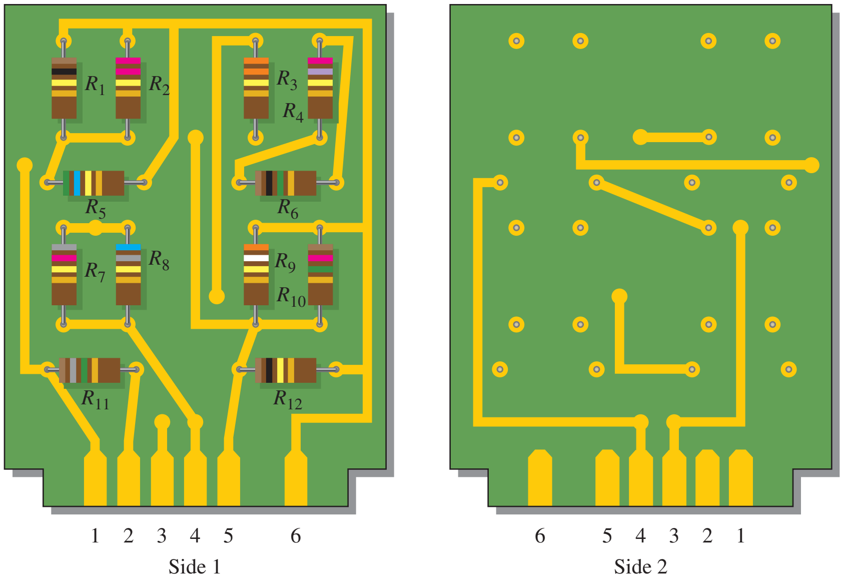
More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 Resistors in Parallel

1. Show how to connect the resistors in Figure 60(a) in parallel across the battery.
2. Determine whether or not all the resistors in Figure 60(b) are connected in parallel on the printed circuit (PC) board.



▲ FIGURE 60


R1
R2
R3
R4
R5
R6
R7
R8
R9
R10
R11
R12
1 2 3 4 5 6
Side 1
6 5 4 3 2 1
Side 2

▲ FIGURE 61

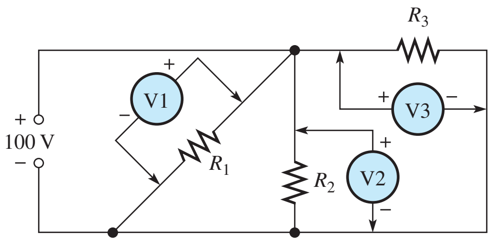
***3.** Identify which groups of resistors are in parallel on the double-sided PC board in Figure 61.

SECTION 2 Voltage in a Parallel Circuit

4. What is the voltage across and the current through each parallel resistor if the total voltage is 12 V and the total resistance is 550 Ω? There are four resistors, all of equal value.

5. The source voltage in Figure 62 is 100 V. How much voltage does each of the meters read?

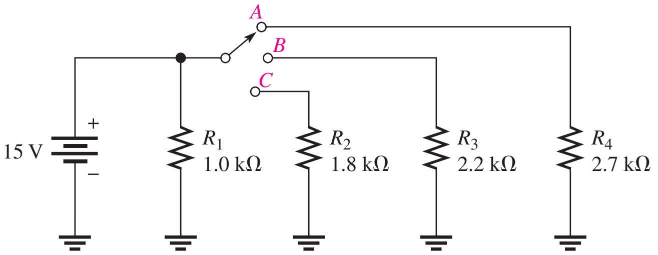
▶ FIGURE 62


R3
+
V1
−
+
V3
−
+
100 V
−
+
R1
R2
V2
−

6. What is the total resistance of the circuit as seen from the voltage source for each position of the switch in Figure 63?

7. What is the voltage across each resistor in Figure 63 for each switch position?

8. What is the total current from the voltage source in Figure 63 for each switch position?


A
B
C
15 V
+
−
R1
1.0 kΩ
R2
1.8 kΩ
R3
2.2 kΩ
R4
2.7 kΩ

▲ FIGURE 63

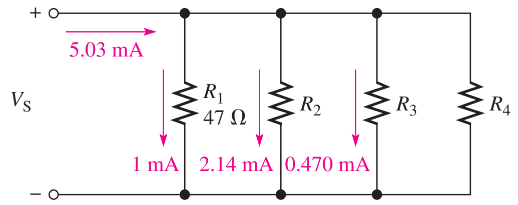
SECTION 3 Kirchhoff's Current Law

9. The following currents are measured in the same direction in a three-branch parallel circuit: 250 mA, 300 mA, and 800 mA. What is the value of the current into the junction of these three branches?

10. There is a total of 500 mA of current into five parallel resistors. The currents through four of the resistors are 50 mA, 150 mA, 25 mA, and 100 mA. What is the current through the fifth resistor?

11. In the circuit of Figure 64, determine the resistance R_2, R_3, and R_4.

▶ **FIGURE 64**



***12.** The electrical circuit in a room has a ceiling lamp that draws 1.25 A and four wall outlets. Two table lamps that each draw 0.833 A are plugged into two outlets, and an electric heater that draws 10 A is connected to the third outlet. When all of these items are in use, how much current is in the main line serving the room? If the main line is protected by a 15 A circuit breaker, how much current can be drawn from the fourth outlet? Draw a schematic of this wiring.

***13.** The total resistance of a parallel circuit is 25 Ω. What is the current through a 220 Ω resistor that makes up part of the parallel circuit if the total current is 100 mA?

14. A travel trailer has four running lights that draw 0.5 A each and two taillights that draw 1.2 A each. What is the current supplied to the trailer when the taillights and running lights are on?

15. Assume the trailer in Problem 14 has two brake lights that draw 1 A each.

(a) When all lights are on, what current is supplied to the trailer?

(b) For this condition, what is the ground return current from the trailer?

SECTION 4 Total Parallel Resistance

16. The following resistors are connected in parallel: 1.0 MΩ, 2.2 MΩ, 5.6 MΩ, 12 MΩ, and 22 MΩ. Determine the total resistance.

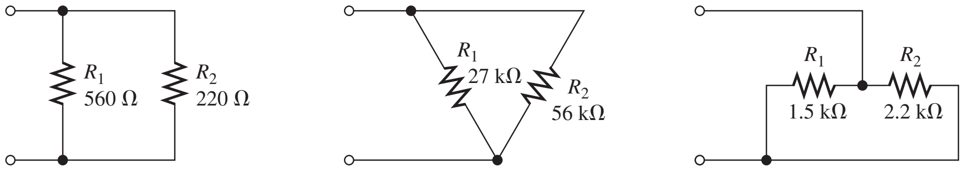
17. Find the total resistance for each of the following groups of parallel resistors:

(a) 560 Ω and 1000 Ω **(b)** 47 Ω and 56 Ω

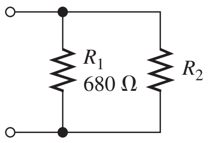
(c) 1.5 kΩ, 2.2 kΩ, 10 kΩ **(d)** 1.0 MΩ, 470 kΩ, 1.0 kΩ, 2.7 MΩ

18. Calculate R_T for each circuit in Figure 65.

▶ **FIGURE 65**



(a) (b) (c)



▲ **FIGURE 66**

19. What is the total resistance of twelve 6.8 kΩ resistors in parallel?

20. Five 470 Ω, ten 1000 Ω, and two 100 Ω resistors are all connected in parallel. What is the total resistance for each of the three groupings?

21. Find the total resistance for the entire parallel circuit in Problem 20.

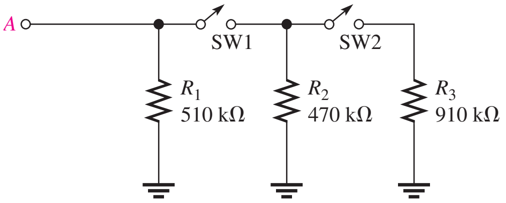
22. If the total resistance in Figure 66 is 389.2 Ω, what is the value of R_2?

23. What is the total resistance between point A and ground in Figure 67 for the following conditions?

(a) SW1 and SW2 open **(b)** SW1 closed, SW2 open

(c) SW1 open, SW2 closed **(d)** SW1 and SW2 closed

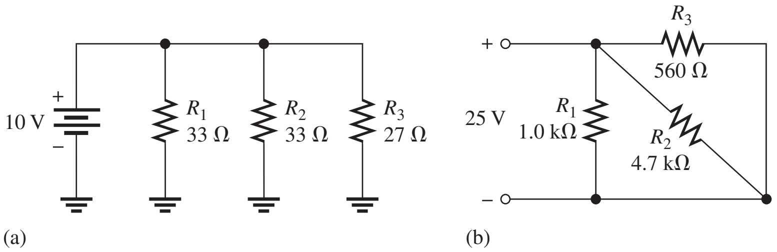
▶ **FIGURE 67**


A
SW1
SW2
R_1
510 kΩ
R_2
470 kΩ
R_3
910 kΩ

SECTION 5 Application of Ohm's Law

24. What is the total current in each circuit of Figure 68?

▶ **FIGURE 68**

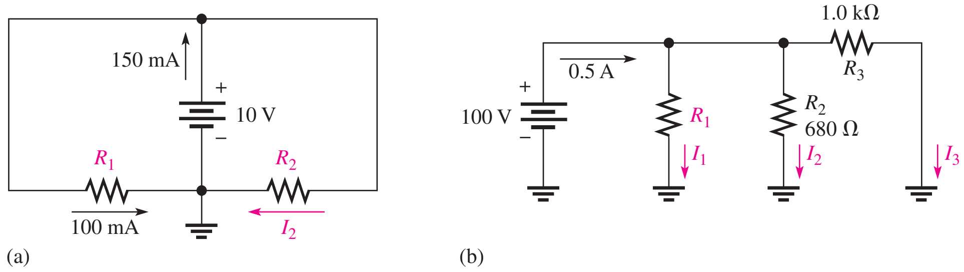

10 V
+
−
R_1
33 Ω
R_2
33 Ω
R_3
27 Ω
(a)
+
−
25 V
R_1
1.0 kΩ
R_3
560 Ω
R_2
4.7 kΩ
(b)

25. The resistance of a 60 W bulb is approximately 240 Ω. What is the current from the source when three bulbs are on in a 120 V parallel circuit?

26. Four equal-value resistors are connected in parallel. Five volts are applied across the parallel circuit, and 1.11 mA are measured from the source. What is the value of each resistor?

27. Many types of decorative lights are connected in parallel. If a set of lights is connected to a 110 V source and the filament of each bulb has a hot resistance of 2.2 kΩ, what is the current through each bulb? Why is it better to have these bulbs in parallel rather than in series?

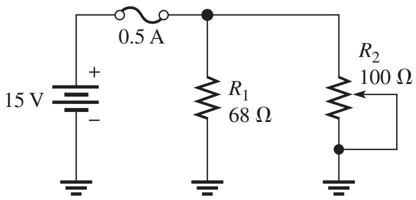
28. Find the values of the unspecified labeled quantities (shown in red) in each circuit of Figure 69.


150 mA
+
10 V
−
R_1
R_2
100 mA
I_2
(a)
1.0 kΩ
R_3
0.5 A
+
100 V
−
R_1
I_1
R_2
680 Ω
I_2
I_3
(b)

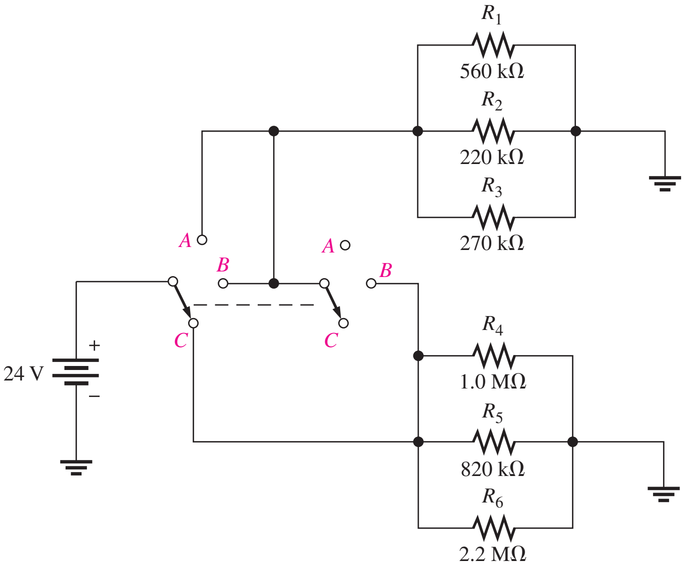
▲ **FIGURE 69**

29. To what minimum value can the 100 Ω rheostat in Figure 70 be adjusted before the 0.5 A fuse blows?

▶ **FIGURE 70**


0.5 A
+
15 V
−
R_1
68 Ω
R_2
100 Ω

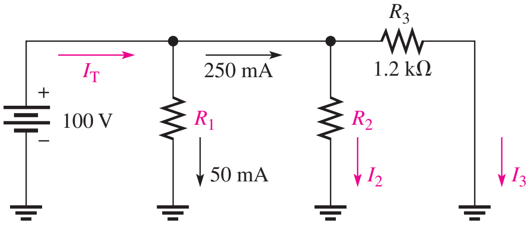
30. Determine the total current from the source and the current through each resistor for each switch position in Figure 71.


R_1
560 kΩ
R_2
220 kΩ
R_3
270 kΩ
A
B
A
B
C
C
+
24 V
−
R_4
1.0 MΩ
R_5
820 kΩ
R_6
2.2 MΩ

▲ **FIGURE 71**

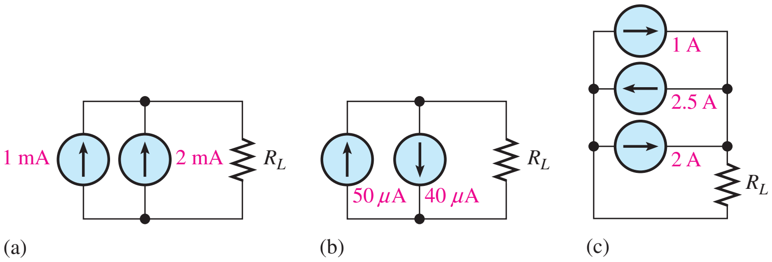
31. Find the values of the unspecified quantities (shown in red) in Figure 72.

▶ **FIGURE 72**


I_T
+
100 V
−
R_1
50 mA
250 mA
R_2
I_2
R_3
1.2 kΩ
I_3

SECTION 6 Current Sources in Parallel

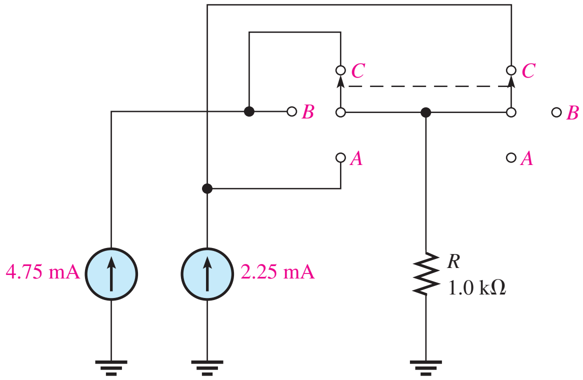
32. Determine the current through R_L in each circuit in Figure 73.


1 mA
2 mA
R_L
(a)
50 μA
40 μA
R_L
(b)
1 A
2.5 A
2 A
R_L
(c)

▲ **FIGURE 73**

33. Find the current through the resistor for each position of the ganged switch in Figure 74.

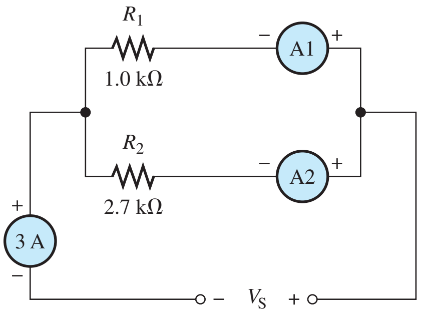
▶ **FIGURE 74**


C
C
B
B
B
A
A
4.75 mA
2.25 mA
R
1.0 kΩ

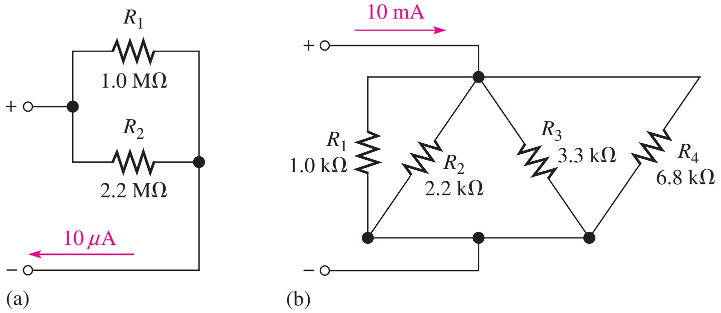
SECTION 7 Current Dividers

34. How much branch current should each meter in Figure 75 indicate?

▶ **FIGURE 75**


R1
1.0 kΩ
A1
R2
2.7 kΩ
A2
3 A
VS

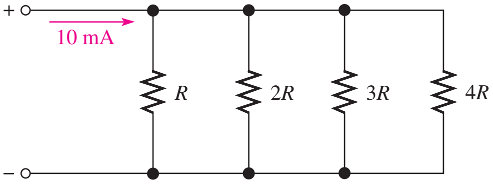
35. Determine the current in each branch of the current dividers of Figure 76.


R1
1.0 MΩ
R2
2.2 MΩ
10 μA
(a)
10 mA
R1
1.0 kΩ
R2
2.2 kΩ
R3
3.3 kΩ
R4
6.8 kΩ
(b)

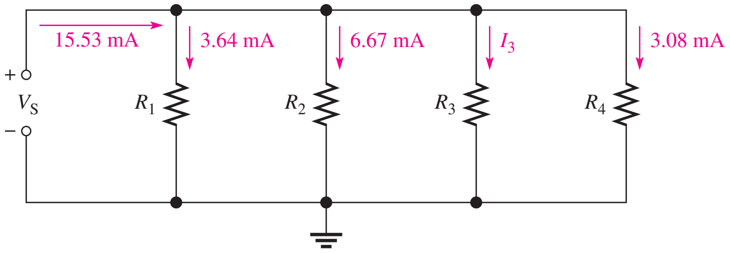
▲ **FIGURE 76**

36. What is the current through each resistor in Figure 77? *R* is the lowest-value resistor, and all others are multiples of that value as indicated.

▶ **FIGURE 77**


10 mA
R
2R
3R
4R

37. Determine all of the resistor values in Figure 78. $R_T = 773\ \Omega$.



▲ FIGURE 78

***38.** **(a)** Determine the required value of the shunt resistor R_{SH1} in the ammeter of Figure 45 if the resistance of the meter movement is 50 Ω.

(b) Find the required value for R_{SH2} in the meter circuit of Figure 46 ($R_M = 50\ \Omega$).

***39.** Special shunt resistors designed to drop 50 mV in high current-measuring applications are available from manufacturers. A 50 mV, 10 kΩ full-scale voltmeter is connected across the shunt to make the measurement.

(a) What value of shunt resistance is required to use a 50 mV meter in a 50 A measurement application?

(b) How much current is through the meter?

SECTION 8 Power in Parallel Circuits

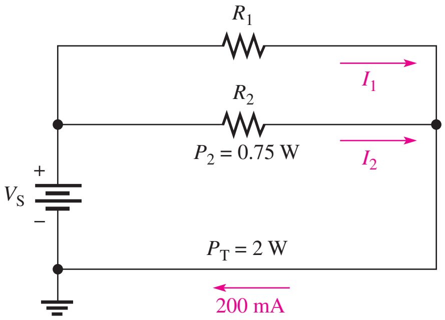
40. Five parallel resistors each handle 250 mW. What is the total power?

41. Determine the total power in each circuit of Figure 76.

42. Six light bulbs are connected in parallel across 120 V. Each bulb is rated at 75 W. What is the current through each bulb, and what is the total current?

***43.** Find the values of the unspecified quantities in Figure 79.

▶ FIGURE 79



***44.** A certain parallel circuit consists of only ½ W resistors. The total resistance is 1.0 kΩ, and the total current is 50 mA. If each resistor is operating at one-half its maximum power level, determine the following:

(a) The number of resistors

(b) The value of each resistor

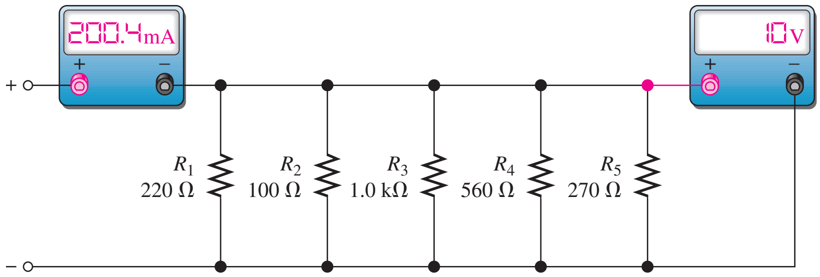
(c) The current in each branch

(d) The applied voltage

SECTION 10 Troubleshooting

45. If one of the bulbs burns out in Problem 42, how much current will be through each of the remaining bulbs? What will the total current be?

▶ FIGURE 80

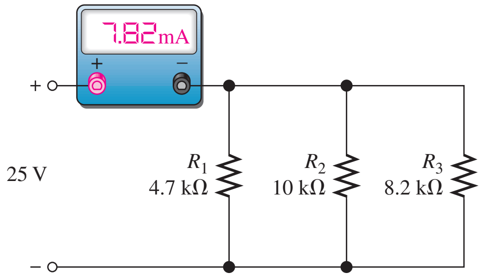

200.4 mA
+
−
10 V
+
−
+
−
R_1 220 Ω
R_2 100 Ω
R_3 1.0 kΩ
R_4 560 Ω
R_5 270 Ω

46. In Figure 80, the current and voltage measurements are indicated. Has a resistor opened, and, if so, which one?

47. What is wrong with the circuit in Figure 81?

48. What is wrong with the circuit in Figure 81 if the meter reads 5.55 mA?

▶ FIGURE 81


7.82 mA
+
−
+
25 V
−
R_1 4.7 kΩ
R_2 10 kΩ
R_3 8.2 kΩ

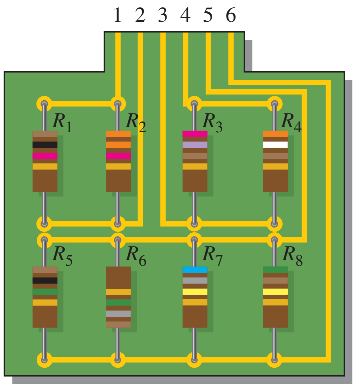
***49.** Develop a test procedure to check the circuit board in Figure 82 to make sure that there are no open components. You must do this test without removing a component from the board. List the procedure in a detailed step-by-step format.

***50.** For the circuit board shown in Figure 83, determine the resistance between the following pins if there is a short between pins 2 and 4:

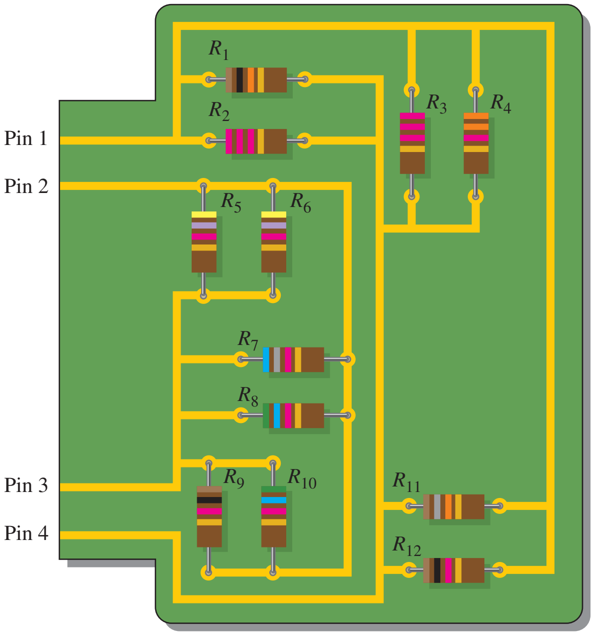
(a) 1 and 2 **(b)** 2 and 3 **(c)** 3 and 4 **(d)** 1 and 4

***51.** For the circuit board shown in Figure 83, determine the resistance between the following pins if there is a short between pins 3 and 4:

(a) 1 and 2 **(b)** 2 and 3 **(c)** 2 and 4 **(d)** 1 and 4


1 2 3 4 5 6
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8

▲ FIGURE 82


R_1
R_2
R_3
R_4
Pin 1
Pin 2
R_5
R_6
R_7
R_8
Pin 3
Pin 4
R_9
R_{10}
R_{11}
R_{12}

▲ FIGURE 83

Multisim Troubleshooting and Analysis

These problems require Multisim.

52. Open file P06-52 and measure the total parallel resistance.

53. Open file P06-53. Determine by measurement if there is an open resistor and, if so, which one.

54. Open file P06-54 and determine the unspecified resistance value.

55. Open file P06-55 and determine the unspecified source voltage.

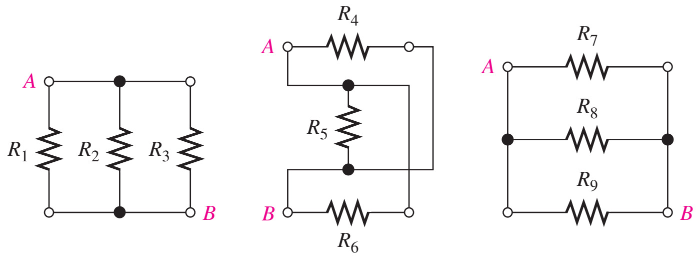
56. Open file P06-56 and find the fault if there is one.

ANSWERS

SECTION CHECKUPS

SECTION 1 Resistors in Parallel

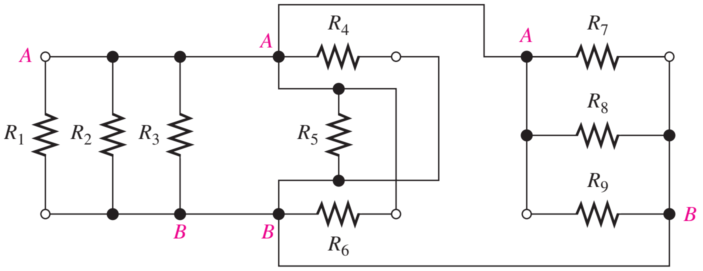
1. Parallel resistors are connected between the same two separate points.
2. A parallel circuit has more than one current path between two given points.
3. See Figure 84.



(a) (b) (c)

▲ FIGURE 84

4. See Figure 85.



▲ FIGURE 85

SECTION 2 Voltage in a Parallel Circuit

1. $V_{10\Omega} = V_{22\Omega} = 5\text{ V}$
2. $V_{R2} = 118\text{ V}; V_S = 118\text{ V}$
3. $V_{R1} = 50\text{ V}$ and $V_{R2} = 50\text{ V}$
4. Voltage is the same across all parallel branches.

SECTION 3 Kirchhoff's Current Law

1. Kirchhoff's law: The algebraic sum of all the currents at a junction is zero; The sum of the currents entering a junction equals the sum of the currents leaving that junction.

2. $I_1 = I_2 = I_3 = I_T = 2.5\text{ mA}$
3. $I_{OUT} = 100\text{ mA} + 300\text{ mA} = 400\text{ mA}$
4. 4 A
5. 10 A

SECTION 4 Total Parallel Resistance

1. R_T decreases with more resistors in parallel.
2. The total parallel resistance is less than the smallest branch resistance.
3. $R_T = \dfrac{1}{(1/R_1) + (1/R_2) + \cdots + (1/R_n)}$
4. $R_T = R_1R_2/(R_1 + R_2)$
5. $R_T = R/n$
6. $R_T = (1.0\text{ k}\Omega)(2.2\text{ k}\Omega)/3.2\text{ k}\Omega = 688\ \Omega$
7. $R_T = 1.0\text{ k}\Omega/4 = 250\ \Omega$
8. $R_T = \dfrac{1}{1/47\ \Omega + 1/150\ \Omega + 1/100\ \Omega} = 26.4\ \Omega$

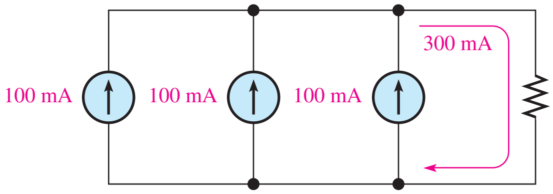
SECTION 5 Application of Ohm's Law

1. $I_T = 10\text{ V}/22.7\ \Omega = 44.1\text{ mA}$
2. $V_S = (20\text{ mA})(222\ \Omega) = 4.44\text{ V}$
3. $I_1 = 4.44\text{ V}/680\ \Omega = 6.53\text{ mA}; I_2 = 4.44\text{ V}/330\ \Omega = 13.5\text{ mA}$
4. $R_T = 12\text{ V}/5.85\text{ mA} = 2.05\text{ k}\Omega; R = (2.05\text{ k}\Omega)(4) = 8.2\text{ k}\Omega$
5. $V = (100\text{ mA})(688\ \Omega) = 68.8\text{ V}$

SECTION 6 Current Sources in Parallel

1. $I_T = 4(0.5\text{ A}) = 2\text{ A}$
2. Three sources; See Figure 86.
3. $I_{R_E} = 10\text{ mA} + 10\text{ mA} = 20\text{ mA}$

▶ FIGURE 86



SECTION 7 Current Dividers

1. $I_x = (R_T/R_x)I_T$
2. $I_1 = \left(\dfrac{R_2}{R_1 + R_2}\right)I_T \qquad I_2 = \left(\dfrac{R_1}{R_1 + R_2}\right)I_T$
3. The 22 kΩ has the most current; the 220 kΩ has the least current.
4. $I_1 = (680\ \Omega/1010\ \Omega)10\text{ mA} = 6.73\text{ mA}; I_2 = (330\ \Omega/1010\ \Omega)10\text{ mA} = 3.27\text{ mA}$
5. $I_3 = (114\ \Omega/470\ \Omega)4\text{ mA} = 970\ \mu\text{A}$

SECTION 8 Power in Parallel Circuits

1. Add the power of each resistor to get total power.
2. $P_T = 238\text{ mW} + 512\text{ mW} + 109\text{ mW} + 876\text{ mW} = 1.74\text{ W}$
3. $P_T = (1\text{ A})^2(615\ \Omega) = 615\text{ W}$
4. $I = P/V = 100\text{ W}/12.6\text{ V} = 7.9\text{ A}$. Choose a 10 A fuse.

SECTION 9 Parallel Circuit Applications

1. To bypass a portion of current around the meter movement.
2. R_{SH} is less than R_M because the shunt resistors must allow currents much greater than the current through the meter movement.

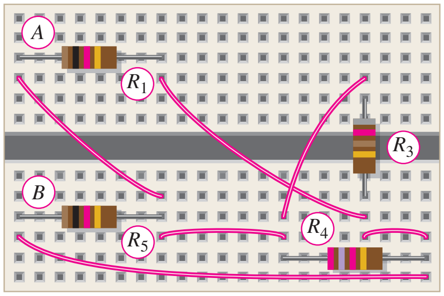
SECTION 10 Troubleshooting

1. When a branch opens, there is no change in voltage; the total current decreases.
2. If a branch opens, total parallel resistance increases.
3. The remaining bulbs continue to glow.
4. All remaining branch currents are 100 mA.
5. The branch with 120 mA is open.

RELATED PROBLEMS FOR EXAMPLES

1 See Figure 87.

▶ FIGURE 87



2 Connect pin 1 to pin 2 and pin 3 to pin 4.
3 25 V
4 20 mA into node *A* and out of node *B*
5 $I_T = 112\text{ mA}, I_2 = 50\text{ mA}$
6 2.5 mA; 5 mA
7 0.5 A
8 9.33 Ω
9 132 Ω
10 4 Ω
11 1044 Ω
12 1.83 mA; 1 mA
13 $I_1 = 20\text{ mA}; I_2 = 9.09\text{ mA}; I_3 = 35.7\text{ mA}, I_4 = 22.0\text{ mA}$
14 1.28 V
15 2.0 Ω
16 30 mA
17 $I_1 = 3.27\text{ mA}; I_2 = 6.73\text{ mA}$
18 $I_1 = 59.4\text{ mA}; I_2 = 40.6\text{ mA}$
19 1.78 W
20 81 W
21 15.4 mA
22 Not correct, R_{10} (68 kΩ) must be open.

TRUE/FALSE QUIZ

1. T **2.** T **3.** F **4.** F **5.** F **6.** T **7.** T **8.** T
9. F **10.** F

SELF-TEST

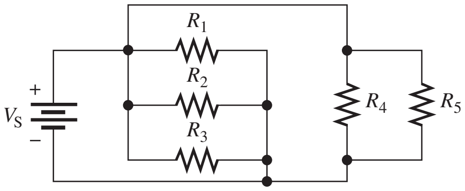
1. (b) **2.** (c) **3.** (b) **4.** (d) **5.** (a) **6.** (c) **7.** (c) **8.** (c)
9. (d) **10.** (b) **11.** (a) **12.** (d) **13.** (c) **14.** (b)

CIRCUIT DYNAMICS QUIZ

1. (c) **2.** (a) **3.** (c) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (c)
9. (b) **10.** (c) **11.** (b) **12.** (c) **13.** (a) **14.** (c) **15.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. See Figure 88.



▲ FIGURE 88

3. $R_1, R_2, R_5, R_9, R_{10}$, and R_{12} are in parallel.

R_4, R_6, R_7, and R_8 are in parallel.

R_3 and R_{11} are in parallel.

5. 100 V

7. Position A: $V_1 = 15\text{ V}, V_2 = 0\text{ V}, V_3 = 0\text{ V}, V_4 = 15\text{ V}$

Position B: $V_1 = 15\text{ V}, V_2 = 0\text{ V}, V_3 = 15\text{ V}, V_4 = 0\text{ V}$

Position C: $V_1 = 15\text{ V}, V_2 = 15\text{ V}, V_3 = 0\text{ V}, V_4 = 0\text{ V}$

9. 1.35 A

11. $R_2 = 22\ \Omega, R_3 = 100\ \Omega, R_4 = 33\ \Omega$

13. 11.4 mA

15. **(a)** 6.4 A **(b)** 6.4 A

17. **(a)** 359 Ω **(b)** 25.6 Ω **(c)** 819 Ω **(d)** 997 Ω

19. 567 Ω

21. 24.6 Ω

23. **(a)** 510 kΩ **(b)** 245 kΩ **(c)** 510 kΩ **(d)** 193 kΩ

25. 1.5 A

27. 50 mA; When one bulb burns out, the others remain on.

29. 53.7 Ω

31. $I_2 = 167\text{ mA}, I_3 = 83.3\text{ mA}, I_T = 300\text{ mA}$,

$R_1 = 2\text{ k}\Omega, R_2 = 600\ \Omega$

33. Position *A:* 2.25 mA

Position *B:* 4.75 mA

Position *C:* 7 mA

35. **(a)** $I_1 = 6.88\ \mu\text{A}, I_2 = 3.12\ \mu\text{A}$

(b) $I_1 = 5.25\text{ mA}, I_2 = 2.39\text{ mA}, I_3 = 1.59\text{ mA}$,

$I_4 = 772\ \mu\text{A}$

37. $R_1 = 3.3\text{ k}\Omega, R_2 = 1.8\text{ k}\Omega, R_3 = 5.6\text{ k}\Omega, R_4 = 3.9\text{ k}\Omega$

39. **(a)** 1 mΩ **(b)** 5 μA

41. **(a)** 68.8 μW **(b)** 52.5 mW

43. $P_1 = 1.25\text{ W}, I_2 = 75\text{ mA}, I_1 = 125\text{ mA}, V_S = 10\text{ V}$,

$R_1 = 80\ \Omega, R_2 = 133\ \Omega$

45. 625 mA, 3.13 A

47. The 8.2 kΩ resistor is open.

49. Connect ohmmeter between the following pins:

Pins 1-2

Correct reading: $R = 1.0\text{ k}\Omega \parallel 3.3\text{ k}\Omega = 767\ \Omega$

R_1 open: $R = 3.3\text{ k}\Omega$

R_2 open: $R = 1.0\text{ k}\Omega$

Pins 3-4

Correct reading: $R = 270\ \Omega \parallel 390\ \Omega = 159.5\ \Omega$

R_3 open: $R = 390\ \Omega$

R_4 open: $R = 270\ \Omega$

Pins 5-6

Correct reading:

$R = 1.0\text{ M}\Omega \parallel 1.8\text{ M}\Omega \parallel 680\text{ k}\Omega \parallel 510\text{ k}\Omega = 201\text{ k}\Omega$

R_5 open: $R = 1.8\text{ M}\Omega \parallel 680\text{ k}\Omega \parallel 510\text{ k}\Omega = 251\text{ k}\Omega$

R_6 open: $R = 1.0\text{ M}\Omega \parallel 680\text{ k}\Omega \parallel 510\text{ k}\Omega = 226\text{ k}\Omega$

R_7 open: $R = 1.0\text{ M}\Omega \parallel 1.8\text{ M}\Omega \parallel 510\text{ k}\Omega = 284\text{ k}\Omega$

R_8 open: $R = 1.0\text{ M}\Omega \parallel 1.8\text{ M}\Omega \parallel 680\text{ k}\Omega = 330\text{ k}\Omega$

51. Short between pins 3 and 4:

(a) $R_{1\text{-}2} = (R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_{11} \parallel R_{12})$

$+ (R_5 \parallel R_6 \parallel R_7 \parallel R_8 \parallel R_9 \parallel R_{10}) = 940\ \Omega$

(b) $R_{2\text{-}3} = R_5 \parallel R_6 \parallel R_7 \parallel R_8 \parallel R_9 \parallel R_{10} = 518\ \Omega$

(c) $R_{2\text{-}4} = R_5 \parallel R_6 \parallel R_7 \parallel R_8 \parallel R_9 \parallel R_{10} = 518\ \Omega$

(d) $R_{1\text{-}4} = R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_{11} \parallel R_{12} = 422\ \Omega$

53. R_2 open

55. $V_S = 3.30\text{ V}$

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SERIES-PARALLEL CIRCUITS

CHAPTER OUTLINE

1 Identifying Series-Parallel Relationships
2 Analysis of Series-Parallel Resistive Circuits
3 Voltage Dividers with Resistive Loads
4 Loading Effect of a Voltmeter
5 Ladder Networks
6 The Wheatstone Bridge
7 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Identify series-parallel relationships
- Analyze series-parallel circuits
- Analyze loaded voltage dividers
- Determine the loading effect of a voltmeter on a circuit
- Analyze ladder networks
- Analyze and apply a Wheatstone bridge
- Troubleshoot series-parallel circuits

KEY TERMS

- Bleeder current
- Wheatstone bridge
- Balanced bridge
- Unbalanced bridge

APPLICATION ACTIVITY PREVIEW

In this application activity, you will see how a Wheatstone bridge in conjunction with a thermistor can be used in a temperature-control application. The circuit in this application is designed to turn a heating element on and off in order to keep the temperature of a liquid in a tank at a desired level.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, both series and parallel resistors are combined into series-parallel circuits. In many practical situations, you will have both series and parallel combinations within the same circuit, and the analysis methods you learned for series circuits and for parallel circuits will apply.

Important types of series-parallel circuits are introduced in this chapter. These circuits include the voltage divider with a resistive load, the ladder network, and the Wheatstone bridge.

The analysis of series-parallel circuits requires the use of Ohm's law, Kirchhoff's voltage and current laws, and methods for finding total resistance and power. The topic of loaded voltage dividers is important because this type of circuit is found in many practical situations, such as the voltage-divider bias circuit for a transistor amplifier. Ladder networks are important in several areas, including a major type of digital-to-analog conversion, which you will study in a digital fundamentals course. The Wheatstone bridge is used in many types of systems for the measurement of unknown parameters, including most electronic scales.

From Chapter 7 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 IDENTIFYING SERIES-PARALLEL RELATIONSHIPS

A series-parallel circuit consists of combinations of both series and parallel current paths. It is important to be able to identify how the components in a circuit are arranged in terms of their series and parallel relationships.

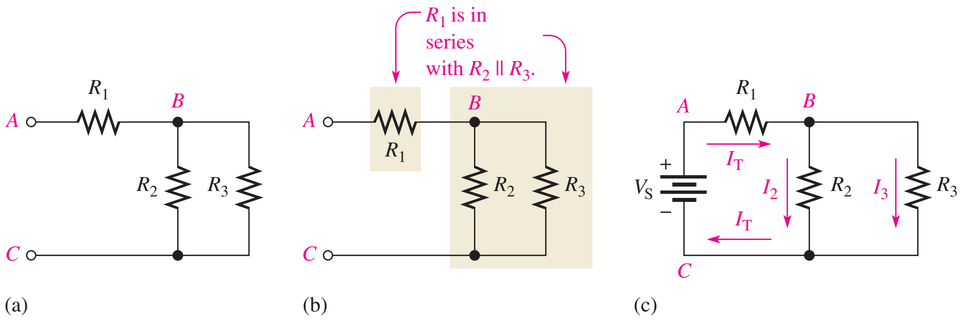
After completing this section, you should be able to

- **Identify series-parallel relationships**
  - Recognize how each resistor in a given circuit is related to the other resistors
  - Determine series and parallel relationships on a PC board

Figure 1(a) shows an example of a simple series-parallel combination of resistors. Notice that the resistance from point *A* to point *B* is R_1. The resistance from point *B* to point *C* is R_2 and R_3 in parallel ($R_2 \| R_3$). The total resistance from point *A* to point *C* is R_1 in series with the parallel combination of R_2 and R_3, as indicated in Figure 1(b). The term *point* can refer to either a node or a terminal. For example, in Figure 1(a), *A* is a terminal because it is an end point, *B* is a node because it is the junction of two or more components, and *C* is both a terminal and a node. So, the term *point* will sometimes be used to represent either or both.

FIGURE 1

A simple series-parallel resistive circuit.

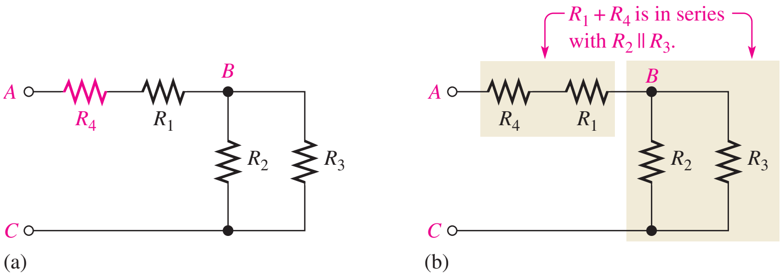


When the circuit of Figure 1(a) is connected to a voltage source as shown in Figure 1(c), the total current is through R_1 and divides at point *B* into the two parallel paths. These two branch currents then recombine, and the total current is into the negative source terminal as shown.

Now, to illustrate series-parallel relationships, let's increase the complexity of the circuit in Figure 1(a) step-by-step. In Figure 2(a), another resistor (R_4) is connected in

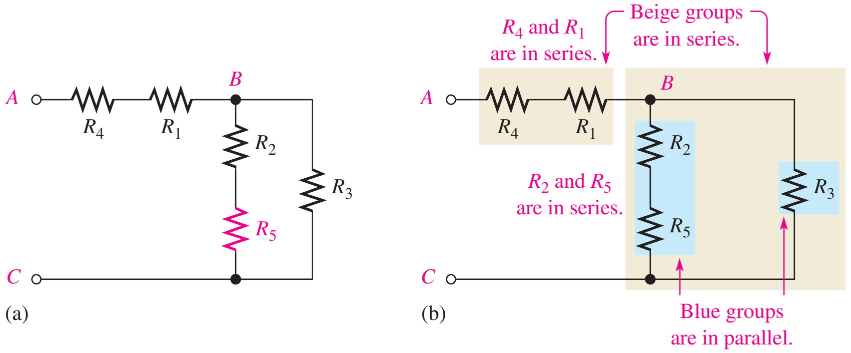
FIGURE 2

R_4 is added to the circuit in series with R_1.



series with R_1. The resistance between points A and B is now $R_1 + R_4$, and this combination is in series with the parallel combination of R_2 and R_3, as illustrated in Figure 2(b).

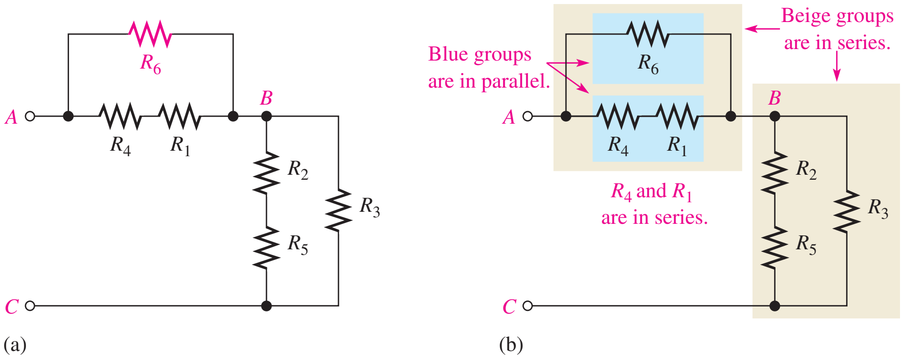
In Figure 3(a), R_5 is connected in series with R_2. The series combination of R_2 and R_5 is in parallel with R_3. This entire series-parallel combination is in series with the series combination of R_1 and R_4, as illustrated in Figure 3(b).


A
B
C
R_4
R_1
R_2
R_3
R_5
(a)
R_4 and R_1 are in series.
Beige groups are in series.
A
B
C
R_4
R_1
R_2
R_5
R_3
R_2 and R_5 are in series.
Blue groups are in parallel.
(b)

▲ **FIGURE 3**

R_5 is added to the circuit in series with R_2.

In Figure 4(a), R_6 is connected in parallel with the series combination of R_1 and R_4. The series-parallel combination of R_1, R_4, and R_6 is in series with the series-parallel combination of R_2, R_3, and R_5, as indicated in Figure 4(b).


R_6
A
B
C
R_4
R_1
R_2
R_3
R_5
(a)
Blue groups are in parallel.
Beige groups are in series.
R_6
A
B
C
R_4
R_1
R_4 and R_1 are in series.
R_2
R_3
R_5
(b)

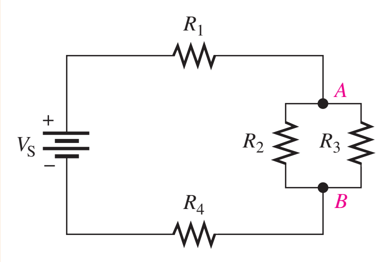
▲ **FIGURE 4**

R_6 is added to the circuit in parallel with the series combination of R_1 and R_4.

EXAMPLE 1

Identify the series-parallel relationships in Figure 5.

▶ **FIGURE 5**

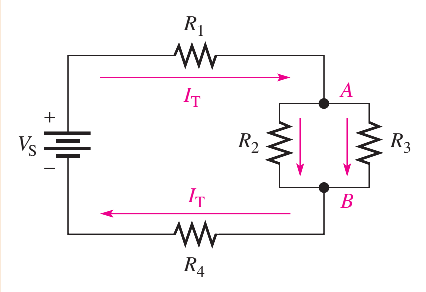

R_1
A
+
V_S
−
R_2
R_3
R_4
B

Solution Starting at the positive terminal of the source, follow the current paths. All of the current produced by the source must go through R_1, which is in series with the rest of the circuit.

The total current takes two paths when it gets to node *A*. Part of it is through R_2, and part of it is through R_3. Resistors R_2 and R_3 are in parallel with each other, and this parallel combination is in series with R_1.

At node *B*, the currents through R_2 and R_3 come together again. Thus, the total current is through R_4. Resistor R_4 is in series with R_1 and the parallel combination of R_2 and R_3. The currents are shown in Figure 6, where I_T is the total current.

▶ FIGURE 6

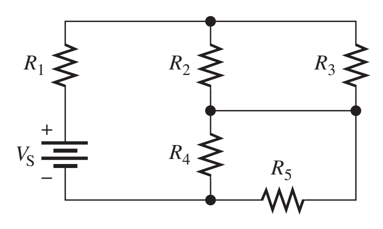


In summary, R_1 and R_4 are in series with the parallel combination of R_2 and R_3 as stated by the following expression:

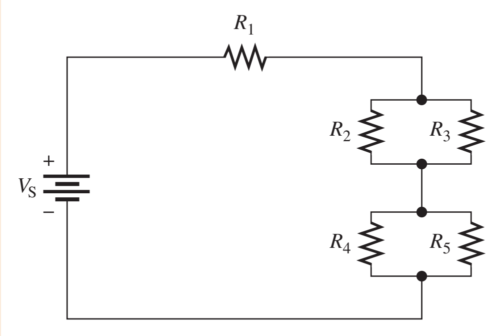
$$R_1 + R_2 \| R_3 + R_4$$

Related Problem* If another resistor, R_5, is connected from node *A* to the negative side of the source in Figure 6, what is its relationship to the other resistors?

EXAMPLE 2 Identify the series-parallel relationships in Figure 7.



▲ FIGURE 7



▲ FIGURE 8

Solution Sometimes it is easier to see a particular circuit arrangement if it is drawn in a different way. In this case, the circuit schematic is redrawn in Figure 8, which better illustrates the series-parallel relationships. Now you can see that R_2 and R_3 are in parallel with each other and also that R_4 and R_5 are in parallel with each other. Both parallel combinations are in series with each other and with R_1 as stated by the following expression:

---

*Answers are at the end of the chapter.

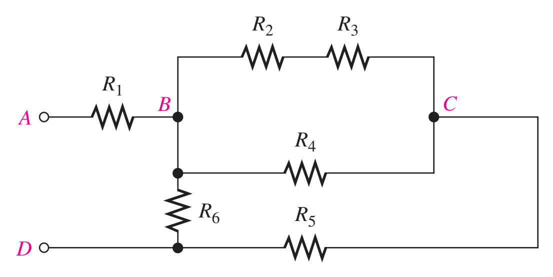
$$R_1 + R_2 \parallel R_3 + R_4 \parallel R_5$$

Related Problem If a resistor is connected from the bottom end of R_3 to the top end of R_5 in Figure 8, what effect does it have on the circuit?

EXAMPLE 3

Describe the series-parallel combination between terminals *A* and *D* in Figure 9.

▶ **FIGURE 9**



Solution Between nodes *B* and *C*, there are two parallel paths. The lower path consists of R_4, and the upper path consists of a series combination of R_2 and R_3. This parallel combination is in series with R_5. The R_2, R_3, R_4, R_5 combination is in parallel with R_6. Resistor R_1 is in series with this entire combination as stated by the following expression:

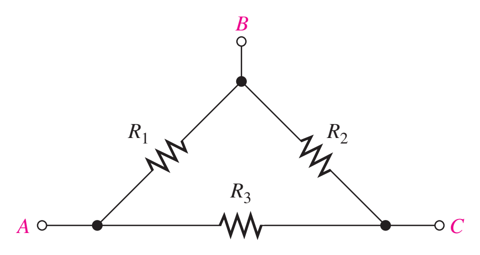
$$R_1 + R_6 \parallel (R_5 + R_4 \parallel (R_2 + R_3))$$

Related Problem If a resistor is connected from *C* to *D* in Figure 9, describe its parallel relationship.

EXAMPLE 4

Describe the total resistance between each pair of terminals in Figure 10.

▶ **FIGURE 10**



Solution

1. From *A* to *B*: R_1 is in parallel with the series combination of R_2 and R_3.

$$R_1 \parallel (R_2 + R_3)$$

2. From *A* to *C*: R_3 is in parallel with the series combination of R_1 and R_2.

$$R_3 \parallel (R_1 + R_2)$$

3. From *B* to *C*: R_2 is in parallel with the series combination of R_1 and R_3.

$$R_2 \parallel (R_1 + R_3)$$

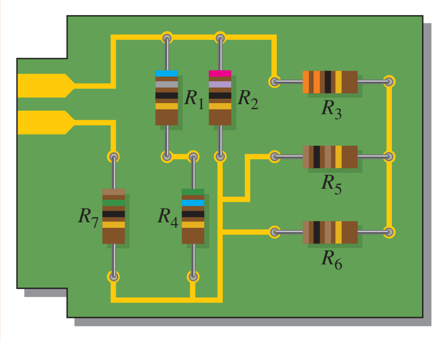
Related Problem In Figure 10, describe the total resistance between each terminal and an added ground if a new resistor, R_4, is connected from C to ground. None of the existing resistors connect directly to the ground.

Usually, the physical arrangement of components on a PC or protoboard bears no resemblance to the actual circuit relationships. By tracing out the circuit and rearranging the components on paper into a recognizable form, you can determine the series-parallel relationships.

EXAMPLE 5

Determine the relationships of the resistors on the PC board in Figure 11.

▶ FIGURE 11



R1
R2
R3
R5
R7
R4
R6

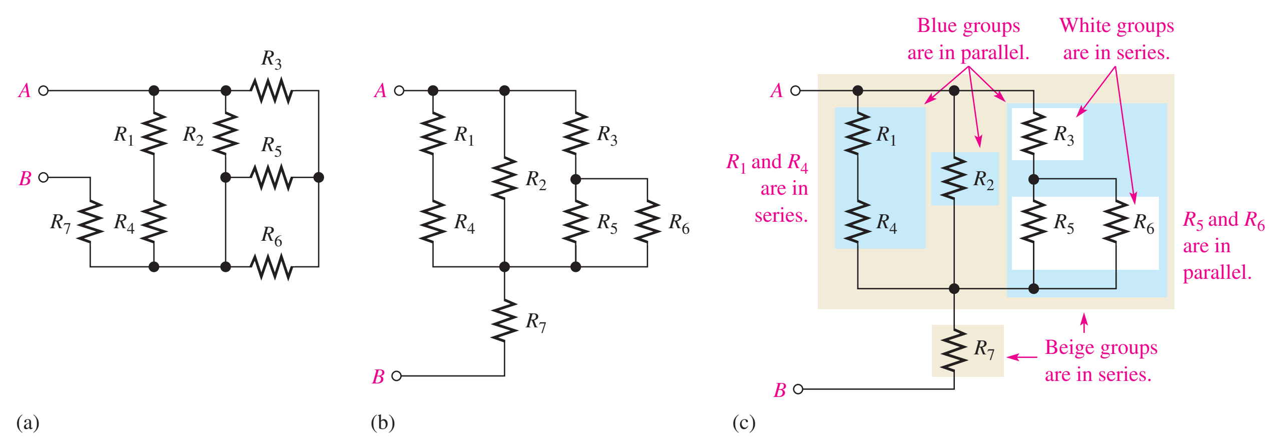
Solution In Figure 12(a), the schematic is drawn in the same arrangement as that of the resistors on the board. In part (b), the resistors are rearranged so that the series-parallel relationships are more obvious.

Resistors R_1 and R_4 are in series; $R_1 + R_4$ is in parallel with R_2; R_5 and R_6 are in parallel and this combination is in series with R_3.

The R_3, R_5, and R_6 series-parallel combination is in parallel with both R_2 and the $R_1 + R_4$ combination. This entire series-parallel combination is in series with R_7.

Figure 12(c) illustrates these relationships. Summarizing in equation form,

$$R_{AB} = (R_5 \| R_6 + R_3) \| R_2 \| (R_1 + R_4) + R_7$$



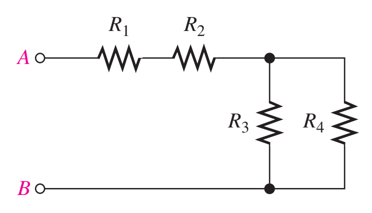
A
B
R3
R1
R2
R5
R7
R4
R6
(a)
A
R1
R3
R2
R4
R5
R6
R7
B
(b)
Blue groups are in parallel.
White groups are in series.
A
R1
R3
R2
R1 and R4 are in series.
R4
R5
R6
R5 and R6 are in parallel.
R7
Beige groups are in series.
B
(c)

▲ FIGURE 12

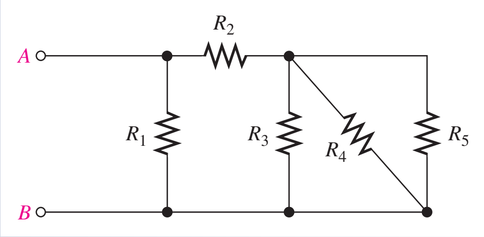
Related Problem If R_5 were removed from the circuit, what would be the relationship of R_3 and R_6?

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. Define *series-parallel resistive circuit.*
2. A certain series-parallel circuit is described as follows: R_1 and R_2 are in parallel. This parallel combination is in series with another parallel combination of R_3 and R_4. Draw the circuit.
3. In the circuit of Figure 13, describe the series-parallel relationships of the resistors.
4. Which resistors are in parallel in Figure 14?



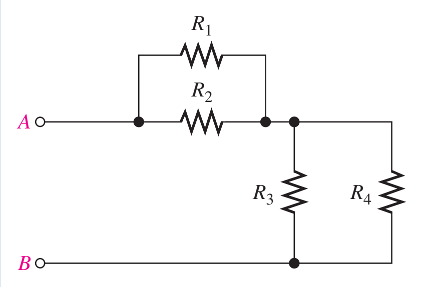
▲ FIGURE 13



▲ FIGURE 14

5. Describe the parallel arrangements in Figure 15.
6. Are the parallel combinations in Figure 15 in series?

▶ FIGURE 15



2 ANALYSIS OF SERIES-PARALLEL RESISTIVE CIRCUITS

The analysis of series-parallel circuits can be approached in many ways, depending on what information you need and what circuit values you know. The examples in this section do not represent an exhaustive coverage, but they give you an idea of how to approach series-parallel circuit analysis.

After completing this section, you should be able to

- **Analyze series-parallel circuits**
  - Determine total resistance
  - Determine all the currents
  - Determine all the voltage drops

If you know Ohm's law, Kirchhoff's laws, the voltage-divider formula, and the current-divider formula, and if you know how to apply these laws, you can solve most resistive circuit analysis problems. The ability to recognize series and parallel combinations is, of course,

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This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

essential. A few circuits, such as the unbalanced Wheatstone bridge, do not have basic series and parallel combinations. Other methods are needed for these cases, as we will discuss later.

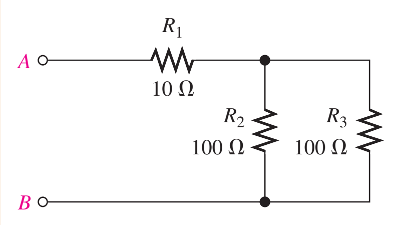
Total Resistance

To find the total resistance (R_T) of a series-parallel combination, simply define the series and parallel relationships; then perform the calculations that you have previously learned. The following two examples illustrate this general approach.

EXAMPLE 6

Determine R_T of the circuit in Figure 16 between terminals *A* and *B*.

▶ FIGURE 16



Solution First, calculate the equivalent parallel resistance of R_2 and R_3. Since R_2 and R_3 are equal in value, you can use this equation:

$$R_{2\|3} = \frac{R}{n} = \frac{100\ \Omega}{2} = 50\ \Omega$$

Notice that the term $R_{2\|3}$ is used here to designate the total resistance of a portion of a circuit in order to distinguish it from the total resistance, R_T, of the complete circuit.

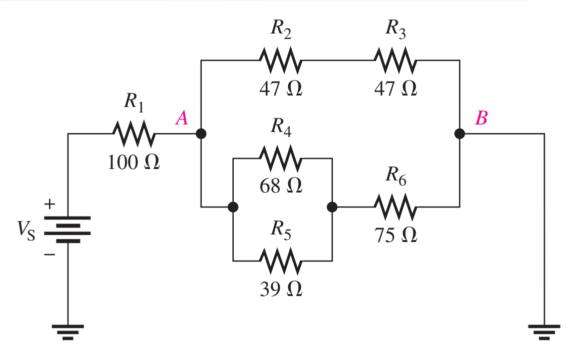
Now, since R_1 is in series with $R_{2\|3}$, add their values as follows:

$$R_T = R_1 + R_{2\|3} = 10\ \Omega + 50\ \Omega = \mathbf{60\ \Omega}$$

Related Problem Determine R_T in Figure 16 if R_3 is changed to 82 Ω.

EXAMPLE 7

Find the total resistance between the positive and negative terminals of the battery in Figure 17.



▲ FIGURE 17

Solution In the upper branch, R_2 is in series with R_3. This series combination is designated R_{2+3} and is equal to $R_2 + R_3$.

$$R_{2+3} = R_2 + R_3 = 47\ \Omega + 47\ \Omega = 94\ \Omega$$

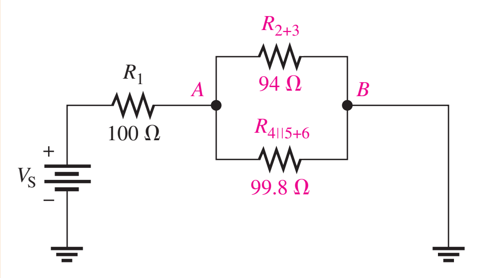
In the lower branch, R_4 and R_5 are in parallel with each other. This parallel combination is designated $R_{4\|5}$.

$$R_{4\|5} = \frac{R_4R_5}{R_4 + R_5} = \frac{(68\ \Omega)(39\ \Omega)}{68\ \Omega + 39\ \Omega} = 24.8\ \Omega$$

Also in the lower branch, the parallel combination of R_4 and R_5 is in series with R_6. This series-parallel combination is designated $R_{4\|5+6}$.

$$R_{4\|5+6} = R_6 + R_{4\|5} = 75\ \Omega + 24.8\ \Omega = 99.8\ \Omega$$

Figure 18 shows the original circuit in a simplified equivalent form.



▲ FIGURE 18

Now you can find the equivalent resistance between A and B. It is R_{2+3} in parallel with $R_{4\|5+6}$. Calculate the equivalent resistance as follows:

$$R_{AB} = \frac{1}{\dfrac{1}{R_{2+3}} + \dfrac{1}{R_{4\|5+6}}} = \frac{1}{\dfrac{1}{94\ \Omega} + \dfrac{1}{99.8\ \Omega}} = 48.4\ \Omega$$

Finally, the total resistance is R_1 in series with R_{AB}.

$$R_T = R_1 + R_{AB} = 100\ \Omega + 48.4\ \Omega = \mathbf{148.4\ \Omega}$$

Related Problem Determine R_T if a 68 Ω resistor is added in parallel from A to B in Figure 17.

Total Current

Once you know the total resistance and the source voltage, you can apply Ohm's law to find the total current in a circuit. Total current is the source voltage divided by the total resistance.

$$I_T = \frac{V_S}{R_T}$$

For example, assuming that the source voltage is 10 V, the total current in the circuit of Example 7 (Figure 17) is

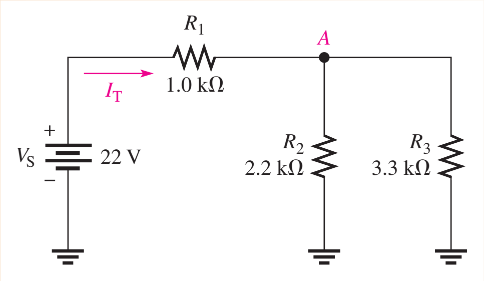
$$I_T = \frac{V_S}{R_T} = \frac{10\ \text{V}}{148.4\ \Omega} = 67.4\ \text{mA}$$

Branch Currents

Using the current-divider formula, Kirchhoff's current law, Ohm's law, or combinations of these, you can find the current in any branch of a series-parallel circuit. In some cases, it may take repeated application of the formula to find a given current. The following two examples will help you understand the procedure. (Notice that the subscripts for the current variables (I) match the R subscripts; for example, current through R_1 is referred to as I_1.)

EXAMPLE 8

Find the current through R_2 and the current through R_3 in Figure 19.



▲ FIGURE 19

Solution First, identify the series and parallel relationship. Next, determine how much current is into node A. This is the total circuit current. To find I_T, you must know R_T.

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} = 1.0\text{ k}\Omega + \frac{(2.2\text{ k}\Omega)(3.3\text{ k}\Omega)}{2.2\text{ k}\Omega + 3.3\text{ k}\Omega} = 1.0\text{ k}\Omega + 1.32\text{ k}\Omega = 2.32\text{ k}\Omega$$

$$I_T = \frac{V_S}{R_T} = \frac{22\text{ V}}{2.32\text{ k}\Omega} = 9.48\text{ mA}$$

Use the current-divider rule for two branches to find the current through R_2.

$$I_2 = \left(\frac{R_3}{R_2 + R_3}\right)I_T = \left(\frac{3.3\text{ k}\Omega}{5.5\text{ k}\Omega}\right)9.48\text{ mA} = \mathbf{5.69\text{ mA}}$$

Now you can use Kirchhoff's current law to find the current through R_3.

$$I_T = I_2 + I_3$$
$$I_3 = I_T - I_2 = 9.48\text{ mA} - 5.69\text{ mA} = \mathbf{3.79\text{ mA}}$$

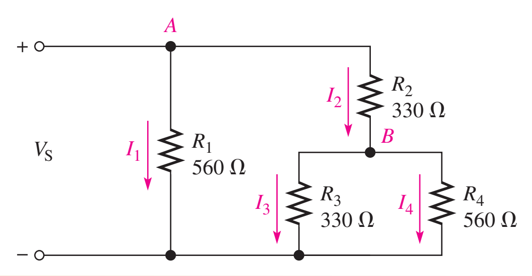
Related Problem A 4.7 kΩ resistor is connected in parallel with R_3 in Figure 19. Determine the current through the new resistor.

Use Multisim file E07-08 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 9

Determine the current through R_4 in Figure 20 if $V_S = 5\text{ V}$.

▶ FIGURE 20



Solution First, find the current (I_2) into node *B*. Once you know this current, use the current-divider formula to find I_4, the current through R_4.

Notice that there are two main branches in the circuit. The left-most branch consists of only R_1. The right-most branch has R_2 in series with the parallel combination of R_3 and R_4. The voltage across both of these main branches is the same and equal to 5 V. Calculate the equivalent resistance ($R_{2+3\|4}$) of the right-most main branch and then apply Ohm's law; I_2 is the total current through this main branch. Thus,

$$R_{2+3\|4} = R_2 + \frac{R_3R_4}{R_3 + R_4} = 330\ \Omega + \frac{(330\ \Omega)(560\ \Omega)}{890\ \Omega} = 538\ \Omega$$

$$I_2 = \frac{V_S}{R_{2+3\|4}} = \frac{5\text{ V}}{538\ \Omega} = 9.3\text{ mA}$$

Use the two-resistor current-divider formula to calculate I_4.

$$I_4 = \left(\frac{R_3}{R_3 + R_4}\right)I_2 = \left(\frac{330\ \Omega}{890\ \Omega}\right)9.3\text{ mA} = \mathbf{3.45\ mA}$$

Related Problem Determine the current through R_1 and R_3 in Figure 20 if $V_S = 2\text{ V}$.

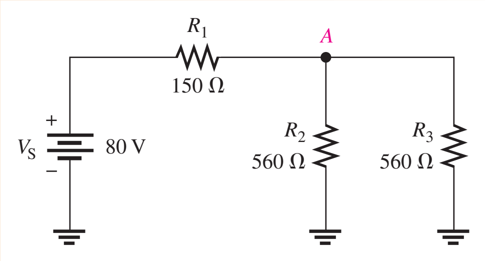
Voltage Drops

To find the voltages across certain parts of a series-parallel circuit, you can use the voltage-divider formula, Kirchhoff's voltage law, Ohm's law, or combinations of each. The following three examples illustrate use of the formulas. (The subscripts for *V* match the subscripts for the corresponding *R*: V_1 is the voltage across R_1; V_2 is the voltage across R_2, etc.)

EXAMPLE 10

Determine the voltage drop from node *A* to ground in Figure 21. Then find the voltage (V_1) across R_1.

▶ FIGURE 21



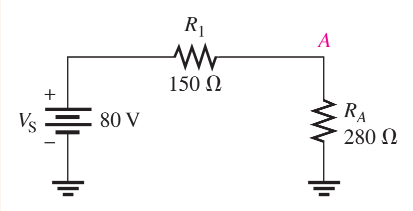
Solution Note that R_2 and R_3 are in parallel in this circuit. Since they are equal in value, their equivalent resistance from node *A* to ground is

$$R_A = \frac{560\ \Omega}{2} = 280\ \Omega$$

In the equivalent circuit shown in Figure 22, R_1 is in series with R_A. The total circuit resistance as seen from the source is

$$R_T = R_1 + R_A = 150\ \Omega + 280\ \Omega = 430\ \Omega$$

▶ **FIGURE 22**



Use the voltage-divider formula to find the voltage across the parallel combination of Figure 21 (between node *A* and ground).

$$V_A = \left(\frac{R_A}{R_T}\right)V_S = \left(\frac{280\ \Omega}{430\ \Omega}\right)80\ \text{V} = \mathbf{52.1\ V}$$

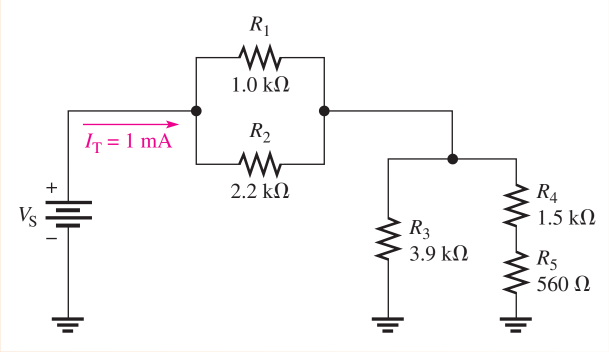
Now use Kirchhoff's voltage law to find V_1.

$$V_S = V_1 + V_A$$
$$V_1 = V_S - V_A = 80\ \text{V} - 52.1\ \text{V} = \mathbf{27.9\ V}$$

Related Problem Determine V_A and V_1 if R_1 is changed to 220 Ω in Figure 21.

Use Multisim file E07-10 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 11 Determine the voltage drop across each resistor in the circuit of Figure 23.



▲ **FIGURE 23**

Solution The source voltage is not given, but you know the total current from the figure. Since R_1 and R_2 are in parallel, they each have the same voltage. The current through R_1 is

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{2.2\text{ k}\Omega}{3.2\text{ k}\Omega}\right)1\text{ mA} = 688\ \mu\text{A}$$

The voltages across R_1 and R_2 are

$$V_1 = I_1R_1 = (688\ \mu\text{A})(1.0\text{ k}\Omega) = \mathbf{688\ mV}$$
$$V_2 = V_1 = \mathbf{688\ mV}$$

The series combination of R_4 and R_5 form the branch resistance, R_{4+5}. Apply the current-divider formula to determine the current through R_3.

$$I_3 = \left(\frac{R_{4+5}}{R_3 + R_{4+5}}\right)I_T = \left(\frac{2.06\text{ k}\Omega}{5.96\text{ k}\Omega}\right)1\text{ mA} = 346\ \mu\text{A}$$

The voltage across R_3 is

$$V_3 = I_3R_3 = (346\ \mu\text{A})(3.9\text{ k}\Omega) = \mathbf{1.35\ V}$$

The currents through R_4 and R_5 are the same because these resistors are in series.

$$I_4 = I_5 = I_T - I_3 = 1\text{ mA} - 346\ \mu\text{A} = 654\ \mu\text{A}$$

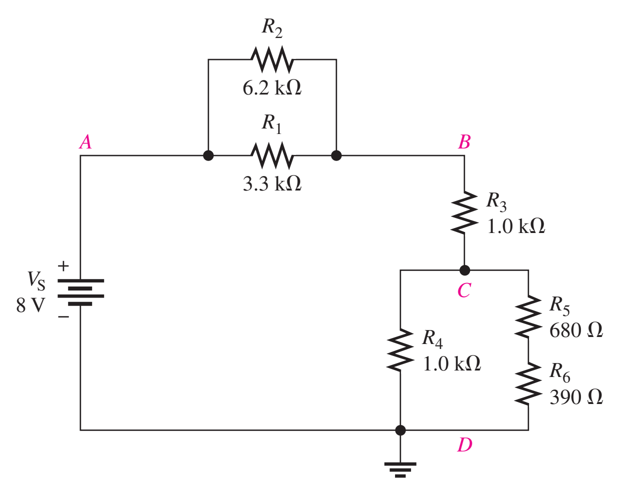
Calculate the voltages across R_4 and R_5 as follows:

$$V_4 = I_4R_4 = (654\ \mu\text{A})(1.5\text{ k}\Omega) = \mathbf{981\ mV}$$
$$V_5 = I_5R_5 = (654\ \mu\text{A})(560\ \Omega) = \mathbf{366\ mV}$$

Related Problem What is the source voltage, V_S, in the circuit of Figure 23?

Use Multisim file E07-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 12 Determine the voltage drop across each resistor in Figure 24.



▲ FIGURE 24

Solution Because the total voltage is given in the figure, you can solve this problem using the voltage-divider formula. First, you need to reduce each parallel combination to an equivalent resistance. Since R_1 and R_2 are in parallel between A and B, combine their values.

$$R_{AB} = \frac{R_1R_2}{R_1 + R_2} = \frac{(3.3\text{ k}\Omega)(6.2\text{ k}\Omega)}{9.5\text{ k}\Omega} = 2.15\text{ k}\Omega$$

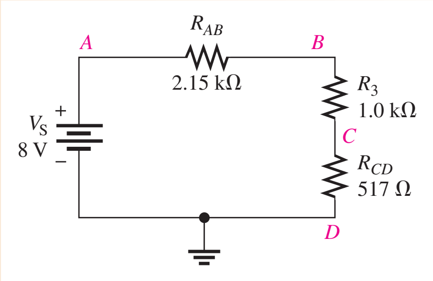
Since R_4 is in parallel with the R_5 and R_6 series combination (R_{5+6}) between C and D, combine these values.

$$R_{CD} = \frac{R_4R_{5+6}}{R_4 + R_{5+6}} = \frac{(1.0\text{ k}\Omega)(1.07\text{ k}\Omega)}{2.07\text{ k}\Omega} = 517\ \Omega$$

The equivalent circuit is drawn in Figure 25. The total circuit resistance is

$$R_{\text{T}} = R_{AB} + R_3 + R_{CD} = 2.15\text{ k}\Omega + 1.0\text{ k}\Omega + 517\ \Omega = 3.67\text{ k}\Omega$$

▶ FIGURE 25



Next, use the voltage-divider formula to determine the voltages in the equivalent circuit.

$$V_{AB} = \left(\frac{R_{AB}}{R_{\text{T}}}\right)V_{\text{S}} = \left(\frac{2.15\text{ k}\Omega}{3.67\text{ k}\Omega}\right)8\text{ V} = 4.69\text{ V}$$

$$V_{CD} = \left(\frac{R_{CD}}{R_{\text{T}}}\right)V_{\text{S}} = \left(\frac{517\ \Omega}{3.67\text{ k}\Omega}\right)8\text{ V} = 1.13\text{ V}$$

$$V_3 = \left(\frac{R_3}{R_{\text{T}}}\right)V_{\text{S}} = \left(\frac{1.0\text{ k}\Omega}{3.67\text{ k}\Omega}\right)8\text{ V} = \mathbf{2.18\text{ V}}$$

Refer to Figure 24. V_{AB} equals the voltage across both R_1 and R_2, so

$$V_1 = V_2 = V_{AB} = \mathbf{4.69\text{ V}}$$

V_{CD} is the voltage across R_4 and across the series combination of R_5 and R_6. Therefore,

$$V_4 = V_{CD} = \mathbf{1.13\text{ V}}$$

Now apply the voltage-divider formula to the series combination of R_5 and R_6 to get V_5 and V_6.

$$V_5 = \left(\frac{R_5}{R_5 + R_6}\right)V_{CD} = \left(\frac{680\ \Omega}{1070\ \Omega}\right)1.13\text{ V} = \mathbf{718\text{ mV}}$$

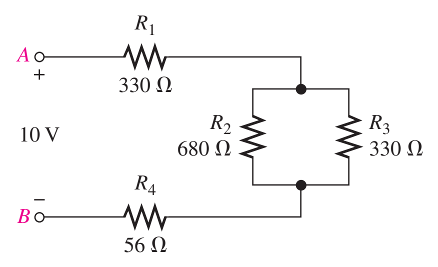
$$V_6 = \left(\frac{R_6}{R_5 + R_6}\right)V_{CD} = \left(\frac{390\ \Omega}{1070\ \Omega}\right)1.13\text{ V} = \mathbf{412\text{ mV}}$$

Related Problem R_2 is removed from the circuit in Figure 24. Calculate V_{AB}, V_{BC}, and V_{CD}.

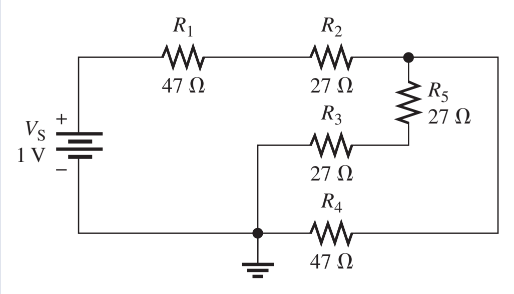
Use Multisim file E07-12 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 2 CHECKUP

1. List four circuit laws and formulas that may be necessary in the analysis of series-parallel circuits.
2. Find the total resistance between *A* and *B* in the circuit of Figure 26.
3. Find the current through R_3 in Figure 26.
4. Find the voltage drop across R_2 in Figure 26.
5. Determine R_T and I_T in Figure 27 as "seen" by the source.



▲ FIGURE 26



▲ FIGURE 27

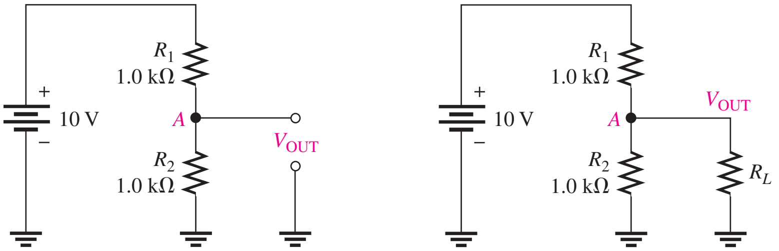
3 VOLTAGE DIVIDERS WITH RESISTIVE LOADS

In this section, you will learn how resistive loads affect the operation of voltage-divider circuits.

After completing this section, you should be able to

- **Analyze loaded voltage dividers**
    - Determine the effect of a resistive load on a voltage-divider circuit
    - Define *bleeder current*

The voltage divider in Figure 28(a) produces an output voltage (V_{OUT}) of 5 V because the two resistors are of equal value. This voltage is the *unloaded output voltage*. When a load resistor, R_L, is connected from the output to ground as shown in Figure 28(b), the output voltage is reduced by an amount that depends on the value of R_L. The load resistor is in parallel with R_2, reducing the resistance from node *A* to ground and, as a result, also



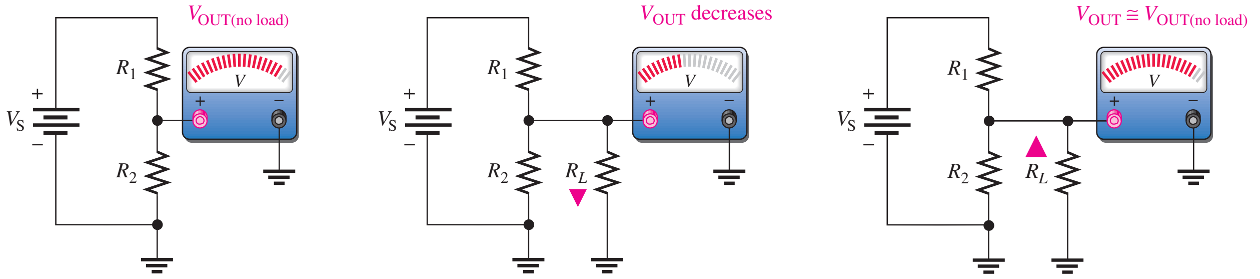
(a) Unloaded (b) Loaded

▲ FIGURE 28

A voltage divider with both unloaded and loaded outputs.

reducing the voltage across the parallel combination. This is one effect of loading a voltage divider. Another effect of a load is that more current is drawn from the source because the total resistance of the circuit is reduced.

The larger R_L is, compared to R_2, the less the output voltage is reduced from its unloaded value, as illustrated in Figure 29. When R_L is large compared to R_2 (at least ten times), the loading effect is small, and the output voltage will change only a small amount from its unloaded value, as shown in Figure 29(c). In this case, the divider is said to be a **stiff voltage divider.**



(a) No load

(b) R_L not significantly greater than R_2

(c) R_L much greater than R_2

▲ FIGURE 29

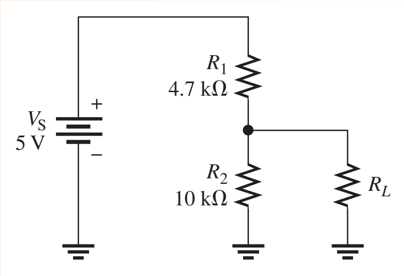
The effect of a load resistor.

EXAMPLE 13

(a) Determine the unloaded output voltage of the voltage divider in Figure 30.

(b) Find the loaded output voltages of the voltage divider in Figure 30 for the following two values of load resistance: $R_L = 10\ \text{k}\Omega$ and $R_L = 100\ \text{k}\Omega$.

▶ FIGURE 30



Solution **(a)** The unloaded output voltage is

$$V_{OUT(unloaded)} = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{10\ \text{k}\Omega}{14.7\ \text{k}\Omega}\right)5\ \text{V} = \mathbf{3.40\ V}$$

(b) With the 10 kΩ load resistor connected, R_L is in parallel with R_2, which gives

$$R_2 \parallel R_L = \frac{R_2 R_L}{R_2 + R_L} = \frac{100\ \text{M}\Omega}{20\ \text{k}\Omega} = 5\ \text{k}\Omega$$

The equivalent circuit is shown in Figure 31(a). The loaded output voltage is

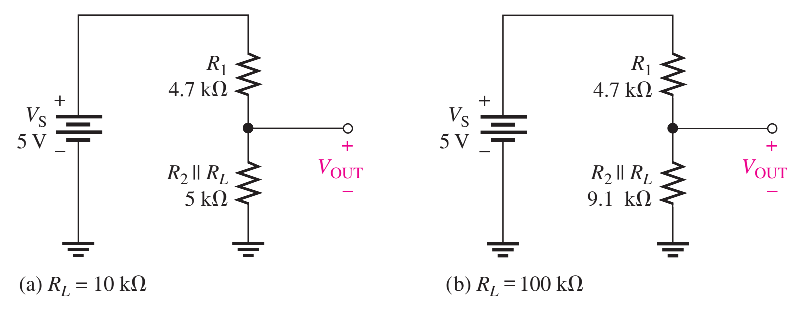
$$V_{OUT(loaded)} = \left(\frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L}\right)V_S = \left(\frac{5\ \text{k}\Omega}{9.7\ \text{k}\Omega}\right)5\ \text{V} = \mathbf{2.58\ V}$$

With the 100 kΩ load, the resistance from output to ground is

$$R_2 \parallel R_L = \frac{R_2 R_L}{R_2 + R_L} = \frac{(10\ \text{k}\Omega)(100\ \text{k}\Omega)}{110\ \text{k}\Omega} = 9.1\ \text{k}\Omega$$

The equivalent circuit is shown in Figure 31(b). The loaded output voltage is

$$V_{\text{OUT(loaded)}} = \left(\frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L}\right)V_S = \left(\frac{9.1\ \text{k}\Omega}{13.8\ \text{k}\Omega}\right)5\ \text{V} = \mathbf{3.30\ V}$$



V_S
5 V
+
−
R_1
4.7 kΩ
$R_2 \parallel R_L$
5 kΩ
+
V_{OUT}
−
V_S
5 V
+
−
R_1
4.7 kΩ
$R_2 \parallel R_L$
9.1 kΩ
+
V_{OUT}
−

(a) $R_L = 10$ kΩ (b) $R_L = 100$ kΩ

▲ FIGURE 31

For the smaller value of R_L, the reduction in V_{OUT} is

$$3.40\ \text{V} - 2.58\ \text{V} = 0.82\ \text{V} \qquad \text{(a 24\% drop in output voltage)}$$

For the larger value of R_L, the reduction in V_{OUT} is

$$3.40\ \text{V} - 3.30\ \text{V} = 0.10\ \text{V} \qquad \text{(a 3\% drop in output voltage)}$$

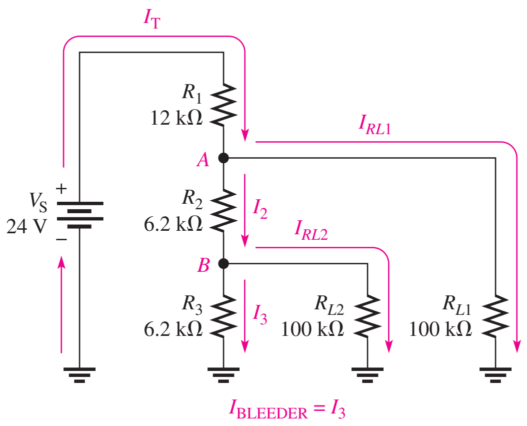
This illustrates the loading effect of R_L on the voltage divider.

Related Problem Determine V_{OUT} in Figure 30 for a 1.0 MΩ load resistance.

Use Multisim file E07-13 to verify the calculated results in this example and to confirm your calculation for the related problem.

Load Current and Bleeder Current

In a multiple-tap loaded voltage-divider circuit, the total current drawn from the source consists of currents through the load resistors, called *load currents*, and the divider resistors. Figure 32 shows a voltage divider with two voltage outputs or two taps. Notice that the total current, I_T, through R_1 enters node A where the current divides into I_{RL1} through R_{L1} and into I_2 through R_2. At node B, the current I_2 divides into I_{RL2} through R_{RL2} and into



I_T
R_1
12 kΩ
I_{RL1}
A
V_S
24 V
+
−
R_2
6.2 kΩ
I_2
I_{RL2}
B
R_3
6.2 kΩ
I_3
R_{L2}
100 kΩ
R_{L1}
100 kΩ
$I_{\text{BLEEDER}} = I_3$

◀ FIGURE 32

Currents in a two-tap loaded voltage divider.

I_3 through R_3. Current I_3 is called the **bleeder current**, which is the current left after the total load current is subtracted from the total current in the circuit.

Equation 1

$$I_{\text{BLEEDER}} = I_{\text{T}} - I_{RL1} - I_{RL2}$$

EXAMPLE 14

Determine the load currents I_{RL1} and I_{RL2} and the bleeder current I_3 in the two-tap loaded voltage divider in Figure 32.

Solution

The equivalent resistance from node A to ground is the 100 kΩ load resistor R_{L1} in parallel with the combination of R_2 in series with the parallel combination of R_3 and R_{L2}. Determine the resistance values first. R_3 in parallel with R_{L2} is designated R_B. The resulting equivalent circuit is shown in Figure 33(a).

$$R_B = \frac{R_3 R_{L2}}{R_3 + R_{L2}} = \frac{(6.2\text{ k}\Omega)(100\text{ k}\Omega)}{106.2\text{ k}\Omega} = 5.84\text{ k}\Omega$$

R_2 in series with R_B is designated R_{2+B}. The resulting equivalent circuit is shown in Figure 33(b).

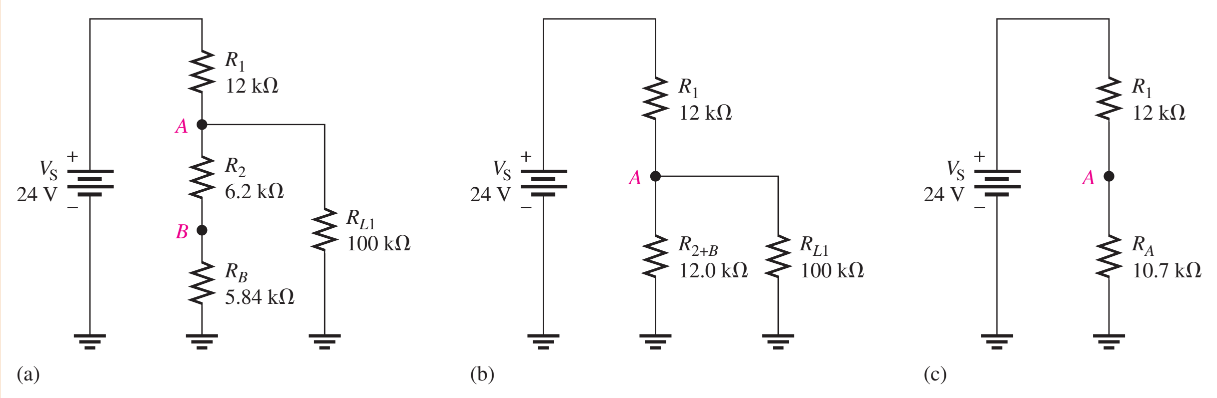
$$R_{2+B} = R_2 + R_B = 6.2\text{ k}\Omega + 5.84\text{ k}\Omega = 12.0\text{ k}\Omega$$

R_{L1} in parallel with R_{2+B} is designated R_A. The resulting equivalent circuit is shown in Figure 33(c).

$$R_A = \frac{R_{L1} R_{2+B}}{R_{L1} + R_{2+B}} = \frac{(100\text{ k}\Omega)(12.0\text{ k}\Omega)}{112\text{ k}\Omega} = 10.7\text{ k}\Omega$$

R_A is the total resistance from node A to ground. The total resistance for the circuit is

$$R_{\text{T}} = R_A + R_1 = 10.7\text{ k}\Omega + 12\text{ k}\Omega = 22.7\text{ k}\Omega$$



▲ FIGURE 33

Determine the voltage across R_{L1} as follows, using the equivalent circuit in Figure 33(c):

$$V_{RL1} = V_A = \left(\frac{R_A}{R_{\text{T}}}\right)V_{\text{S}} = \left(\frac{10.7\text{ k}\Omega}{22.7\text{ k}\Omega}\right)24\text{ V} = 11.3\text{ V}$$

The load current through R_{L1} is

$$I_{RL1} = \frac{V_{RL1}}{R_{L1}} = \left(\frac{11.3\text{ V}}{100\text{ k}\Omega}\right) = \mathbf{113\ \mu A}$$

Determine the voltage at node B by using the equivalent circuit in Figure 33(a) and the voltage at node A.

$$V_B = \left(\frac{R_B}{R_{2+B}}\right)V_A = \left(\frac{5.84\text{ k}\Omega}{12.0\text{ k}\Omega}\right)11.3\text{ V} = 5.50\text{ V}$$

The load current through R_{L2} is

$$I_{RL2} = \frac{V_{RL2}}{R_{L2}} = \frac{V_B}{R_{L2}} = \frac{5.50\text{ V}}{100\text{ k}\Omega} = \mathbf{55\ \mu A}$$

The bleeder current is

$$I_3 = \frac{V_B}{R_3} = \frac{5.50\text{ V}}{6.2\text{ k}\Omega} = \mathbf{887\ \mu A}$$

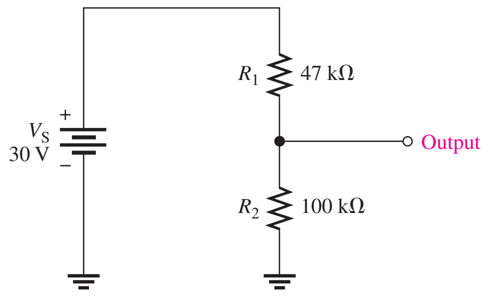
Related Problem What will happen to the load current in R_{L2} if R_{L1} is disconnected?

Use Multisim file E07-14 to verify the calculated results in this example.

SECTION 3 CHECKUP

1. A load resistor is connected to an output tap on a voltage divider. What effect does the load resistor have on the output voltage at this tap?
2. A larger-value load resistor will cause the output voltage to change less than a smaller-value one will. (T or F)
3. For the voltage divider in Figure 34, determine the unloaded output voltage with respect to ground. Also determine the output voltage with a 10 kΩ load resistor connected across the output.

▶ FIGURE 34



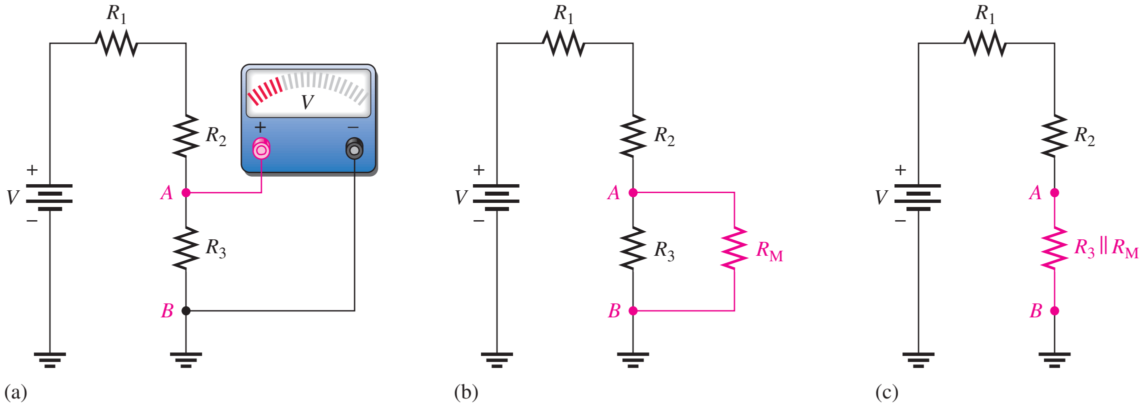
4 LOADING EFFECT OF A VOLTMETER

As you have learned, voltmeters must be connected in parallel with a resistor in order to measure the voltage across the resistor. Because of its internal resistance, a voltmeter puts a load on the circuit and will affect, to a certain extent, the voltage that is being measured. Until now, we have ignored the loading effect because the internal resistance of a voltmeter is very high, and normally it has negligible effect on the circuit that is being measured. However, if the internal resistance of the voltmeter is not sufficiently greater than the circuit resistance across which it is connected, the loading effect will cause the measured voltage to be less than its actual value. You should always be aware of this effect.

After completing this section, you should be able to

- **Determine the loading effect of a voltmeter on a circuit**
    - Explain why a voltmeter can load a circuit
    - Discuss the internal resistance of a voltmeter

When a voltmeter is connected to a circuit as shown, for example, in Figure 35(a), its internal resistance appears in parallel with R_3, as shown in part (b). The resistance from A to B is altered by the loading effect of the voltmeter's internal resistance, R_M, and is equal to $R_3 \parallel R_M$, as indicated in part (c).



▲ FIGURE 35

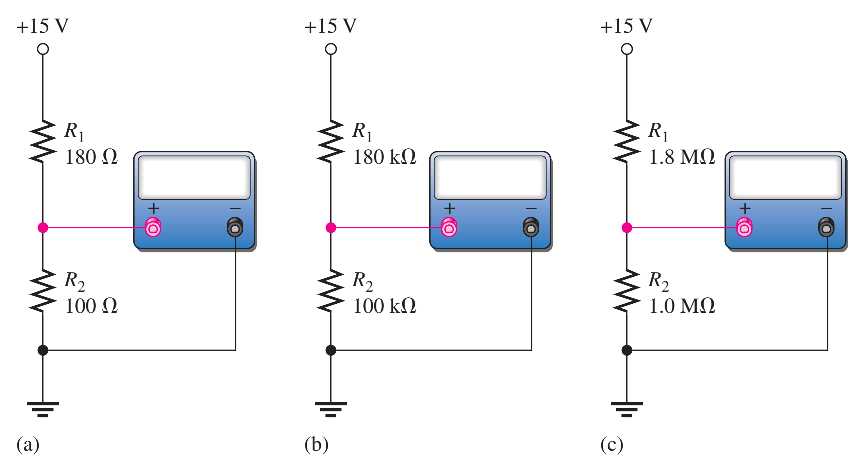
The loading effect of a voltmeter.

If R_M is much greater than R_3, the resistance from A to B changes very little, and the meter indicates the actual voltage. If R_M is not sufficiently greater than R_3, the resistance from A to B is reduced significantly, and the voltage across R_3 is altered by the loading effect of the meter. A good rule of thumb for troubleshooting work is that *if the loading effect is less than 10%, it can usually be neglected, depending on the accuracy required.*

Most voltmeters are part of a multifunction instrument such as the DMM or the analog multimeter. The voltmeter in a DMM will typically have an internal resistance of 10 MΩ or more, so the loading effect is important only in very high-resistance circuits. DMMs have a constant resistance on all ranges because the input is connected to an internal fixed voltage divider. For analog multimeters, the internal resistance depends on the range selected for making a measurement. To determine the loading effect, you need to know the sensitivity of the meter, a value given by the manufacturer on the meter or in the manual. Sensitivity is expressed in ohms/volt and is typically about 20,000 Ω/V. To determine the internal series resistance, multiply the sensitivity by the maximum voltage on the range selected. For example, a 20,000 Ω/V meter will have an internal resistance of 20,000 Ω on the 1 V range and 200,000 Ω on the 10 V range. As you can see, there is a smaller loading effect for higher voltage ranges than for lower ones on the analog multimeter.

EXAMPLE 15

How much does the digital voltmeter affect the voltage being measured for each circuit indicated in Figure 36? Assume the meter has an input resistance (R_M) of 10 MΩ.



▲ FIGURE 36

Solution To show the small differences more clearly, the results are expressed in more than three significant figures in this example.

(a) Refer to Figure 36(a). The unloaded voltage across R_2 in the voltage-divider circuit is

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{100\ \Omega}{280\ \Omega}\right)15\text{ V} = 5.357\text{ V}$$

The meter's resistance in parallel with R_2 is

$$R_2 \parallel R_M = \left(\frac{R_2R_M}{R_2 + R_M}\right) = \frac{(100\ \Omega)(10\text{ M}\Omega)}{10.0001\text{ M}\Omega} = 99.999\ \Omega$$

The voltage actually measured by the meter is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{99.999\ \Omega}{279.999\ \Omega}\right)15\text{ V} = 5.357\text{ V}$$

The voltmeter has no measurable loading effect.

(b) Refer to Figure 36(b).

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{100\text{ k}\Omega}{280\text{ k}\Omega}\right)15\text{ V} = 5.357\text{ V}$$

$$R_2 \parallel R_M = \frac{R_2R_M}{R_2 + R_M} = \frac{(100\text{ k}\Omega)(10\text{ M}\Omega)}{10.1\text{ M}\Omega} = 99.01\text{ k}\Omega$$

The voltage actually measured by the meter is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{99.01\text{ k}\Omega}{279.01\text{ k}\Omega}\right)15\text{ V} = 5.323\text{ V}$$

The loading effect of the voltmeter reduces the voltage by a very small amount.

(c) Refer to Figure 36(c).

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{1.0\text{ M}\Omega}{2.8\text{ M}\Omega}\right)15\text{ V} = 5.357\text{ V}$$

$$R_2 \parallel R_M = \frac{R_2 R_M}{R_2 + R_M} = \frac{(1.0\text{ M}\Omega)(10\text{ M}\Omega)}{11\text{ M}\Omega} = 909.09\text{ k}\Omega$$

The voltage actually measured is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{909.09\text{ k}\Omega}{2.709\text{ M}\Omega}\right)15\text{ V} = 5.034\text{ V}$$

The loading effect of the voltmeter reduces the voltage by a noticeable amount. As you can see, the higher the resistance across which a voltage is measured, the more the loading effect.

Related Problem Calculate the voltage across R_2 in Figure 36(c) if the meter resistance is 20 MΩ.

Open Multisim file E07-15 and verify the calculated results by using a 10 MΩ resistor to simulate the meter load.

SECTION 4 CHECKUP

1. Explain why a voltmeter can potentially load a circuit.
2. If a voltmeter with a 10 MΩ internal resistance is measuring the voltage across a 10 kΩ resistor, should you normally be concerned about the loading effect?
3. If a voltmeter with a 10 MΩ resistance is measuring the voltage across a 3.3 MΩ resistor, should you be concerned about the loading effect?
4. What is the internal series resistance of a 20,000 Ω/V VOM if it is on the 200 V range?

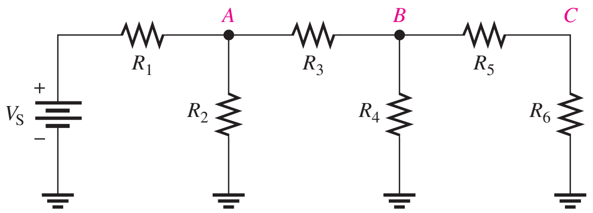
5 LADDER NETWORKS

A resistive ladder network is a special type of series-parallel circuit. The $R/2R$ ladder network is commonly used to scale down voltages to certain weighted values for digital-to-analog conversion, which is a process that you will study in another course.

After completing this section, you should be able to

- **Analyze ladder networks**
  - Determine the voltages in a three-step ladder network
  - Analyze an $R/2R$ ladder

One approach to the analysis of a ladder network such as the one shown in Figure 37 is to simplify it one step at a time, starting at the side farthest from the source. In this way, you can determine the current in any branch or the voltage at any node, as illustrated in Example 16.

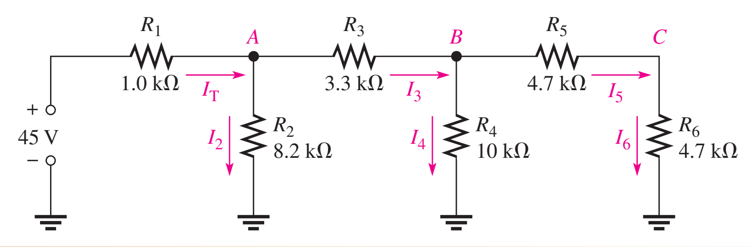

A
B
C
R_1
R_3
R_5
+
V_S
−
R_2
R_4
R_6

▲ FIGURE 37

Basic three-step ladder network.

EXAMPLE 16

Determine the current through each resistor and the voltage at each labeled node with respect to ground in the ladder network of Figure 38.


R_1
A
R_3
B
R_5
C
1.0 kΩ
I_T
3.3 kΩ
I_3
4.7 kΩ
I_5
+
45 V
−
I_2
R_2
8.2 kΩ
I_4
R_4
10 kΩ
I_6
R_6
4.7 kΩ

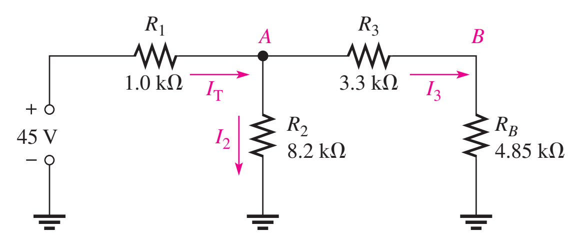
▲ FIGURE 38

Solution To find the current through each resistor, you must know the total current from the source (I_T). To obtain I_T, you must find the total resistance "seen" by the source.

Determine R_T in a step-by-step process, starting at the right of the circuit diagram. First, notice that R_5 and R_6 are in series across R_4. Neglecting the circuit to the left of node B, the resistance from node B to ground is

$$R_B = \frac{R_4(R_5 + R_6)}{R_4 + (R_5 + R_6)} = \frac{(10\text{ k}\Omega)(9.4\text{ k}\Omega)}{19.4\text{ k}\Omega} = 4.85\text{ k}\Omega$$

Using R_B, you can draw the equivalent circuit as shown in Figure 39.


R_1
A
R_3
B
1.0 kΩ
I_T
3.3 kΩ
I_3
+
45 V
−
I_2
R_2
8.2 kΩ
R_B
4.85 kΩ

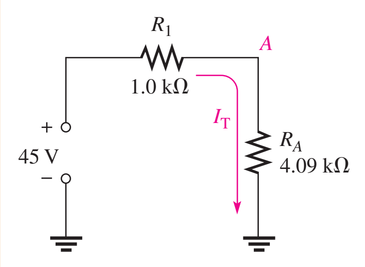
▲ FIGURE 39

Next, neglecting the circuit to the left of node A, the resistance from node A to ground (R_A) is R_2 in parallel with the series combination of R_3 and R_B. Calculate resistance R_A.

$$R_A = \frac{R_2(R_3 + R_B)}{R_2 + (R_3 + R_B)} = \frac{(8.2\text{ k}\Omega)(8.15\text{ k}\Omega)}{16.35\text{ k}\Omega} = 4.09\text{ k}\Omega$$

Using R_A, you can further simplify the equivalent circuit of Figure 39 as shown in Figure 40.

▶ FIGURE 40



Finally, the total resistance "seen" by the source is R_1 in series with R_A.

$$R_T = R_1 + R_A = 1.0\,\text{k}\Omega + 4.09\,\text{k}\Omega = 5.09\,\text{k}\Omega$$

The total circuit current is

$$I_T = \frac{V_S}{R_T} = \frac{45\text{ V}}{5.09\,\text{k}\Omega} = \mathbf{8.84\,mA}$$

As indicated in Figure 39, I_T is into node A and divides between R_2 and the branch containing $R_3 + R_B$. Since the branch resistances are approximately equal in this particular example, half the total current is through R_2 and half into node B. So the currents through R_2 and R_3 are

$$I_2 = \mathbf{4.42\,mA}$$
$$I_3 = \mathbf{4.42\,mA}$$

If the branch resistances are not equal, use the current-divider formula. As indicated in Figure 38. I_3 is into node B and is divided between R_4 and the branch containing $R_5 + R_6$. Therefore, the currents through R_4, R_5, and R_6 can be calculated.

$$I_4 = \left(\frac{R_5 + R_6}{R_4 + (R_5 + R_6)}\right)I_3 = \left(\frac{9.4\,\text{k}\Omega}{19.4\,\text{k}\Omega}\right)4.42\text{ mA} = \mathbf{2.14\,mA}$$
$$I_5 = I_6 = I_3 - I_4 = 4.42\text{ mA} - 2.14\text{ mA} = \mathbf{2.28\,mA}$$

To determine V_A, V_B, and V_C, apply Ohm's law.

$$V_A = I_2R_2 = (4.42\text{ mA})(8.2\,\text{k}\Omega) = \mathbf{36.2\,V}$$
$$V_B = I_4R_4 = (2.14\text{ mA})(10\,\text{k}\Omega) = \mathbf{21.4\,V}$$
$$V_C = I_6R_6 = (2.28\text{ mA})(4.7\,\text{k}\Omega) = \mathbf{10.7\,V}$$

Related Problem Recalculate the currents through each resistor and the voltages at each node in Figure 38 if R_1 is increased to 2.2 kΩ.

Use Multisim file E07-16 to verify the calculated results in this example and to confirm your calculations for the related problem.

The *R*/2*R* Ladder Network

A basic $R/2R$ ladder network is shown in Figure 41. As you can see, the name comes from the relationship of the resistor values. R represents a common value, and one set of resistors has twice the value of the others. This type of ladder network is used in applications

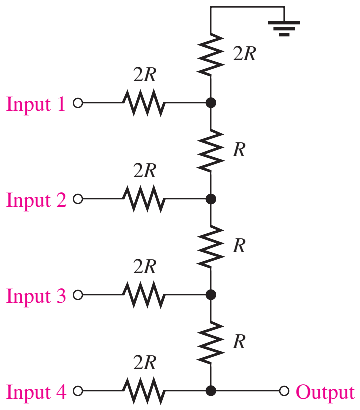


FIGURE 41

A basic four-step $R/2R$ ladder network.

where digital codes are converted to speech, music, or other types of analog signals as found, for example, in the area of digital recording and reproduction. This application is called *digital-to-analog* (D/A) *conversion.*

Let's examine the general operation of a basic $R/2R$ ladder using the four-step circuit in Figure 42. In a later course in digital fundamentals, you will learn specifically how this type of circuit is used in D/A conversion.

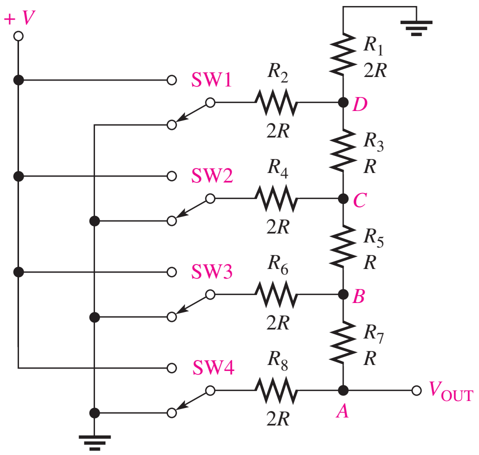


FIGURE 42

$R/2R$ ladder with switch inputs to simulate a two-level (digital) code.

The switches used in this illustration simulate the digital (two-level) inputs. One switch position is connected to ground (0 V), and the other position is connected to a positive voltage (V). The analysis is as follows: Start by assuming that switch SW4 in Figure 42 is at the V position and the others are at ground so that the inputs are as shown in Figure 43(a).

The total resistance from node A to ground is found by first combining R_1 and R_2 in parallel from node D to ground. The simplified circuit is shown in Figure 43(b).

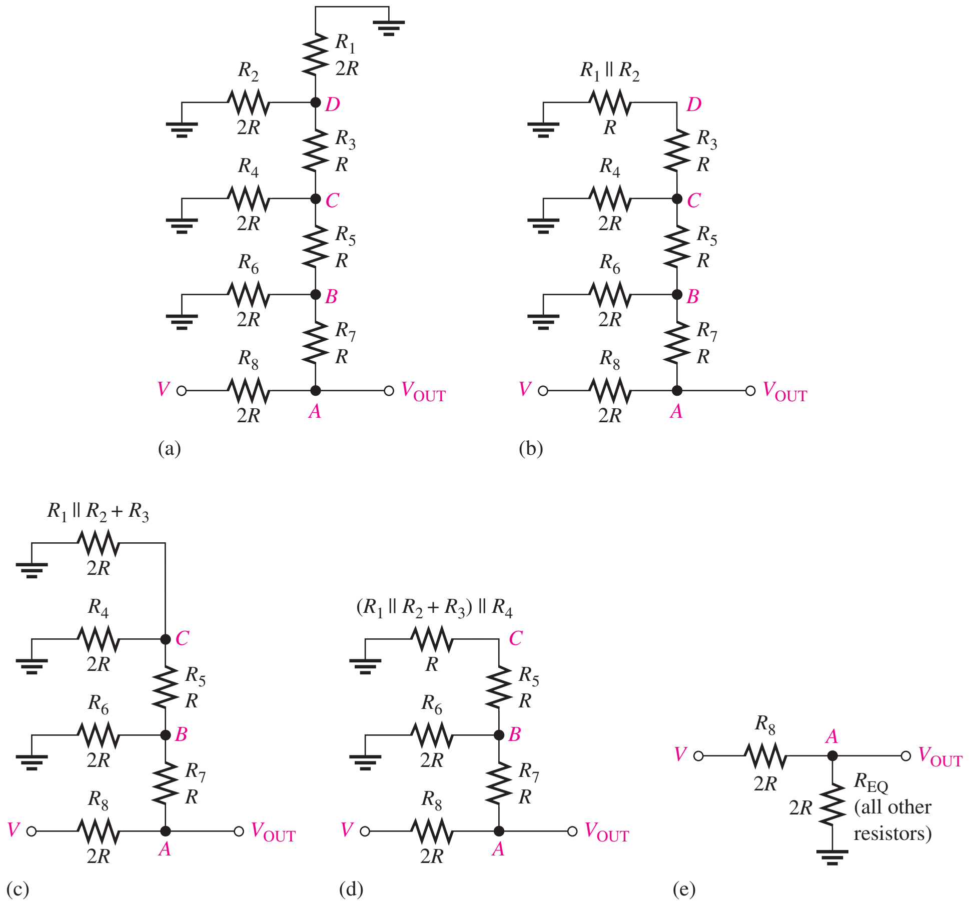
$$R_1 \| R_2 = \frac{2R}{2} = R$$

$R_1 \| R_2$ is in series with R_3 from node C to ground as illustrated in part (c).

$$R_1 \| R_2 + R_3 = R + R = 2R$$

Next, this combination is in parallel with R_4 from node C to ground as shown in part (d).

$$(R_1 \| R_2 + R_3) \| R_4 = 2R \| 2R = \frac{2R}{2} = R$$


R_1
2R
R_2
2R
D
R_3
R
R_4
2R
C
R_5
R
R_6
2R
B
R_7
R
R_8
2R
V
A
V_{OUT}
(a)
$R_1 \| R_2$
R
(b)
$R_1 \| R_2 + R_3$
(c)
$(R_1 \| R_2 + R_3) \| R_4$
(d)
R_{EQ}
(all other resistors)
(e)

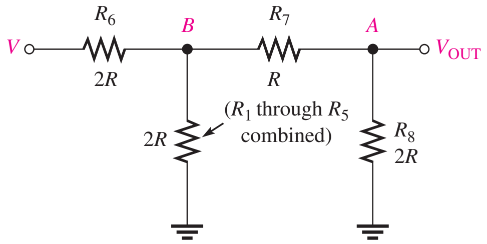
▲ **FIGURE 43**

Simplification of *R*/2*R* ladder for analysis.

Continuing this simplification process results in the circuit in part (e) in which the output voltage can be expressed using the voltage-divider formula as

$$V_{OUT} = \left(\frac{2R}{4R}\right)V = \frac{V}{2}$$

A similar analysis, except with switch SW3 in Figure 42 connected to *V* and the other switches connected to ground, results in the simplified circuit shown in Figure 44.


R_6
2R
B
R_7
R
A
V
V_{OUT}
2R
(R_1 through R_5 combined)
R_8
2R

▲ **FIGURE 44**

Simplified ladder with only *V* input at SW3 in Figure 42.

The analysis for this case is as follows: The resistance from node B to ground is

$$R_B = (R_7 + R_8) \parallel 2R = 3R \parallel 2R = \frac{6R}{5}$$

Using the voltage-divider formula, we can express the voltage at node B with respect to ground as

$$V_B = \left(\frac{R_B}{R_6 + R_B}\right)V = \left(\frac{6R/5}{2R + 6R/5}\right)V = \left(\frac{6R/5}{10R/5 + 6R/5}\right)V = \left(\frac{6R/5}{16R/5}\right)V$$

$$= \left(\frac{6R}{16R}\right)V = \frac{3V}{8}$$

The output voltage is, therefore,

$$V_{\text{OUT}} = \left(\frac{R_8}{R_7 + R_8}\right)V_B = \left(\frac{2R}{3R}\right)\left(\frac{3V}{8}\right) = \frac{V}{4}$$

Notice that the output voltage in this case ($V/4$) is one half the output voltage ($V/2$) for the case where V is connected at switch SW4.

A similar analysis for each of the remaining switch inputs in Figure 42 results in output voltages as follows: For SW2 connected to V and the other switches connected to ground,

$$V_{\text{OUT}} = \frac{V}{8}$$

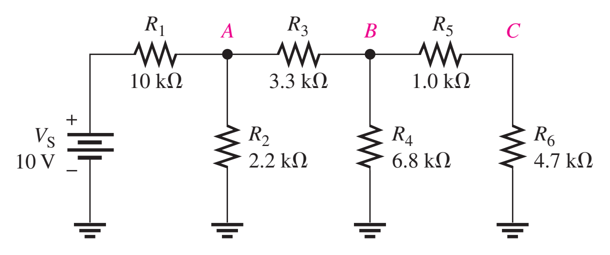
For SW1 connected to V and the other switches connected to ground,

$$V_{\text{OUT}} = \frac{V}{16}$$

When more than one input at a time are connected to V, the total output is the sum of the individual outputs, according to the superposition theorem. These particular relationships among the output voltages for the various levels of inputs are important in the application of $R/2R$ ladder networks to digital-to-analog conversion.

SECTION 5 CHECKUP

1. Draw a basic four-step ladder network.
2. Determine the total circuit resistance presented to the source by the ladder network of Figure 45.
3. What is the total current in Figure 45?
4. What is the current through R_2 in Figure 45?
5. What is the voltage at node A with respect to ground in Figure 45?



▲ FIGURE 45

6 THE WHEATSTONE BRIDGE

The Wheatstone bridge circuit can be used to precisely measure resistance. However, the bridge is most commonly used in conjunction with transducers to measure physical quantities such as strain, temperature, and pressure. **Transducers** are devices that sense a change in a physical parameter and convert that change into an electrical quantity such as a change in resistance. For example, a strain gauge exhibits a change in resistance when it is exposed to mechanical factors such as force, pressure, or displacement. A thermistor exhibits a change in its resistance when it is exposed to a change in temperature. The Wheatstone bridge can be operated in a balanced or an unbalanced condition. The condition of operation depends on the type of application.

After completing this section, you should be able to

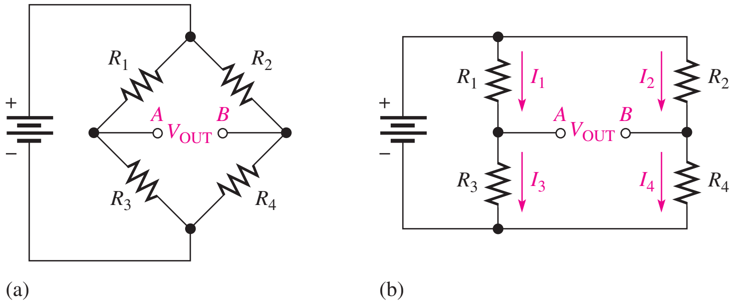
- **Analyze and apply a Wheatstone bridge**
  - Determine when a bridge is balanced
  - Determine an unknown resistance with a balanced bridge
  - Determine when a bridge is unbalanced
  - Discuss measurements using an unbalanced bridge

HISTORY NOTE

Sir Charles Wheatstone 1802–1875

Wheatstone was a British scientist and inventor of many scientific breakthroughs, including the English concertina, the stereoscope (a device for displaying three-dimensional images), and an encryption cipher technique. Wheatstone is best known for his contributions in the development of the Wheatstone bridge, originally invented by Samuel Hunter Christie, and as a major figure in the development of telegraphy. Wheatstone gave full credit for the invention to Christie, but because of his work in developing and applying the bridge, it became known as the Wheatstone bridge. (Photo credit: AIP Emilio Segrè Visual Archives, Brittle Books Collection.)

A **Wheatstone bridge** circuit is shown in its most common "diamond" configuration in Figure 46(a). It consists of four resistors and a dc voltage source connected across the top and bottom points of the "diamond." The output voltage is taken across the left and right points of the "diamond" between *A* and *B*. In part (b), the circuit is drawn in a slightly different way to more clearly show its series-parallel configuration.



▲ FIGURE 46

Wheatstone bridge. Notice that the bridge forms two back-to-back voltage dividers.

The Balanced Wheatstone Bridge

The Wheatstone bridge in Figure 46 is in the **balanced bridge** condition when the output voltage (V_{OUT}) between terminals *A* and *B* is equal to zero.

$$V_{OUT} = 0\text{ V}$$

When the bridge is balanced, the voltages across R_1 and R_2 are equal ($V_1 = V_2$) and the voltages across R_3 and R_4 are equal ($V_3 = V_4$). Therefore, the voltage ratios can be written as

$$\frac{V_1}{V_3} = \frac{V_2}{V_4}$$

Substituting IR for V by Ohm's law gives

$$\frac{I_1R_1}{I_3R_3} = \frac{I_2R_2}{I_4R_4}$$

Since $I_1 = I_3$ and $I_2 = I_4$, all the current terms cancel, leaving the resistor ratios.

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Solving for R_1 results in the following formula:

$$R_1 = R_3\left(\frac{R_2}{R_4}\right)$$

This formula allows you to find the value of resistor R_1 in terms of the other resistor values when the bridge is balanced. You can also find the value of any other resistor in a similar way.

Using the Balanced Wheatstone Bridge to Find an Unknown Resistance Assume that R_1 in Figure 46 has an unknown value, which we call R_X. Resistors R_2 and R_4 have fixed values so that their ratio, R_2/R_4, also has a fixed value. Since R_X can be any value, R_3 must be adjusted to make $R_1/R_3 = R_2/R_4$ in order to create a balanced condition. Therefore, R_3 is a variable resistor, which we will call R_V. When R_X is placed in the bridge, R_V is adjusted until the bridge is balanced as indicated by a zero output voltage. Then, the unknown resistance is found as

$$R_X = R_V\left(\frac{R_2}{R_4}\right)$$

Equation 2

The ratio R_2/R_4 is the scale factor.

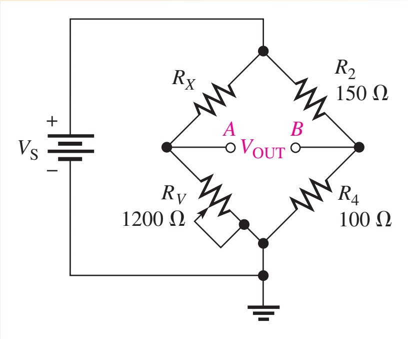
An older type of measuring instrument called a *galvanometer* can be connected between the output terminals A and B to detect a balanced condition. The galvanometer is essentially a very sensitive ammeter that senses current in either direction. It differs from a regular ammeter in that the midscale point is zero. In modern instruments, an amplifier connected across the bridge output indicates a balanced condition when its output is 0 V. Also, high precision micro-adjustable resistors can be used in demanding applications. The micro-adjustable resistors enable fine adjustment of the bridge resistors during manufacture for applications such as medical sensors, scales, and precision measurements.

From Equation 2, the value of R_V at balance multiplied by the scale factor R_2/R_4 is the actual resistance value of R_X. If $R_2/R_4 = 1$, then $R_X = R_V$, if $R_2/R_4 = 0.5$, then $R_X = 0.5R_V$, and so on. In a practical bridge circuit, the position of the R_V adjustment can be calibrated to indicate the actual value of R_X on a scale or with some other method of display.

EXAMPLE 17

Determine the value of R_X in the balanced bridge shown in Figure 47.

▶ FIGURE 47



Solution The scale factor is

$$\frac{R_2}{R_4} = \frac{150\ \Omega}{100\ \Omega} = 1.5$$

The bridge is balanced ($V_{OUT} = 0$ V) when R_V is set at 1200 Ω, so the unknown resistance is

$$R_X = R_V\left(\frac{R_2}{R_4}\right) = (1200\ \Omega)(1.5) = \mathbf{1800\ \Omega}$$

Related Problem If R_V must be adjusted to 2.2 kΩ to balance the bridge in Figure 47, what is R_X?

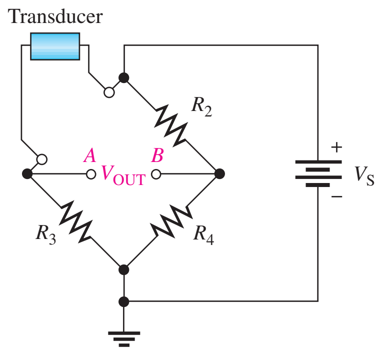
Use Multisim file E07-17 to verify the calculated results in this example and to confirm your calculation for the related problem.

The Unbalanced Wheatstone Bridge

An **unbalanced bridge** condition occurs when V_{OUT} is not equal to zero. The unbalanced bridge is used to measure several types of physical quantities such as mechanical strain, temperature, or pressure. This can be done by connecting a transducer in one leg of the bridge, as shown in Figure 48. The resistance of the transducer changes proportionally to the changes in the parameter that it is measuring. If the bridge is balanced at a known point, then the amount of deviation from the balanced condition, as indicated by the output voltage, indicates the amount of change in the parameter being measured. Therefore, the value of the parameter being measured can be determined by the amount that the bridge is unbalanced.

▶ FIGURE 48

A bridge Circuit for measuring a physical parameter using a transducer.



A Bridge Circuit for Measuring Temperature If temperature is to be measured, the transducer can be a thermistor, which is a temperature-sensitive resistor. The thermistor resistance changes in a predictable way as the temperature changes. A change in temperature causes a change in thermistor resistance, which causes a corresponding change in the output voltage of the bridge as it becomes unbalanced. The output voltage is proportional to the temperature; therefore, either a voltmeter connected across the output can be calibrated to show the temperature or the output voltage can be amplified and converted to digital form to drive a readout display of the temperature.

A bridge circuit used to measure temperature is designed so that it is balanced at a reference temperature and becomes unbalanced at a measured temperature. For example, let's say the bridge is to be balanced at 25°C. A thermistor will have a known value of resistance at 25°C. For simplicity, let's assume the other three bridge resistors are equal to the thermistor resistance at 25°C, so $R_{therm} = R_2 = R_3 = R_4$. For this particular case, the change in output voltage (ΔV_{OUT}) can be shown to be related to the change in R_{therm} by the following formula:

$$\Delta V_{OUT} \cong \Delta R_{therm}\left(\frac{V_S}{4R}\right)$$

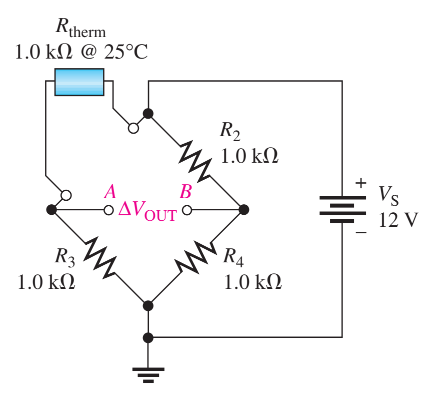
Equation 3

The Δ (Greek letter delta) in front of a variable means a change in the variable. This formula applies only to the case where all resistances in the bridge are equal when the bridge is balanced. Keep in mind that the bridge can be initially balanced without having all the resistors equal as long as $R_1 = R_2$ and $R_3 = R_4$ (see Figure 46), but the formula for ΔV_{OUT} would be more complicated.

EXAMPLE 18

Determine the output voltage of the temperature-measuring bridge circuit in Figure 49 if the thermistor is exposed to a temperature of 50°C and its resistance at 25°C is 1.0 kΩ. Assume the resistance of the thermistor decreases to 900 Ω at 50°C.

▶ FIGURE 49



Solution

$$\Delta R_{therm} = 1.0\text{ k}\Omega - 900\ \Omega = 100\ \Omega$$

$$\Delta V_{OUT} \cong \Delta R_{therm}\left(\frac{V_S}{4R}\right) = 100\ \Omega\left(\frac{12\text{ V}}{4\text{ k}\Omega}\right) = 0.3\text{ V}$$

Since $V_{OUT} = 0$ V when the bridge is balanced at 25°C and it changes 0.3 V, then

$$V_{OUT} = \mathbf{0.3\ V}$$

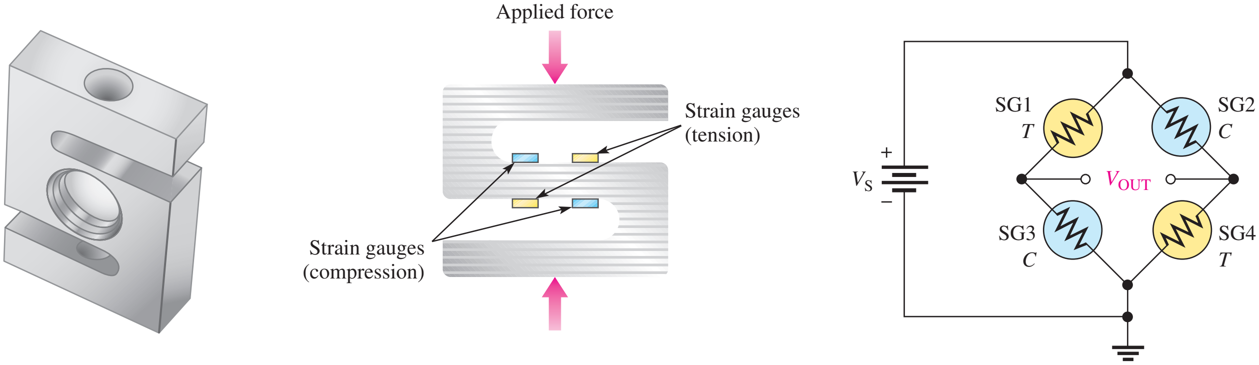
when the temperature is 50°C.

Related Problem If the temperature is increased to 60°C, causing the thermistor resistance in Figure 49 to decrease to 850 Ω, what is V_{OUT}?

Other Unbalanced Wheatstone Bridge Applications A Wheatstone bridge with a strain gauge can be used to measure forces. A strain gauge is a device that exhibits a change in resistance when it is compressed or stretched by the application of an external force. As the resistance of the strain gauge changes, the previously balanced bridge becomes unbalanced. This unbalance causes the output voltage to change from zero, and this change can be measured to determine the amount of strain. In strain gauges, the resistance change is extremely small. This tiny change unbalances a Wheatstone bridge and can be detected because of its high sensitivity. For example, Wheatstone bridges with strain gauges are commonly used in weight scales.

Some resistive transducers have extremely small resistance changes, and these changes are difficult to measure accurately with a direct measurement. In particular, strain gauges are one of the most useful resistive transducers that convert the stretching or compression of a fine wire into a change in resistance. When strain causes the wire in the gauge to stretch, the resistance increases a small amount; and when it compresses, the resistance of the wire decreases.

Strain gauges are used in many types of scales, from those that are used for weighing small parts to those for weighing huge trucks. Typically, the gauges are mounted on a special block of aluminum that deforms when a weight is on the scale. The strain gauges are extremely delicate and must be mounted properly, so the entire assembly is generally prepared as a single unit called a *load cell*. A **load cell** is a transducer that uses strain gauges to convert mechanical force into an electrical signal. A wide variety of load cells with different shapes and sizes are available from manufacturers depending on the application. A typical S-type load cell for a weighing application is shown in Figure 50(a). The load cell has four strain gauges as illustrated in Figure 50(b). The gauges are mounted so that two of the gauges stretch (tension) when a load is placed on the scale and two of the gauges compress.



(a) A typical S-type load cell

(b) A load cell with four active strain gauges (two compression, two tension)

(c) Wheatstone bridge

▲ **FIGURE 50**

Wheatstone bridge in a scale application.

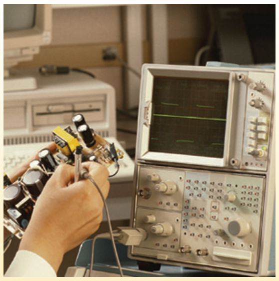
Load cells are usually connected to a Wheatstone bridge as shown in Figure 50(c) with strain gauges (SG) in tension (T) and compression (C) in opposite diagonal legs as shown. The output of the bridge is normally digitized and converted to a reading for a display or sent to a computer for processing. The major advantage of the Wheatstone bridge circuit is that it is capable of accurately measuring very small differences in resistance. The use of four active transducers increases the sensitivity of the measurement and makes the bridge the ideal circuit for instrumentation. The Wheatstone bridge circuit has the added benefit of compensating for temperature variations and wire resistance of connecting wires that would otherwise contribute to inaccuracies.

In addition to scales, strain gauges are used with Wheatstone bridges in other types of measurements including pressure measurements, displacement and acceleration measurements to name a few. In pressure measurements, the strain gauges are bonded to a flexible diaphragm that stretches when pressure is applied to the transducer. The amount of flexing is related to the pressure, which again converts to a very small resistance change.

SECTION 6 CHECKUP

1. Draw a basic Wheatstone bridge circuit.
2. Under what condition is a bridge balanced?
3. What is the unknown resistance in Figure 47 when $R_V = 3.3\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, and $R_4 = 2.2\ \text{k}\Omega$?
4. How is a Wheatstone bridge used in the unbalanced condition?
5. What is a load cell?

7 TROUBLESHOOTING



As you know, troubleshooting is the process of identifying and locating a failure or problem in a circuit. Some troubleshooting techniques and the application of logical thought have already been discussed in relation to both series circuits and parallel circuits. A basic premise of troubleshooting is that you must know what to look for before you can successfully troubleshoot a circuit.

After completing this section, you should be able to

- **Troubleshoot series-parallel circuits**
  - Determine the effects of an open in a circuit
  - Determine the effects of a short in a circuit
  - Locate opens and shorts

Opens and shorts are typical problems that occur in electric circuits. If a resistor burns out, it will normally produce an open. Bad solder connections, broken wires, and poor contacts can also be causes of open paths. Pieces of foreign material, such as solder splashes, broken insulation on wires, and so on, can lead to shorts in a circuit. A short is considered to be a zero resistance path between two points.

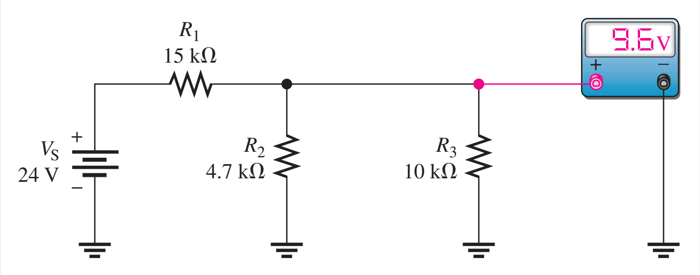
In addition to complete opens or shorts, partial opens or partial shorts can develop in a circuit. A partial open would be a much higher than normal resistance, but not infinitely large. A partial short would be a much lower than normal resistance, but not zero.

The following three examples illustrate troubleshooting series-parallel circuits.

EXAMPLE 19

From the indicated voltmeter reading in Figure 51, determine if there is a fault by applying the APM approach. If there is a fault, identify it as either a short or an open.

▶ FIGURE 51



Solution **Step 1:** Analysis

Determine what the voltmeter should be indicating as follows. Since R_2 and R_3 are in parallel, their combined resistance is

$$R_{2\|3} = \frac{R_2R_3}{R_2 + R_3} = \frac{(4.7\text{ k}\Omega)(10\text{ k}\Omega)}{14.7\text{ k}\Omega} = 3.20\text{ k}\Omega$$

Determine the voltage across the parallel combination by the voltage-divider formula.

$$V_{2\|3} = \left(\frac{R_{2\|3}}{R_1 + R_{2\|3}}\right)V_S = \left(\frac{3.2\text{ k}\Omega}{18.2\text{ k}\Omega}\right)24\text{ V} = 4.22\text{ V}$$

This calculation shows that 4.22 V is the voltage reading that you should get on the meter. However, the meter reads 9.6 V across $R_{2\|3}$. This value is incorrect, and, because it is higher than it should be, either R_2 or R_3 is probably open. Why? Because if either of these two resistors is open, the resistance across which the meter is connected is larger than expected. A higher resistance will drop a higher voltage in this circuit.

Step 2: Planning

Start trying to find the open resistor by assuming that R_2 is open. If it is, the voltage across R_3 is

$$V_3 = \left(\frac{R_3}{R_1 + R_3}\right)V_S = \left(\frac{10\text{ k}\Omega}{25\text{ k}\Omega}\right)24\text{ V} = 9.6\text{ V}$$

Since the measured voltage is also 9.6 V, this calculation shows that R_2 is probably open.

Step 3: Measurement

Disconnect power and remove R_2. Measure its resistance to verify it is open. If it is not, inspect the wiring, solder, or connections around R_2, looking for the open.

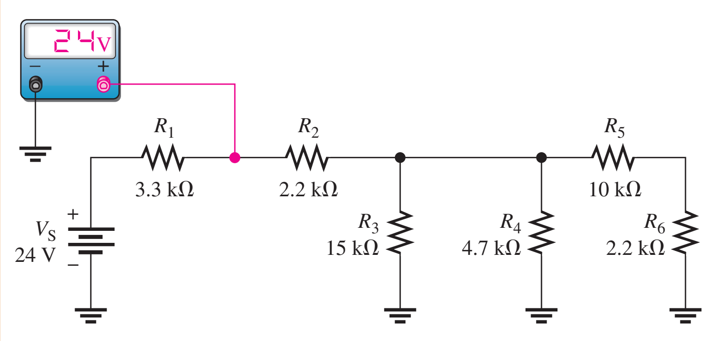
Related Problem What would be the voltmeter reading if R_3 were open in Figure 51? If R_1 were open?

Open Multisim file E07-19 and verify the result in this example by removing R_2 to simulate an open and observe the meter reading.

EXAMPLE 20

Suppose that you measure 24 V with the voltmeter in Figure 52. Determine if there is a fault, and, if there is, identify it.

▶ FIGURE 52



Solution **Step 1:** Analysis

There is no voltage drop across R_1 because both sides of the resistor are at +24 V. Either there is no current through R_1 from the source, which tells you that R_2 is open in the circuit, or R_1 is shorted.

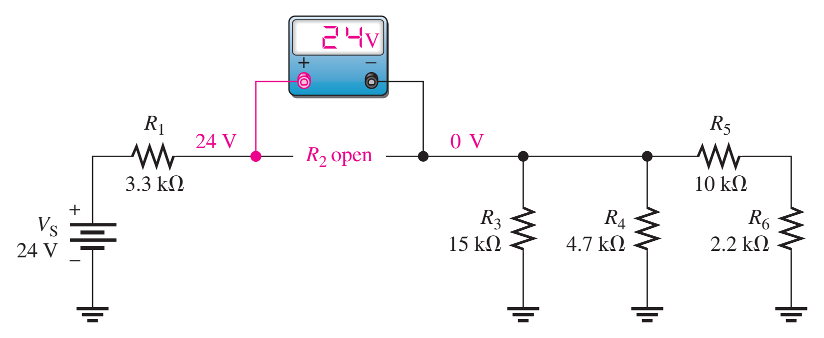
Step 2: Planning

The most probable failure is an open R_2. If it is open, then there will be no current from the source. To verify this, measure across R_2 with the voltmeter. If R_2 is open, the meter will indicate 24 V. The right side of R_2 will be at zero volts because there is no current through any of the other resistors to cause a voltage drop across them.

Step 3: Measurement

The measurement to verify that R_2 is open is shown in Figure 53.

FIGURE 53

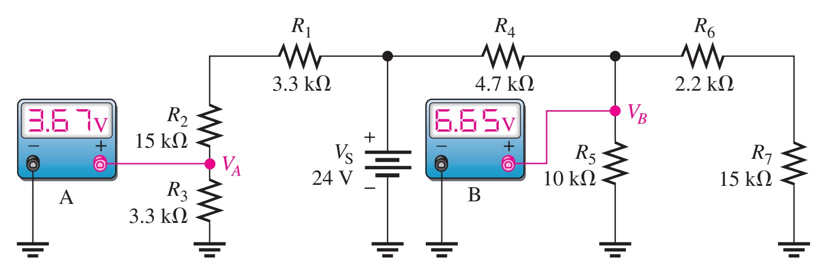


Related Problem What would be the voltage across an open R_5 in Figure 52 assuming no other faults?

Open Multisim file E07-20 and confirm the fault by measurement.

EXAMPLE 21 The two voltmeters in Figure 54 indicate the voltages shown. Apply logical thought and your knowledge of circuit operation to determine if there are any opens or shorts in the circuit and, if so, where they are located.

FIGURE 54



Solution **Step 1:** Determine if the voltmeter readings are correct. R_1, R_2, and R_3 act as a voltage divider. Calculate the voltage (V_A) across R_3 as follows:

$$V_A = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{3.3\text{ k}\Omega}{21.6\text{ k}\Omega}\right)24\text{ V} = 3.67\text{ V}$$

The voltmeter A reading is correct. This indicates that R_1, R_2, and R_3 are connected and are not faulty.

Step 2: See if the voltmeter B reading is correct. $R_6 + R_7$ is in parallel with R_5. The series-parallel combination of R_5, R_6, and R_7 is in series with R_4. Calculate the resistance of the R_5, R_6, and R_7 combination as follows:

$$R_{5\|(6+7)} = \frac{R_5(R_6 + R_7)}{R_5 + R_6 + R_7} = \frac{(10\text{ k}\Omega)(17.2\text{ k}\Omega)}{27.2\text{ k}\Omega} = 6.32\text{ k}\Omega$$

$R_{5\|(6+7)}$ and R_4 form a voltage divider, and voltmeter B measures the voltage across $R_{5\|(6+7)}$. Is it correct? Check as follows:

$$V_B = \left(\frac{R_{5\|(6+7)}}{R_4 + R_{5\|(6+7)}}\right)V_S = \left(\frac{6.32\text{ k}\Omega}{11\text{ k}\Omega}\right)24\text{ V} = 13.8\text{ V}$$

Thus, the actual measured voltage (6.65 V) at this point is incorrect. Some logical thinking will help to isolate the problem.

Step 3: R_4 is not open, because if it were, the meter would read 0 V. If there were a short across it, the meter would read 24 V. Since the actual voltage is much less than it should be, $R_{5\|(6+7)}$ must be less than the calculated value of 6.32 kΩ. The most likely problem is a short across R_7. If there is a short from the top of R_7 to ground, R_6 is effectively in parallel with R_5. In this case,

$$R_5 \parallel R_6 = \frac{R_5 R_6}{R_5 + R_6} = \frac{(10\text{ k}\Omega)(2.2\text{ k}\Omega)}{12.2\text{ k}\Omega} = 1.80\text{ k}\Omega$$

Then V_B is

$$V_B = \left(\frac{1.80\text{ k}\Omega}{6.5\text{ k}\Omega}\right)24\text{ V} = 6.65\text{ V}$$

This value for V_B agrees with the voltmeter B reading. So there is a short across R_7. If this were an actual circuit, you would try to find the physical cause of the short.

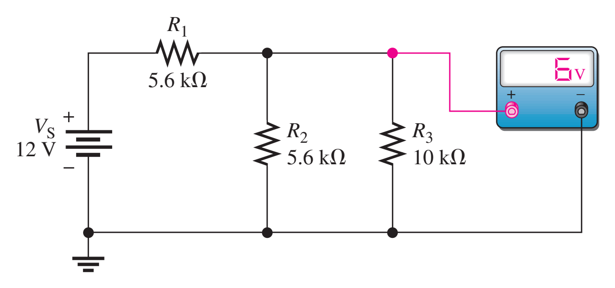
Related Problem If the only fault in Figure 54 is that R_2 is shorted, what will voltmeter A read? What will voltmeter B read?

Open Multisim file E07-21 and verify the fault by measurement.

SECTION 7 CHECKUP

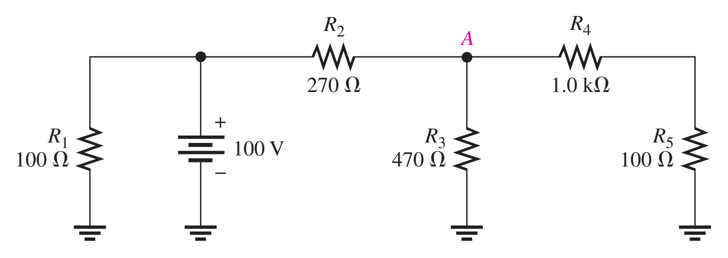
1. Name two types of common circuit faults.
2. In Figure 55, one of the resistors in the circuit is open. Based on the meter reading, determine which is the open resistor.

▶ FIGURE 55

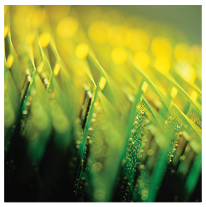


3. For the following faults in Figure 56, what voltage would be measured at node *A* with respect to ground?
   (a) No faults
   (b) R_1 open
   (c) Short across R_5
   (d) R_3 and R_4 open
   (e) R_2 open

▶ FIGURE 56



Application Activity



The Wheatstone bridge is widely used in measurement applications that use sensors to convert a physical parameter to a change in resistance. Modern Wheatstone bridges are automated; with intelligent interface modules, the output can be conditioned and converted to any desired unit for display or processing (for example, the output might be displayed in pounds for a scale application).

The Wheatstone bridge provides a null measurement, which enables it to have great sensitivity. It can also be designed to compensate for changes in temperature, a great advantage for many resistive measurements, particularly when the resistance change of the sensor is very small. Usually, the output voltage of the bridge is increased by an amplifier that has a minimum loading effect on the bridge.

Temperature Controller

In this application activity, a Wheatstone bridge circuit is used in a temperature controller. The sensor is a thermistor ("thermal resistor"), which is a resistive sensor that changes resistance as temperature changes. Thermistors are available with positive or negative resistance characteristics as a function of temperature. The thermistor in this circuit is one of the resistors in a Wheatstone bridge but is located a short distance from the circuit board for sensing the temperature at a point off the board. The threshold voltage for the output to change is controlled by the 10 kΩ potentiometer, R_3.

The amplifier and LEDs are shown only to provide a context for this wheatstone bridge application and are not intended to be the focus. You will study operational amplifiers in a later course. The amplifier is an operational amplifier ("op-amp"), which is an integrated circuit. The term *amplifier* is used in electronics to describe a device that produces a larger replica of the input voltage or current at its output. The term *gain* refers to the amount of amplification. The op-amp in this circuit is configured as a *comparator,* which is be used to compare the voltage on one side of the bridge with the voltage on the other.

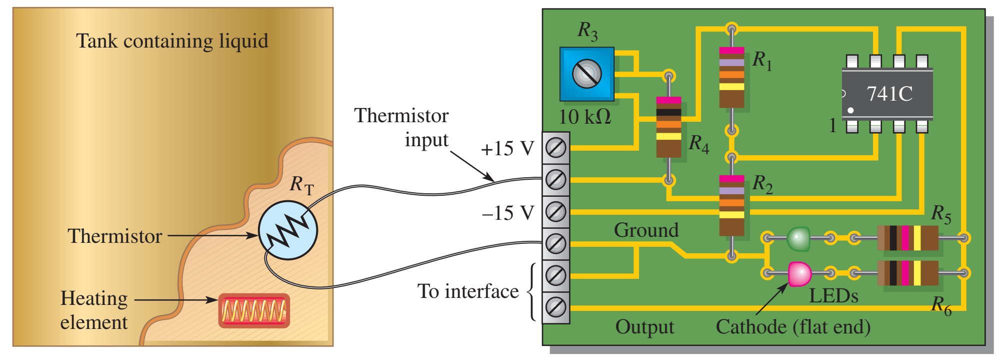
The advantage of a comparator is that it is extremely sensitive to an unbalanced bridge and produces a large output when the bridge is unbalanced. In fact, it is so sensitive, it is virtually impossible to adjust the bridge for perfect balance. Even the tiniest imbalance will cause the output to go to a voltage near either the maximum or minimum possible (the power supply voltages). This is handy for turning on a heater or other device based on the temperature.

The Control Circuit

This application has a tank containing a liquid that needs to be held at a warm temperature, as illustrated in Figure 57(a). The circuit board for the temperature controller is shown in Figure 57(b). The circuit board controls a heating unit (through an interface, which is not shown) when the temperature is too cold. The thermistor, which is located in the tank, is connected between one of the amplifier inputs and ground as illustrated.

The op-amp has two inputs, an output, and connections for positive and negative supply voltages. The schematic symbol for the op-amp with light-emitting diodes (LEDs) connected to the output is shown in Figure 58. The red LED is on when the op-amp output is a positive voltage, indicating the heater is on. The green LED is on when the output is a negative voltage, indicating the heater is off.

1. Using the circuit board as a guide, complete the schematic in Figure 58. The op-amp inputs are connected to a Wheatstone bridge. Show the values for all resistors.



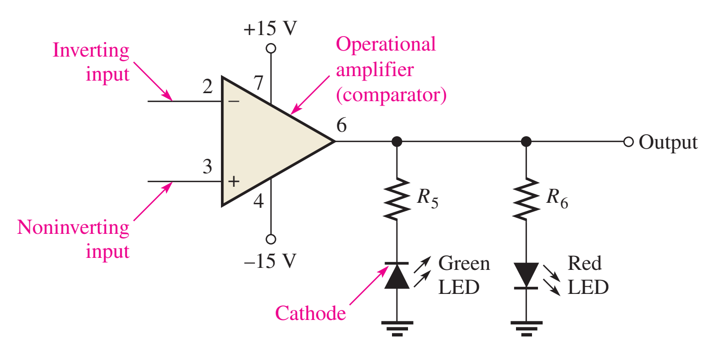
(a) Tank with heating element and thermistor

(b) Control board

▲ FIGURE 57

FIGURE 58

The op-amp and output LED indicators.



The Thermistor

The thermistor is a mixture of two metal-oxides, which exhibit a large resistance change as a function of temperature. The thermistor in the temperature controller circuit is located off the board near the point where temperature is to be sensed in the tank and is connected between the thermistor input and ground.

Thermistors have a nonlinear resistance-temperature characteristic described by the exponential equation:

$$R_T = R_0 e^{\beta\left(\frac{T_0 - T}{T_0 T}\right)}$$

Where:

R_T = the resistance at a given temperature

R_0 = the resistance at a reference temperature

T_0 = the reference temperature in Kelvin (K), typically 298 K, which is 25°C

T = temperature in K

β = a constant (K) provided by the manufacturer

This exponential equation where e is the base of natural logarithms can be solved easily on a scientific calculator.

The thermistor in this application is a Thermometrics RL2006-13.3K-140-D1 thermistor with a specified resistance of 25 kΩ at 25°C and a β of 4615 K. For convenience, the resistance of this thermistor is plotted as a function of temperature in Figure 59. Notice that the negative slope indicates this thermistor has a negative temperature coefficient (NTC); that is, its resistance decreases as the temperature increases.

As an example, the calculation for finding the resistance at $T = 50°\text{C}$ is shown. First, convert 50°C to K.

$$T = °\text{C} + 273 = 50°\text{C} + 273 = 323\text{ K}$$

Also,

$$T_0 = °\text{C} + 273 = 25°\text{C} + 273 = 298\text{ K}$$

$$R_0 = 25\text{ k}\Omega$$

$$R_T = R_0 e^{\beta\left(\frac{T_0 - T}{T_0 T}\right)}$$

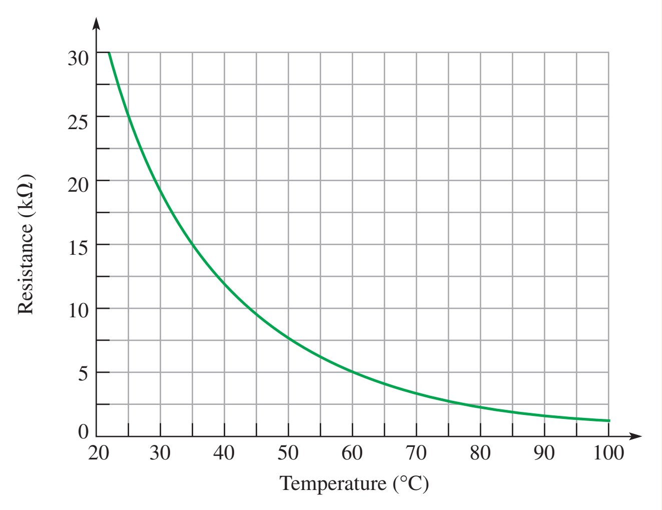
$$= (25\text{ k}\Omega)e^{4615\left(\frac{298-323}{298\times 323}\right)}$$

$$= (25\text{ k}\Omega)e^{-1.198}$$

$$= (25\text{ k}\Omega)(0.302)$$

$$= 7.54\text{ k}\Omega$$

FIGURE 59



Using your calculator, first determine the value of the exponent $\beta(T_0 - T)/(T_0T)$. Next determine the value of the term $e^{\beta\left(\frac{T_0-T}{T_0T}\right)}$. Finally, multiply by R_0. On many calculators, e^x is a secondary function.

2. Calculate the resistance of the thermistor at a temperature of 40°C using the exponential equation and confirm that your calculation is correct by comparing your result with Figure 59. Remember that temperatures in the equation are in kelvin. (K = °C + 273).
3. Calculate the resistance setting of R_3 to balance the bridge at 25°C.
4. Calculate the output voltage of the bridge (input to the op-amp) when the temperature of the thermistor is 40°C. Assume the bridge was balanced at 25°C and that the only change is the resistance of the thermistor.
5. If you needed to set the reference temperature to 0°C, what simple change would you make to the circuit? Show with a calculation that your change will work and draw the revised schematic.

Review

6. At 25°C, the thermistor will have about 7.5 V across it. Calculate the power it dissipates.
7. As the temperature increases, does the thermistor resistance increase or decrease?
8. Can ⅛ W resistors be used in this application? Explain your answer.
9. Why is only one LED on at a time at the output?

SUMMARY

- A series-parallel circuit is a combination of both series and parallel current paths.
- To determine total resistance in a series-parallel circuit, identify the series and parallel relationships, and then apply the formulas for series resistance and parallel resistance.
- To find the total current, apply Ohm's law and divide the total voltage by the total resistance.
- To determine branch currents, apply the current-divider formula, Kirchhoff's current law, or Ohm's law. Consider each circuit problem individually to determine the most appropriate method.
- To determine voltage drops across any portion of a series-parallel circuit, use the voltage-divider formula, Kirchhoff's voltage law, or Ohm's law. Consider each circuit problem individually to determine the most appropriate method.
- When a load resistor is connected across a voltage-divider output, the output voltage decreases.
- The load resistor should be large compared to the resistance across which it is connected, in order that the loading effect may be minimized.
- To find total resistance of a ladder network, start at the point farthest from the source and reduce the resistance in steps.
- A balanced Wheatstone bridge can be used to measure an unknown resistance.
- A bridge is balanced when the output voltage is zero. The balanced condition produces zero current through a load connected across the output terminals of the bridge.
- An unbalanced Wheatstone bridge can be used to measure physical quantities using transducers.
- Opens and shorts are typical circuit faults.
- Resistors normally open when they burn out.

KEY TERMS

Balanced bridge A bridge circuit that is in the balanced state is indicated by 0 V across the output.

Bleeder current The current left after the total load current is subtracted from the total current into the circuit.

Unbalanced bridge A bridge circuit that is in the unbalanced state as indicated by a voltage across the output that is proportional to the amount of deviation from the balanced state.

Wheatstone bridge A 4-legged type of bridge circuit with which an unknown resistance can be accurately measured using the balanced state. Deviations in resistance can be measured using the unbalanced state.

FORMULAS

1	$I_{\mathrm{BLEEDER}} = I_{\mathrm{T}} - I_{RL1} - I_{RL2}$	Bleeder current
2	$R_X = R_V\left(\frac{R_2}{R_4}\right)$	Unknown resistance in a Wheatstone bridge
3	$\Delta V_{\mathrm{OUT}} = \Delta R_{\mathrm{therm}}\left(\frac{V_{\mathrm{S}}}{4R}\right)$	Thermistor bridge output

TRUE/FALSE QUIZ

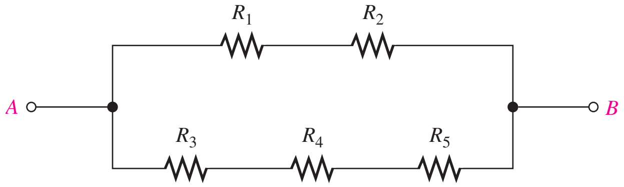
Answers are at the end of the chapter.

1. Parallel resistors are always connected between the same pair of nodes.
2. If one resistor is connected in series with a parallel combination, the series resistor will always have a larger voltage drop than the parallel resistors.
3. In a series-parallel combinational circuit, the same current will always be in parallel resistors.
4. A larger-value load resistor has a smaller loading effect on a circuit.
5. Normally, a DMM will have a small loading effect on a circuit.
6. When measuring dc voltage, the input resistance of a DMM is the same no matter what scale it is used on.
7. When measuring dc voltage, the input resistance of an analog multimeter is the same no matter what scale it is used on.
8. An $R/2R$ ladder is used in analog-to-digital converters.
9. A Wheatstone bridge is balanced when its output voltage is negative.
10. A Wheatstone bridge can be used to precisely measure resistance.

SELF-TEST

Answers are at the end of the chapter.

1. Which of the following statements are true concerning Figure 60?
   (a) R_1 and R_2 are in series with R_3, R_4, and R_5.
   (b) R_1 and R_2 are in series.
   (c) R_3, R_4, and R_5 are in parallel.
   (d) The series combination of R_1 and R_2 is in parallel with the series combination of R_3, R_4, and R_5.
   (e) answers (b) and (d)
2. The total resistance of Figure 60 can be found with which of the following formulas?
   (a) $R_1 + R_2 + R_3 \| R_4 \| R_5$ **(b)** $R_1 \| R_2 + R_3 \| R_4 \| R_5$
   (c) $(R_1 + R_2) \| (R_3 + R_4 + R_5)$ **(d)** none of these answers
3. If all of the resistors in Figure 60 have the same value, when voltage is applied across terminals A and B, the current is
   (a) greatest in R_5 **(b)** greatest in R_3, R_4, and R_5
   (c) greatest in R_1 and R_2 **(d)** the same in all the resistors



▲ FIGURE 60

4. Two 1.0 kΩ resistors are in series and this series combination is in parallel with a 2.2 kΩ resistor. The voltage across one of the 1.0 kΩ resistors is 6 V. The voltage across the 2.2 kΩ resistor is

(a) 6 V **(b)** 3 V **(c)** 12 V **(d)** 13.2 V

5. The parallel combination of a 330 Ω resistor and a 470 Ω resistor is in series with the parallel combination of four 1.0 kΩ resistors. A 100 V source is connected across the circuit. The resistor with the most current has a value of

(a) 1.0 kΩ **(b)** 330 Ω **(c)** 470 Ω

6. In the circuit described in Question 5, the resistor(s) with the most voltage has (have) a value of

(a) 1.0 kΩ **(b)** 470 Ω **(c)** 330 Ω

7. In the circuit of Question 5, the percentage of the total current through any single 1.0 kΩ resistor is

(a) 100% **(b)** 25% **(c)** 50% **(d)** 31.3%

8. The output of a certain voltage divider is 9 V with no load. When a load is connected, the output voltage

(a) increases **(b)** decreases **(c)** remains the same **(d)** becomes zero

9. A certain voltage divider consists of two 10 kΩ resistors in series. Which of the following load resistors will have the most effect on the output voltage?

(a) 1.0 MΩ **(b)** 20 kΩ **(c)** 100 kΩ **(d)** 10 kΩ

10. When a load resistance is connected to the output of a voltage-divider circuit, the current drawn from the source

(a) decreases **(b)** increases **(c)** remains the same **(d)** is cut off

11. In a ladder network, simplification should begin at

(a) the source **(b)** the resistor farthest from the source

(c) the center **(d)** the resistor closest to the source

12. In a certain four-step $R/2R$ ladder network, the smallest resistor value is 10 kΩ. The largest value is

(a) indeterminable **(b)** 20 kΩ **(c)** 50 kΩ **(d)** 100 kΩ

13. The output voltage of a balanced Wheatstone bridge is

(a) equal to the source voltage

(b) equal to zero

(c) dependent on all of the resistor values in the bridge

(d) dependent on the value of the unknown resistor

14. A certain Wheatstone bridge has the following resistor values: $R_V = 8\text{ k}\Omega, R_2 = 680\ \Omega$, and $R_4 = 2.2\text{ k}\Omega$. The unknown resistance is

(a) 2473 Ω **(b)** 25.9 kΩ **(c)** 187 Ω **(d)** 2890 Ω

15. You are measuring the voltage at a given point in a circuit that has very high resistance values and the measured voltage is a little lower than it should be. This is possibly because of

(a) one or more of the resistance values being off

(b) the loading effect of the voltmeter

(c) the source voltage is too low

(d) all of these answers

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 61(b).

1. If R_2 opens, the total current

(a) increases **(b)** decreases **(c)** stays the same

2. If R_3 opens, the current in R_2

(a) increases **(b)** decreases **(c)** stays the same

3. If R_4 opens, the voltage across it

(a) increases **(b)** decreases **(c)** stays the same

4. If R_4 is shorted, the total current

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 63.

5. If R_{10} opens, with 10 V applied between terminals A and B, the total current

(a) increases **(b)** decreases **(c)** stays the same

6. If R_1 opens with 10 V applied between terminals A and B, the voltage across R_1

(a) increases **(b)** decreases **(c)** stays the same

7. If there is a short between the left contact of R_3 and the bottom contact of R_5, the total resistance between A and B

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 67.

8. If R_4 opens, the voltage at point C

(a) increases **(b)** decreases **(c)** stays the same

9. If there is a short from point D to ground, the voltage from A to B

(a) increases **(b)** decreases **(c)** stays the same

10. If R_5 opens, the current through R_1

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 73.

11. If a 10 kΩ load resistor is connected across the output terminals A and B, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

12. If the 10 kΩ load resistor mentioned in Question 11 is replaced by a 100 kΩ load resistor, V_{OUT}

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 74.

13. If there is a short between the V_2 and V_3 terminals of the switch, the voltage V_1 with respect to ground

(a) increases **(b)** decreases **(c)** stays the same

14. If the switch is in the position shown and if the V_3 terminal of the switch is shorted to ground, the voltage across R_L

(a) increases **(b)** decreases **(c)** stays the same

15. If R_4 opens with the switch in the position shown, the voltage across R_L

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 79.

16. If R_4 opens, V_{OUT}

(a) increases **(b)** decreases **(c)** stays the same

17. If R_7 is shorted to ground, V_{OUT}

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 Identifying Series-Parallel Relationships

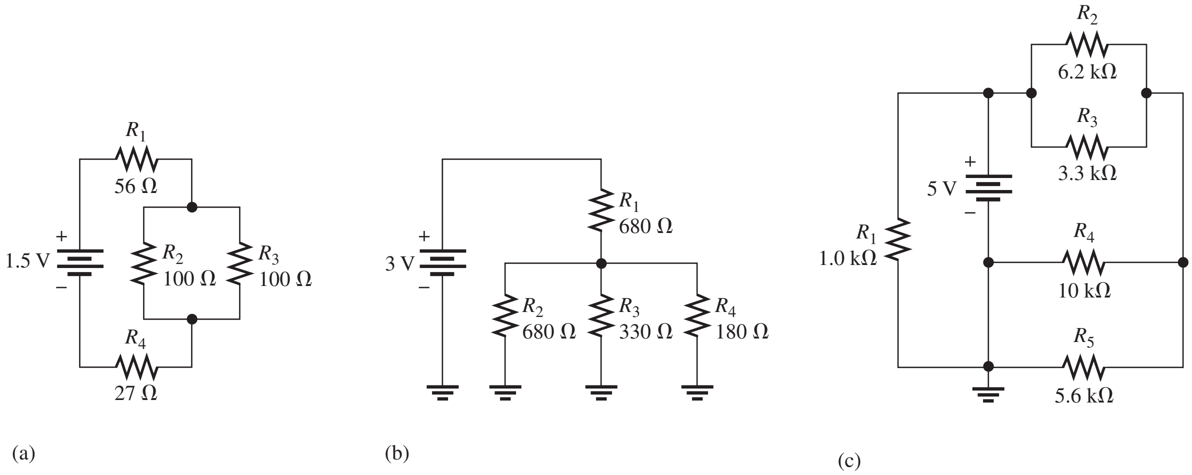
1. Visualize and draw the following series-parallel combinations:

(a) R_1 in series with the parallel combination of R_2 and R_3

  - **(b)** R_1 in parallel with the series combination of R_2 and R_3
  - **(c)** R_1 in parallel with a branch containing R_2 in series with a parallel combination of four other resistors

2. Visualize and draw the following series-parallel circuits:
  - **(a)** A parallel combination of three branches, each containing two series resistors
  - **(b)** A series combination of three parallel circuits, each containing two resistors

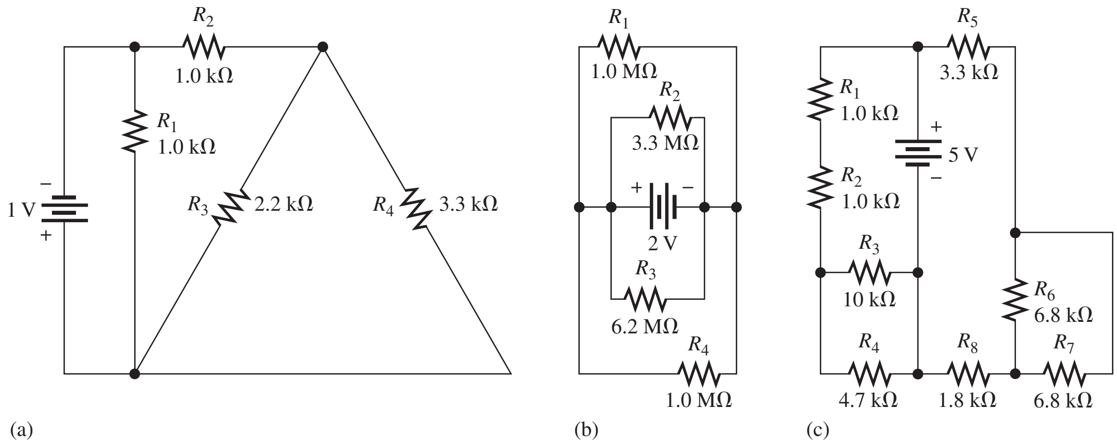
3. In each circuit of Figure 61, identify the series and parallel relationships of the resistors viewed from the source.



R_1
56 Ω
+
1.5 V
–
R_2
100 Ω
R_3
100 Ω
R_4
27 Ω
(a)
+
3 V
–
R_1
680 Ω
R_2
680 Ω
R_3
330 Ω
R_4
180 Ω
(b)
R_2
6.2 kΩ
R_3
3.3 kΩ
+
5 V
–
R_1
1.0 kΩ
R_4
10 kΩ
R_5
5.6 kΩ
(c)

▲ **FIGURE 61**

4. For each circuit in Figure 62, identify the series and parallel relationships of the resistors viewed from the source.

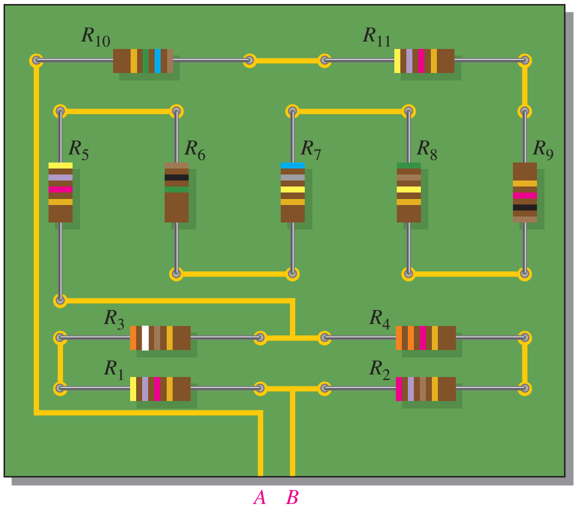


R_2
1.0 kΩ
R_1
1.0 kΩ
–
1 V
+
R_3
2.2 kΩ
R_4
3.3 kΩ
(a)
R_1
1.0 MΩ
R_2
3.3 MΩ
+
–
2 V
R_3
6.2 MΩ
R_4
1.0 MΩ
(b)
R_5
3.3 kΩ
R_1
1.0 kΩ
+
5 V
–
R_2
1.0 kΩ
R_3
10 kΩ
R_6
6.8 kΩ
R_4
4.7 kΩ
R_8
1.8 kΩ
R_7
6.8 kΩ
(c)

▲ **FIGURE 62**

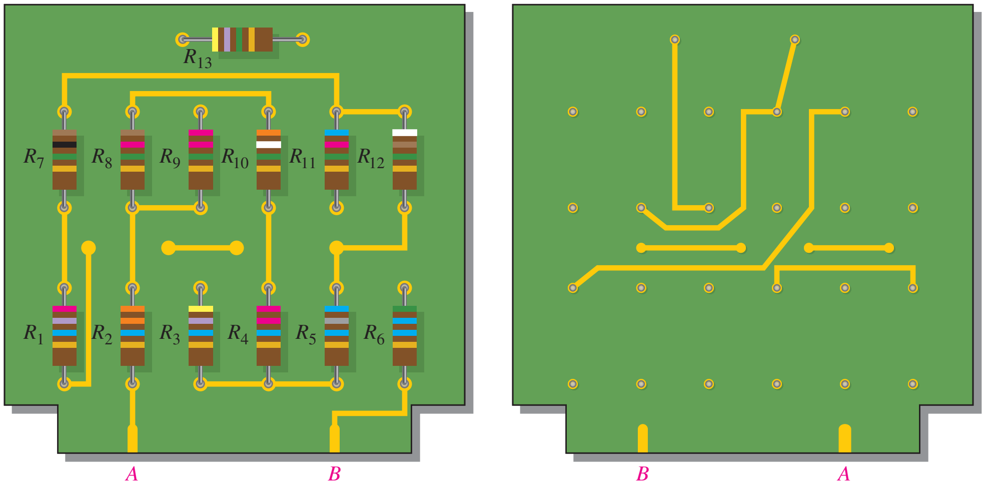
5. Draw the schematic of the PC board layout in Figure 63 showing resistor values and identify the series-parallel relationships.

▶ **FIGURE 63**



***6.** Develop a schematic for the double-sided PC board in Figure 64 and label the resistor values.

***7.** Lay out a PC board for the circuit in Figure 62(c). The battery is to be connected external to the board.



Side 1

Side 2

▲ **FIGURE 64**

SECTION 2 Analysis of Series-Parallel Resistive Circuits

8. A certain circuit is composed of two parallel resistors. The total resistance is 667 Ω. One of the resistors is 1.0 kΩ. What is the other resistor?

9. For each circuit in Figure 61, determine the total resistance presented to the source.

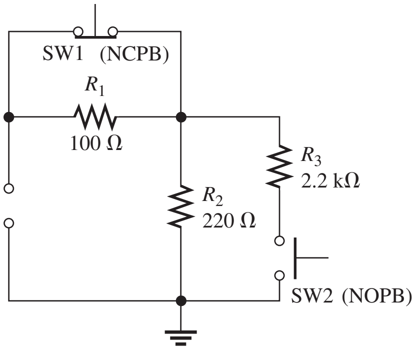
10. Repeat Problem 9 for each circuit in Figure 62.

11. Determine the current through each resistor in each circuit in Figure 61; then calculate each voltage drop.

12. Determine the current through each resistor in each circuit in Figure 62; then calculate each voltage drop.

13. Find R_T for all combinations of the switches in Figure 65.

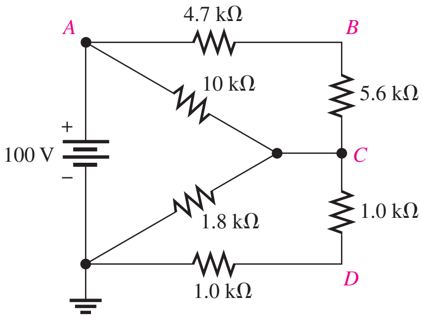
▶ **FIGURE 65**


SW1 (NCPB)
R_1
100 Ω
R_3
2.2 kΩ
R_2
220 Ω
SW2 (NOPB)

14. Determine the resistance between *A* and *B* in Figure 66 with the source removed.

15. Determine the voltage at each node with respect to ground in Figure 66.

▶ **FIGURE 66**


A
4.7 kΩ
B
10 kΩ
5.6 kΩ
+
100 V
–
C
1.8 kΩ
1.0 kΩ
1.0 kΩ
D

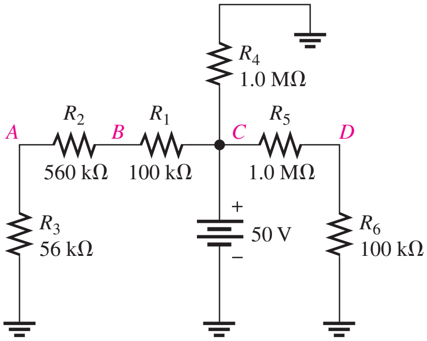
16. Determine the voltage at each node with respect to ground in Figure 67.

17. In Figure 67, how would you determine the voltage across R_2 by measuring without connecting a meter directly across the resistor?

18. Determine the resistance of the circuit in Figure 66 as seen from the voltage source.

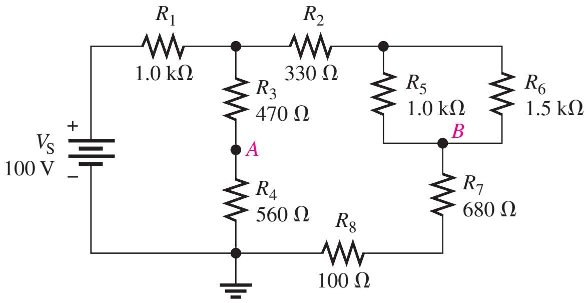
19. Determine the resistance of the circuit in Figure 67 as seen from the voltage source.

▶ **FIGURE 67**


R_4
1.0 MΩ
A
R_2
B
R_1
C
R_5
D
560 kΩ
100 kΩ
1.0 MΩ
+
R_3
56 kΩ
50 V
–
R_6
100 kΩ

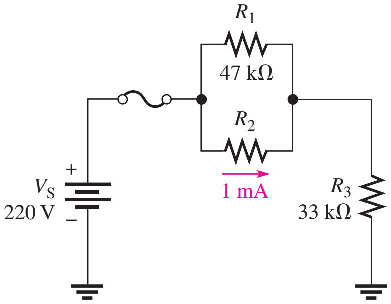
20. Determine the voltage, V_{AB}, in Figure 68.

▶ **FIGURE 68**


R_1
R_2
1.0 kΩ
330 Ω
R_3
470 Ω
R_5
1.0 kΩ
R_6
1.5 kΩ
B
+
V_S
100 V
–
A
R_4
560 Ω
R_7
680 Ω
R_8
100 Ω

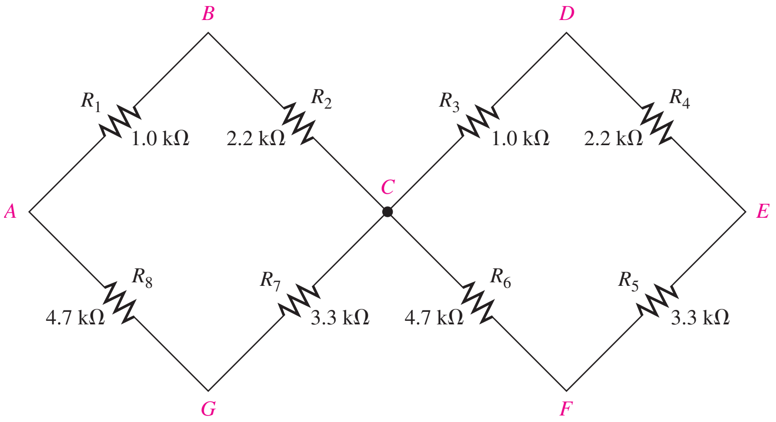
*__21.__ (a) Find the value of R_2 in Figure 69. (b) Determine the power in R_2.

▶ **FIGURE 69**



R_1
47 kΩ
R_2
1 mA
V_S
220 V
+
–
R_3
33 kΩ

***22.** Find the resistance between node *A* and each of the other nodes (R_{AB}, R_{AC}, R_{AD}, R_{AE}, R_{AF}, and R_{AG}) in Figure 70.

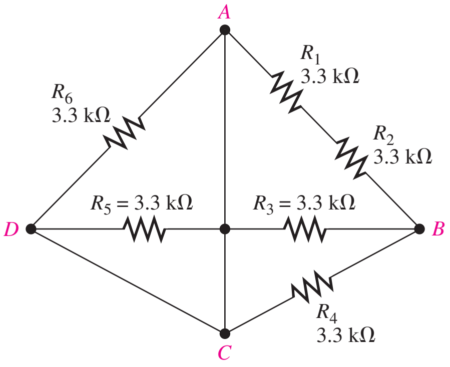


B
D
R_1
1.0 kΩ
R_2
2.2 kΩ
R_3
1.0 kΩ
R_4
2.2 kΩ
C
A
E
R_8
4.7 kΩ
R_7
3.3 kΩ
R_6
4.7 kΩ
R_5
3.3 kΩ
G
F

▲ **FIGURE 70**

***23.** Find the resistance between each of the following sets of nodes in Figure 71: *AB*, *BC*, and *CD*.

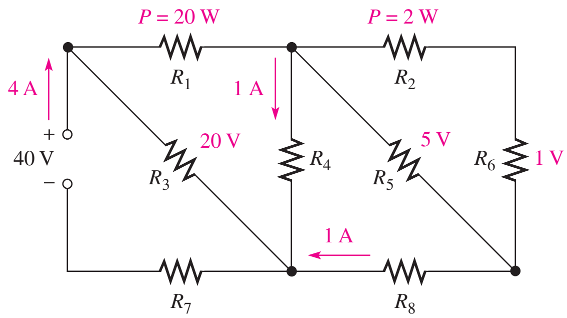
▶ **FIGURE 71**



A
R_1
3.3 kΩ
R_6
3.3 kΩ
R_2
3.3 kΩ
R_5 = 3.3 kΩ
R_3 = 3.3 kΩ
D
B
R_4
3.3 kΩ
C

***24.** Determine the value of each resistor in Figure 72.

▶ **FIGURE 72**



SECTION 3 Voltage Dividers with Resistive Loads

25. A voltage divider consists of two 56 kΩ resistors and a 15 V source. Calculate the unloaded output voltage. What will the output voltage be if a load resistor of 1.0 MΩ is connected to the output?

26. A 12 V battery output is divided down to obtain two output voltages. Three 3.3 kΩ resistors are used to provide the two taps. Determine the output voltages. If a 10 kΩ load is connected to the higher of the two outputs, what will its loaded value be?

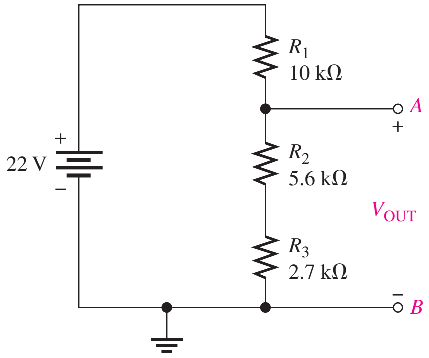
27. Which will cause a smaller decrease in output voltage for a given voltage divider, a 10 kΩ load or a 47 kΩ load?

28. In Figure 73, determine the output voltage with no load across the output terminals. With a 100 kΩ load connected from *A* to *B*, what is the output voltage?

29. In Figure 73, determine the output voltage with a 33 kΩ load connected between *A* and *B*.

30. In Figure 73, determine the continuous current drawn from the source with no load across the output terminals. With a 33 kΩ load, what is the current drain?

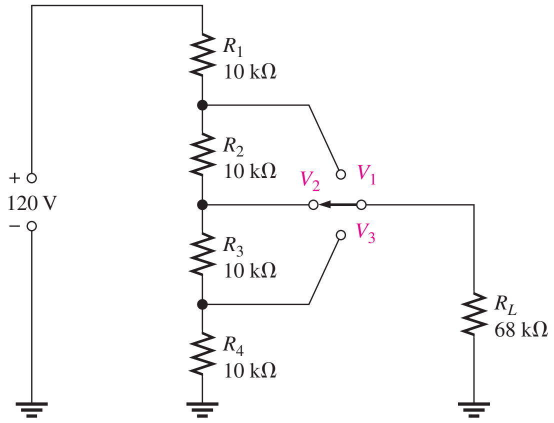
▶ **FIGURE 73**



***31.** Determine the resistance values for a voltage divider that must meet the following specifications: The current drawn from the source under unloaded condition is not to exceed 5 mA. The source voltage is to be 10 V, and the required outputs are to be 5 V and 2.5 V. Sketch the circuit. Determine the effect on the output voltages if a 1.0 kΩ load is connected to each tap one at a time.

32. The voltage divider in Figure 74 has a switched load. Determine the voltage at each tap (V_1, V_2, and V_3) for each position of the switch.

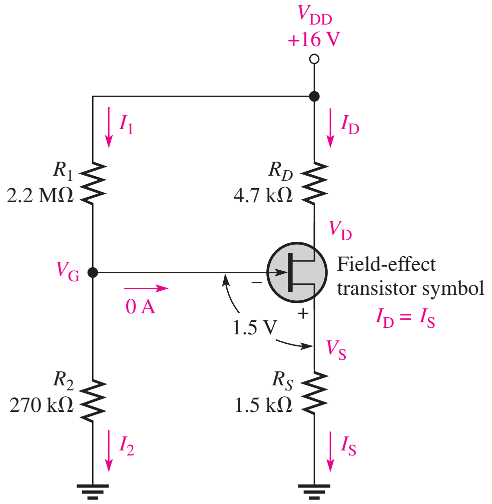
▶ **FIGURE 74**



***33.** Figure 75 shows a dc biasing arrangement for a field-effect transistor amplifier. Biasing is a common method for setting up certain dc voltage levels required for proper amplifier operation. Although you are not expected to be familiar with transistor amplifiers at this point, the dc voltages and currents in the circuit can be determined using methods that you already know.

(a) Find V_G and V_S **(b)** Determine I_1, I_2, I_D, and I_S **(c)** Find V_{DS} and V_{DG}

▶ **FIGURE 75**



***34.** Design a voltage divider to provide a 6 V output with no load and a minimum of 5.5 V across a 1.0 kΩ load. The source voltage is 24 V, and the unloaded current drain is not to exceed 100 mA.

SECTION 4 Loading Effect of a Voltmeter

35. On which one of the following voltage range settings will a voltmeter present the minimum load on a circuit?

(a) 1 V **(b)** 10 V **(c)** 100 V **(d)** 1000 V

36. Determine the internal resistance of a 20,000 Ω/V voltmeter on each of the following range settings.

(a) 0.5 V **(b)** 1 V **(c)** 5 V **(d)** 50 V **(e)** 100 V **(f)** 1000 V

37. The voltmeter described in Problem 36 is used to measure the voltage across R_4 in Figure 61(a).

(a) What range should be used?

(b) How much less is the voltage measured by the meter than the actual voltage?

38. Repeat Problem 37 if the voltmeter is used to measure the voltage across R_4 in the circuit of Figure 61(b).

39. A 10,000 Ω/V analog multimeter is used on the 10 V scale to measure the output of a voltage divider. If the divider consists of two series 100 kΩ resistors, what fraction of the source voltage will be measured across one of the resistors?

40. If a DMM with 10 MΩ input resistance is used instead of the analog multimeter in Problem 39, what percentage of the source voltage will be measured by the DMM?

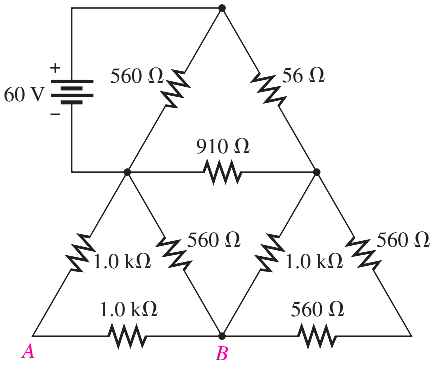
SECTION 5 Ladder Networks

41. For the circuit shown in Figure 76, calculate the following:

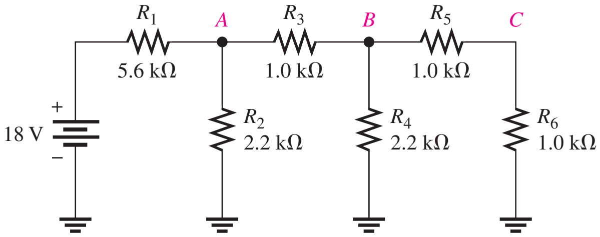
(a) Total resistance across the source **(b)** Total current from the source

(c) Current through the 910 Ω resistor **(d)** Voltage from *A* to point *B*

42. Determine the total resistance and the voltage at nodes *A*, *B*, and *C* in the ladder network of Figure 77.


60 V
+
−
560 Ω
56 Ω
910 Ω
1.0 kΩ
560 Ω
1.0 kΩ
560 Ω
1.0 kΩ
560 Ω
A
B

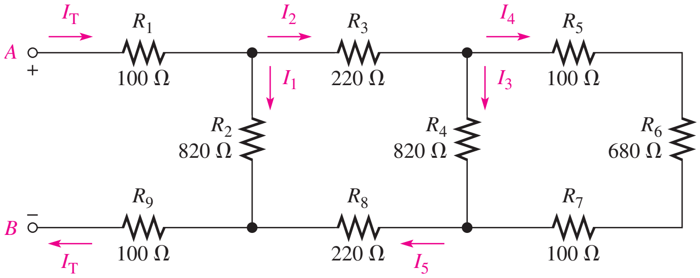
▲ FIGURE 76


R_1
5.6 kΩ
A
R_3
1.0 kΩ
B
R_5
1.0 kΩ
C
18 V
+
−
R_2
2.2 kΩ
R_4
2.2 kΩ
R_6
1.0 kΩ

▲ FIGURE 77

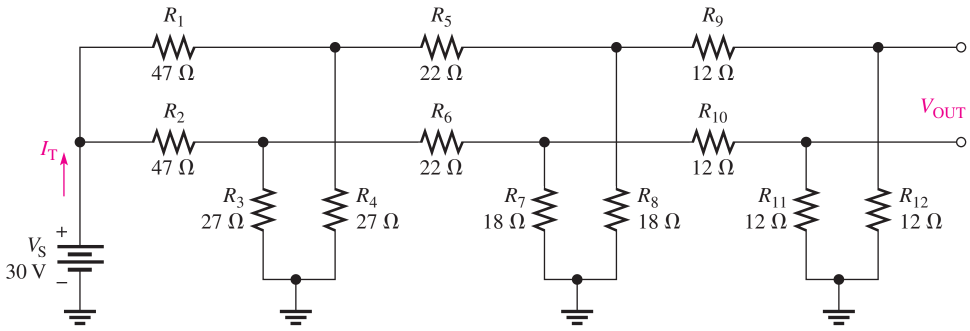
***43.** Determine the total resistance between terminals *A* and *B* of the ladder network in Figure 78. Also calculate the current in each branch with 10 V between *A* and *B*.

44. What is the voltage across each resistor in Figure 78 with 10 V between *A* and *B*?


I_T
R_1
100 Ω
A
+
I_2
R_3
220 Ω
I_4
R_5
100 Ω
I_1
I_3
R_2
820 Ω
R_4
820 Ω
R_6
680 Ω
R_9
100 Ω
B
−
I_T
R_8
220 Ω
I_5
R_7
100 Ω

▲ FIGURE 78

***45.** Find I_T and V_{OUT} in Figure 79.


R_1
47 Ω
R_5
22 Ω
R_9
12 Ω
R_2
47 Ω
R_6
22 Ω
R_{10}
12 Ω
V_{OUT}
I_T
R_3
27 Ω
R_4
27 Ω
R_7
18 Ω
R_8
18 Ω
R_{11}
12 Ω
R_{12}
12 Ω
V_S
30 V
+
−

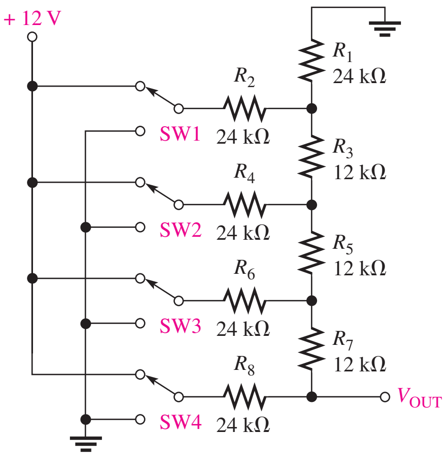
▲ FIGURE 79

46. Determine V_{OUT} for the $R/2R$ ladder network in Figure 80 for the following conditions:

(a) Switch SW2 connected to +12 V and the others connected to ground

(b) Switch SW1 connected to +12 V and the others connected to ground

▶ **FIGURE 80**


+ 12 V
R_1
24 kΩ
R_2
SW1
24 kΩ
R_3
12 kΩ
R_4
SW2
24 kΩ
R_5
12 kΩ
R_6
SW3
24 kΩ
R_7
12 kΩ
R_8
V_{OUT}
SW4
24 kΩ

47. Repeat Problem 46 for the following conditions:

(a) SW3 and SW4 to +12 V, SW1 and SW2 to ground

(b) SW3 and SW1 to +12 V, SW2 and SW4 to ground

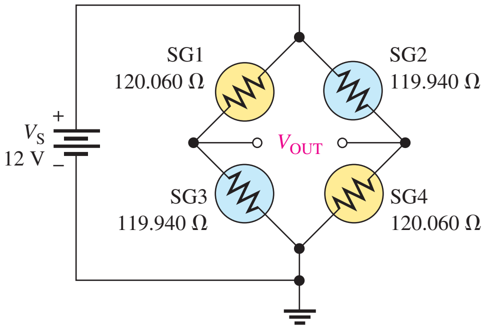
(c) All switches to +12 V

SECTION 6 The Wheatstone Bridge

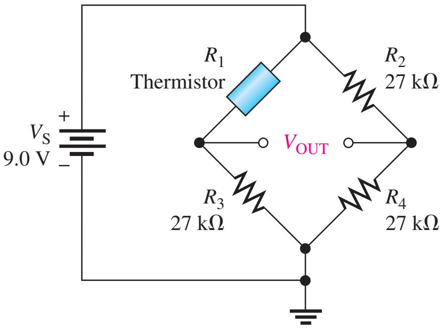
48. A resistor of unknown value is connected to a Wheatstone bridge circuit. The bridge parameters for a balanced condition are set as follows: $R_V = 18\ \text{k}\Omega$ and $R_2/R_4 = 0.02$. What is R_X?

49. A load cell has four identical strain gauges with an unstrained resistance of 120.000 Ω for each gauge (a standard value). When a load is added, the gauges in tension increase their resistance by 60 mΩ to 120.060 Ω and the gauges in compression decrease their resistance by 60 mΩ to 119.940 Ω as shown in Figure 81. What is the output voltage under load?

50. Determine the output voltage for the unbalanced bridge in Figure 82 for a temperature of 60°C. The temperature resistance characteristic for the thermistor is shown in Figure 59.


SG1
120.060 Ω
SG2
119.940 Ω
V_S
12 V
+
–
V_{OUT}
SG3
119.940 Ω
SG4
120.060 Ω

▲ **FIGURE 81**

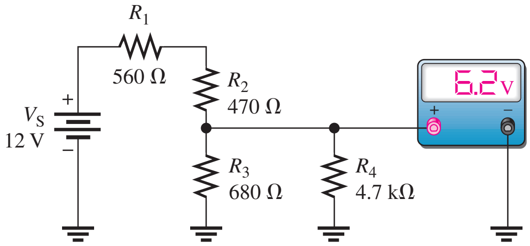

R_1
Thermistor
R_2
27 kΩ
V_S
9.0 V
+
–
V_{OUT}
R_3
27 kΩ
R_4
27 kΩ

▲ **FIGURE 82**

SECTION 7 Troubleshooting

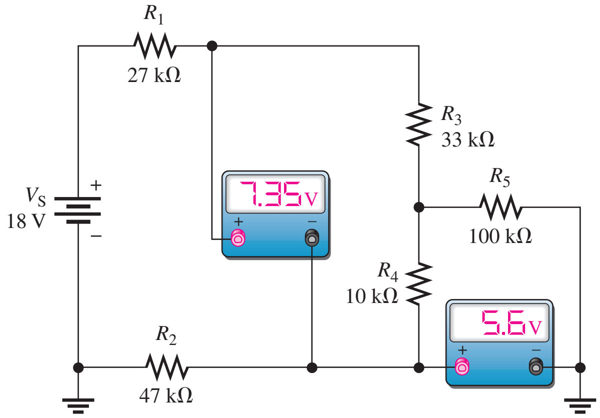
51. Is the voltmeter reading in Figure 83 correct?

52. Are the meter readings in Figure 84 correct?



R_1
560 Ω
V_S
12 V
+
−
R_2
470 Ω
R_3
680 Ω
R_4
4.7 kΩ
6.2V
+
−

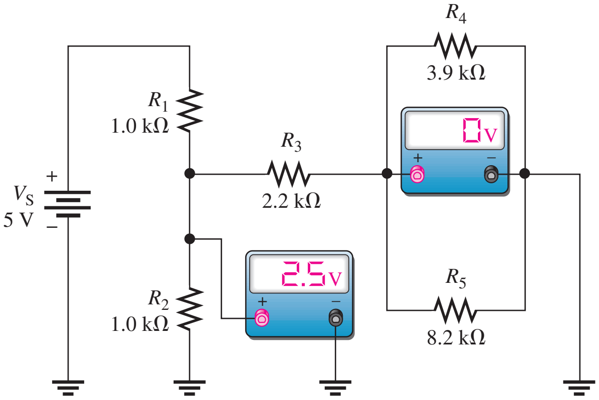
▲ FIGURE 83



R_1
27 kΩ
V_S
18 V
+
−
7.35V
+
−
R_3
33 kΩ
R_5
100 kΩ
R_4
10 kΩ
R_2
47 kΩ
5.6V
+
−

▲ FIGURE 84

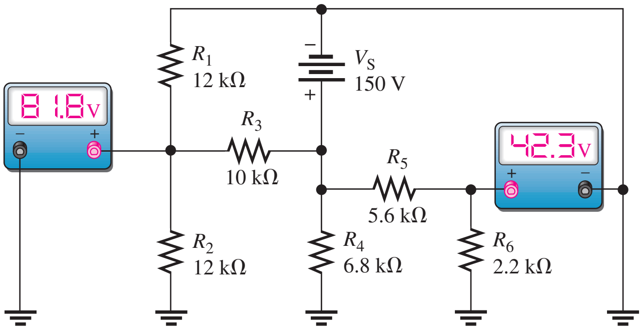
53. There is one fault in Figure 85. Based on the meter indications, determine what the fault is.



R_4
3.9 kΩ
R_1
1.0 kΩ
R_3
2.2 kΩ
0V
+
−
V_S
5 V
+
−
2.5V
+
−
R_2
1.0 kΩ
R_5
8.2 kΩ

▲ FIGURE 85

54. Look at the meters in Figure 86 and determine if there is a fault in the circuit. If there is a fault, identify it.

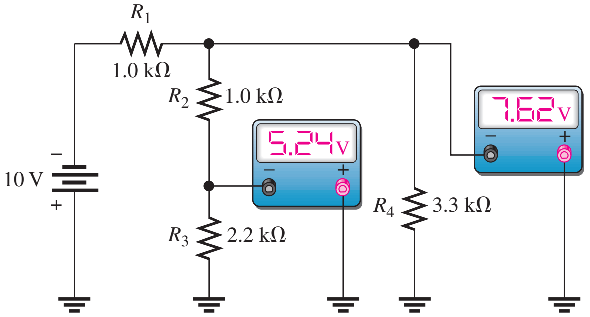


−
V_S
150 V
+
R_1
12 kΩ
81.8V
−
+
R_3
10 kΩ
R_5
5.6 kΩ
42.3V
+
−
R_2
12 kΩ
R_4
6.8 kΩ
R_6
2.2 kΩ

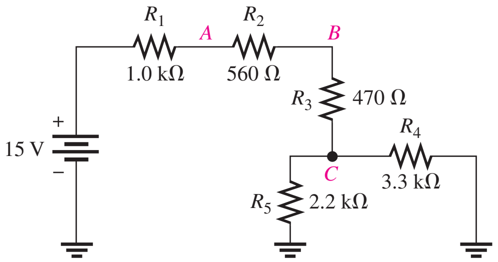
▲ FIGURE 86

55. Check the meter readings in Figure 87 and locate any fault that may exist.

56. If R_2 in Figure 88 opens, what voltages will be read at points A, B, and C?



▲ FIGURE 87



▲ FIGURE 88

Multisim Troubleshooting and Analysis

These problems require Multisim.

57. Open file P07-57 and measure the total resistance.

58. Open file P07-58. Determine by measurement if there is an open resistor and, if so, which one.

59. Open file P07-59 and determine the unspecified resistance value.

60. Open file P07-60 and determine how much the load resistance affects each of the resistor voltages.

61. Open file P07-61 and find the shorted resistor, if there is one.

62. Open file P07-62 and adjust the value of R_X until the bridge is approximately balanced.

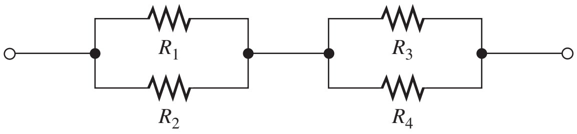
ANSWERS

SECTION CHECKUPS

SECTION 1 Identifying Series-Parallel Relationships

1. A series-parallel resistive circuit is a circuit consisting of both series and parallel connections.
2. See Figure 89.
3. Resistors R_1 and R_2 are in series with the parallel combination of R_3 and R_4.
4. R_3, R_4, and R_5 are in parallel. Also the series-parallel combination $R_2 + (R_3 \parallel R_4 \parallel R_5)$ is in parallel with R_1.
5. Resistors R_1 and R_2 are in parallel; R_3 and R_4 are in parallel.
6. Yes, the parallel combinations are in series.

▶ FIGURE 89



SECTION 2 Analysis of Series-Parallel Resistive Circuits

1. Voltage-divider and current-divider formulas, Kirchhoff's laws, and Ohm's law can be used in series-parallel analysis.
2. $R_T = R_1 + R_2 \parallel R_3 + R_4 = 608\ \Omega$
3. $I_3 = [R_2/(R_2 + R_3)]I_T = 11.1\text{ mA}$
4. $V_2 = I_2R_2 = 3.65\text{ V}$
5. $R_T = 47\ \Omega + 27\ \Omega + (27\ \Omega + 27\ \Omega) \parallel 47\ \Omega = 99.1\ \Omega$; $I_T = 1\text{ V}/99.1\ \Omega = 10.1\text{ mA}$

SECTION 3 Voltage Dividers with Resistive Loads

1. The load resistor decreases the output voltage.
2. True
3. $V_{\text{OUT(unloaded)}} = (100\text{ k}\Omega/147\text{ k}\Omega)30\text{ V} = 20.4\text{ V}$; $V_{\text{OUT(loaded)}} = (9.1\text{ k}\Omega/56.1\text{ k}\Omega)30\text{ V} = 4.87\text{ V}$

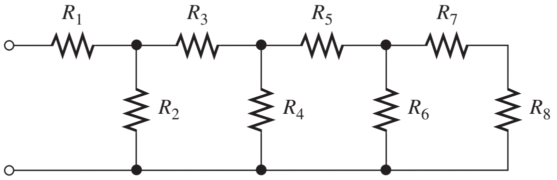
SECTION 4 Loading Effect of a Voltmeter

1. A voltmeter loads a circuit because the internal resistance of the meter appears in parallel with the circuit resistance across which it is connected, reducing the resistance between those two points of the circuit and drawing current from the circuit.
2. No, because the meter resistance is much larger than 1.0 kΩ.
3. Yes.
4. 40 MΩ

SECTION 5 Ladder Networks

1. See Figure 90.
2. $R_T = 11.6\text{ k}\Omega$
3. $I_T = 10\text{ V}/11.6\text{ k}\Omega = 859\ \mu\text{A}$
4. $I_2 = 640\ \mu\text{A}$
5. $V_A = 1.41\text{ V}$

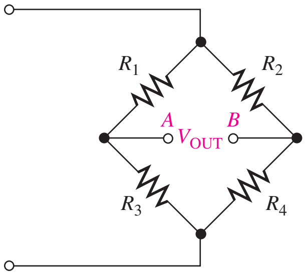
▶ FIGURE 90



SECTION 6 The Wheatstone Bridge

1. See Figure 91.
2. The bridge is balanced when $V_A = V_B$; that is, when $V_{\text{OUT}} = 0$
3. $R_X = 15\text{ k}\Omega$
4. An unbalanced bridge is used to measure transducer-sensed quantities.
5. A load cell is a transducer that uses strain gauges to convert mechanical force into an electrical signal.

▶ FIGURE 91



SECTION 7 Troubleshooting

1. Common circuit faults are opens and shorts.
2. The 10 kΩ resistor (R_3) is open.
3. **(a)** $V_A = 55\text{ V}$ **(b)** $V_A = 55\text{ V}$ **(c)** $V_A = 54.2\text{ V}$ **(d)** $V_A = 100\text{ V}$ **(e)** $V_A = 0\text{ V}$

RELATED PROBLEMS FOR EXAMPLES

1 The new resistor is in parallel with $R_4 + R_2 \| R_3$.

2 The resistor has no effect because it is shorted.

3 The new resistor is in parallel with R_5.

4 A to gnd: $R_T = R_4 + R_3 \| (R_1 + R_2)$
B to gnd: $R_T = R_4 + R_2 \| (R_1 + R_3)$
C to gnd: $R_T = R_4$

5 R_3 and R_6 are in series.

6 55.1 Ω

7 128.3 Ω

8 2.38 mA

9 $I_1 = 3.57$ mA; $I_3 = 2.34$ mA

10 $V_A = 44.8$ V; $V_1 = 35.2$ V

11 2.04 V

12 $V_{AB} = 5.48$ V; $V_{BC} = 1.66$ V; $V_{CD} = 0.86$ V

13 3.39 V

14 The current will increase to 59 μA.

15 5.19 V

16 $I_1 = 7.16$ mA; $I_2 = 3.57$ mA; $I_3 = 3.57$ mA; $I_4 = 1.74$ mA; $I_5 = 1.85$ mA; $I_6 = 1.85$ mA; $V_A = 29.3$ V; $V_B = 17.4$ V; $V_C = 8.70$ V

17 3.3 kΩ

18 0.45 V

19 5.73 V; 0 V

20 9.46 V

21 $V_A = 12$ V; $V_B = 13.8$ V

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** T **6.** T **7.** F **8.** T **9.** F **10.** T

SELF-TEST

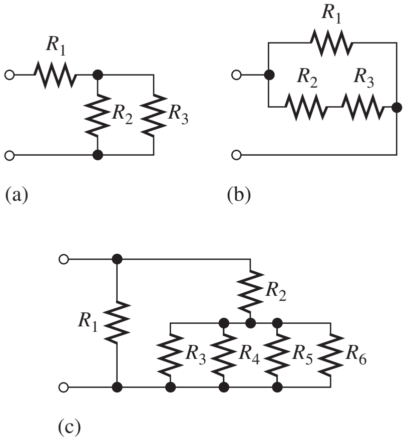
1. (e) **2.** (c) **3.** (c) **4.** (c) **5.** (b) **6.** (a) **7.** (b) **8.** (b)
9. (d) **10.** (b) **11.** (b) **12.** (b) **13.** (b) **14.** (a) **15.** (d)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (a) **3.** (a) **4.** (a) **5.** (b) **6.** (a)
7. (b) **8.** (c) **9.** (c) **10.** (c) **11.** (b) **12.** (a)
13. (b) **14.** (b) **15.** (a) **16.** (a) **17.** (a)

Answers to Odd-Numbered Problems

1. See Figure 92.


R_1
R_2
R_3
(a)
R_1
R_2
R_3
(b)
R_2
R_1
R_3
R_4
R_5
R_6
(c)

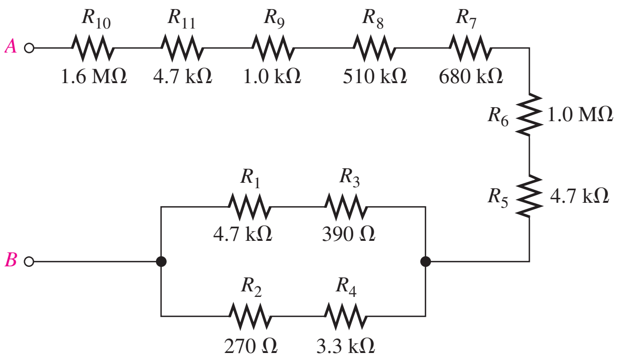
▲ FIGURE 92

3. **(a)** R_1 and R_4 are in series with the parallel combination of R_2 and R_3.

(b) R_1 is in series with the parallel combination of R_2, R_3, and R_4.

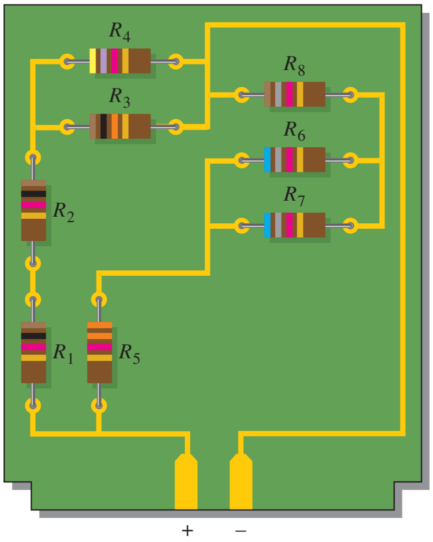
(c) The parallel combination of R_2 and R_3 is in series with the parallel combination of R_4 and R_5. This is all in parallel with R_1.

5. See Figure 93.


R_{10}
R_{11}
R_9
R_8
R_7
A
1.6 MΩ
4.7 kΩ
1.0 kΩ
510 kΩ
680 kΩ
R_6
1.0 MΩ
R_1
R_3
R_5
4.7 kΩ
4.7 kΩ
390 Ω
B
R_2
R_4
270 Ω
3.3 kΩ

▲ FIGURE 93

7. See Figure 94.


R_4
R_8
R_3
R_6
R_2
R_7
R_1
R_5
+
−

▲ FIGURE 94

9. **(a)** 133 Ω **(b)** 779 Ω **(c)** 852 Ω

11. **(a)** $I_1 = I_4 = 11.3\text{ mA}, I_2 = I_3 = 5.64\text{ mA},$
$V_1 = 633\text{ mV}, V_2 = V_3 = 564\text{ mV},$
$V_4 = 305\text{ mV}$

(b) $I_1 = 3.85\text{ mA}, I_2 = 563\ \mu\text{A},$
$I_3 = 1.16\text{ mA}, I_4 = 2.13\text{ mA}, V_1 = 2.62\text{ V},$
$V_2 = V_3 = V_4 = 383\text{ mV}$

(c) $I_1 = 5\text{ mA}, I_2 = 303\ \mu\text{A},$
$I_3 = 568\ \mu\text{A}, I_4 = 313\ \mu\text{A},$
$I_5 = 558\ \mu\text{A}, V_1 = 5\text{ V},$
$V_2 = V_3 = 1.88\text{ V}, V_4 = V_5 = 3.13\text{ V}$

13. SW1 closed, SW2 open: 220 Ω
SW1 closed, SW2 closed: 200 Ω
SW1 open, SW2 open: 320 Ω
SW1 open, SW2 closed: 300 Ω

15. $V_A = 100\text{ V}, V_B = 61.5\text{ V}, V_C = 15.7\text{ V}, V_D = 7.87\text{ V}$

17. Measure the voltage at A with respect to ground and the voltage at B with respect to ground. The difference is V_{R2}.

19. 303 kΩ

21. **(a)** 110 kΩ **(b)** 110 mW

23. $R_{AB} = 1.32\text{ k}\Omega$

$R_{BC} = 1.32\text{ k}\Omega$

$R_{CD} = 0\ \Omega$

25. 7.5 V unloaded, 7.29 V loaded

27. 47 kΩ

29. 8.77 V

31. $R_1 = 1000\ \Omega$; $R_2 = R_3 = 500\ \Omega$;

lower tap loaded: $V_{\text{lower}} = 1.82$ V, $V_{\text{upper}} = 4.55$ V

upper tap loaded: $V_{\text{lower}} = 1.67$ V, $V_{\text{upper}} = 3.33$ V

33. **(a)** $V_G = 1.75$ V, $V_S = 3.25$ V

(b) $I_1 = I_2 = 6.48\ \mu$A, $I_D = I_S = 2.17$ mA

(c) $V_{DS} = 2.55$ V, $V_{DG} = 4.05$ V

35. 1000 V

37. **(a)** 0.5 V range **(b)** Approximately 1 mV

39. 33.3%

41. **(a)** 271 Ω **(b)** 221 mA **(c)** 58.7 mA **(d)** 12 V

43. 621 Ω, $I_1 = I_9 = 16.1$ mA, $I_2 = 8.27$ mA,

$I_3 = I_8 = 7.84$ mA, $I_4 = 4.06$ mA, $I_5 = I_6 = I_7 = 3.78$ mA

45. 971 mA

47. **(a)** 9 V **(b)** 3.75 V **(c)** 11.25 V

49. 6 mV (right side positive with respect to left side)

51. No, it should be 4.39 V.

53. The 2.2 kΩ resistor (R_3) is open.

55. The 3.3 kΩ resistor (R_4) is open.

57. $R_T = 296.7\ \Omega$

59. $R_3 = 560\text{ k}\Omega$

61. R_5 shorted

CIRCUIT THEOREMS AND CONVERSIONS

CHAPTER OUTLINE

1 The DC Voltage Source
2 The Current Source
3 Source Conversions
4 The Superposition Theorem
5 Thevenin's Theorem
6 Norton's Theorem
7 Maximum Power Transfer Theorem
8 Delta-to-Wye (Δ-to-Y) and Wye-to-Delta (Y-to-Δ) Conversions
Application Activity

CHAPTER OBJECTIVES

- Describe the characteristics of a dc voltage source
- Describe the characteristics of a current source
- Perform source conversions
- Apply the superposition theorem to circuit analysis
- Apply Thevenin's theorem to simplify a circuit for analysis
- Apply Norton's theorem to simplify a circuit
- Apply the maximum power transfer theorem
- Perform Δ-to-Y and Y-to-Δ conversions

KEY TERMS

- Terminal equivalency
- Superposition theorem
- Thevenin's theorem
- Norton's theorem
- Maximum power transfer

APPLICATION ACTIVITY PREVIEW

In this application activity, a temperature-measurement and control circuit uses a Wheatstone bridge. You will utilize Thevenin's theorem as well as other techniques in the evaluation of this circuit.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

Some types of circuits are difficult to analyze using only Ohm's law and Kirchhoff's laws and require additional methods in order to simplify the analysis.

The theorems and conversions in this chapter make analysis easier for certain types of circuits. These methods do not replace Ohm's law and Kirchhoff's laws, but they are normally used in conjunction with the laws in certain situations.

Because all electric circuits are driven by either voltage sources or current sources, it is important to understand how to work with these elements. The superposition theorem will help you to deal with circuits that have multiple sources. Thevenin's and Norton's theorems provide methods for reducing a circuit to a simple equivalent form for ease of analysis. The maximum power transfer theorem is used in applications where it is important for a given circuit to provide maximum power to a load. An example of this is an audio amplifier that provides maximum power to a speaker. Delta-to-wye and wye-to-delta conversions are sometimes useful when you analyze bridge circuits that are commonly found in systems that measure physical parameters such as temperature, pressure, and strain.

From Chapter 8 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 THE DC VOLTAGE SOURCE

The dc voltage source is one of the principal types of energy source in electronic applications, so it is important to understand its characteristics. The dc voltage source ideally provides constant voltage to a load even when the load resistance varies.

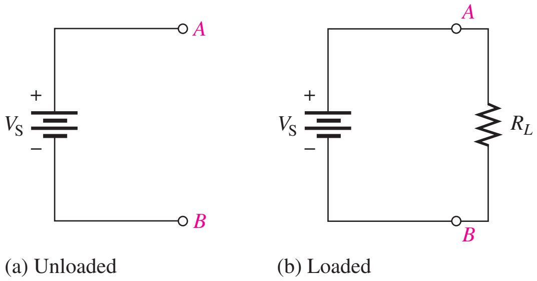
After completing this section, you should be able to

- **Describe the characteristics of a dc voltage source**
  - Compare a practical voltage source to an ideal source
  - Discuss the effect of loading on a practical voltage source

Figure 1(a) is the familiar symbol for an ideal dc voltage source. The voltage across its terminals, *A* and *B*, remains fixed regardless of the value of load resistance that may be connected across its output. Figure 1(b) shows a load resistor, R_L, connected. All of the source voltage, V_S, is dropped across R_L. Ideally, R_L can be changed to any value except zero, and the voltage will remain fixed. The ideal voltage source has an internal resistance of zero.

▶ FIGURE 1

Ideal dc voltage source.

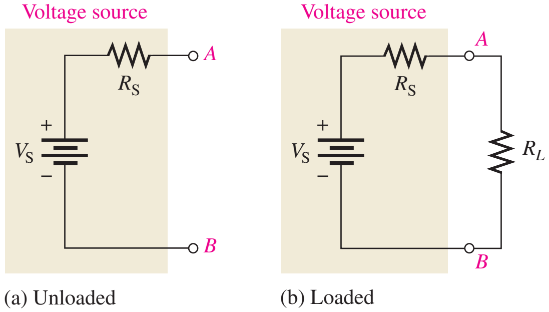


(a) Unloaded (b) Loaded

In reality, no voltage source is ideal; however, regulated power supplies can approach ideal when operated within the specified output current. All voltage sources have some inherent internal resistance as a result of their physical and/or chemical makeup, which can be represented by a resistor in series with an ideal source, as shown in Figure 2(a). R_S is the internal source resistance and V_S is the source voltage. With no load, the output voltage (voltage from *A* to *B*) is V_S. This voltage is sometimes called the *open circuit voltage.*

▶ FIGURE 2

Practical voltage source.



(a) Unloaded (b) Loaded

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Loading of the Voltage Source

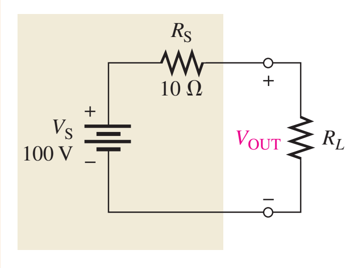
When a load resistor is connected across the output terminals, as shown in Figure 2(b), all of the source voltage does not appear across R_L. Some of the voltage is dropped across R_S because R_S and R_L are in series.

If R_S is very small compared to R_L, the source approaches ideal because almost all of the source voltage, V_S, appears across the larger resistance, R_L. Very little voltage is dropped across the internal resistance, R_S. If R_L changes, most of the source voltage remains across the output as long as R_L is much larger than R_S. As a result, very little change occurs in the output voltage. The larger R_L is, compared to R_S, the less change there is in the output voltage. Example 1 illustrates the effect of changes in R_L on the output voltage when R_L is much greater than R_S.

EXAMPLE 1

Calculate the voltage output of the source in Figure 3 for the following values of R_L: 100 Ω, 560 Ω, and 1.0 kΩ.

▶ FIGURE 3



Solution For $R_L = 100\ \Omega$, the voltage output is

$$V_{OUT} = \left(\frac{R_L}{R_S + R_L}\right)V_S = \left(\frac{100\ \Omega}{110\ \Omega}\right)100\ \text{V} = \mathbf{90.9\ V}$$

For $R_L = 560\ \Omega$,

$$V_{OUT} = \left(\frac{560\ \Omega}{570\ \Omega}\right)100\ \text{V} = \mathbf{98.2\ V}$$

For $R_L = 1.0\ \text{k}\Omega$,

$$V_{OUT} = \left(\frac{1000\ \Omega}{1010\ \Omega}\right)100\ \text{V} = \mathbf{99.0\ V}$$

Notice that the output voltage is within 10% of the source voltage, V_S, for all three values of R_L because R_L is at least ten times R_S.

*Related Problem** Determine V_{OUT} in Figure 3 if $R_S = 50\ \Omega$ and $R_L = 10\ \text{k}\Omega$.

Use Multisim file E08-01 to verify the calculated results in this example and to confirm your calculation for the related problem.

---

*Answers are at the end of the chapter.

The output voltage decreases significantly as the load resistance is made smaller compared to the internal source resistance. Example 2 illustrates the effect of a smaller R_L and confirms the requirement that R_L must be much larger than R_S (at least 10 times) in order to maintain the output voltage near its open circuit value.

EXAMPLE 2

Determine V_{OUT} for $R_L = 10\ \Omega$ and for $R_L = 1.0\ \Omega$ in Figure 3.

Solution For $R_L = 10\ \Omega$, the voltage output is

$$V_{OUT} = \left(\frac{R_L}{R_S + R_L}\right)V_S = \left(\frac{10\ \Omega}{20\ \Omega}\right)100\text{ V} = \mathbf{50\ V}$$

For $R_L = 1.0\ \Omega$,

$$V_{OUT} = \left(\frac{1.0\ \Omega}{11\ \Omega}\right)100\text{ V} = \mathbf{9.09\ V}$$

Related Problem What is V_{OUT} with no load resistor in Figure 3?

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. What is the symbol for the ideal voltage source?
2. Draw a practical voltage source.
3. What is the internal resistance of the ideal voltage source?
4. What effect does the load have on the output voltage of the practical voltage source?

2 THE CURRENT SOURCE

The current source is another type of energy source that ideally provides a constant current to a variable load. The concept of the current source is important in certain types of transistor circuits.

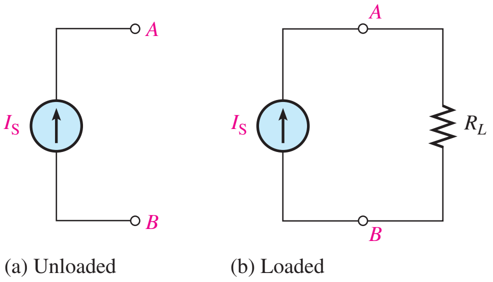
After completing this section, you should be able to

- **Describe the characteristics of a current source**
  - Compare a practical current source to an ideal source
  - Discuss the effect of loading on a practical current source

Figure 4(a) shows a symbol for the ideal current source. The arrow indicates the direction of source current, I_S. An ideal current source produces a constant value of current through a load, regardless of the value of the load. This concept is illustrated in Figure 4(b), where a load resistor is connected to the current source between terminals *A* and *B*. The ideal current source has an infinitely large internal parallel resistance.

FIGURE 4

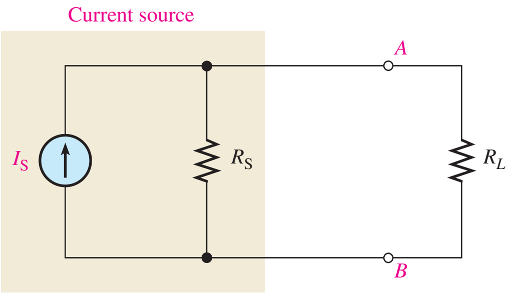
Ideal current source.



(a) Unloaded (b) Loaded

Transistors act basically as current sources, and for this reason, knowledge of the current source concept is important. You will find that the equivalent model of a transistor does contain a current source.

Although the ideal current source can be used in most analysis work, no actual device is ideal. A practical current source representation is shown in Figure 5. Here the internal resistance appears in parallel with the ideal current source.



◀ **FIGURE 5**

Practical current source with load.

If the internal source resistance, R_S, is much larger than a load resistor, the practical source approaches ideal. The reason is illustrated in the practical current source shown in Figure 5. Part of the current, I_S, is through R_S, and part is through R_L. The internal source resistance, R_S, and the load resistor, R_L, act as a current divider. If R_S is much larger than R_L, most of the current is through R_L and very little through R_S. As long as R_L remains much smaller than R_S, the current through R_L will stay almost constant, no matter how much R_L changes.

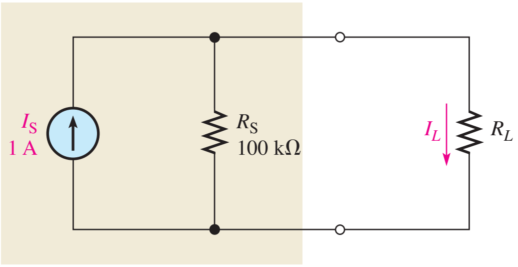
If there is a constant-current source, you can normally assume that R_S is so much larger than the load resistance that R_S can be neglected. This simplifies the source to ideal, making the analysis easier.

Example 3 illustrates the effect of changes in R_L on the load current when R_L is much smaller than R_S. Generally, R_L should be at least ten times smaller than R_S ($10R_L \leq R_S$) for a source to act as a reasonable current source.

EXAMPLE 3

Calculate the load current (I_L) in Figure 6 for the following values of R_L: 1 kΩ, 5.6 kΩ, and 10 kΩ.

▶ **FIGURE 6**



Solution For $R_L = 1\text{ k}\Omega$, the load current is

$$I_L = \left(\frac{R_S}{R_S + R_L}\right)I_S = \left(\frac{100\text{ k}\Omega}{101\text{ k}\Omega}\right)1\text{ A} = \mathbf{990\text{ mA}}$$

For $R_L = 5.6\text{ k}\Omega$,

$$I_L = \left(\frac{100\text{ k}\Omega}{105.6\text{ k}\Omega}\right)1\text{ A} = \mathbf{947\text{ mA}}$$

For $R_L = 10\text{ k}\Omega$,

$$I_L = \left(\frac{100\text{ k}\Omega}{110\text{ k}\Omega}\right)1\text{ A} = \mathbf{909\text{ mA}}$$

Notice that the load current, I_L, is within 10% of the source current for each value of R_L because R_L is at least ten times smaller than R_S in each case.

Related Problem At what value of R_L in Figure 6 will the load current equal 750 mA?

SECTION 2 CHECKUP

1. What is the symbol for an ideal current source?
2. Draw the practical current source.
3. What is the internal resistance of the ideal current source?
4. What effect does the load have on the load current of the practical current source?

3 SOURCE CONVERSIONS

In circuit analysis, it is sometimes useful to convert a voltage source to an equivalent current source, or vice versa.

After completing this section, you should be able to

- **Perform source conversions**
  - Convert a voltage source to a current source
  - Convert a current source to a voltage source
  - Define *terminal equivalency*

Converting a Voltage Source to a Current Source

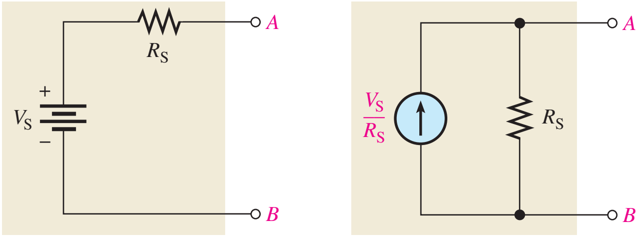
The source voltage, V_S, divided by the internal source resistance, R_S, gives the value of the equivalent source current.

$$I_S = \frac{V_S}{R_S}$$

The value of R_S is the same for both the voltage and current sources. As illustrated in Figure 7, the directional arrow for the current points from minus to plus. The equivalent current source is in parallel with R_S.

▶ FIGURE 7

Conversion of voltage source to equivalent current source.

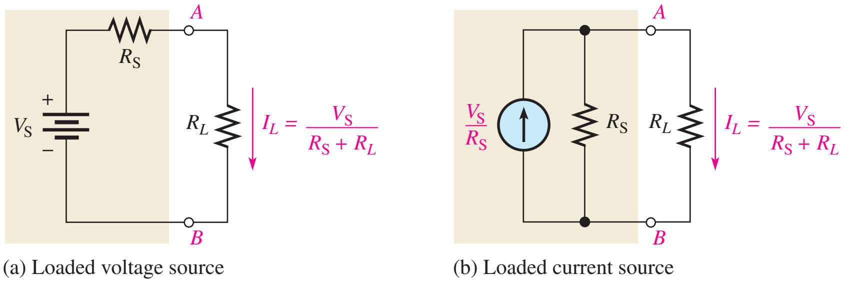


(a) Voltage source (b) Current source

Equivalency of two sources means that for any given load resistance connected to the two sources, the same load voltage and load current are produced by both sources. This concept is called **terminal equivalency**.

You can show that the voltage source and the current source in Figure 7 are equivalent by connecting a load resistor to each, as shown in Figure 8, and then calculating the load current. For the voltage source, the load current is

$$I_L = \frac{V_S}{R_S + R_L}$$



(a) Loaded voltage source

(b) Loaded current source

▲ **FIGURE 8**

Equivalent sources with loads.

For the current source,

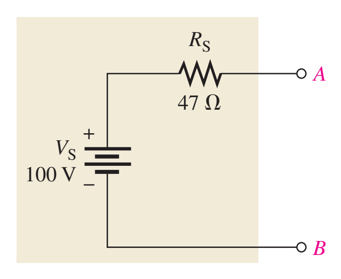
$$I_L = \left(\frac{R_S}{R_S + R_L}\right)\frac{V_S}{R_S} = \frac{V_S}{R_S + R_L}$$

As you see, both expressions for I_L are the same. These equations show that the sources are equivalent as far as the load or terminals *A* and *B* are concerned.

EXAMPLE 4

Convert the voltage source in Figure 9 to an equivalent current source and show the equivalent circuit.

► **FIGURE 9**

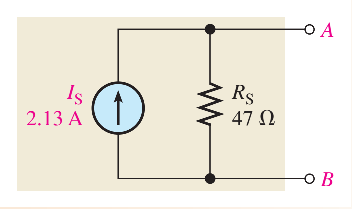


Solution The value of the internal resistance, R_S, of the equivalent current is the same as the internal resistance of the voltage source. Therefore, the equivalent current source is

$$I_S = \frac{V_S}{R_S} = \frac{100\text{ V}}{47\ \Omega} = 2.13\text{ A}$$

Figure 10 shows the equivalent circuit.

▶ FIGURE 10



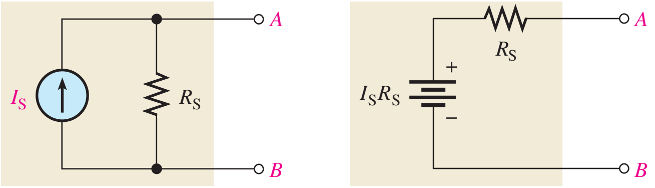
Related Problem Determine I_S and R_S of a current source equivalent to a voltage source with $V_S = 12$ V and $R_S = 10\ \Omega$.

Converting a Current Source to a Voltage Source

The source current, I_S, multiplied by the internal source resistance, R_S, gives the value of the equivalent source voltage.

$$V_S = I_S R_S$$

Again, R_S remains the same. The polarity of the voltage source is minus to plus in the direction of the current. The equivalent voltage source is the voltage in series with R_S, as illustrated in Figure 11.



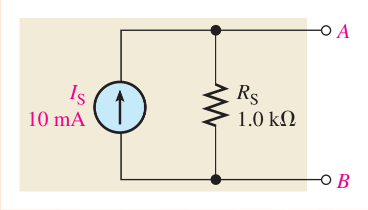
(a) Current source (b) Voltage source

▲ FIGURE 11

Conversion of current source to equivalent voltage source.

EXAMPLE 5 Convert the current source in Figure 12 to an equivalent voltage source and show the equivalent circuit.

▶ FIGURE 12

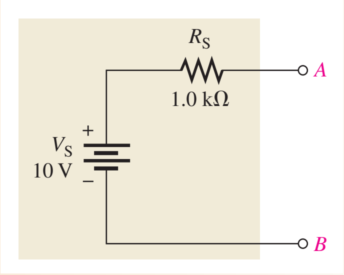


Solution The value of R_S is the same as with a current source. Therefore, the equivalent voltage source is

$$V_S = I_S R_S = (10\text{ mA})(1.0\text{ k}\Omega) = 10\text{ V}$$

Figure 13 shows the equivalent circuit.

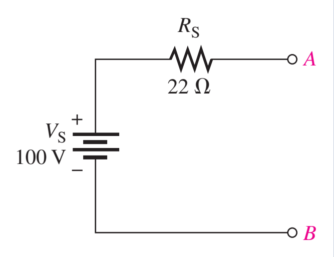
▶ FIGURE 13



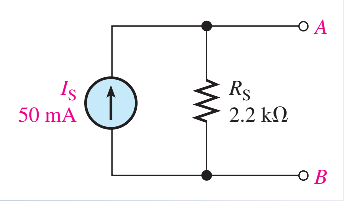
Related Problem Determine V_S and R_S of a voltage source equivalent to a current source with $I_S = 500$ mA and $R_S = 600\ \Omega$.

SECTION 3 CHECKUP

1. Write the formula for converting a voltage source to a current source.
2. Write the formula for converting a current source to a voltage source.
3. Convert the voltage source in Figure 14 to an equivalent current source.
4. Convert the current source in Figure 15 to an equivalent voltage source.



▲ FIGURE 14



▲ FIGURE 15

4 THE SUPERPOSITION THEOREM

Some circuits use more than one voltage or current source. For example, most amplifiers operate with two voltage sources: an ac and a dc source. Additionally, some amplifiers require both a positive and a negative dc voltage source for proper operation. When multiple sources are used in a circuit, the superposition theorem provides a method for analysis.

After completing this section, you should be able to

- **Apply the superposition theorem to circuit analysis**
  - State the superposition theorem
  - List the steps in applying the theorem

The superposition method is a way to determine currents in a circuit with multiple sources by leaving one source at a time and replacing the other sources by their internal resistances. Recall that an ideal voltage source has a zero internal resistance and an ideal current source has infinite internal resistance. All sources will be treated as ideal in order to simplify the coverage.

A general statement of the **superposition theorem** is as follows:

> **The current in any given branch of a multiple-source circuit can be found by determining the currents in that particular branch produced by each source acting alone, with all other sources replaced by their internal resistances. The total current in the branch is the algebraic sum of the individual currents in that branch.**

The steps in applying the superposition method are as follows:

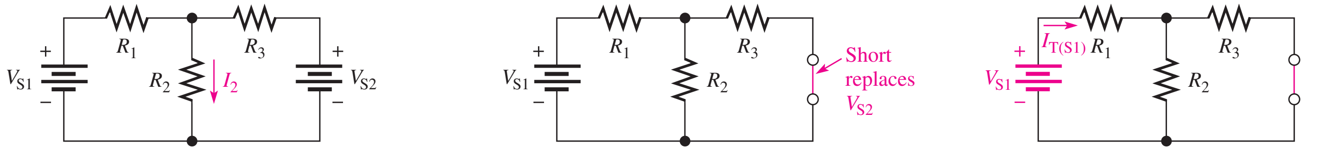
Step 1: Leave one voltage (or current) source at a time in the circuit and replace each of the other voltage (or current) sources with its internal resistance. For ideal sources a short represents zero internal resistance and an open represents infinite internal resistance.

Step 2: Determine the particular current (or voltage) that you want just as if there were only one source in the circuit.

Step 3: Take the next source in the circuit and repeat Steps 1 and 2. Do this for each source.

Step 4: To find the actual current in a given branch, algebraically sum the currents due to each individual source. Once you find the current, you can determine the voltage using Ohm's law.

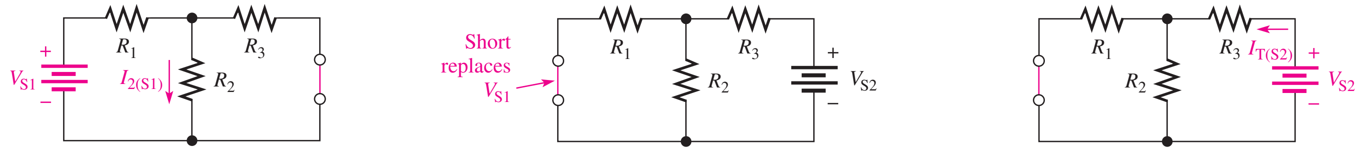
The approach to superposition is demonstrated in Figure 16 for a series-parallel circuit with two ideal voltage sources. Study the steps in this figure.



(a) Problem: Find I_2.

(b) Replace V_{S2} with zero resistance (short).

(c) Find R_T and I_T looking from V_{S1}:
$R_{T(S1)} = R_1 + R_2 \parallel R_3$
$I_{T(S1)} = V_{S1}/R_{T(S1)}$

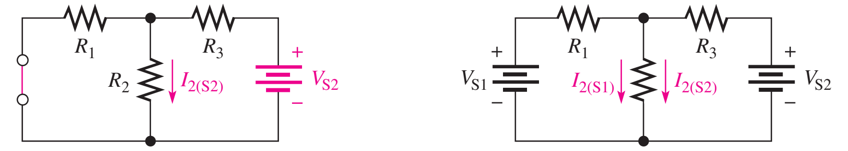


(d) Find I_2 due to V_{S1} (current divider):

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)}$$

(e) Replace V_{S1} with zero resistance (short).

(f) Find R_T and I_T looking from V_{S2}:
$R_{T(S2)} = R_3 + R_1 \parallel R_2$
$I_{T(S2)} = V_{S2}/R_{T(S2)}$



(g) Find I_2 due to V_{S2}:

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)}$$

(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):
$I_2 = I_{2(S1)} + I_{2(S2)}$

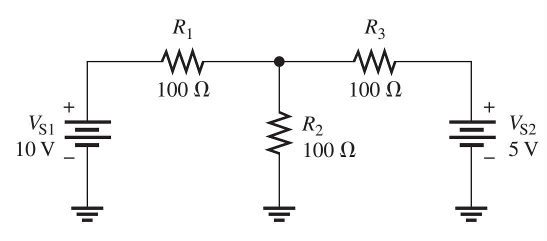
▲ FIGURE 16

Demonstration of the superposition method.

EXAMPLE 6

Use the superposition theorem to find the current through R_2 of Figure 17.

▶ FIGURE 17



Solution **Step 1:** Replace V_{S2} with a short and find the current through R_2 due to voltage source V_{S1}, as shown in Figure 18. To find I_2, use the current-divider formula. Looking from V_{S1},

$$R_{T(S1)} = R_1 + \frac{R_3}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

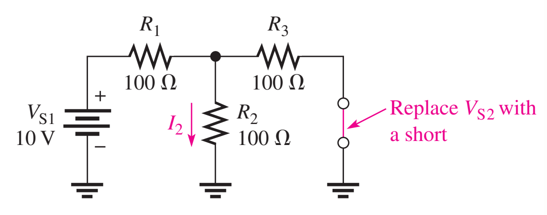
$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{10\text{ V}}{150\ \Omega} = 66.7\text{ mA}$$

The current through R_2 due to V_{S1} is

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right)I_{T(S1)} = \left(\frac{100\ \Omega}{200\ \Omega}\right)66.7\text{ mA} = 33.3\text{ mA}$$

Note that this current is downward through R_2.

▶ FIGURE 18



Step 2: Find the current through R_2 due to voltage source V_{S2} by replacing V_{S1} with a short, as shown in Figure 19. Looking from V_{S2},

$$R_{T(S2)} = R_3 + \frac{R_1}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

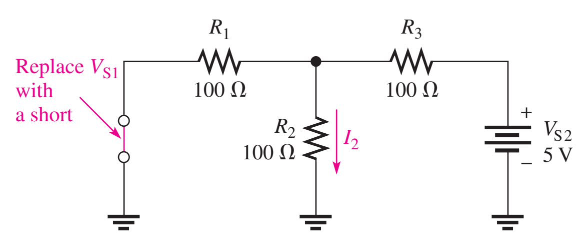
$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{5\text{ V}}{150\ \Omega} = 33.3\text{ mA}$$

The current through R_2 due to V_{S2} is

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right)I_{T(S2)} = \left(\frac{100\ \Omega}{200\ \Omega}\right)33.3\text{ mA} = 16.7\text{ mA}$$

Note that this current is downward through R_2.

▶ FIGURE 19


Replace V_{S1}
with
a short
R_1
100 Ω
R_3
100 Ω
R_2
100 Ω
I_2
+
V_{S2}
5 V
–

Step 3: Both component currents are downward through R_2, so they have the same algebraic sign. Therefore, add the values to get the total current through R_2.

$$I_{2(\mathrm{tot})} = I_{2(\mathrm{S1})} + I_{2(\mathrm{S2})} = 33.3\text{ mA} + 16.7\text{ mA} = \mathbf{50\text{ mA}}$$

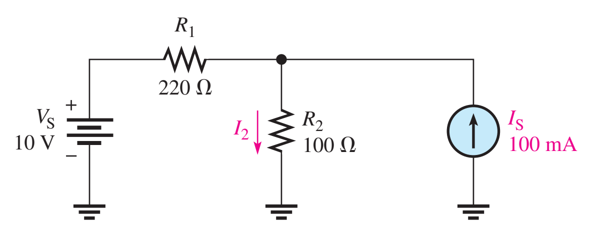
Related Problem Determine the total current through R_2 if the polarity of V_{S2} in Figure 17 is reversed.

Use Multisim file E08-06 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 7

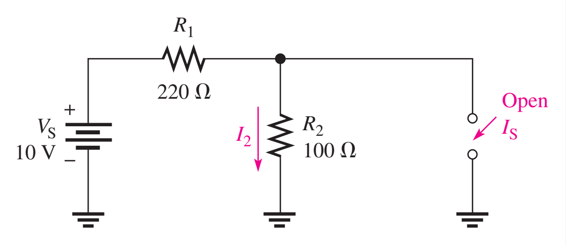
Find the current through R_2 in the circuit of Figure 20.

▶ FIGURE 20


R_1
220 Ω
+
V_S
10 V
–
I_2
R_2
100 Ω
I_S
100 mA

Solution **Step 1:** Find the current through R_2 due to V_S by replacing I_S with an open, as shown in Figure 21.

▶ FIGURE 21


R_1
220 Ω
+
V_S
10 V
–
I_2
R_2
100 Ω
Open
I_S

Notice that all of the current produced by V_S is through R_2. Looking from V_S,

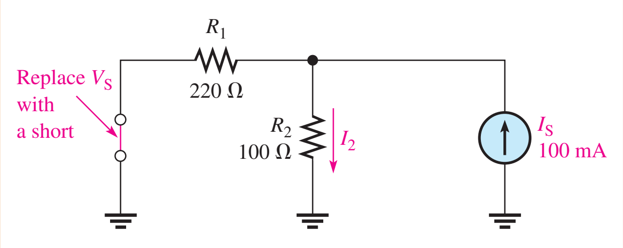
$$R_T = R_1 + R_2 = 320\ \Omega$$

The current through R_2 due to V_S is

$$I_{2(V_S)} = \frac{V_S}{R_T} = \frac{10\text{ V}}{320\ \Omega} = 31.2\text{ mA}$$

Note that this current is downward through R_2.

Step 2: Find the current through R_2 due to I_S by replacing V_S with a short, as shown in Figure 22.



▲ FIGURE 22

Use the current-divider formula to determine the current through R_2 due to I_S.

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2}\right)I_S = \left(\frac{220\ \Omega}{320\ \Omega}\right)100\text{ mA} = 68.8\text{ mA}$$

Note that this current also is downward through R_2.

Step 3: Both currents are in the same direction through R_2, so add them to get the total.

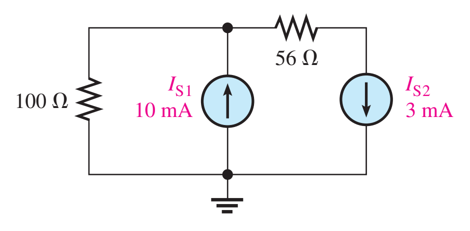
$$I_{2(\text{tot})} = I_{2(V_S)} + I_{2(I_S)} = 31.2\text{ mA} + 68.8\text{ mA} = \mathbf{100\ mA}$$

Related Problem If the polarity of V_S in Figure 20 is reversed, how is the value of I_S affected?

EXAMPLE 8

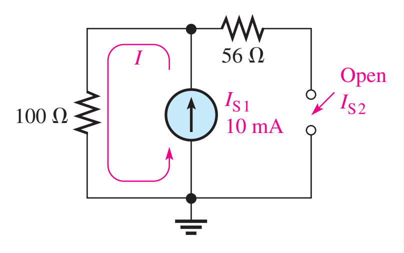
Find the current through the 100 Ω resistor in Figure 23.

▶ FIGURE 23

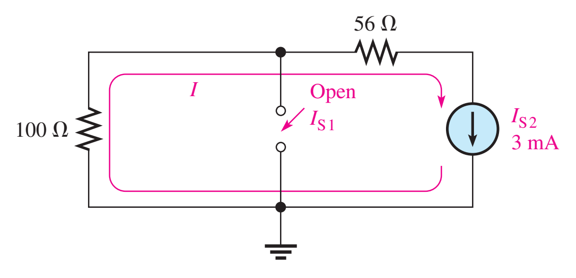


Solution **Step 1:** Find the current through the 100 Ω resistor due to current source I_{S1} by replacing source I_{S2} with an open, as shown in Figure 24. As you can see, the entire 10 mA from the current source I_{S1} is downward through the 100 Ω resistor.

▶ FIGURE 24



Step 2: Find the current through the 100 Ω resistor due to source I_{S2} by replacing source I_{S1} with an open, as indicated in Figure 25. Notice that all of the 3 mA from source I_{S2} is upward through the 100 Ω resistor.



▲ FIGURE 25

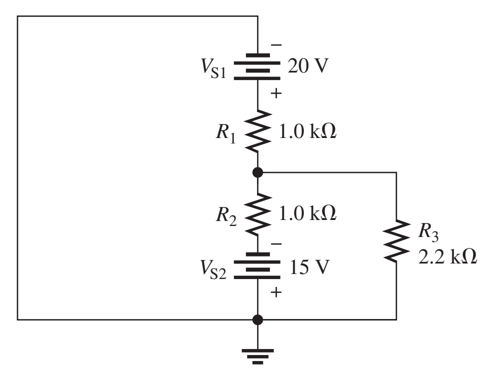
Step 3: To get the total current through the 100 Ω resistor, subtract the smaller current from the larger because they are in opposite directions. The resulting total current is in the direction of the larger current from source I_{S1}.

$$\begin{aligned} I_{100\Omega(\text{tot})} &= I_{100\Omega(I_{S1})} - I_{100\Omega(I_{S2})} \\ &= 10\text{ mA} - 3\text{ mA} = \mathbf{7\text{ mA}} \end{aligned}$$

The resulting current is downward through the resistor.

Related Problem If the 100 Ω resistor in Figure 23 is changed to 68 Ω, what will be the current through it?

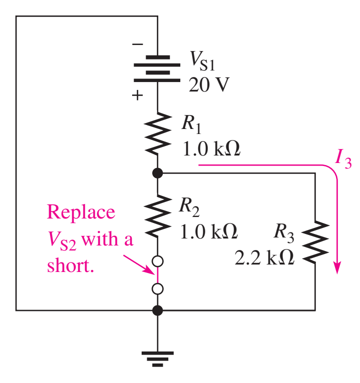
EXAMPLE 9 Find the total current through R_3 in Figure 26.



▲ FIGURE 26

Solution **Step 1:** Find the current through R_3 due to source V_{S1} by replacing source V_{S2} with a short, as shown in Figure 27.

FIGURE 27



V_{S1}
20 V
R_1
1.0 kΩ
I_3
R_2
1.0 kΩ
R_3
2.2 kΩ
Replace V_{S2} with a short.

Looking from V_{S1},

$$R_{T(S1)} = R_1 + \frac{R_2R_3}{R_2 + R_3} = 1.0\,\text{k}\Omega + \frac{(1.0\,\text{k}\Omega)(2.2\,\text{k}\Omega)}{3.2\,\text{k}\Omega} = 1.69\,\text{k}\Omega$$

$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{20\text{ V}}{1.69\,\text{k}\Omega} = 11.8\text{ mA}$$

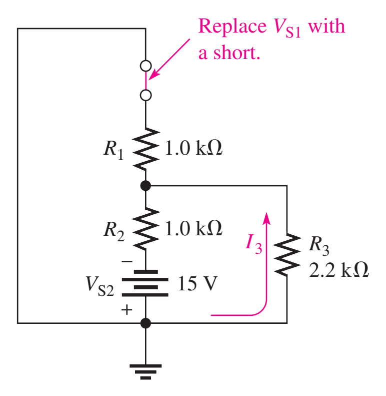
Now apply the current-divider formula to get the current through R_3 due to source V_{S1}.

$$I_{3(S1)} = \left(\frac{R_2}{R_2 + R_3}\right)I_{T(S1)} = \left(\frac{1.0\,\text{k}\Omega}{3.2\,\text{k}\Omega}\right)11.8\text{ mA} = 3.69\text{ mA}$$

Notice that this current is downward through R_3.

Step 2: Find I_3 due to source V_{S2} by replacing source V_{S1} with a short, as shown in Figure 28.

FIGURE 28



Replace V_{S1} with a short.
R_1
1.0 kΩ
R_2
1.0 kΩ
I_3
R_3
2.2 kΩ
V_{S2}
15 V

Looking from V_{S2},

$$R_{T(S2)} = R_2 + \frac{R_1R_3}{R_1 + R_3} = 1.0\,\text{k}\Omega + \frac{(1.0\,\text{k}\Omega)(2.2\,\text{k}\Omega)}{3.2\,\text{k}\Omega} = 1.69\,\text{k}\Omega$$

$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{15\text{ V}}{1.69\,\text{k}\Omega} = 8.88\text{ mA}$$

Now apply the current-divider formula to find the current through R_3 due to source V_{S2}.

$$I_{3(S2)} = \left(\frac{R_1}{R_1 + R_3}\right)I_{T(S2)} = \left(\frac{1.0\,\text{k}\Omega}{3.2\,\text{k}\Omega}\right)8.88\text{ mA} = 2.78\text{ mA}$$

Notice that this current is upward through R_3.

Step 3: Calculate the total current through R_3.

$$I_{3(\text{tot})} = I_{3(\text{S1})} - I_{3(\text{S2})} = 3.69\text{ mA} - 2.78\text{ mA} = 0.91\text{ mA} = \mathbf{910\ \mu A}$$

This current is downward through R_3.

Related Problem Find $I_{3(\text{tot})}$ in Figure 26 if V_{S1} is changed to 12 V and its polarity reversed.

Use Multisim file E08-09 to verify the calculated results in this example and to confirm your calculation for the related problem.

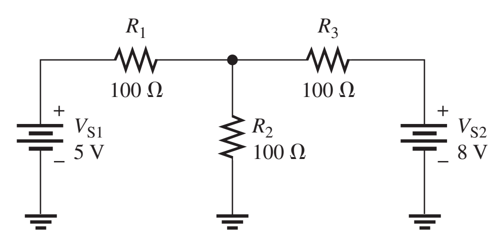
Although regulated dc power supplies are close to ideal voltage sources, many ac sources are not. For example, function generators generally have 50 Ω or 600 Ω of internal resistance, which appears as a resistance in series with an ideal source. Also, batteries can look ideal when they are fresh; but as they age, the internal resistance increases. When applying the superposition theorem, it is important to recognize when a source is not ideal and replace it with its equivalent internal resistance.

Current sources are not as common as voltage sources and are also not always ideal. If a current source is not ideal, as in the case of many transistors, it should be replaced by its equivalent internal resistance when the superposition theorem is applied.

SECTION 4 CHECKUP

1. State the superposition theorem.
2. Why is the superposition theorem useful for analysis of multiple-source circuits?
3. Why is an ideal voltage source shorted and an ideal current source opened when the superposition theorem is applied?
4. Using the superposition theorem, find the current through R_1 in Figure 29.
5. If, as a result of applying the superposition theorem, two currents are in opposing directions through a branch of a circuit, in which direction is the net current?

▶ FIGURE 29



5 THEVENIN'S THEOREM

Thevenin's theorem provides a method for simplifying a circuit to a standard equivalent form with respect to two output terminals. This theorem can be used to simplify the analysis of complex circuits.

After completing this section, you should be able to

- **Apply Thevenin's theorem to simplify a circuit for analysis**
  - Describe the form of a Thevenin equivalent circuit
  - Obtain the Thevenin equivalent voltage source

- Obtain the Thevenin equivalent resistance
- Explain terminal equivalency in the context of Thevenin's theorem
- Thevenize a portion of a circuit
- Thevenize a bridge circuit

The Thevenin equivalent form of any two-terminal resistive circuit consists of an equivalent voltage source (V_{TH}) and an equivalent resistance (R_{TH}), arranged as shown in Figure 30. The values of the equivalent voltage and resistance depend on the values in the original circuit. Any resistive circuit can be simplified regardless of its complexity with respect to two output terminals.

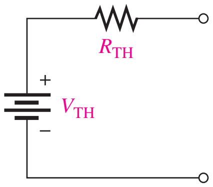


FIGURE 30

The general form of a thevenin equivalent circuit is a voltage source in series with a resistance.

The equivalent voltage, V_{TH}, is one part of the complete Thevenin equivalent circuit. The other part is R_{TH}.

> **The Thevenin equivalent voltage (V_{TH}) is the open circuit (no-load) voltage between two output terminals in a circuit.**

Any component connected between these two terminals effectively "sees" V_{TH} in series with R_{TH}. As defined by **Thevenin's theorem**,

> **The Thevenin equivalent resistance (R_{TH}) is the total resistance appearing between two terminals in a given circuit with all sources replaced by their internal resistances.**

Although a Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of the output voltage and current. Try the following demonstration as illustrated in Figure 31. Place a resistive circuit of any complexity in a box with only the

HISTORY NOTE

Léon Charles Thèvenin
1857–1926

Thevenin was a French telegraph engineer who became increasingly interested in the problems of measurement in electrical circuits. As a result of studying Kirchhoff's circuit laws and Ohm's law, in 1882 he developed a method, now known as Thévenin's theorem, which made it possible to analyze a complex electrical circuit by reducing it to a simple equivalent circuit.

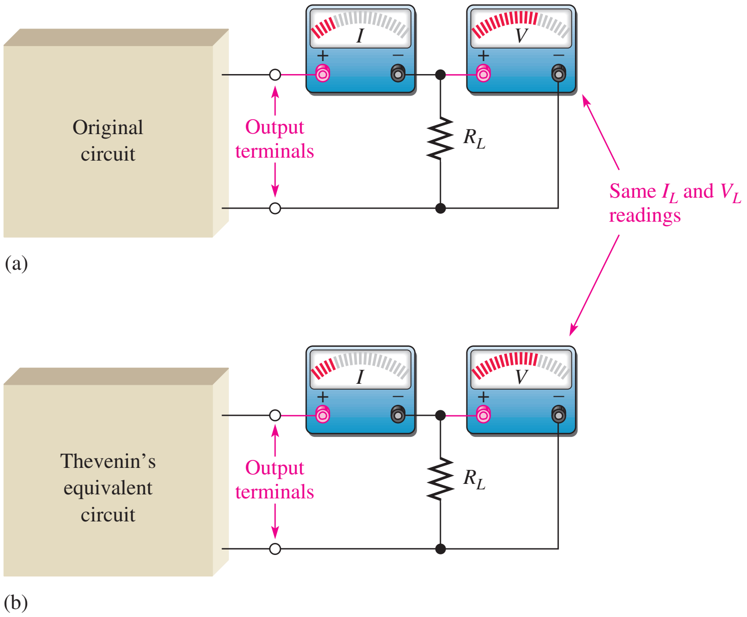
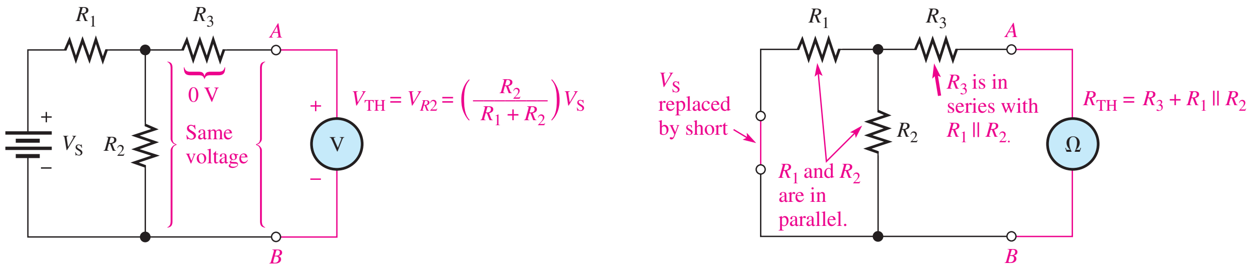


FIGURE 31

Which box contains the original circuit and which contains the Thevenin equivalent circuit? You cannot tell by observing the meters.

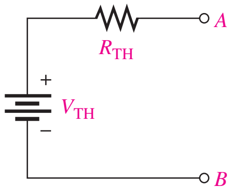
output terminals exposed. Then place the Thevenin equivalent of that circuit in an identical box with, again, only the output terminals exposed. Connect identical load resistors across the output terminals of each box. Next connect a voltmeter and an ammeter to measure the voltage and current for each load as shown in the figure. The measured values will be identical (neglecting tolerance variations), and you will not be able to determine which box contains the original circuit and which contains the Thevenin equivalent. That is, in terms of your observations based on any electrical measurements, both circuits appear to be the same. This condition is sometimes known as *terminal equivalency* because both circuits look the same from the "viewpoint" of the two output terminals.

To find the Thevenin equivalent of any circuit, determine the equivalent voltage, V_{TH}, and the equivalent resistance, R_{TH}, looking from the output terminals. As an example, the Thevenin equivalent for the circuit between terminals A and B is developed in Figure 32.


R_1
R_3
A
+
V_S
R_2
−
0 V
Same voltage
+
V
−
$V_{TH} = V_{R2} = \left(\frac{R_2}{R_1 + R_2}\right)V_S$
B
V_S replaced by short
R_1
R_3
A
R_3 is in series with $R_1 \parallel R_2$.
R_2
R_1 and R_2 are in parallel.
$R_{TH} = R_3 + R_1 \parallel R_2$
Ω
B

(a) Finding V_{TH}

(b) Finding R_{TH}



(c) Thevenin equivalent circuit

▲ **FIGURE 32**

Example of the simplification of a circuit by Thevenin's theorem.

In Figure 32(a), the voltage across the designated terminals A and B is the Thevenin equivalent voltage. In this particular circuit, the voltage from A to B is the same as the voltage across R_2 because there is no current through R_3 and, therefore, no voltage drop across it. The Thevenin voltage is expressed as follows for this particular example:

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_S$$

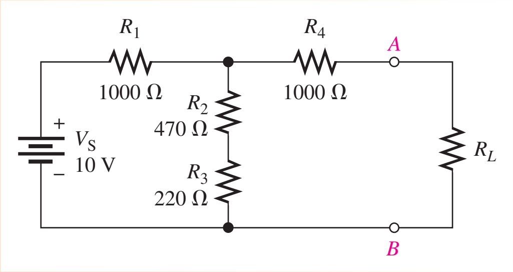
In Figure 32(b), the resistance between terminals A and B with the source replaced by a short (zero internal resistance) is the Thevenin equivalent resistance. In this particular circuit, the resistance from A to B is R_3 in series with the parallel combination of R_1 and R_2. Therefore, R_{TH} is expressed as follows:

$$R_{TH} = R_3 + \frac{R_1R_2}{R_1 + R_2}$$

The Thevenin equivalent circuit is shown in Figure 32(c).

EXAMPLE 10 Find the Thevenin equivalent circuit between A and B of the circuit in Figure 33.

▶ **FIGURE 33**


R_1
1000 Ω
R_4
1000 Ω
A
R_2
470 Ω
R_3
220 Ω
+
V_S
10 V
−
R_L
B

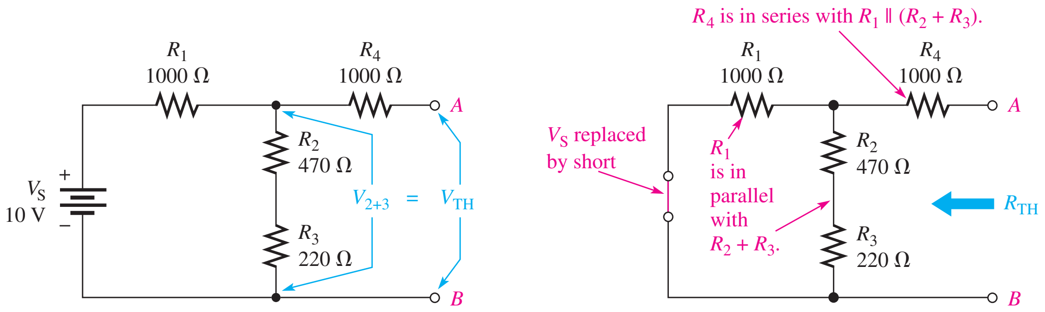
Solution First, remove R_L. Then V_{TH} equals the voltage across $R_2 + R_3$, as shown in Figure 34(a), because $V_4 = 0$ V since there is no current through it.

$$V_{TH} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{690\ \Omega}{1690\ \Omega}\right)10\ \text{V} = \mathbf{4.08\ V}$$

To find R_{TH}, first replace the source with a short to simulate a zero internal resistance. Then R_1 appears in parallel with $R_2 + R_3$, and R_4 is in series with the series-parallel combination of R_1, R_2, and R_3, as indicated in Figure 34(b).

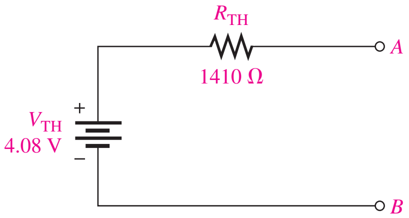
$$R_{TH} = R_4 + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 1000\ \Omega + \frac{(1000\ \Omega)(690\ \Omega)}{1690\ \Omega} = \mathbf{1410\ \Omega}$$

The resulting Thevenin equivalent circuit is shown in Figure 34(c).


R_1
1000 Ω
R_4
1000 Ω
A
R_2
470 Ω
R_3
220 Ω
V_S
10 V
+
−
V_{2+3} = V_{TH}
B
R_4 is in series with $R_1 \parallel (R_2 + R_3)$.
V_S replaced by short
R_1 is in parallel with $R_2 + R_3$.
R_{TH}

(a) The voltage from A to B is V_{TH} and equals V_{2+3}.

(b) Looking from terminals A and B, R_4 appears in series with the combination of R_1 in parallel with $(R_2 + R_3)$.


R_{TH}
1410 Ω
A
+
V_{TH}
4.08 V
−
B

(c) Thevenin equivalent circuit

▲ **FIGURE 34**

Related Problem Determine V_{TH} and R_{TH} if a 560 Ω resistor is connected in parallel across R_2 and R_3.

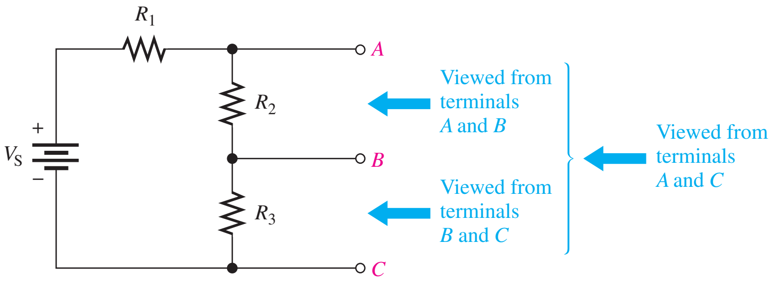
Use Multisim file E08-10 to verify the calculated results in this example and to confirm your calculations for the related problem.

Thevenin Equivalency Depends on the Viewpoint

The Thevenin equivalent for any circuit depends on the location of the two output terminals from which the circuit is "viewed." In Figure 33, you viewed the circuit from between the two terminals labeled *A* and *B*. Any given circuit can have more than one Thevenin equivalent, depending on how the output terminals are designated. For example, if you view the circuit in Figure 35 from between terminals *A* and *C*, you obtain a completely different result than if you viewed it from between terminals *A* and *B* or from between terminals *B* and *C*.

FIGURE 35

Thevenin's equivalent depends on the output terminals from which the circuit is viewed.


R_1
A
R_2
V_S
+
−
B
R_3
C
Viewed from terminals A and B
Viewed from terminals B and C
Viewed from terminals A and C

In Figure 36(a), when viewed from between terminals *A* and *C*, V_{TH} is the voltage across $R_2 + R_3$ and can be expressed using the voltage-divider formula as

$$V_{TH(AC)} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S$$

Also, as shown in Figure 36(b), the resistance between terminals *A* and *C* is $R_2 + R_3$ in parallel with R_1 (the source is replaced by a short) and can be expressed as

$$R_{TH(AC)} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

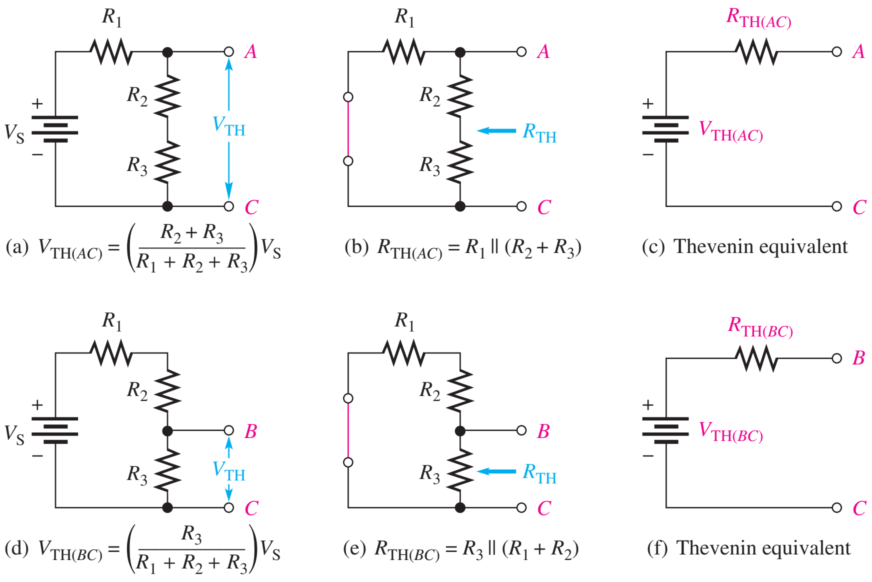
The resulting Thevenin equivalent circuit is shown in Figure 36(c).

When viewed from between terminals *B* and *C* as indicated in Figure 36(d), V_{TH} is the voltage across R_3 and can be expressed as

$$V_{TH(BC)} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S$$

FIGURE 36

Example of a circuit thevenized from two different sets of terminals. Parts (a), (b), and (c) illustrate one set of terminals and parts (d), (e), and (f) illustrate another set of terminals. (The V_{TH} and R_{TH} values are different for each case.)



(a) $V_{TH(AC)} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S$

(b) $R_{TH(AC)} = R_1 \parallel (R_2 + R_3)$

(c) Thevenin equivalent

(d) $V_{TH(BC)} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S$

(e) $R_{TH(BC)} = R_3 \parallel (R_1 + R_2)$

(f) Thevenin equivalent

As shown in Figure 36(e), the resistance between terminals *B* and *C* is R_3 in parallel with the series combination of R_1 and R_2.

$$R_{\mathrm{TH}(BC)} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

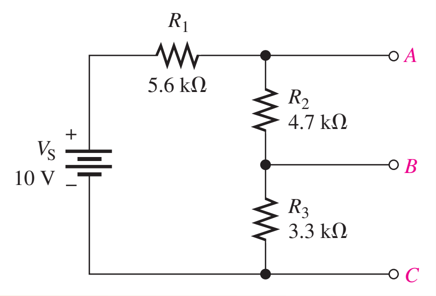
The resulting Thevenin equivalent is shown in Figure 36 (f).

EXAMPLE 11

(a) Determine the Thevenin equivalent circuit for the circuit in Figure 37 viewed from terminals *A* and *C*.

(b) Determine the Thevenin equivalent circuit for the circuit in Figure 37 viewed from terminals *B* and *C*.

▶ FIGURE 37



Solution **(a)** $V_{\mathrm{TH}(AC)} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega}}{5.6\,\mathrm{k\Omega} + 4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega}}\right)10\,\mathrm{V} = \mathbf{5.88\,V}$

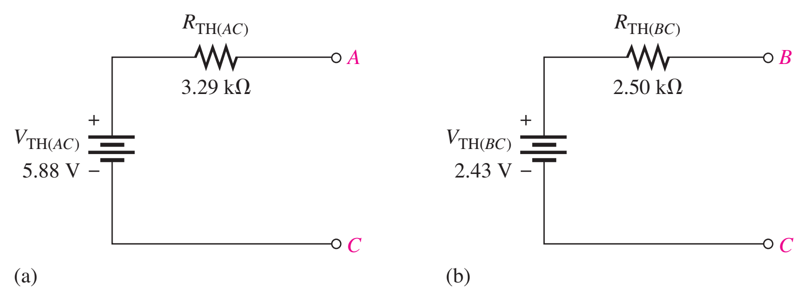
$R_{\mathrm{TH}(AC)} = R_1 \parallel (R_2 + R_3) = 5.6\,\mathrm{k\Omega} \parallel (4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega}) = \mathbf{3.29\,k\Omega}$

The Thevenin equivalent circuit is shown in Figure 38(a).

(b) $V_{\mathrm{TH}(BC)} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{3.3\,\mathrm{k\Omega}}{5.6\,\mathrm{k\Omega} + 4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega}}\right)10\,\mathrm{V} = \mathbf{2.43\,V}$

$R_{\mathrm{TH}(BC)} = R_3 \parallel (R_1 + R_2) = 3.3\,\mathrm{k\Omega} \parallel (5.6\,\mathrm{k\Omega} + 4.7\,\mathrm{k\Omega}) = \mathbf{2.50\,k\Omega}$

The Thevenin equivalent circuit is shown in Figure 38(b).



▲ FIGURE 38

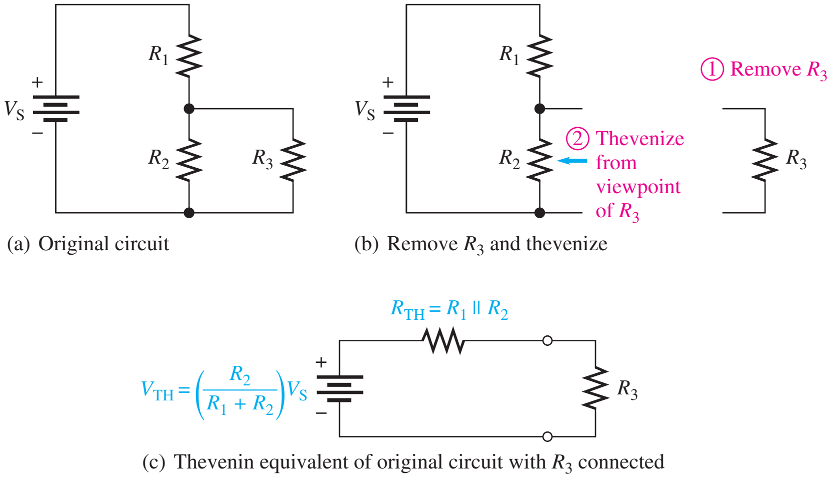
Related problem Determine the Thevenin equivalent circuit viewed from terminals *A* and *B* in Figure 37.

Thevenizing a Portion of a Circuit

In many cases, it helps to thevenize only a portion of a circuit. For example, when you need to know the equivalent circuit as viewed by one particular resistor in the circuit, you can remove the resistor and apply Thevenin's theorem to the remaining part of the circuit as viewed from the points between which that resistor was connected. Figure 39 illustrates the thevenizing of part of a circuit.

▶ **FIGURE 39**

Example of thevenizing a portion of a circuit. In this case, the circuit is thevenized from the viewpoint of the load resistor, R_3.


V_S
+
−
R_1
R_2
R_3
① Remove R_3
② Thevenize from viewpoint of R_3
$R_{TH} = R_1 \parallel R_2$
$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_S$

(a) Original circuit

(b) Remove R_3 and thevenize

(c) Thevenin equivalent of original circuit with R_3 connected

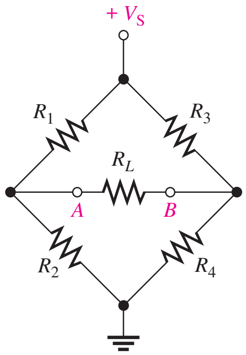
Using this type of approach, you can easily find the voltage and current for a specified resistor for any number of resistor values using only Ohm's law. This method eliminates the necessity of reanalyzing the original circuit for each different resistance value.

Thevenizing a Bridge Circuit

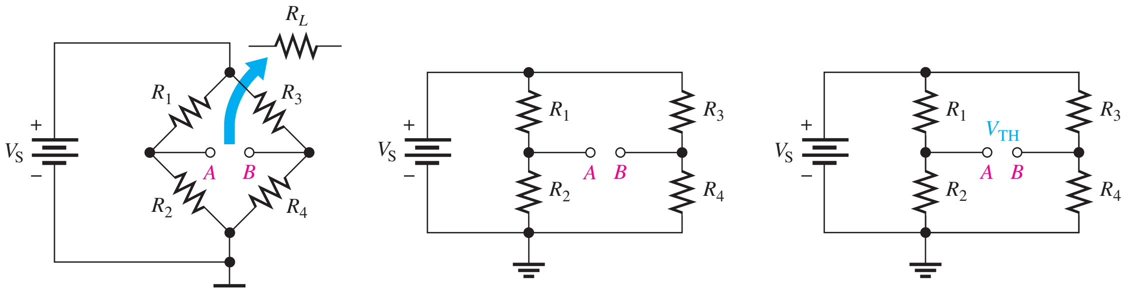
The usefulness of Thevenin's theorem is perhaps best illustrated when it is applied to a Wheatstone bridge circuit. For example, when a load resistor is connected to the output terminals of a Wheatstone bridge, as shown in Figure 40, the circuit is difficult to analyze because it is not a straightforward series-parallel arrangement. There are no resistors that are in series or in parallel with another resistor.

▶ **FIGURE 40**

A Wheatstone bridge with a load resistor connected between the output terminals is not a straightforward series-parallel circuit.



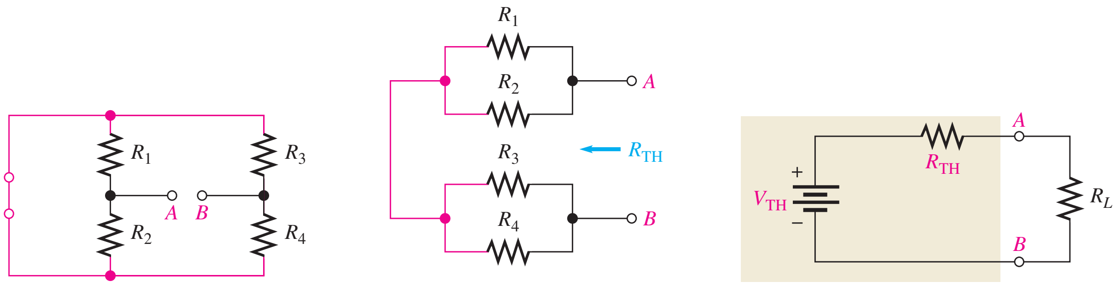
Using Thevenin's theorem, you can simplify the bridge circuit to an equivalent circuit viewed from the load resistor as shown step-by-step in Figure 41. Study carefully the steps in this figure. Once the equivalent circuit for the bridge is found, the voltage and current for any value of load resistor can easily be determined.



(a) Remove R_L.

(b) Redraw to find V_{TH}.

(c) $V_{TH} = V_A - V_B = \left(\frac{R_2}{R_1 + R_2}\right)V_S - \left(\frac{R_4}{R_3 + R_4}\right)V_S$



(d) Replace V_S with a short. *Note*: The red lines represent the same electrical point as the red lines in Part (e).

(e) Redraw to find R_{TH}: $R_{TH} = R_1 \parallel R_2 + R_3 \parallel R_4$

(f) Thevenin's equivalent with R_L reconnected

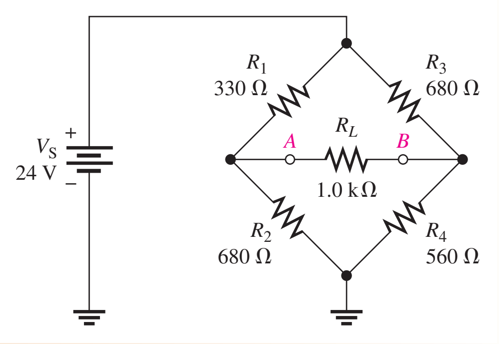
▲ FIGURE 41

Simplifying a Wheatstone bridge with Thevenin's Theorem.

EXAMPLE 12

Determine the voltage and current for the load resistor, R_L, in the bridge circuit of Figure 42.

▶ FIGURE 42



Solution

Step 1: Remove R_L.

Step 2: To thevenize the bridge as viewed from between terminals A and B, as was shown in Figure 41, first determine V_{TH}.

$$V_{TH} = V_A - V_B = \left(\frac{R_2}{R_1 + R_2}\right)V_S - \left(\frac{R_4}{R_3 + R_4}\right)V_S$$

$$= \left(\frac{680\ \Omega}{1010\ \Omega}\right)24\ \text{V} - \left(\frac{560\ \Omega}{1240\ \Omega}\right)24\ \text{V} = 16.16\ \text{V} - 10.84\ \text{V} = 5.32\ \text{V}$$

Step 3: Determine R_{TH}.

$$R_{\text{TH}} = \frac{R_1R_2}{R_1 + R_2} + \frac{R_3R_4}{R_3 + R_4}$$

$$= \frac{(330\ \Omega)(680\ \Omega)}{1010\ \Omega} + \frac{(680\ \Omega)(560\ \Omega)}{1240\ \Omega} = 222\ \Omega + 307\ \Omega = 529\ \Omega$$

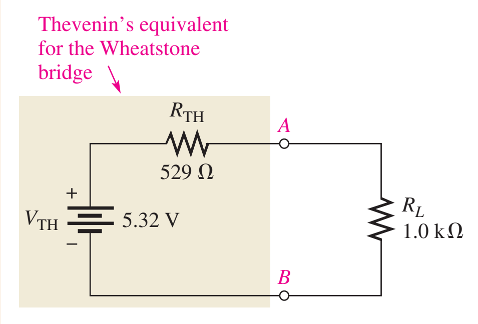
Step 4: Place V_{TH} and R_{TH} in series to form the Thevenin equivalent circuit.

Step 5: Connect the load resistor between terminals *A* and *B* of the equivalent circuit, and determine the load voltage and current as illustrated in Figure 43.

$$V_L = \left(\frac{R_L}{R_L + R_{\text{TH}}}\right)V_{\text{TH}} = \left(\frac{1.0\ \text{k}\Omega}{1.529\ \text{k}\Omega}\right)5.32\ \text{V} = \mathbf{3.48\ V}$$

$$I_L = \frac{V_L}{R_L} = \frac{3.48\ \text{V}}{1.0\ \text{k}\Omega} = \mathbf{3.48\ mA}$$

▶ FIGURE 43


Thevenin's equivalent for the Wheatstone bridge

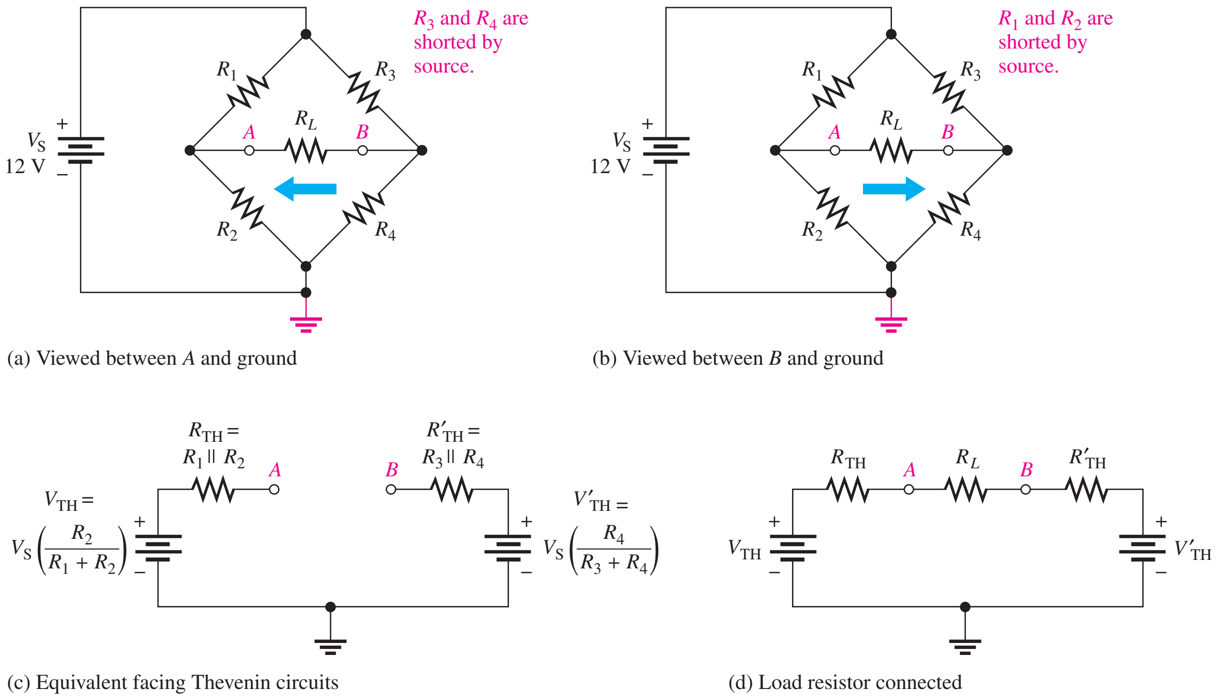
Related Problem Calculate I_L for $R_1 = 2.2\ \text{k}\Omega$, $R_2 = 3.3\ \text{k}\Omega$, $R_3 = 3.9\ \text{k}\Omega$, and $R_4 = 2.7\ \text{k}\Omega$.

Use Multisim files E08-12A and E08-12B to verify the calculated results in this example and to confirm your calculation for the related problem.

An Alternate Approach An alternate way to thevenize the Wheatstone bridge is to consider a different viewpoint. Instead of viewing it from between the *A* and *B* terminals, you can view it from terminal *A* to ground and from terminal *B* to ground, as illustrated in Figure 44(a) and (b). The resulting equivalent circuit is simplified into two facing Thevenin circuits that still include ground, as illustrated in Figure 44(c). When calculating the Thevenin resistance, replace the voltage source by a short; thus, two of the bridge resistors are shorted out. In Figure 44(a), R_3 and R_4 are shorted and in part (b) R_1 and R_2 are shorted. In each case, the remaining two resistors appear in parallel to form the Thevenin resistance. The load resistor can be replaced as in Figure 44(d), which is seen to be a simple series circuit with two opposing sources. The advantage of this method is that ground is still shown in this equivalent circuit, so it is easy to find the voltage at terminal *A* or *B* with respect to ground by applying the superposition theorem to the equivalent circuit.

Summary of Thevenin's Theorem

Remember, the Thevenin equivalent circuit is *always* in the form of an equivalent voltage source in series with an equivalent resistance regardless of the original circuit that it replaces.


R_3 and R_4 are shorted by source.
V_S 12 V
R_1
R_3
R_L
A
B
R_2
R_4
R_1 and R_2 are shorted by source.
$R_{TH} = R_1 \parallel R_2$
$R'_{TH} = R_3 \parallel R_4$
$V_{TH} = V_S\left(\frac{R_2}{R_1 + R_2}\right)$
$V'_{TH} = V_S\left(\frac{R_4}{R_3 + R_4}\right)$
V_{TH}
V'_{TH}

(a) Viewed between *A* and ground

(b) Viewed between *B* and ground

(c) Equivalent facing Thevenin circuits

(d) Load resistor connected

▲ **FIGURE 44**

The significance of Thevenin's theorem is that the equivalent circuit can replace the original circuit as far as any external load is concerned. Any load resistor connected between the terminals of a Thevenin equivalent circuit will have the same current through it and the same voltage across it as if it were connected to the terminals of the original circuit.

A summary of steps for applying Thevenin's theorem is as follows:

Step 1: Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.

Step 2: Determine the voltage (V_{TH}) across the two open terminals.

Step 3: Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).

Step 4: Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.

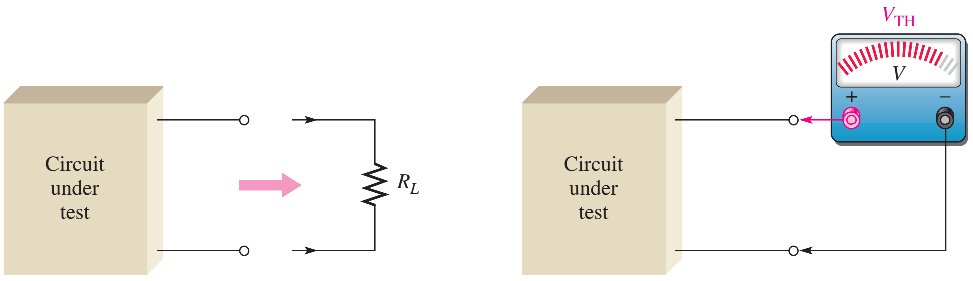
Step 5: Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

Determining V_{TH} and R_{TH} by Measurement

Thevenin's theorem is largely an analytical tool that is applied theoretically in order to simplify circuit analysis. However, you can find Thevenin's equivalent for an actual circuit by the following general measurement methods. These steps are illustrated in Figure 45.

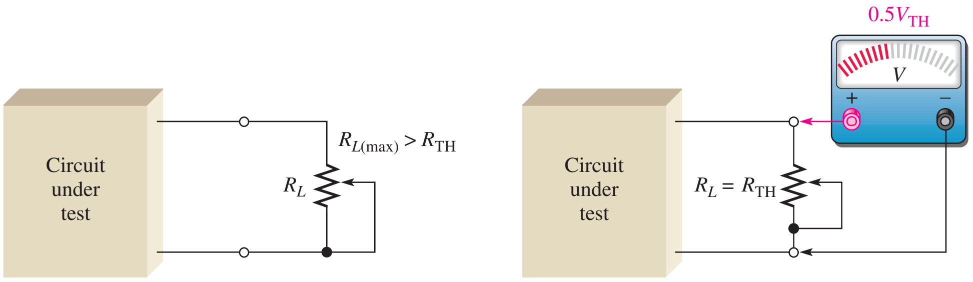
Step 1: Remove any load from the output terminals of the circuit.

Step 2: Measure the open terminal voltage. The voltmeter used must have an internal resistance much greater (at least 10 times greater) than the R_{TH} of the circuit so that it has negligible loading effect. (V_{TH} is the open terminal voltage.)



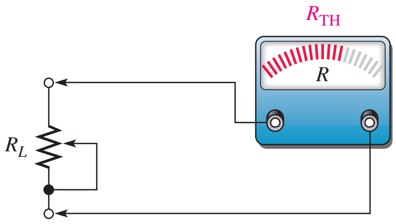
Step 1: Open the output terminals (remove load).

Step 2: Measure V_{TH}.



Step 3: Connect variable load resistance set to its maximum value across the terminals.

Step 4: Adjust R_L until $V_L = 0.5V_{TH}$. When $V_L = 0.5V_{TH}$, $R_L = R_{TH}$.



Step 5: Remove R_L from the circuit under test and measure its resistance to get R_{TH}.

▲ FIGURE 45

Determination of Thevenin's equivalent by measurement.

Step 3: Connect a variable resistor (rheostat) across the output terminals. Set it at its maximum value, which must be greater than R_{TH}.

Step 4: Adjust the rheostat until the terminal voltage equals $0.5V_{TH}$. At this point, the resistance of the rheostat is equal to R_{TH}.

Step 5: Disconnect the rheostat from the terminals and measure its resistance with an ohmmeter. This measured resistance is equal to R_{TH}.

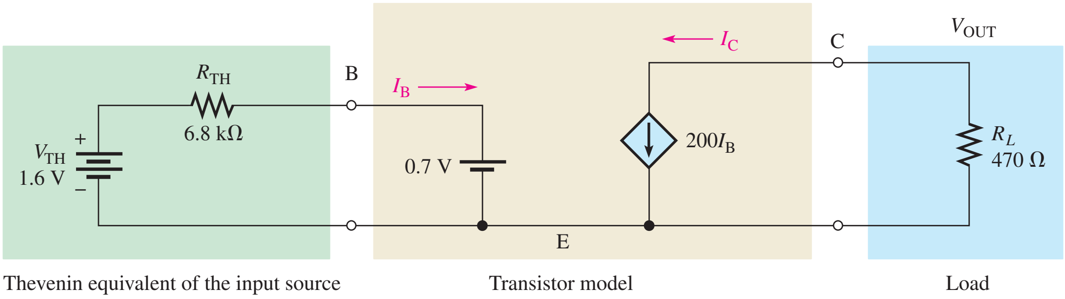
This procedure for determining R_{TH} differs from the theoretical procedure because it is impractical to short voltage sources or open current sources in an actual circuit. Also, when measuring R_{TH}, be certain that the circuit is capable of providing the required current to the variable resistor load and that the variable resistor can handle the required power. These considerations may make the procedure impractical in some cases.

An Example of a Practical Application

Although you have not studied transistor circuits, a basic amplifier can be used to illustrate the usefulness of Thevenin's equivalent circuit. A transistor circuit can be modeled with basic components including a dependent current source and a Thevenin equivalent circuit. Modeling is generally a mathematical simplification of a complex circuit, retaining only the most important parts of the circuit and eliminating those that have only a minimum effect.

A typical dc model of a transistor is shown in Figure 46. This type of transistor (bipolar junction transistor) has three terminals, labeled base (B), collector (C), and emitter (E). In this case, the emitter terminal is both an input and an output, so it is common. The dependent current source (diamond-shaped symbol) is controlled by the base current, I_B. For this example, the current from the dependent source is 200 times larger than the base current as expressed by the term βI_B, where β is a transistor gain parameter and, in this case, $\beta = 200$.

The transistor is part of a dc amplifier circuit, and you can use the basic model to predict the output current. The output current is larger than the input circuit can provide by itself. For example, the source can represent a small transducer, such as a solar cell with an internal resistance of 6.8 kΩ. It is shown as an equivalent Thevenin voltage and Thevenin resistance. The load could be any device that requires higher current than the source can provide directly.



▲ **FIGURE 46**

DC transistor circuit. The diamond-shaped symbol indicates a dependent current source.

EXAMPLE 13

(a) Write KVL around the left part of the circuit in Figure 46. Solve for I_B.

(b) Determine the current from the dependent current source. This current is I_C.

(c) Calculate the output voltage and the power in the load resistor R_L.

(d) Compare the power determined in (c) with the power delivered to the load if the load resistor were connected directly to the Thevenin circuit.

Solution

(a) $V_{TH} - R_{TH}I_B - 0.7\text{ V} = 0$

$$I_B = \frac{V_{TH} - 0.7\text{ V}}{R_{TH}} = \frac{1.6\text{ V} - 0.7\text{ V}}{6.8\text{ k}\Omega} = \mathbf{132\ \mu A}$$

(b) $I_C = \beta I_B = 200(132\ \mu\text{A}) = \mathbf{26.5\ mA}$

(c) $V_{OUT} = I_C R_L = (26.5\text{ mA})(470\ \Omega) = \mathbf{12.4\ V}$

$$P_L = \frac{V_{OUT}^2}{R_L} = \frac{(12.4\text{ V})^2}{470\ \Omega} = \mathbf{327\ mW}$$

(d) $P_L = I_B^2 R_L = (132\ \mu\text{A})^2(470\ \Omega) = 8.19\ \mu\text{W}$

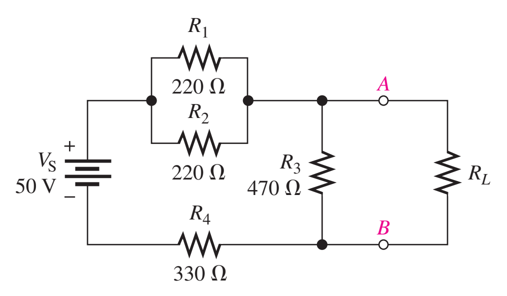
The power in the load resistor is 327 mW/8.19 μW = 39,927 times greater than the power that the Thevenin input circuit could deliver to the same load. This illustrates that the transistor can operate as a power amplifier.

Related Problem Determine the input voltage at the base (B) of the transistor. Compare this value to V_{OUT}. By how much does the amplifier increase the input voltage?

SECTION 5 CHECKUP

1. What are the two components of a Thevenin equivalent circuit?
2. Draw the general form of a Thevenin equivalent circuit.
3. How is V_{TH} defined?
4. How is R_{TH} defined?
5. For the original circuit in Figure 47, draw the Thevenin equivalent circuit as viewed from the output terminals *A* and *B*.

▶ FIGURE 47

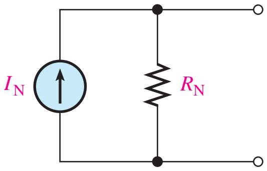


6 NORTON'S THEOREM

Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form. The basic difference is that Norton's theorem results in an equivalent current source in parallel with an equivalent resistance.

After completing this section, you should be able to

- **Apply Norton's theorem to simplify a circuit**
  - Describe the form of a Norton equivalent circuit
  - Obtain the Norton equivalent current source
  - Obtain the Norton equivalent resistance



▲ FIGURE 48

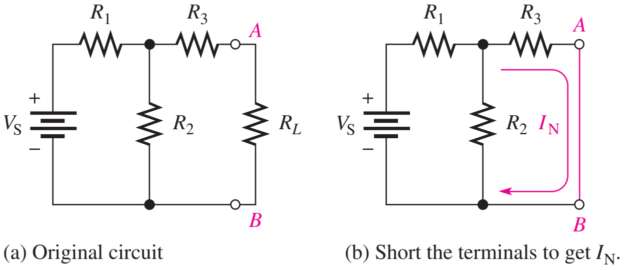
Form of Norton's equivalent circuit.

Norton's theorem is a method for simplifying a two-terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistor. The form of Norton's equivalent circuit is shown in Figure 48. Regardless of how complex the original two-terminal circuit is, it can always be reduced to this equivalent form. The equivalent current source is designated I_N, and the equivalent resistance is designated R_N. To apply Norton's theorem, you must know how to find the two quantities I_N and R_N. Once you know them for a given circuit, simply connect them in parallel to get the complete Norton circuit.

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.

Any component connected between these two terminals effectively "sees" a current source I_N in parallel with R_N. To illustrate, suppose that a resistive circuit of some kind has a resistor (R_L) connected between two output terminals in the circuit, as shown in Figure 49(a). You want to find the Norton circuit that is equivalent to the one shown as "seen" by R_L. To find I_N, calculate the current between terminals *A* and *B* with these two terminals shorted, as shown in Figure 49(b). Example 14 demonstrates how to find I_N.



(a) Original circuit (b) Short the terminals to get I_N.

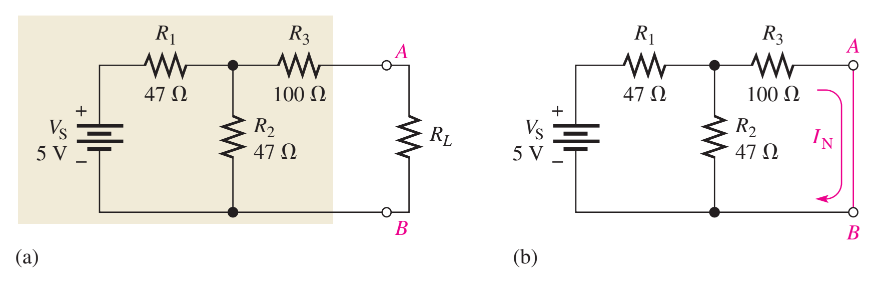
FIGURE 49

Determining the Norton equivalent current, I_N.

EXAMPLE 14

Determine I_N for the circuit within the beige area in Figure 50(a).

FIGURE 50



Solution Short terminals *A* and *B* as shown in Figure 50(b). I_N is the current through the short. First, the total resistance seen by the voltage source is

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 47\ \Omega + \frac{(47\ \Omega)(100\ \Omega)}{147\ \Omega} = 79\ \Omega$$

The total current from the source is

$$I_T = \frac{V_S}{R_T} = \frac{5\text{ V}}{79\ \Omega} = 63.3\text{ mA}$$

Now apply the current-divider formula to find I_N (the current through the short).

$$I_N = \left(\frac{R_2}{R_2 + R_3}\right) I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right) 63.3\text{ mA} = \mathbf{20.2\ mA}$$

This is the value for the equivalent Norton current source.

Related Problem Determine I_N in Figure 50(a) if the value of R_2 is doubled.

Use Multisim file E08-14 to verify the calculated results in this example and to confirm your calculation for the related problem.

Norton's Equivalent Resistance (R_N)

Norton's equivalent resistance (R_N) is defined in the same way as R_{TH}.

> **The Norton equivalent resistance, R_N, is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.**

Example 15 demonstrates how to find R_N.

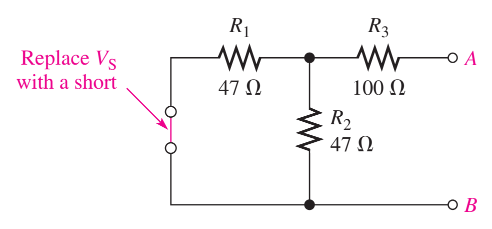
EXAMPLE 15

Find R_N for the circuit within the beige area of Figure 50(a) (see Example 14).

Solution First reduce V_S to zero by shorting it, as shown in Figure 51. Looking in at terminals *A* and *B*, you can see that the parallel combination of R_1 and R_2 is in series with R_3. Thus,

$$R_N = R_3 + \frac{R_1}{2} = 100\ \Omega + \frac{47\ \Omega}{2} = \mathbf{124\ \Omega}$$

FIGURE 51



Related Problem Determine R_N in Figure 50(a) if the value of R_2 is doubled.

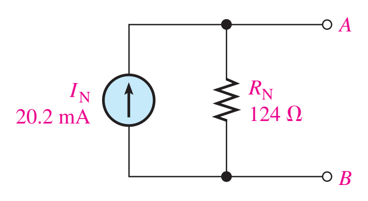
Examples 14 and 15 have shown how to find the two equivalent components of a Norton equivalent circuit, I_N and R_N. Keep in mind that these values can be found for any linear circuit. Once these are known, they must be connected in parallel to form the Norton equivalent circuit, as illustrated in Example 16.

EXAMPLE 16

Draw the complete Norton equivalent circuit for the original circuit in Figure 50(a) (Example 14).

Solution In Examples 14 and 15 it was found that $I_N = 20.2$ mA and $R_N = 124\ \Omega$. The Norton equivalent circuit is shown in Figure 52.

FIGURE 52



Related Problem Find R_N for the circuit in Figure 50(a) if all the resistor values are doubled.

Summary of Norton's Theorem

Any load resistor connected between the output terminals of a Norton equivalent circuit will have the same current through it and the same voltage across it as if it were connected to the output terminals of the original circuit. A summary of steps for theoretically applying Norton's theorem is as follows:

Step 1: Short the two terminals between which you want to find the Norton equivalent circuit.

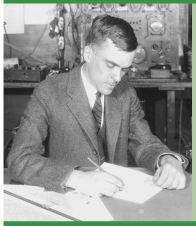
Step 2: Determine the current (I_N) through the shorted terminals.

Step 3: Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

Step 4: Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

Norton's equivalent circuit can also be derived from Thevenin's equivalent circuit by use of the source conversion method discussed in Section 3.

HISTORY NOTE

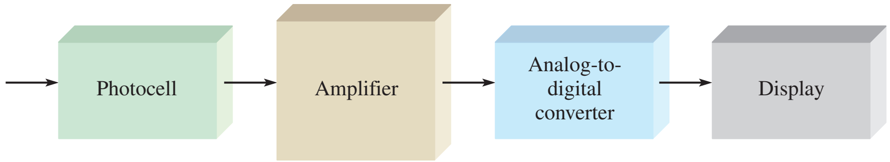


Edward Lawry Norton
1898–1983

Norton started work as an electrical engineer in 1922 at the Western Electric Corporation, which became Bell Laboratories in 1925. Norton's theorem is basically an extension of Thévenin's theorem and was developed in 1926 and documented in a Bell Labs internal technical report. A German telecommunications engineer, Hans Mayer, published the same result in the *same month* as Norton's technical memorandum. In Europe, it is known as the Mayer-Norton equivalent. (Photo credit: Courtesy of AT&T Archives and History Center.)

An Example of a Practical Application

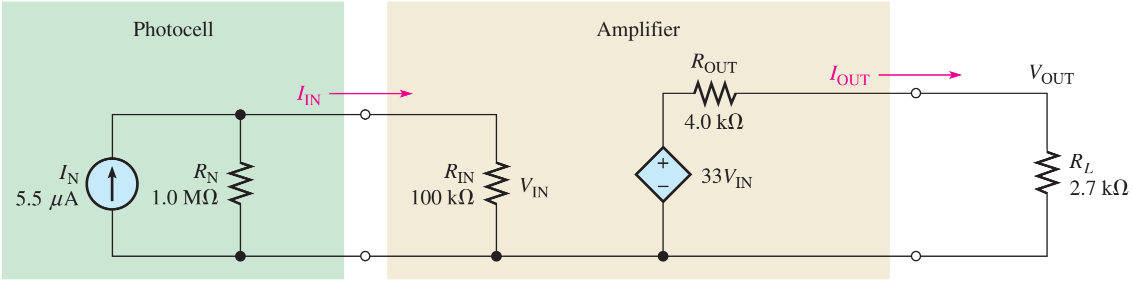
A voltage amplifier in a digital light meter is modeled using a Norton equivalent circuit and a dependent voltage source. A block diagram of the light meter is shown in Figure 53. The light meter uses a photocell as a sensor. The photocell is a current source that produces a very small current proportional to the incident light. Because it is a current source, a Norton circuit can be used to model the photocell. The very small amount of current from the photocell is converted to a small input voltage across R_N. A dc amplifier is used to increase the voltage to a level sufficient for driving the analog-to-digital converter.



◀ FIGURE 53

Light meter block diagram.

In this application, only with the first two blocks in the light meter diagram are of interest. These have been modeled as shown in Figure 54. The photocell has been modeled as a Norton circuit on the input. The output of the Norton circuit is fed to the amplifier's input resistance, which converts the current I_N to a small voltage V_{IN}. The amplifier increases this voltage by 33 to drive the analog-to-digital converter which, for simplicity, is modeled simply as a load resistor, R_L. The value of 33 is the gain of this particular amplifier.



▲ FIGURE 54

Photocell and amplifier model. The diamond-shaped symbol indicates a dependent voltage source.

EXAMPLE 17

Refer to Figure 54.

(a) Apply the current-divider rule to the input Norton circuit to calculate I_{IN}.

(b) Use Ohm's law to calculate V_{IN}.

(c) Determine the voltage from the dependent voltage source. This gain is 33.

(d) Apply the voltage-divider rule to calculate V_{OUT}.

Solution

(a) $I_{\text{IN}} = I_{\text{N}}\left(\dfrac{R_{\text{N}}}{R_{\text{N}} + R_{\text{IN}}}\right) = (5.5\ \mu\text{A})\left(\dfrac{1.0\ \text{M}\Omega}{1.1\ \text{M}\Omega}\right) = \mathbf{5\ \mu A}$

(b) $V_{\text{IN}} = I_{\text{IN}}R_{\text{IN}} = (5\ \mu\text{A})(100\ \text{k}\Omega) = \mathbf{0.5\ V}$

(c) $33V_{\text{IN}} = (33)(0.5\ \text{V}) = \mathbf{16.5\ V}$

(d) $V_{\text{OUT}} = (33\ V_{\text{IN}})\left(\dfrac{R_L}{R_L + R_{\text{OUT}}}\right) = (16.5\ \text{V})(0.403) = \mathbf{6.65\ V}$

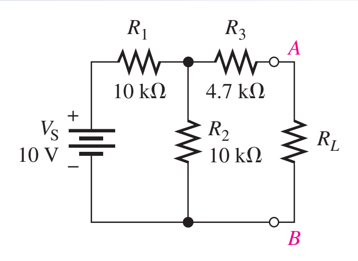
Related Problem

If the photocell is replaced by one having the same current but a Norton equivalent resistance of 2.0 MΩ, what is the output voltage?

SECTION 6 CHECKUP

1. What are the two components of a Norton equivalent circuit?
2. Draw the general form of a Norton equivalent circuit.
3. How is I_{N} defined?
4. How is R_{N} defined?
5. Find the Norton circuit as seen by R_L in Figure 55.

▶ FIGURE 55



7 MAXIMUM POWER TRANSFER THEOREM

The maximum power transfer theorem is important when you need to know the value of the load at which the most power is delivered from the source.

After completing this section, you should be able to

- **Apply the maximum power transfer theorem**
  - State the theorem
  - Determine the value of load resistance for which maximum power is transferred from a given circuit

The **maximum power transfer** theorem is stated as follows:

For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.

The source resistance, R_S, of a circuit is the equivalent resistance as viewed from the output terminals using Thevenin's theorem. A Thevenin equivalent circuit with its output resistance and load is shown in Figure 56. When $R_L = R_S$, the maximum power possible is transferred from the voltage source to R_L for a given value of V_S.

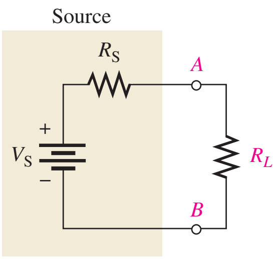


FIGURE 56

Maximum power is transferred to the load when $R_L = R_S$.

Practical applications of the maximum power transfer theorem include audio systems such as stereo, radio, and public address. In these systems the resistance of the speaker is the load. The circuit that drives the speaker is a power amplifier. The systems are typically optimized for maximum power to the speakers. Thus, the resistance of the speaker must equal the internal source resistance of the amplifier.

Example 18 shows that maximum power occurs when $R_L = R_S$.

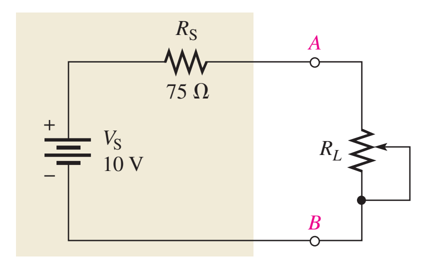
EXAMPLE 18

The source in Figure 57 has an internal source resistance of 75 Ω. Determine the load power for each of the following values of load resistance:

(a) 0 Ω **(b)** 25 Ω **(c)** 50 Ω **(d)** 75 Ω **(e)** 100 Ω **(f)** 125 Ω

Draw a graph showing the load power versus the load resistance.

FIGURE 57



Solution Use Ohm's law ($I = V/R$) and the power formula ($P = I^2R$) to find the load power, P_L, for each value of load resistance.

(a) For $R_L = 0\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 0\ \Omega} = 133\text{ mA}$$

$$P_L = I^2R_L = (133\text{ mA})^2(0\ \Omega) = \mathbf{0\ mW}$$

(b) For $R_L = 25\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 25\ \Omega} = 100\text{ mA}$$

$$P_L = I^2R_L = (100\text{ mA})^2(25\ \Omega) = \mathbf{250\ mW}$$

(c) For $R_L = 50\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{125\ \Omega} = 80\text{ mA}$$

$$P_L = I^2R_L = (80\text{ mA})^2(50\ \Omega) = \mathbf{320\ mW}$$

(d) For $R_L = 75\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{150\ \Omega} = 66.7\text{ mA}$$

$$P_L = I^2R_L = (66.7\text{ mA})^2(75\ \Omega) = \mathbf{334\ mW}$$

(e) For $R_L = 100\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{175\ \Omega} = 57.1\text{ mA}$$

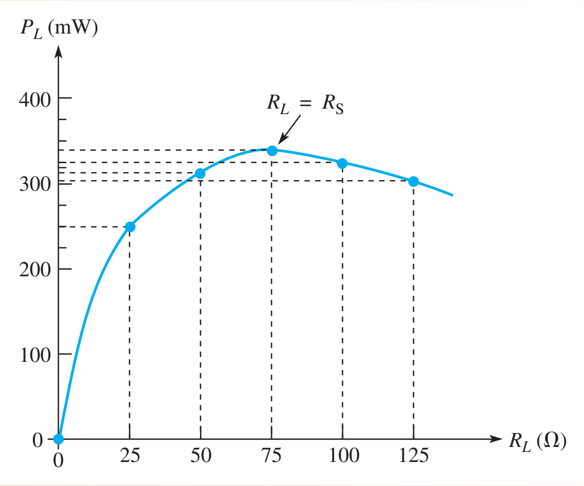
$$P_L = I^2R_L = (57.1\text{ mA})^2(100\ \Omega) = \mathbf{326\ mW}$$

(f) For $R_L = 125\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{200\ \Omega} = 50\text{ mA}$$

$$P_L = I^2R_L = (50\text{ mA})^2(125\ \Omega) = \mathbf{313\ mW}$$

Notice that the load power is greatest when $R_L = 75\ \Omega$, which is the same as the internal source resistance. When the load resistance is less than or greater than this value, the power drops off, as the curve in Figure 58 graphically illustrates.



▲ **FIGURE 58**

Curve showing that the load power is maximum when $R_L = R_S$.

Related Problem If the source resistance in Figure 57 is 600 Ω, what is the maximum power that can be delivered to a load?

Open Multisim file E08-18 and measure the current in each value of load resistance to verify the calculations in the example.

SECTION 7 CHECKUP

1. State the maximum power transfer theorem.
2. When is maximum power delivered from a source to a load?
3. A given circuit has an internal source resistance of 50 Ω. What will be the value of the load to which the maximum power is delivered?

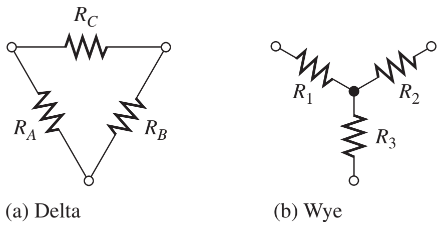
8 DELTA-TO-WYE (Δ-TO-Y) AND WYE-TO-DELTA (Y-TO-Δ) CONVERSIONS

Conversions between delta-type and wye-type circuit arrangements are useful in certain specialized three-terminal applications. One example is in the analysis of a loaded Wheatstone bridge circuit.

After completing this section, you should be able to

- **Perform Δ-to-Y and Y-to-Δ conversions**
  - Apply Δ-to-Y conversion to a bridge circuit

A resistive delta (Δ) circuit is a three-terminal arrangement as shown in Figure 59(a). A wye (Y) circuit is shown in Figure 59(b). Notice that letter subscripts are used to designate resistors in the delta circuit and that numerical subscripts are used to designate resistors in the wye circuit.



(a) Delta (b) Wye

FIGURE 59

Delta and wye circuits.

Δ-to-Y Conversion

It is convenient to think of the wye positioned within the delta, as shown in Figure 60. To convert from delta to wye, you need R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. The conversion rule is as follows:

Each resistor in the wye is equal to the product of the resistors in two adjacent delta branches, divided by the sum of all three delta resistors.

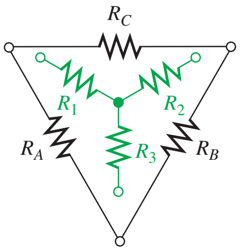


FIGURE 60

"Y within Δ" aid for conversion formulas.

In Figure 60, R_A and R_C are adjacent to R_1; therefore,

Equation 1

$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

Also, R_B and R_C are adjacent to R_2, so

Equation 2

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

and R_A and R_B are adjacent to R_3, so

Equation 3

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Y-to-Δ Conversion

To convert from wye to delta, you need R_A, R_B, and R_C in terms of R_1, R_2, and R_3. The conversion rule is as follows:

Each resistor in the delta is equal to the sum of all possible products of wye resistors taken two at a time, divided by the opposite wye resistor.

In Figure 60, R_2 is opposite to R_A; therefore,

Equation 4

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

Also, R_1 is opposite to R_B, so

Equation 5

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

and R_3 is opposite to R_C, so

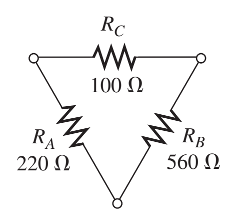
Equation 6

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

EXAMPLE 19

Convert the delta circuit in Figure 61 to a wye circuit.

▶ FIGURE 61



Solution Use Equations 1, 2, and 3.

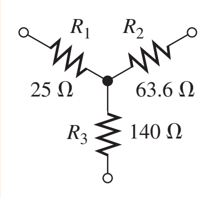
$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(220\ \Omega)(100\ \Omega)}{220\ \Omega + 560\ \Omega + 100\ \Omega} = \mathbf{25\ \Omega}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(560\ \Omega)(100\ \Omega)}{880\ \Omega} = \mathbf{63.6\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(220\ \Omega)(560\ \Omega)}{880\ \Omega} = \mathbf{140\ \Omega}$$

The resulting wye circuit is shown in Figure 62.

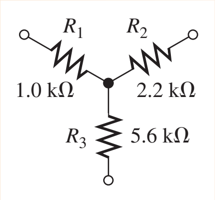
▶ FIGURE 62



Related Problem Convert the delta circuit to a wye network for $R_A = 2.2\text{ k}\Omega$, $R_B = 1.0\text{ k}\Omega$, and $R_C = 1.8\text{ k}\Omega$.

EXAMPLE 20 Convert the wye circuit in Figure 63 to a delta circuit.

▶ FIGURE 63



Solution Use Equations 4, 5, and 6.

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$= \frac{(1.0\text{ k}\Omega)(2.2\text{ k}\Omega) + (1.0\text{ k}\Omega)(5.6\text{ k}\Omega) + (2.2\text{ k}\Omega)(5.6\text{ k}\Omega)}{2.2\text{ k}\Omega} = \mathbf{9.15\text{ k}\Omega}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

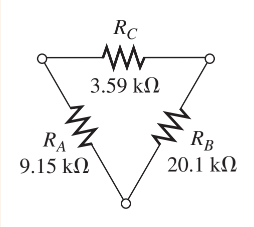
$$= \frac{(1.0\text{ k}\Omega)(2.2\text{ k}\Omega) + (1.0\text{ k}\Omega)(5.6\text{ k}\Omega) + (2.2\text{ k}\Omega)(5.6\text{ k}\Omega)}{1.0\text{ k}\Omega} = \mathbf{20.1\text{ k}\Omega}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

$$= \frac{(1.0\text{ k}\Omega)(2.2\text{ k}\Omega) + (1.0\text{ k}\Omega)(5.6\text{ k}\Omega) + (2.2\text{ k}\Omega)(5.6\text{ k}\Omega)}{5.6\text{ k}\Omega} = \mathbf{3.59\text{ k}\Omega}$$

The resulting delta circuit is shown in Figure 64.

▶ FIGURE 64

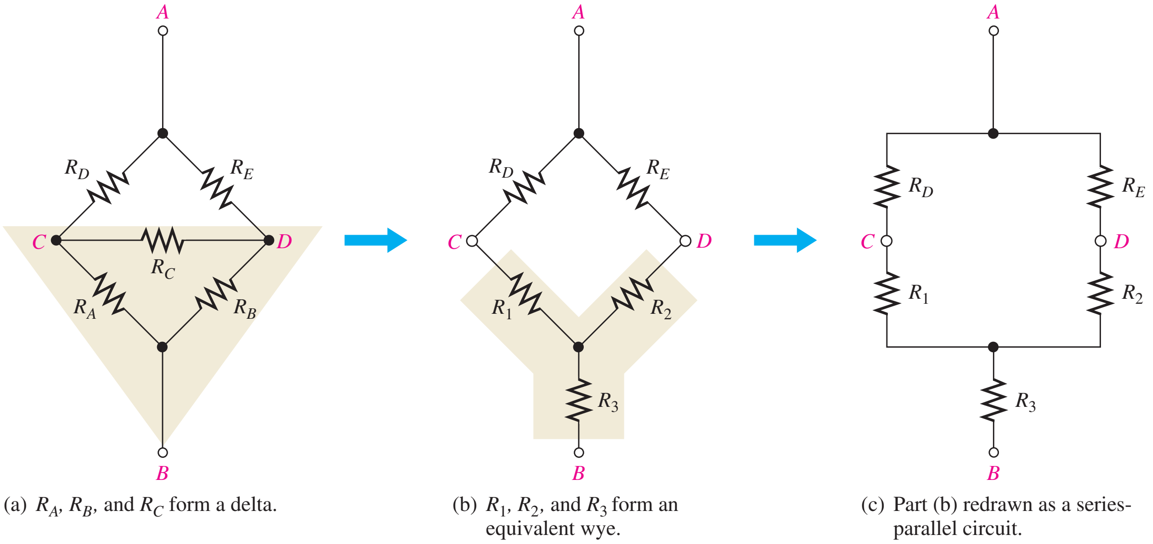


Related Problem Convert the wye circuit to a delta circuit for $R_1 = 100\ \Omega$, $R_2 = 330\ \Omega$, and $R_3 = 470\ \Omega$.

Application of Δ-to-Y Conversion to a Bridge Circuit

In Section 5 you learned how Thevenin's theorem can be used to simplify a bridge circuit. Now you will see how Δ-to-Y conversion can be used for converting a bridge circuit to a series-parallel form for easier analysis.

Figure 65 illustrates how the delta (Δ) formed by R_A, R_B, and R_C can be converted to a wye (Y), thus creating an equivalent series-parallel circuit. Equations 1, 2, and 3 are used in this conversion.



(a) R_A, R_B, and R_C form a delta.

(b) R_1, R_2, and R_3 form an equivalent wye.

(c) Part (b) redrawn as a series-parallel circuit.

▲ **FIGURE 65**

Conversion of a bridge circuit to a series-parallel configuration.

In a bridge circuit, the load is connected between terminals C and D. In Figure 65(a), R_C represents the load resistor. When voltage is applied across terminals A and B, the voltage from C to D (V_{CD}) can be determined using the equivalent series-parallel circuit in Figure 65(c) as follows. The total resistance from terminal A to terminal B is

$$R_T = \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)} + R_3$$

Then,

$$I_T = \frac{V_{AB}}{R_T}$$

The resistance of the parallel portion of the circuit in Figure 65(c) is

$$R_{T(p)} = \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)}$$

The current through the left branch is

$$I_{AC} = \left(\frac{R_{T(p)}}{R_1 + R_D}\right)I_T$$

The current through the right branch is

$$I_{AD} = \left(\frac{R_{T(p)}}{R_2 + R_E}\right)I_T$$

The voltage at terminal C with respect to terminal A is

$$V_{CA} = V_A - I_{AC}R_D$$

The voltage at terminal D with respect to terminal A is

$$V_{DA} = V_A - I_{AD}R_E$$

The voltage from terminal C to terminal D is

$$\begin{aligned} V_{CD} &= V_{CA} - V_{DA} \\ &= (V_A - I_{AC}R_D) - (V_A - I_{AD}R_E) = I_{AD}R_E - I_{AC}R_D \end{aligned}$$

V_{CD} is the voltage across the load (R_C) in the bridge circuit of Figure 65(a).

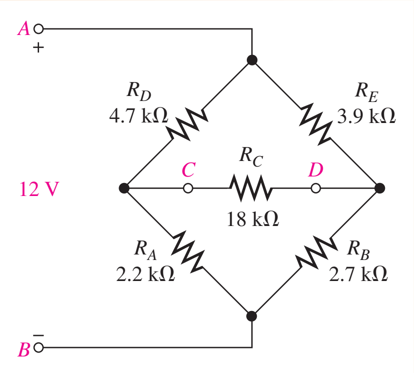
The load current through R_C can be found by Ohm's law.

$$I_{R_C} = \frac{V_{CD}}{R_C}$$

EXAMPLE 21

Determine the voltage across the load resistor and the current through the load resistor in the bridge circuit in Figure 66. Notice that the resistors are labeled for convenient conversion using Equations 1, 2, and 3. R_C is the load resistor.

▶ FIGURE 66



Solution First, convert the delta formed by R_A, R_B, and R_C to a wye.

$$R_1 = \frac{R_AR_C}{R_A + R_B + R_C} = \frac{(2.2\text{ k}\Omega)(18\text{ k}\Omega)}{2.2\text{ k}\Omega + 2.7\text{ k}\Omega + 18\text{ k}\Omega} = 1.73\text{ k}\Omega$$

$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C} = \frac{(2.7\text{ k}\Omega)(18\text{ k}\Omega)}{22.9\text{ k}\Omega} = 2.12\text{ k}\Omega$$

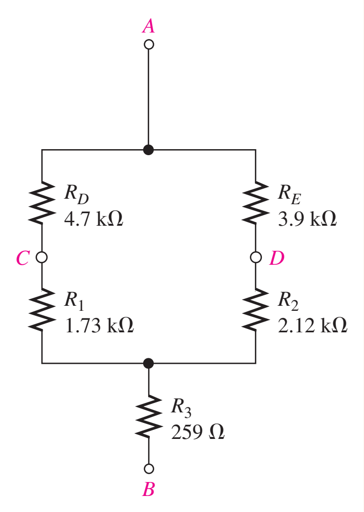
$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C} = \frac{(2.2\text{ k}\Omega)(2.7\text{ k}\Omega)}{22.9\text{ k}\Omega} = 259\ \Omega$$

The resulting equivalent series-parallel circuit is shown in Figure 67.

Next, determine R_T and the branch currents in Figure 67.

$$\begin{aligned} R_T &= \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)} + R_3 \\ &= \frac{(6.43\text{ k}\Omega)(6.02\text{ k}\Omega)}{6.43\text{ k}\Omega + 6.02\text{ k}\Omega} + 259\ \Omega = 3.11\text{ k}\Omega + 259\ \Omega = 3.37\text{ k}\Omega \end{aligned}$$

$$I_T = \frac{V_{AB}}{R_T} = \frac{12\text{ V}}{3.37\text{ k}\Omega} = 3.56\text{ mA}$$



▲ FIGURE 67

The total resistance of the parallel part of the circuit, $R_{T(p)}$, is 3.11 kΩ.

$$I_{AC} = \left(\frac{R_{T(p)}}{R_1 + R_D}\right)I_T = \left(\frac{3.11\text{ k}\Omega}{1.73\text{ k}\Omega + 4.7\text{ k}\Omega}\right)3.56\text{ mA} = 1.72\text{ mA}$$

$$I_{AD} = \left(\frac{R_{T(p)}}{R_2 + R_E}\right)I_T = \left(\frac{3.11\text{ k}\Omega}{2.12\text{ k}\Omega + 3.9\text{ k}\Omega}\right)3.56\text{ mA} = 1.84\text{ mA}$$

The voltage from terminal C to terminal D is

$$V_{CD} = I_{AD}R_E - I_{AC}R_D = (1.84\text{ mA})(3.9\text{ k}\Omega) - (1.72\text{ mA})(4.7\text{ k}\Omega)$$
$$= 7.18\text{ V} - 8.08\text{ V} = \mathbf{-0.9\ V}$$

V_{CD} is the voltage across the load (R_C) in the bridge circuit shown in Figure 66. The load current through R_C is

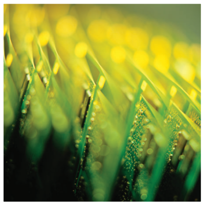
$$I_{R_C} = \frac{V_{CD}}{R_C} = \frac{-0.9\text{ V}}{18\text{ k}\Omega} = \mathbf{-50\ \mu A}$$

Related Problem Determine the load current, I_{R_C}, in Figure 66 for the following resistor values: $R_A = 27\text{ k}\Omega$, $R_B = 33\text{ k}\Omega$, $R_D = 39\text{ k}\Omega$, $R_E = 47\text{ k}\Omega$, and $R_C = 100\text{ k}\Omega$.

Use Multisim file E08-21 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 8 CHECKUP

1. Draw a delta circuit.
2. Draw a wye circuit.
3. Write the formulas for delta-to-wye conversion.
4. Write the formulas for wye-to-delta conversion.



Application Activity

The Wheatstone bridge circuit was expanded in this chapter to include the use of Thevenin's theorem. You may have encountered a circuit application that used a thermistor in one arm of the bridge to sense temperature. The bridge was used to compare the resistance of the thermistor with the resistance of a rheostat, which set the emperature at which the output would switch from one polarity to the opposite for turning on a heater in a tank filled with a liquid. In this application activity, you will work with a similar circuit, but this time it will be used to monitor the temperature in the tank to provide a visual indication that the temperature is within a specified range.

The Temperature Monitor

The basic measuring circuit in the temperature monitor is a Wheatstone bridge with an ammeter and a series resistor acting as the load. The meter is an analog panel meter with a sensitivity of 50 μA full scale. The Wheatstone bridge temperature-measuring circuit is shown in Figure 68(a), and the meter panel is shown in Figure 68(b).

The Thermistor

The thermistor is a Thermometrics RL2006-13.3K-140-D1 thermistor with a specified resistance of 25 kΩ at 25°C and a β of 4615K. Recall that β is a constant supplied by the manufacturer that indicates the shape of the temperature-resistance characteristic. The exponential equation for the resistance of a thermistor is approximated by

$$R_\text{T} = R_0 e^{\beta\left(\frac{T_0 - T}{T_0 T}\right)}$$

where:

R_T = the resistance at a given temperature

R_0 = the resistance at a reference temperature

T_0 = the reference temperature in K (typically 298 K, which is 25°C)

T = temperature (K)

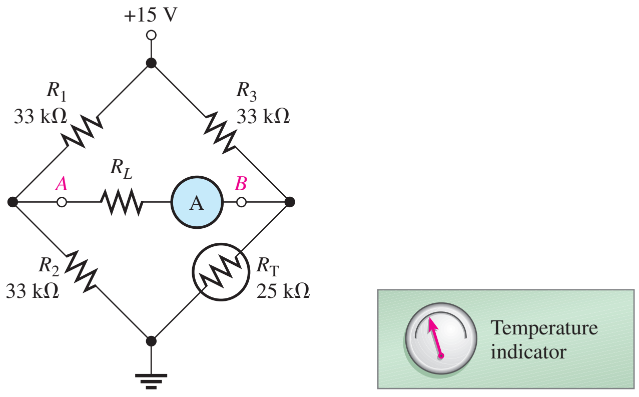
β = a constant (K) provided by the manufacturer

A plot of this equation is given in Figure 69. You can confirm that your thermistor resistance calculations in this circuit application are in reasonable agreement with this plot.

Temperature Measuring Circuit

The Wheatstone bridge is designed to be balanced at 20°C. The resistance of the thermistor is approximately 33 kΩ at this temperature. You can confirm this value by substituting the temperature (in Kelvin) into the equation for R_T. Remember that the temperature in K is °C + 273.

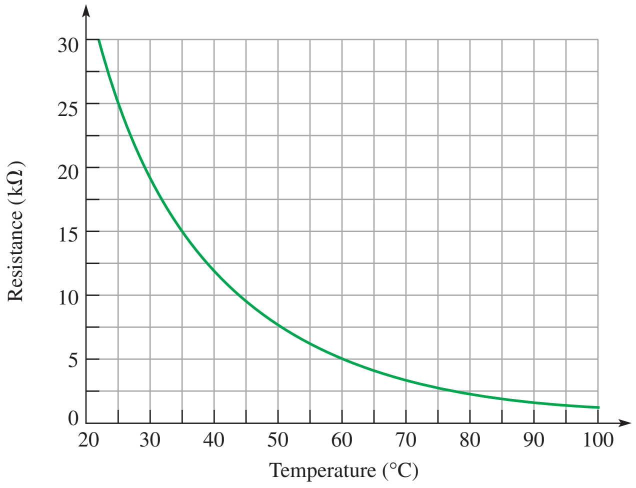
1. Substituting into the equation for R_T, calculate the resistance of the thermistor at a temperature of 50°C (full-scale deflection of the meter).
2. Thevenize the bridge between terminals A and B by keeping the ground reference and forming two facing Thevenin circuits as was illustrated in Figure 44. Assume the thermistor temperature is 50°C and its resistance is the value calculated previously. Draw the Thevenin circuit for this temperature but do not show a load.
3. Show the load resistor for the Thevenin circuit you drew. The load is a resistor in series with the ammeter, which will have a current of 50 μA at full scale (50°C). You can find the value of the required load resistor by applying the



(a) Temperature-measuring circuit with thermistor

(b) Meter panel

▲ FIGURE 68



▲ FIGURE 69

superposition theorem to the two sources and calculating the total resistance from Ohm's law (using the full-scale deflection as the current). Subtract the Thevenin resistance of each arm from the total resistance to obtain the required load resistance. Neglect the meter resistance. Show the value calculated on the Thevenin circuit.

4. Calculate the thermistor resistance for the lower and upper limits of temperature (30°C and 40°C). Draw Thevenin circuits for each temperature and calculate the current through the load resistor.

The Meter Scale

A requirement for the temperature monitor is to mark three color bands on the meter to indicate that the temperature is within the desired range. The desired range is between a low of 30°C and 40°C. The meter should indicate a too-cold range from 20°C to 30°C, a proper operating range from 30°C to 40°C, and a too hot range from 40°C and 50°C. Full-scale deflection of the meter should be set for 50°C

5. Indicate how you would mark the meter to have a quick visual indication of the temperature in the tank.

Review

6. At 35°C, what is the current in the meter?
7. What change is needed if a 100 μA meter is used instead of a 50 μA meter?

SUMMARY

- An ideal voltage source has zero internal resistance. It provides a constant voltage across its terminals regardless of the load resistance.
- A practical voltage source has a nonzero internal resistance.
- An ideal current source has infinite internal resistance. It provides a constant current regardless of the load resistance.
- A practical current source has a finite internal resistance.
- The superposition theorem is useful for multiple-source circuits.
- Thevenin's theorem provides for the reduction of any two-terminal linear resistive circuit to an equivalent form consisting of an equivalent voltage source in series with an equivalent resistance.
- The term *equivalency,* as used in Thevenin's and Norton's theorems, means that when a given load resistance is connected to the equivalent circuit, it will have the same voltage across it and the same current through it as when it was connected to the original circuit.
- Norton's theorem provides for the reduction of any two-terminal linear resistive circuit to an equivalent form consisting of an equivalent current source in parallel with an equivalent resistance.
- Maximum power is transferred to a load from a source when the load resistance equals the internal source resistance.

KEY TERMS

Maximum power transfer	For a given source voltage, a transfer of maximum power from a source to a load occurs when the load resistance equals the internal source resistance.
Norton's theorem	A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistance.
Superposition theorem	A method for the analysis of circuits with more than one source.
Terminal equivalency	The concept that when any given load resistance is connected to two sources, the same load voltage and load current are produced by both sources.
Thevenin's theorem	A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a voltage source in series with a resistance.

FORMULAS

Δ-to-Y Conversions

1 $$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

2 $$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

3 $$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Y-to-Δ Conversions

4 $$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

5 $$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

6 $$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. A dc voltage source has a very high internal resistance.
2. A dc current source has a very high internal resistance.
3. The superposition theorem can be applied to circuits with multiple sources.
4. When applying the superposition theorem, replace all sources by their internal resistances.
5. A Thevenin equivalent circuit consists of a voltage source with a parallel resistor.
6. The Thevenin voltage is equal to the voltage across the open terminals of a circuit.
7. A Norton equivalent circuit consists of a current source with a series resistor.
8. The Norton current is equal to the current through the shorted output terminals of a circuit.
9. Maximum power is transferred when the load resistance and source resistance are not equal.
10. Δ-to-Y conversion can be applied to the analysis of a Wheatstone bridge.

SELF-TEST

Answers are at the end of the chapter.

1. A 100 Ω load is connected across an ideal voltage source with $V_S = 10$ V. The voltage across the load is

   (a) 0 V **(b)** 10 V **(c)** 100 V

2. A 100 Ω load is connected across a voltage source with $V_S = 10$ V and $R_S = 10\ \Omega$. The voltage across the load is

   (a) 10 V **(b)** 0 V **(c)** 9.09 V **(d)** 0.909 V

3. A certain voltage source has the values $V_S = 25$ V and $R_S = 5\ \Omega$. The values for an equivalent current source are

   (a) 5 A, 5 Ω **(b)** 25 A, 5 Ω **(c)** 5 A, 125 Ω

4. A certain current source has the values $I_S = 3\ \mu$A and $R_S = 1.0$ MΩ. The values for an equivalent voltage source are

   (a) 3 μV, 1.0 MΩ **(b)** 3 V, 1.0 MΩ **(c)** 1 V, 3.0 MΩ

5. In a two-source circuit, one source acting alone produces 10 mA through a given branch. The other source acting alone produces 8 mA in the opposite direction through the same branch. The actual current through the branch is

   (a) 10 mA **(b)** 18 mA **(c)** 8 mA **(d)** 2 mA

6. Thevenin's theorem converts a circuit to an equivalent form consisting of
- **(a)** a current source and a series resistance
- **(b)** a voltage source and a parallel resistance
- **(c)** a voltage source and a series resistance
- **(d)** a current source and a parallel resistance

7. The Thevenin equivalent voltage for a given circuit is found by
- **(a)** shorting the output terminals
- **(b)** opening the output terminals
- **(c)** shorting the voltage source
- **(d)** removing the voltage source and replacing it with a short

8. A certain circuit produces 15 V across its open output terminals, and when a 10 kΩ load is connected across its output terminals, it produces 12 V. The Thevenin equivalent for this circuit is
- **(a)** 15 V in series with 10 kΩ
- **(b)** 12 V in series with 10 kΩ
- **(c)** 12 V in series with 2.5 kΩ
- **(d)** 15 V in series with 2.5 kΩ

9. Maximum power is transferred from a source to a load when
- **(a)** the load resistance is very large
- **(b)** the load resistance is very small
- **(c)** the load resistance is twice the source resistance
- **(d)** the load resistance equals the source resistance

10. For the circuit described in Question 8, maximum power is transferred to a

(a) 10 kΩ load **(b)** 2.5 kΩ load **(c)** an infinitely large resistance load

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 71.

1. If a short develops across R_4, the voltage across R_5

(a) increases **(b)** decreases **(c)** stays the same

2. If the 2 V source opens, the voltage across R_1

(a) increases **(b)** decreases **(c)** stays the same

3. If R_2 opens, the current through R_1

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 79.

4. If R_L opens, the voltage at the output terminal with respect to ground

(a) increases **(b)** decreases **(c)** stays the same

5. If either of the 5.6 kΩ resistors are shorted, the current through the load resistor

(a) increases **(b)** decreases **(c)** stays the same

6. If either of the 5.6 kΩ resistors are shorted, current from the source

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 81.

7. If the input to the amplifier becomes shorted to ground, the current drawn from both voltage sources

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 84.

8. If R_1 is actually 1.0 kΩ instead of 10 kΩ, the expected voltage between A and B

(a) increases **(b)** decreases **(c)** stays the same

9. If a 10 MΩ load resistor is connected from A to B, the voltage between A and B

(a) increases **(b)** decreases **(c)** stays the same

10. If a short develops across R_4, the magnitude of the voltage between A and B

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 86.

11. If the 220 Ω resistor opens, V_{AB}

(a) increases **(b)** decreases **(c)** stays the same

12. If a short develops across the 330 Ω resistor, V_{AB}

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 87(d).

13. If the 680 Ω resistor opens, the current through R_L

(a) increases **(b)** decreases **(c)** stays the same

14. If the 47 Ω resistor becomes shorted, the voltage across R_L

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

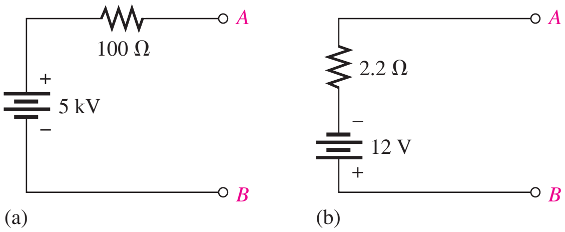
More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 3 Source Conversions

1. A voltage source has the values $V_S = 300$ V and $R_S = 50\ \Omega$. Convert it to an equivalent current source.

2. Convert the practical voltage sources in Figure 70 to equivalent current sources.

▶ FIGURE 70



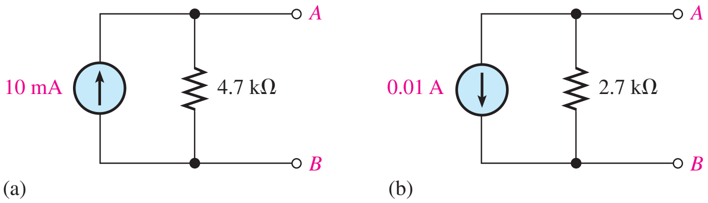
3. A fresh D cell battery has a terminal voltage of 1.6 V and can supply up to 8.0 A into a short for a very short time. What is the internal resistance of the battery?

4. Draw the voltage and current source equivalent circuits for the D cell in Problem 3.

5. A current source has an I_S of 600 mA and an R_S of 1.2 kΩ. Convert it to an equivalent voltage source.

6. Convert the practical current sources in Figure 71 to equivalent voltage sources.

▶ FIGURE 71

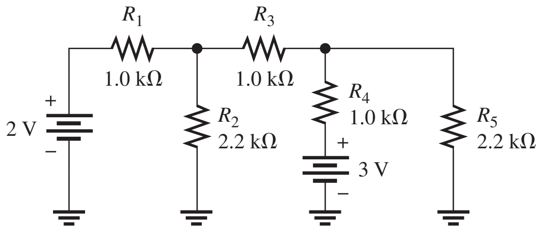


SECTION 4 The Superposition Theorem

7. Using the superposition method, calculate the current through R_5 in Figure 72.

8. Use the superposition theorem to find the current in and the voltage across the R_2 branch of Figure 72.

9. Determine the current through R_4 in Figure 72.

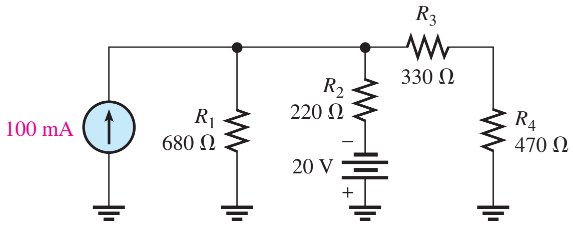


R1
1.0 kΩ
R3
1.0 kΩ
+
2 V
–
R2
2.2 kΩ
R4
1.0 kΩ
+
3 V
–
R5
2.2 kΩ

▶ FIGURE 72

10. Determine the current through R_2 in Figure 73.

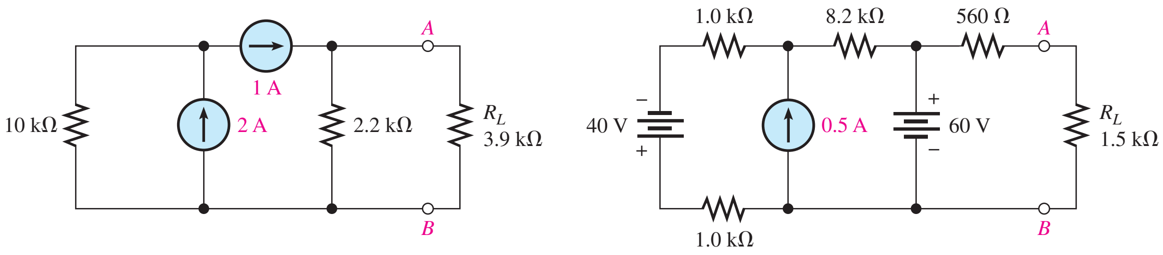
11. Using the superposition theorem, solve for the current through R_3 in Figure 73.



R3
330 Ω
100 mA
R1
680 Ω
R2
220 Ω
–
20 V
+
R4
470 Ω

▶ FIGURE 73

12. Using the superposition theorem, find the load current in each circuit of Figure 74.



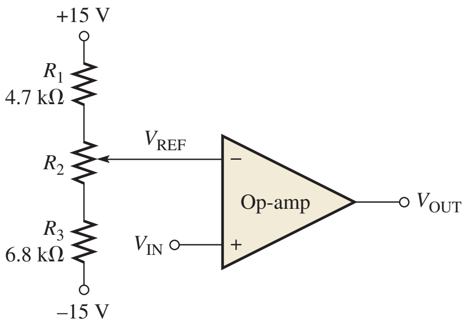
A
1 A
10 kΩ
2 A
2.2 kΩ
RL
3.9 kΩ
B
1.0 kΩ
8.2 kΩ
560 Ω
A
–
40 V
+
0.5 A
+
60 V
–
RL
1.5 kΩ
1.0 kΩ
B

(a)

(b)

▲ FIGURE 74

13. A comparator circuit is shown in Figure 75. The input voltage, V_{IN}, is compared to the reference voltage, V_{REF}, and an output is generated that is negative if $V_{REF} > V_{IN}$; otherwise it is positive. The comparator does not load either input. If R_2 is 1.0 kΩ, what is the range of the reference voltage?



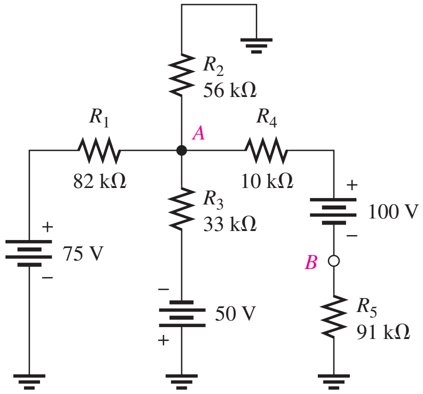
+15 V
R1
4.7 kΩ
VREF
R2
–
Op-amp
VOUT
R3
6.8 kΩ
VIN
+
–15 V

▶ FIGURE 75

14. Repeat Problem 13 if R_2 is 10 kΩ.

***15.** Determine the voltage from point *A* to point *B* in Figure 76.

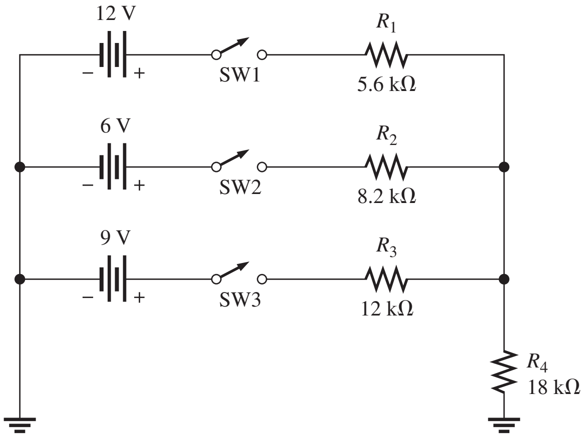
▶ **FIGURE 76**



R_2
56 kΩ
R_1
82 kΩ
A
R_4
10 kΩ
R_3
33 kΩ
+
100 V
−
+
75 V
−
B
−
50 V
+
R_5
91 kΩ

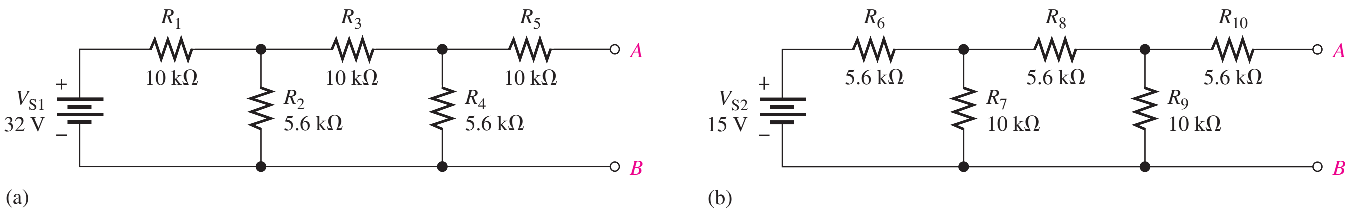
16. The switches in Figure 77 are closed in sequence, SW1 first. Find the current through R_4 after each switch closure.

▶ **FIGURE 77**



12 V
SW1
R_1
5.6 kΩ
6 V
SW2
R_2
8.2 kΩ
9 V
SW3
R_3
12 kΩ
R_4
18 kΩ

***17.** Figure 78 shows two ladder networks. Determine the current provided by each of the batteries when terminals *A* are connected (*A* to *A*) and terminals *B* are connected (*B* to *B*).



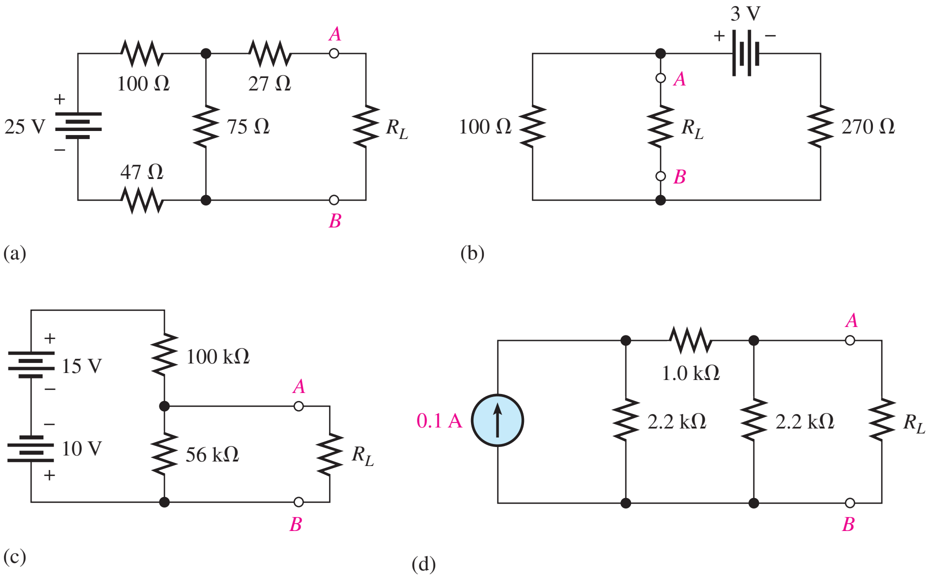
R_1
10 kΩ
R_3
10 kΩ
R_5
10 kΩ
A
V_{S1}
32 V
R_2
5.6 kΩ
R_4
5.6 kΩ
B
(a)
R_6
5.6 kΩ
R_8
5.6 kΩ
R_{10}
5.6 kΩ
A
V_{S2}
15 V
R_7
10 kΩ
R_9
10 kΩ
B
(b)

▲ **FIGURE 78**

SECTION 5 Thevenin's Theorem

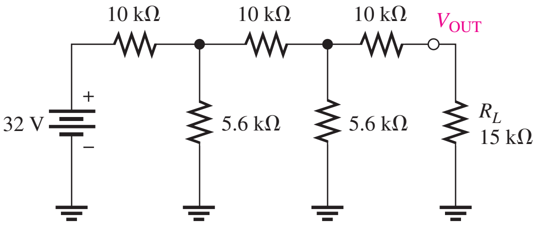
18. For each circuit in Figure 79, determine the Thevenin equivalent as seen from terminals *A* and *B*.

▶ **FIGURE 79**

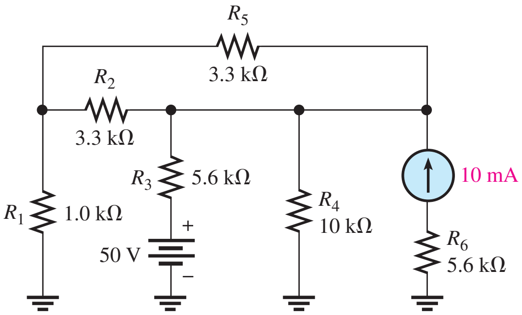

A
100 Ω
27 Ω
+
25 V
−
75 Ω
R_L
47 Ω
B
(a)
3 V
+
−
A
100 Ω
R_L
270 Ω
B
(b)
+
15 V
−
100 kΩ
A
−
10 V
+
56 kΩ
R_L
B
(c)
A
1.0 kΩ
0.1 A
2.2 kΩ
2.2 kΩ
R_L
B
(d)

19. Using Thevenin's theorem, determine the current through the load R_L in Figure 80.

***20.** Using Thevenin's theorem, find the voltage across R_4 in Figure 81.


10 kΩ
10 kΩ
10 kΩ
V_{OUT}
+
32 V
−
5.6 kΩ
5.6 kΩ
R_L
15 kΩ

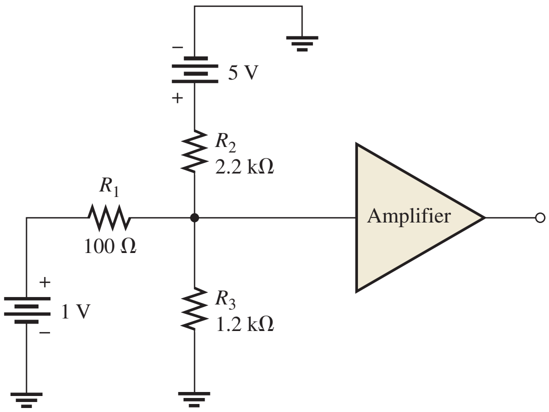
▲ **FIGURE 80**


R_5
3.3 kΩ
R_2
3.3 kΩ
R_3
5.6 kΩ
10 mA
R_1
1.0 kΩ
+
50 V
−
R_4
10 kΩ
R_6
5.6 kΩ

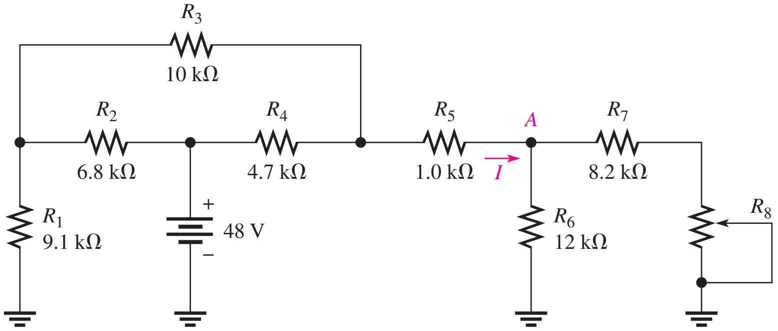
▲ **FIGURE 81**

21. Find the Thevenin equivalent for the circuit external to the amplifier in Figure 82.

▶ **FIGURE 82**


−
5 V
+
R_2
2.2 kΩ
R_1
100 Ω
Amplifier
+
1 V
−
R_3
1.2 kΩ

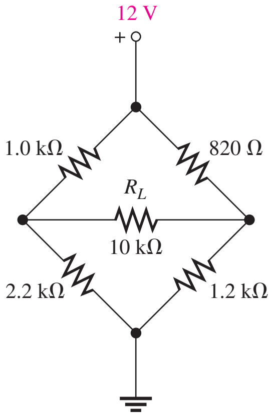
22. Determine the current into point *A* when R_8 is 1.0 kΩ, 5 kΩ, and 10 kΩ in Figure 83.



R_3
10 kΩ
R_2
6.8 kΩ
R_4
4.7 kΩ
R_5
1.0 kΩ
A
I
R_7
8.2 kΩ
R_1
9.1 kΩ
+
48 V
–
R_6
12 kΩ
R_8

▲ FIGURE 83

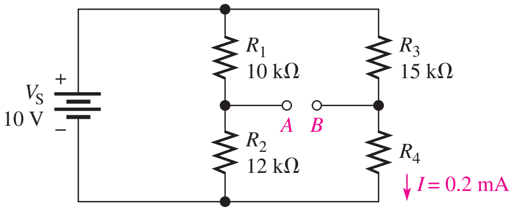
***23.** Find the current through the load resistor in the bridge circuit of Figure 84.



12 V
+
1.0 kΩ
820 Ω
R_L
10 kΩ
2.2 kΩ
1.2 kΩ

▲ FIGURE 84

24. Determine the Thevenin equivalent looking from terminals *A* and *B* for the circuit in Figure 85.



R_1
10 kΩ
R_3
15 kΩ
+
V_S
10 V
–
A
B
R_2
12 kΩ
R_4
I = 0.2 mA

▲ FIGURE 85

SECTION 6 Norton's Theorem

25. For each circuit in Figure 79, determine the Norton equivalent as seen by R_L.

26. Using Norton's theorem, find the current through the load resistor R_L in Figure 80.

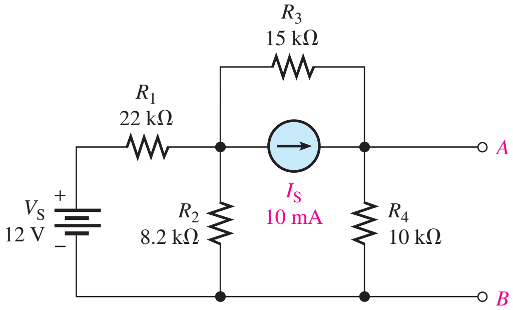
***27.** Using Norton's theorem, find the voltage across R_5 in Figure 81.

28. Using Norton's theorem, find the current through R_1 in Figure 83 when $R_8 = 8\ \text{k}\Omega$.

29. Determine the Norton equivalent circuit for the bridge in Figure 84 with R_L removed.

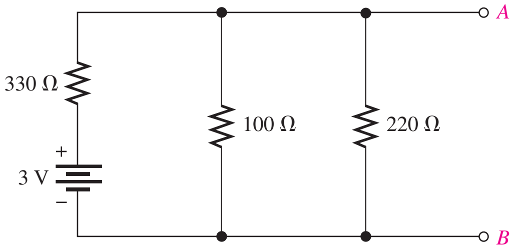
30. Reduce the circuit between terminals *A* and *B* in Figure 86 to its Norton equivalent.

▶ **FIGURE 86**


R_3
15 kΩ
R_1
22 kΩ
A
V_S
12 V
+
–
R_2
8.2 kΩ
I_S
10 mA
R_4
10 kΩ
B

31. Apply Norton's theorem to the circuit of Figure 87.

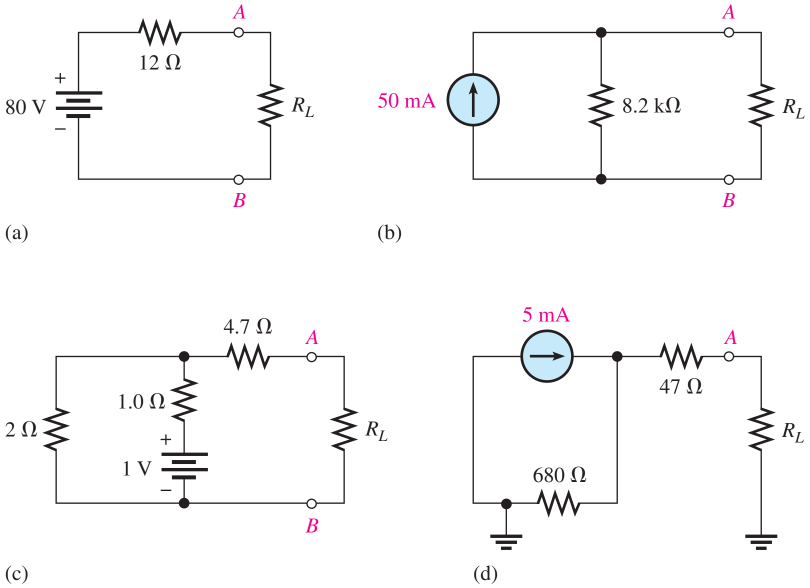
▶ **FIGURE 87**


A
330 Ω
100 Ω
220 Ω
+
3 V
–
B

SECTION 7 Maximum Power Transfer Theorem

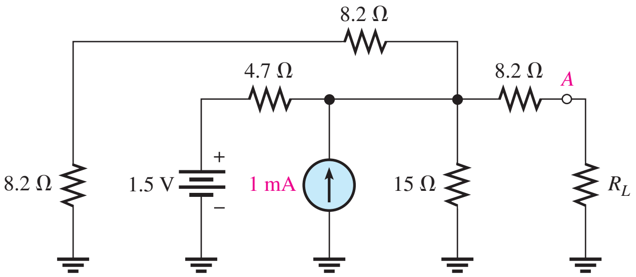
32. For each circuit in Figure 88, maximum power is to be transferred to the load R_L. Determine the appropriate value for R_L in each case.

▶ **FIGURE 88**


A
12 Ω
80 V
+
–
R_L
B
(a)
A
50 mA
8.2 kΩ
R_L
B
(b)
4.7 Ω
A
1.0 Ω
2 Ω
+
1 V
–
R_L
B
(c)
5 mA
A
47 Ω
R_L
680 Ω
(d)

33. Determine the value of R_L for maximum power in Figure 89.

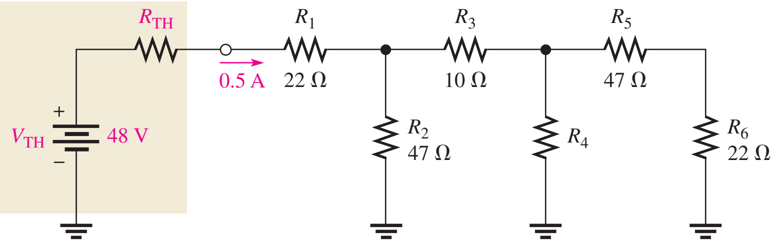
▶ **FIGURE 89**


8.2 Ω
4.7 Ω
8.2 Ω
A
8.2 Ω
1.5 V
+
–
1 mA
15 Ω
R_L

***34.** How much power is delivered to the load when R_L is 10% higher than its value for maximum power in Figure 89?

***35.** What are the values of R_4 and R_{TH} when the maximum power is transferred from the thevenized source to the ladder network in Figure 90?

▶ **FIGURE 90**

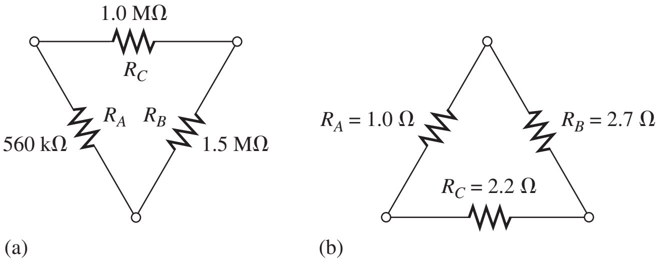


R_{TH}
R_1
R_3
R_5
0.5 A
22 Ω
10 Ω
47 Ω
+
V_{TH}
48 V
–
R_2
47 Ω
R_4
R_6
22 Ω

SECTION 8 Delta-to-Wye (Δ-to-Y) and Wye-to-Delta (Y-to-Δ) Conversions

36. In Figure 91, convert each delta network to a wye network.

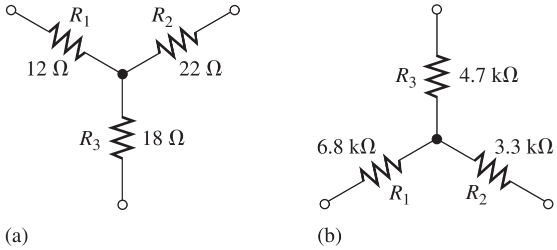
▶ **FIGURE 91**



1.0 MΩ
R_C
R_A
R_B
560 kΩ
1.5 MΩ
(a)
R_A = 1.0 Ω
R_B = 2.7 Ω
R_C = 2.2 Ω
(b)

37. In Figure 92, convert each wye network to a delta network.

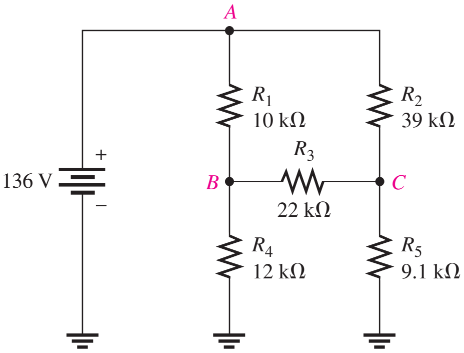
▶ **FIGURE 92**



R_1
R_2
12 Ω
22 Ω
R_3
18 Ω
(a)
R_3
4.7 kΩ
6.8 kΩ
3.3 kΩ
R_1
R_2
(b)

***38.** Find all currents in the circuit of Figure 93.

▶ **FIGURE 93**



A
R_1
10 kΩ
R_2
39 kΩ
R_3
+
136 V
–
B
22 kΩ
C
R_4
12 kΩ
R_5
9.1 kΩ

Multisim Troubleshooting and Analysis

These problems require Multisim.

39. Open file P08-39 and verify that the current through each resistor is correct and, if not, determine the fault.

40. Open file P08-40 and determine by measurement the Thevenin equivalent for the circuit between terminal *A* and ground.

41. Open file P08-41 and determine by measurement the Norton equivalent for the circuit between terminal *A* and ground.

42. Open file P08-42 and determine the fault, if any.

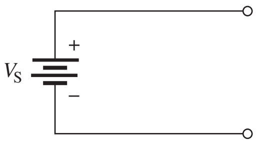
43. Open file P08-43 and determine the value of a load resistor to be connected between terminals *A* and *B* to achieve maximum power transfer.

ANSWERS

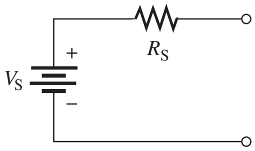
SECTION CHECKUPS

SECTION 1 The DC Voltage Source

1. For ideal voltage source, see Figure 94.
2. For practical voltage source, see Figure 95.
3. The internal resistance of an ideal voltage source is zero ohms.
4. Output voltage of a voltage source varies directly with load resistance.



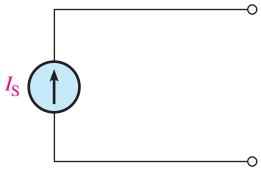
▲ FIGURE 94



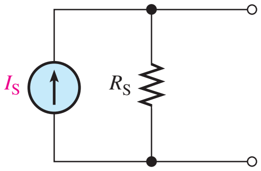
▲ FIGURE 95

SECTION 2 The Current Source

1. For ideal current source, see Figure 96.
2. For practical current source, see Figure 97.
3. An ideal current source has infinite internal resistance.
4. Load current from a current source varies inversely with load resistance.



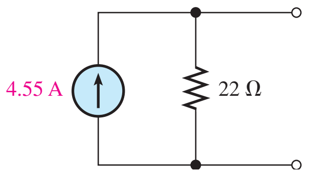
▲ FIGURE 96



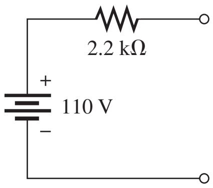
▲ FIGURE 97

SECTION 3 Source Conversions

1. $I_S = V_S/R_S$
2. $V_S = I_S R_S$
3. See Figure 98.
4. See Figure 99.



▲ FIGURE 98



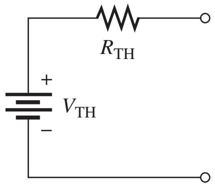
▲ FIGURE 99

SECTION 4 The Superposition Theorem

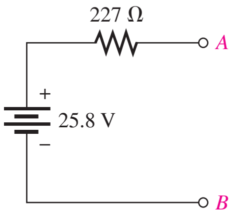
1. The superposition theorem states that the total current in any branch of a multiple-source linear circuit is equal to the algebraic sum of the currents due to the individual sources acting alone, with the other sources replaced by their internal resistances.
2. The superposition theorem allows each source to be treated independently.
3. A short simulates the internal resistance of an ideal voltage source; an open simulates the internal resistance of an ideal current source.
4. $I_{R1} = 6.67$ mA
5. The net current is in the direction of the larger current.

SECTION 5 Thevenin's Theorem

1. A Thevenin equivalent circuit consists of V_{TH} and R_{TH}.
2. See Figure 100 for the general form of a Thevenin equivalent circuit.
3. V_{TH} is the open circuit voltage between two terminals in a circuit.
4. R_{TH} is the resistance as viewed from two terminals in a circuit, with all sources replaced by their internal resistances.
5. See Figure 101.



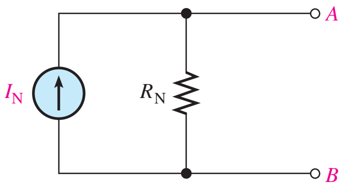
▲ FIGURE 100



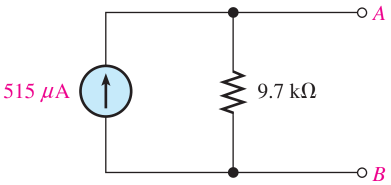
▲ FIGURE 101

SECTION 6 Norton's Theorem

1. A Norton equivalent circuit consists of I_{N} and R_{N}.
2. See Figure 102 for the general form of a Norton equivalent circuit.
3. I_{N} is the short circuit current between two terminals in a circuit.
4. R_{N} is the resistance as viewed from the two open terminals in a circuit.
5. See Figure 103.



▲ FIGURE 102



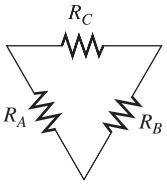
▲ FIGURE 103

SECTION 7 Maximum Power Transfer Theorem

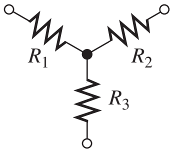
1. For a given source voltage, the maximum power transfer theorem states that maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.
2. Maximum power is delivered to a load when $R_L = R_S$.
3. $R_L = R_S = 50\ \Omega$

SECTION 8 Delta-to-Wye (Δ-to-Y) and Wye-to-Delta (Y-to-Δ) Conversions

1. For a delta circuit, see Figure 104.
2. For a wye circuit, see Figure 105.



▲ FIGURE 104



▲ FIGURE 105

3. The delta-to-wye conversion equations are

$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$
$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

4. The wye-to-delta conversion equations are

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$
$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$
$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

RELATED PROBLEMS FOR EXAMPLES

1 99.5 V
2 100 V
3 33.3 kΩ
4 1.2 A; 10 Ω
5 300 V; 600 Ω
6 16.6 mA
7 I_S is not affected.
8 7 mA
9 5 mA
10 2.36 V; 1240 Ω
11 $V_{TH(AB)} = 3.46$ V; $R_{TH(AB)} = 3.08$ kΩ
12 1.17 mA
13 0.7 V; V_{OUT} is 17.7 greater than V_{IN} at the base (B).

14 25.4 mA
15 131 Ω
16 $R_N = 248\ \Omega$
17 6.93 V
18 41.7 mW
19 $R_1 = 792\ \Omega, R_2 = 360\ \Omega, R_3 = 440\ \Omega$
20 $R_A = 712\ \Omega, R_B = 2.35\ \text{k}\Omega, R_C = 500\ \Omega$
21 0.3 μA

TRUE/FALSE QUIZ

1. F **2.** T **3.** T **4.** F **5.** F
6. T **7.** F **8.** T **9.** F **10.** T

SELF-TEST

1. (b) **2.** (c) **3.** (a) **4.** (b) **5.** (d) **6.** (c) **7.** (b) **8.** (d)
9. (d) **10.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (b) **4.** (a) **5.** (b) **6.** (a) **7.** (a) **8.** (a)
9. (b) **10.** (a) **11.** (a) **12.** (a) **13.** (a) **14.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. $I_S = 6\text{ A}, R_S = 50\ \Omega$

3. $200\text{ m}\Omega$

5. $V_S = 720\text{ V}, R_S = 1.2\text{ k}\Omega$

7. $845\ \mu\text{A}$

9. 1.13 mA

11. 1.6 mA

13. $V_{max} = 3.72\text{ V}; V_{min} = 1.32\text{ V}$

15. 90.7 V

17. $I_{S1} = 2.28\text{ mA}, I_{S2} = 1.35\text{ mA}$

19. $116\ \mu\text{A}$

21. $R_{TH} = 88.6\ \Omega, V_{TH} = 1.09\text{ V}$

23. $100\ \mu\text{A}$

25. **(a)** $I_N = 110\text{ mA}, R_N = 76.7\ \Omega$
  (b) $I_N = 11.1\text{ mA}, R_N = 73\ \Omega$
  (c) $I_N = 50\ \mu\text{A}, R_N = 35.9\text{ k}\Omega$
  (d) $I_N = 68.8\text{ mA}, R_N = 1.3\text{ k}\Omega$

27. 17.9 V

29. $I_N = 953\ \mu\text{A}, R_N = 1175\ \Omega$

31. $I_N = -48.2\text{ mA}, R_N = 56.9\ \Omega$

33. $11.1\ \Omega$

35. $R_{TH} = 48\ \Omega, R_4 = 160\ \Omega$

37. **(a)** $R_A = 39.8\ \Omega, R_B = 73\ \Omega, R_C = 48.7\ \Omega$
  (b) $R_A = 21.2\text{ k}\Omega, R_B = 10.3\text{ k}\Omega, R_C = 14.9\text{ k}\Omega$

39. R_1 leaky

41. $I_N = 0.383\text{ mA}; R_N = 9.674\text{ k}\Omega$

43. $I_{AB} = 1.206\text{ mA}; V_{AB} = 3.432\text{ V};$
  $R_L = 2.846\text{ k}\Omega$

BRANCH, LOOP, AND NODE ANALYSES

From Chapter 9 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.

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BRANCH, LOOP, AND NODE ANALYSES

CHAPTER OUTLINE

1 Simultaneous Equations in Circuit Analysis
2 Branch Current Method
3 Loop Current Method
4 Node Voltage Method
Application Activity

CHAPTER OBJECTIVES

- Discuss three methods to solve simultaneous equations
- Use the branch current method to find unknown quantities in a circuit
- Use loop analysis to find unknown quantities in a circuit
- Use node analysis to find unknown quantities in a circuit

KEY TERMS

- Simultaneous equations
- Matrix
- Determinant
- Loop
- Node
- Branch

APPLICATION ACTIVITY PREVIEW

In this application activity, you will analyze a model of an amplifier using the methods covered in this chapter.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, three circuit analysis methods are introduced. These methods, based on Ohm's law and Kirchhoff's laws, are particularly useful in the analysis of multiple-loop circuits having two or more voltage or current sources. The methods presented here can be used alone or in conjunction with the techniques covered in the previous chapters. With experience, you will learn which method is best for a particular problem or you may develop a preference for one of them.

In the branch current method, you will apply Kirchhoff's laws to solve for current in various branches of a multiple-loop circuit. A loop is a complete current path within a circuit. In the loop current method, you will solve for loop currents rather than branch currents. In the node voltage method, you will find the voltages at the independent nodes in a circuit. As you know, a node is the junction of two or more components.

1 SIMULTANEOUS EQUATIONS IN CIRCUIT ANALYSIS

The circuit analysis methods covered in this chapter allow you to find two or more unknown currents or voltages by solving simultaneous equations. These analysis methods, which include branch current, loop current, and node voltage methods, result in a number of equations equal to the number of unknowns. Coverage is limited to equations with two unknowns (2nd order) and equations with three unknowns (3rd order). These equations can then be solved simultaneously for the unknown quantities using one of the methods covered in this section.

After completing this section, you should be able to

- **Discuss three methods to solve simultaneous equations**
  - Write a set of simultaneous equations in standard form
  - Solve simultaneous equations using algebraic substitution
  - Solve simultaneous equations using determinants
  - Solve simultaneous equations using a calculator

Simultaneous equations consist of a set of n equations containing n unknowns, where n is a number with a value of 2 or more. The number of equations in the set must equal the number of unknowns. For example, to solve for two unknown variables, you must have two equations; to solve for three unknowns you must have three equations, and so on.

Standard Form Equations

An equation with two variables is called a *second-order equation*. In circuit analysis, the variables can represent unknowns such as current or voltage. In order to solve for variables x_1 and x_2, there must be a set of two equations containing those variables expressed in standard form.

In standard form, the x_1 variables are in the first position in each equation, and the x_2 variables are in the second position in each equation. The variables with their coefficients are on the left side of the equation, and the constants are on the right side.

The set of two simultaneous second-order equations written in standard form is

$$\mathrm{a}_{1,1}x_1 + \mathrm{a}_{1,2}x_2 = \mathrm{b}_1$$
$$\mathrm{a}_{2,1}x_1 + \mathrm{a}_{2,2}x_2 = \mathrm{b}_2$$

In these simultaneous equations, the "a" is the coefficient of the variables x_1 and x_2 and can represent values of circuit components, such as resistors. Notice that the subscripts of the coefficients contain two numbers. For example, $\mathrm{a}_{1,1}$ appears in the *first* equation as the coefficient of x_1 and $\mathrm{a}_{2,1}$ appears in the *second* equation as the coefficient of x_1. The "b" is the constant and can represent a voltage source. This notation will be useful when you use a calculator to solve the equations.

EXAMPLE 1

Assume the following two equations describe a particular circuit with two unknown currents I_1 and I_2. The coefficients are resistance values and the constants are voltages in the circuit. Write the equations in standard form.

$$2I_1 = 8 - 5I_2$$
$$4I_2 - 5I_1 + 6 = 0$$

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Solution Rearrange the equations in standard form as follows:

$$2I_1 + 5I_2 = 8$$
$$5I_1 + 4I_2 = -6$$

*Related Problem** Convert these two equations to standard form:

$$20x_1 + 15 = 11x_2$$
$$10 = 25x_2 + 18x_1$$

---

*Answers are at the end of the chapter.

A third-order equation contains three variables and a constant term. Just as in second-order equations, each variable has a coefficient. In order to solve for the variables x_1, x_2, and x_3, there must be a set of three simultaneous equations containing those variables. The general form for three simultaneous third-order equations written in standard form is

$$\mathrm{a}_{1,1}x_1 + \mathrm{a}_{1,2}x_2 + \mathrm{a}_{1,3}x_3 = \mathrm{b}_1$$
$$\mathrm{a}_{2,1}x_1 + \mathrm{a}_{2,2}x_2 + \mathrm{a}_{2,3}x_3 = \mathrm{b}_2$$
$$\mathrm{a}_{3,1}x_1 + \mathrm{a}_{3,2}x_2 + \mathrm{a}_{3,3}x_3 = \mathrm{b}_3$$

EXAMPLE 2

Assume the following three equations describe a particular circuit with three unknown currents I_1, I_2, and I_3. The coefficients are resistance values and the constants are known voltages in the circuit. Write the equations in standard form.

$$4I_3 + 2I_2 + 7I_1 = 0$$
$$5I_1 + 6I_2 + 9I_3 - 7 = 0$$
$$8 = 1I_1 + 2I_2 + 5I_3$$

Solution The equations are rearranged to put them in standard form as follows:

$$7I_1 + 2I_2 + 4I_3 = 0$$
$$5I_1 + 6I_2 + 9I_3 = 7$$
$$1I_1 + 2I_2 + 5I_3 = 8$$

Related Problem Convert these three equations to standard form:

$$10V_1 + 15 = 21V_2 + 50V_3$$
$$10 + 12V_3 = 25V_2 + 18V_1$$
$$12V_3 - 25V_2 + 18V_1 = 9$$

Solutions of Simultaneous Equations

Three ways for solving simultaneous equations are algebraic substitution, the determinant method, and using a calculator.

Solving by Substitution You can solve two or three simultaneous equations in standard form using algebraic substitution by first solving one of the variables in terms of the others.

However, because the process can become quite lengthy, we will restrict this method to second-order equations. Consider the following set of simultaneous equations:

$$\begin{aligned} 2x_1 + 6x_2 &= 8 \quad \text{(Eq. 1)} \\ 3x_1 + 6x_2 &= 2 \quad \text{(Eq. 2)} \end{aligned}$$

Step 1: Solve for x_1 in terms of x_2 in Eq. 1.

$$\begin{aligned} 2x_1 &= 8 - 6x_2 \\ x_1 &= 4 - 3x_2 \end{aligned}$$

Step 2: Substitute the result for x_1 into Eq. 2 and solve for x_2.

$$\begin{aligned} 3x_1 + 6x_2 &= 2 \\ 3(4 - 3x_2) + 6x_2 &= 2 \\ 12 - 9x_2 + 6x_2 &= 2 \\ -3x_2 &= -10 \\ x_2 &= \frac{-10}{-3} = 3.33 \end{aligned}$$

Step 3: Substitute the value for x_2 into the equation for x_1 in Step 1.

$$x_1 = 4 - 3x_2 = 4 - 3(3.33) = 4 - 9.99 = -5.99$$

Solving by Determinants The determinant method is a part of matrix algebra and provides a "cookbook" approach for solving simultaneous equations with two or three variables. A **matrix** is an array of numbers, and a **determinant** is effectively the solution to a matrix, resulting in a specific value. Second-order determinants are used for two variables and third-order determinants are used for three variables. The equations must be in standard form for a solution.

To illustrate the determinant method for second-order equations, let's find the values of I_1 and I_2 in the following two equations expressed in standard form. The coefficients are resistance values in ohms, and the constants are voltage values in volts.

$$\begin{aligned} 10I_1 + 5I_2 &= 15 \\ 2I_1 + 4I_2 &= 8 \end{aligned}$$

First, form the characteristic determinant from the matrix of the coefficients of the unknown currents. The first column in the determinant consists of the coefficients of I_1, and the second column consists of the coefficients of I_2. The resulting determinant is

$$\begin{array}{cc} \text{1st column} \downarrow & \downarrow \text{2nd column} \\ \left|\begin{matrix} 10 \\ 2 \end{matrix}\right. & \left.\begin{matrix} 5 \\ 4 \end{matrix}\right| \end{array}$$

An evaluation of this characteristic determinant requires three steps.

Step 1: Multiply the first number in the left column by the second number in the right column.

$$\begin{vmatrix} 10 & & 5 \\ & \times & \\ 2 & & 4 \end{vmatrix} = 10 \times 4 = 40$$

Step 2: Multiply the second number in the left column by the first number in the right column.

$$\begin{vmatrix} 10 & & 5 \\ & \times & \\ 2 & & 4 \end{vmatrix} = 2 \times 5 = 10$$

Step 3: Subtract the product in Step 2 from the product in Step 1.

$$40 - 10 = 30$$

This difference is the value of the characteristic determinant (30 in this case).

Next, replace the coefficients of I_1 in the first column of the characteristic determinant with the constants (fixed numbers) on the right side of the equations to form another determinant:

$$\begin{vmatrix} 15 & 5 \\ 8 & 4 \end{vmatrix}$$

Replace coefficients of I_1 with constants from right sides of equations.

Evaluate this I_1 determinant as follows:

$$\begin{vmatrix} 15 & & 5 \\ & \times & \\ 8 & & 4 \end{vmatrix} = 15 \times 4 = 60$$

$$\begin{vmatrix} 15 & & 5 \\ & \times & \\ 8 & & 4 \end{vmatrix} = 60 - (8 \times 5) = 60 - 40 = 20$$

The value of this determinant is 20.

Now solve for I_1 by dividing the I_1 determinant by the characteristic determinant as follows:

$$I_1 = \frac{\begin{vmatrix} 15 & 5 \\ 8 & 4 \end{vmatrix}}{\begin{vmatrix} 10 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{20}{30} = 0.667\text{ A}$$

To find I_2, form another determinant by substituting the constants on the right side of the given equations for the coefficients of I_2 in the second column of the characteristic determinant.

$$\begin{vmatrix} 10 & 15 \\ 2 & 8 \end{vmatrix}$$

Replace coefficients of I_2 with constants from right sides of equations.

Solve for I_2 by dividing this determinant by the characteristic determinant previously found.

$$I_2 = \frac{\begin{vmatrix} 10 & 15 \\ 2 & 8 \end{vmatrix}}{30} = \frac{(10 \times 8) - (2 \times 15)}{30} = \frac{80 - 30}{30} = \frac{50}{30} = 1.67\text{ A}$$

EXAMPLE 3

Solve the following set of equations for the unknown currents:

$$2I_1 - 5I_2 = 10$$
$$6I_1 + 10I_2 = 20$$

Solution Given that the coefficients are in ohms and the constants are in volts, evaluate the characteristic determinant as follows:

$$\begin{vmatrix} 2 & -5 \\ 6 & 10 \end{vmatrix} = (2)(10) - (-5)(6) = 20 - (-30) = 20 + 30 = 50$$

Solving for I_1 yields

$$I_1 = \frac{\begin{vmatrix} 10 & -5 \\ 20 & 10 \end{vmatrix}}{50} = \frac{(10)(10) - (-5)(20)}{50} = \frac{100 - (-100)}{50} = \frac{200}{50} = \mathbf{4\ A}$$

Solving for I_2 yields

$$I_2 = \frac{\begin{vmatrix} 2 & 10 \\ 6 & 20 \end{vmatrix}}{50} = \frac{(2)(20) - (6)(10)}{50} = \frac{40 - 60}{50} = \mathbf{-0.4\ A}$$

In a circuit problem, a result with a negative sign indicates that the direction of actual current is opposite to the assigned direction.

Note that multiplication can be expressed either by the multiplication sign such as 2×10 or by parentheses such as (2)(10).

Related Problem Solve the following set of equations for I_1:

$$\begin{aligned} 5I_1 + 3I_2 &= 4 \\ I_1 + 2I_2 &= -6 \end{aligned}$$

Third-order determinants can be evaluated by the expansion method. We will illustrate this method by finding the unknown current values in the following three equations expressed in standard form:

$$\begin{aligned} 1I_1 + 3I_2 - 2I_3 &= 7 \\ 0I_1 + 4I_2 + 1I_3 &= 8 \\ -5I_1 + 1I_2 + 6I_3 &= 9 \end{aligned}$$

The characteristic determinant for the matrix of coefficients for this set of equations is formed in a similar way to that used earlier for the second-order determinant. The first column consists of the coefficients of I_1, the second column consists of the coefficients of I_2, and the third column consists of the coefficients of I_3, as shown below.

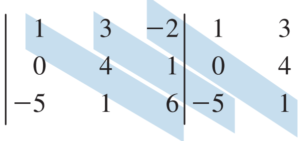
$$\begin{array}{|ccc|} \multicolumn{1}{c}{\text{Coefficients of } I_1} & \multicolumn{1}{c}{\text{Coefficients of } I_2} & \multicolumn{1}{c}{\text{Coefficients of } I_3} \\ \multicolumn{1}{c}{\downarrow} & \multicolumn{1}{c}{\downarrow} & \multicolumn{1}{c}{\downarrow} \\ 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{array}$$

This third-order determinant is evaluated by the expansion method as follows:

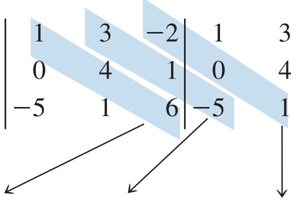
Step 1: Rewrite the first two columns immediately to the right of the determinant.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix} \begin{matrix} 1 & 3 \\ 0 & 4 \\ -5 & 1 \end{matrix}$$

Step 2: Identify the three downward diagonal groups of three coefficients each.

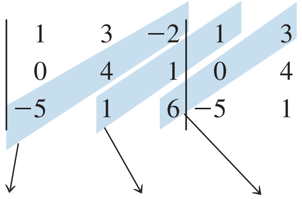


Step 3: Multiply the numbers in each diagonal and add the products.



$$(1)(4)(6) + (3)(1)(-5) + (-2)(0)(1) = 24 + (-15) + 0 = 9$$

Step 4: Repeat Steps 2 and 3 for the three upward diagonal groups of three coefficients.



$$(-5)(4)(-2) + (1)(1)(1) + (6)(0)(3) = 40 + 1 + 0 = 41$$

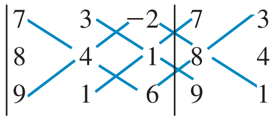
Step 5: Subtract the result in Step 4 from the result in Step 3 to get the value of the characteristic determinant.

$$9 - 41 = -32$$

Next, form another determinant by substituting the constants on the right of the equations for the coefficients of I_1 in the characteristic determinant.

$$\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}$$

Evaluate this determinant using the method described in the previous steps.



$$= [(7)(4)(6) + (3)(1)(9) + (-2)(8)(1)] - [(9)(4)(-2) + (1)(1)(7) + (6)(8)(3)]$$
$$= (168 + 27 - 16) - (-72 + 7 + 144) = 179 - 79 = 100$$

Solve for I_1 by dividing this determinant by the characteristic determinant. The negative result indicates that the actual current is in a direction opposite to the original assumption.

$$I_1 = \frac{\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix}} = \frac{100}{-32} = -3.125 \text{ A}$$

You can find I_2 and I_3 in a similar way.

EXAMPLE 4

Given that the coefficients are in ohms and the constants are in volts, determine the value of I_2 from the following set of equations:

$$\begin{aligned} 2I_1 + 0.5I_2 + 1I_3 &= 0 \\ 0.75I_1 + 0I_2 \quad + 2I_3 &= 1.5 \\ 3I_1 + 0.2I_2 + 0I_3 &= -1 \end{aligned}$$

Solution Evaluate the characteristic determinant as follows:

$$\left|\begin{array}{ccc|cc} 2 & 0.5 & 1 & 2 & 0.5 \\ 0.75 & 0 & 2 & 0.75 & 0 \\ 3 & 0.2 & 0 & 3 & 0.2 \end{array}\right.$$

$$= [(2)(0)(0) + (0.5)(2)(3) + (1)(0.75)(0.2)] - [(3)(0)(1) + (0.2)(2)(2) + (0)(0.75)(0.5)]$$
$$= (0 + 3 + 0.15) - (0 + 0.8 + 0) = 3.15 - 0.8 = 2.35$$

Evaluate the determinant for I_2 is as follows:

$$\left|\begin{array}{ccc|cc} 2 & 0 & 1 & 2 & 0 \\ 0.75 & 1.5 & 2 & 0.75 & 1.5 \\ 3 & -1 & 0 & 3 & -1 \end{array}\right.$$

$$= [(2)(1.5)(0) + (0)(2)(3) + (1)(0.75)(-1)] - [(3)(1.5)(1) + (-1)(2)(2) + (0)(0.75)(0)]$$
$$= [0 + 0 + (-0.75)] - [4.5 + (-4) + 0] = -0.75 - 0.5 = -1.25$$

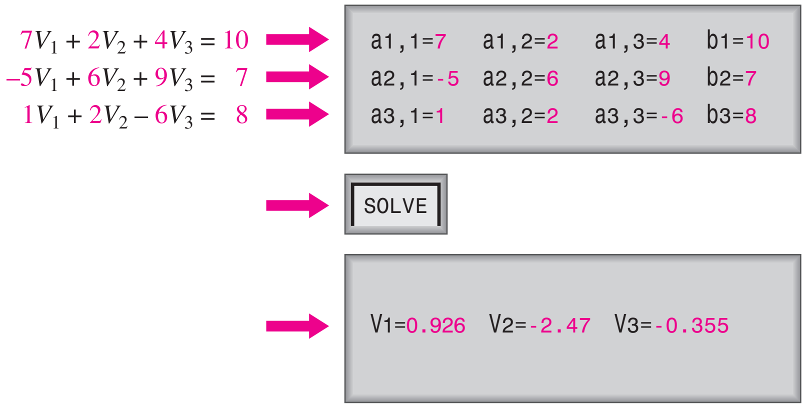
Finally, divide the two determinants.

$$I_2 = \frac{-1.25}{2.35} = -0.532\text{ A} = \mathbf{-532\ mA}$$

Related Problem Determine the value of I_1 in the set of equations used in this example.

Solving by Calculator Calculators generally employ matrix algorithms for simultaneous equation solution and make the results much easier to obtain. As with the two "manual" methods, it is important to first get the equations into the standard form before entering data on a calculator. Calculators that offer simultaneous equation solutions generally employ the notation discussed earlier in relation to the general form of equations. Variables are labeled as x_1, x_2, etc, coefficients are designated as $a_{1,1}$, $a_{1,2}$, $a_{2,1}$, $a_{2,2}$, etc., and constants are designated as b_1, b_2, etc.

A typical sequence for entering the data for a specific set of equations into a calculator is illustrated in a generic way for three simultaneous equations in Figure 1.



$7V_1 + 2V_2 + 4V_3 = 10$
$-5V_1 + 6V_2 + 9V_3 = 7$
$1V_1 + 2V_2 - 6V_3 = 8$
a1,1=7 a1,2=2 a1,3=4 b1=10
a2,1=-5 a2,2=6 a2,3=9 b2=7
a3,1=1 a3,2=2 a3,3=-6 b3=8
SOLVE
V1=0.926 V2=-2.47 V3=-0.355

◀ **FIGURE 1**

The TI-86 and the TI-89 calculators are used to illustrate the procedure in the following two examples, although other scientific calculators can be used. If your calculator has simultaneous equation capability, consult the user's manual for the correct procedure.

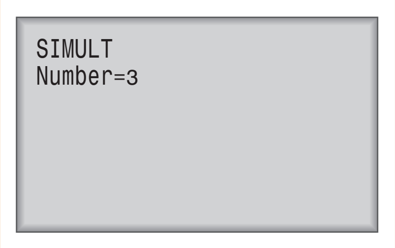
EXAMPLE 5

Use the TI-86 calculator to solve the following three simultaneous equations for the three unknowns.

$$\begin{aligned} 8I_1 + 4I_2 + 1I_3 &= 7 \\ 2I_1 - 5I_2 + 6I_3 &= 3 \\ 3I_1 + 3I_2 - 2I_3 &= -5 \end{aligned}$$

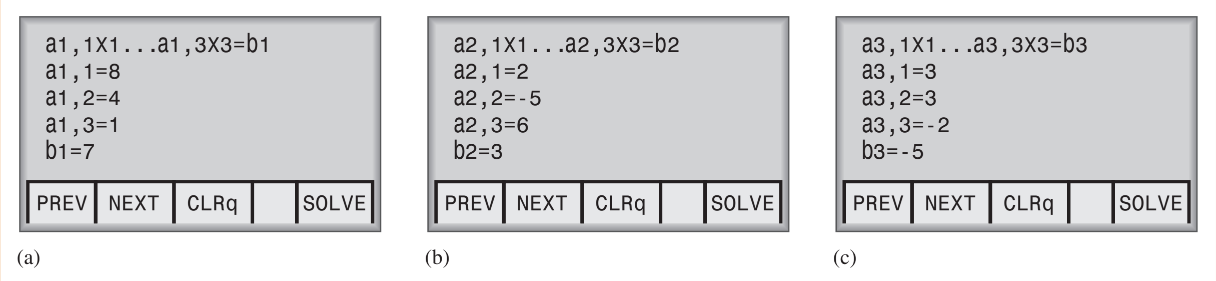
Solution Press the 2nd key, then SIMULT to enter the number of equations as shown in Figure 2.

▶ FIGURE 2



```
SIMULT
Number=3
```

After you enter 3 and press ENTER, the first equation screen comes up. Enter the coefficients 8, 4, 1 and the constant 7 by pressing each number key followed by the ENTER key, which results in the screen shown in Figure 3(a). After you enter the last number and press ENTER, the second equation screen appears. Enter the coefficients 2, −5, 6 and the constant 3 as shown in Figure 3(b) (A negative value is entered by first pressing the (−) key.). Finally, enter the coefficients of the third equation (3, 3, −2) and the constant −5 as shown in Figure 3(c).



(a)

```
a1,1X1...a1,3X3=b1
a1,1=8
a1,2=4
a1,3=1
b1=7
```

(b)

```
a2,1X1...a2,3X3=b2
a2,1=2
a2,2=-5
a2,3=6
b2=3
```

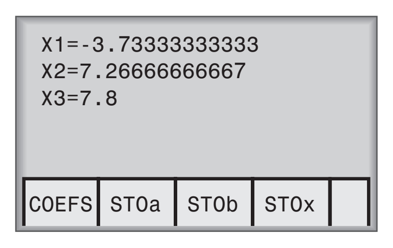
(c)

```
a3,1X1...a3,3X3=b3
a3,1=3
a3,2=3
a3,3=-2
b3=-5
```

▲ FIGURE 3

Selecting SOLVE, which is the F5 key, produces the results displayed in Figure 9-4. X1 is I_1, X2 is I_2, and X3 is I_3.

▶ FIGURE 4



```
X1=-3.73333333333
X2=7.26666666667
X3=7.8
```

Related Problem Edit the equations to change a1,2 from 4 to −3, a2,3 from 6 to 2.5, and b3 from −5 to 8 and solve the modified equations.

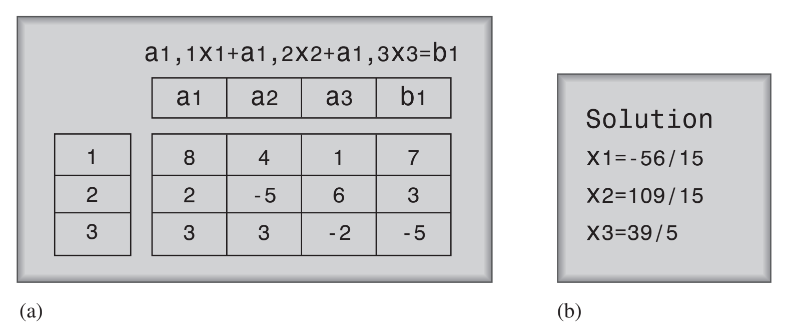
EXAMPLE 6 Use the TI-89 Titanium calculator to solve the same three simultaneous equations that were given in Example 5.

$$\begin{aligned} 8I_1 + 4I_2 + 1I_3 &= 7 \\ 2I_1 - 5I_2 + 6I_3 &= 3 \\ 3I_1 + 3I_2 - 2I_3 &= -5 \end{aligned}$$

Solution From the Home screen select the simultaneous equation icon

A|b
Simultaneo…

Press ENTER. Select 3:New, then press ENTER again. Next specify the number of equations and the number of unknowns and press ENTER. On the simultaneous equation screen enter the coefficients and the constants as shown in Figure 5(a). Press ENTER after each number.



```
a1,1x1+a1,2x2+a1,3x3=b1
```

	a1	a2	a3	b1
1	8	4	1	7
2	2	-5	6	3
3	3	3	-2	-5

(a)

```
Solution
x1=-56/15
x2=109/15
x3=39/5
```

(b)

▲ FIGURE 5

After you have entered the coefficients and constants, press the F5 key to solve. As indicated by the screen in part (b), the results are returned as fractions. They agree with the TI-86 results, which were returned as decimal numbers.

Related Problem Repeat the Related Problem from Example 5 using the TI-89.

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. Evaluate the following determinants:

(a) $\begin{vmatrix} 0 & -1 \\ 4 & 8 \end{vmatrix}$ (b) $\begin{vmatrix} 0.25 & 0.33 \\ -0.5 & 1 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 3 & 7 \\ 2 & -1 & 7 \\ -4 & 0 & -2 \end{vmatrix}$

2. Set up the characteristic determinant for the following set of simultaneous equations:

$$2I_1 + 3I_2 = 0$$
$$5I_1 + 4I_2 = 1$$

3. Find I_2 in Question 2.
4. Use your calculator to solve the following set of simultaneous equations for I_1, I_2, I_3, and I_4.

$$\begin{aligned}
100I_1 + 220I_2 + 180I_3 + 330I_4 &= 0 \\
470I_1 + 390I_2 + 100I_3 + 100I_4 &= 12 \\
120I_1 - 270I_2 + 150I_3 - 180I_4 &= -9 \\
560I_1 + 680I_2 - 220I_3 + 390I_4 &= 0
\end{aligned}$$

5. Modify the equations in Question 4 by changing the constant in the first equation to 8.5, the coefficient of I_3 in the second equation to 220, and the coefficient of I_1 in the fourth equation to 330. Solve the new set of equations for the currents.

2 BRANCH CURRENT METHOD

The branch current method is a circuit analysis method using Kirchhoff's voltage and current laws to find the current in each branch of a circuit by generating simultaneous equations. Once you know the branch currents, you can determine voltages.

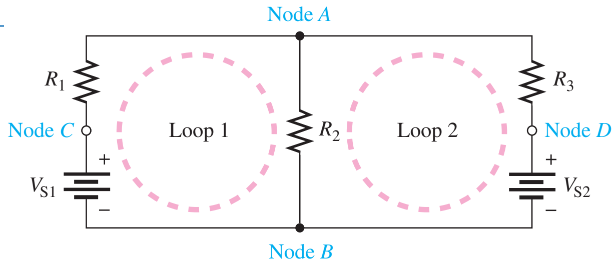
After completing this section, you should be able to

- **Use the branch current method to find unknown quantities in a circuit**
  - Identify loops and nodes in a circuit
  - Develop a set of branch current equations
  - Solve the branch current equations

Figure 6 shows a circuit that will be used as the basic model throughout the chapter to illustrate each of the three circuit analysis methods. In this circuit, there are only two nonredundant closed loops. A **loop** is a complete current path within a circuit, and you can view a set of nonredundant closed loops as a set of "windowpanes," where each windowpane represents one nonredundant loop. Also, there are four nodes as indicated by the letters *A*, *B*, *C*, and *D*. A **node** is a point where two or more components are connected. A **branch** is a path that connects two nodes, and there are three branches in this circuit: one containing R_1, one containing R_2, and one containing R_3.

FIGURE 6

Circuit showing loops, nodes, and branches.



The following are the general steps used in applying the branch current method.

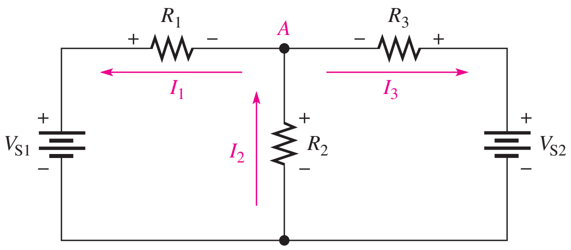
Step 1: Assign a current in each circuit branch in an arbitrary direction.

Step 2: Show the polarities of the resistor voltages according to the assigned branch current directions.

Step 3: Apply Kirchhoff's voltage law around each closed loop (algebraic sum of voltages is equal to zero).

Step 4: Apply Kirchhoff's current law at the minimum number of nodes so that all branch currents are included (algebraic sum of currents at a node equals zero).

Step 5: Solve the equations resulting from Steps 3 and 4 for the branch current values.



◀ FIGURE 7

Circuit for demonstrating branch current analysis.

These steps are demonstrated with the aid of Figure 7. First, the **branch currents** I_1, I_2, and I_3 are assigned in the direction shown. Don't worry about the actual current directions at this point. Second, the polarities of the voltage drops across R_1, R_2, and R_3 are indicated in the figure according to the assigned current directions. Third, Kirchhoff's voltage law applied to the two loops gives the following equations where the resistance values are the coefficients for the unknown currents:

Equation 1: $R_1I_1 + R_2I_2 - V_{S1} = 0$ for loop 1

Equation 2: $R_2I_2 + R_3I_3 - V_{S2} = 0$ for loop 2

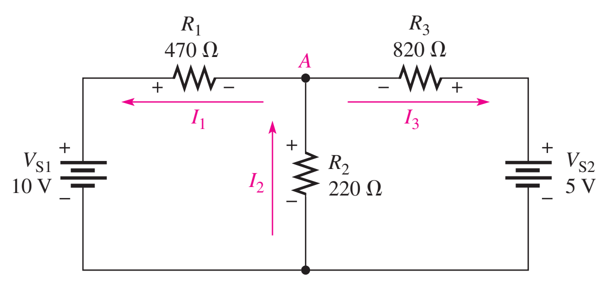
Fourth, Kirchhoff's current law is applied to node A, including all branch currents as follows:

Equation 3: $I_1 - I_2 + I_3 = 0$

The negative sign indicates that I_2 is into the node. Fifth and last, the three equations must be solved for the three unknown currents, I_1, I_2, and I_3. Example 7 shows how to solve equations by the *substitution* method.

EXAMPLE 7

Use the branch current method to find each branch current in Figure 8.



▲ FIGURE 8

Solution

Step 1: Assign branch currents as shown in Figure 8. Keep in mind that you can assume any current direction at this point and that the final solution will have a negative sign if the actual current is opposite to the assigned current.

Step 2: Mark the polarities of the resistor voltage drops in accordance with the assigned current directions as shown in the figure.

Step 3: Applying Kirchhoff's voltage law around the left loop gives

$$470I_1 + 220I_2 - 10 = 0$$

Around the right loop gives

$$220I_2 + 820I_3 - 5 = 0$$

where all resistance values are in ohms and voltage values are in volts. For simplicity, the units are not shown.

Step 4: At node A, the current equation is

$$I_1 - I_2 + I_3 = 0$$

Step 5: The equations are solved by substitution as follows. First, find I_1 in terms of I_2 and I_3.

$$I_1 = I_2 - I_3$$

Now, substitute $I_2 - I_3$ for I_1 in the left loop equation.

$$\begin{aligned} 470(I_2 - I_3) + 220I_2 &= 10 \\ 470I_2 - 470I_3 + 220I_2 &= 10 \\ 690I_2 - 470I_3 &= 10 \end{aligned}$$

Next, take the right loop equation and solve for I_2 in terms of I_3.

$$\begin{aligned} 220I_2 &= 5 - 820I_3 \\ I_2 &= \frac{5 - 820I_3}{220} \end{aligned}$$

Substituting this expression for I_2 into $820I_2 - 470I_3 = 10$ yields

$$\begin{aligned} 690\left(\frac{5 - 820I_3}{220}\right) - 470I_3 &= 10 \\ \frac{3450 - 565800I_3}{220} - 470I_3 &= 10 \\ 15.68 - 2571.8I_3 - 470I_3 &= 10 \\ -3041.8I_3 &= -5.68 \\ I_3 = \frac{5.68}{3041.8} &= 0.00187\text{ A} = \mathbf{1.87\text{ mA}} \end{aligned}$$

Now, substitute the value of I_3 in amps into the right loop equation.

$$220I_2 + 820(0.00187) = 5$$

Solve for I_2.

$$I_2 = \frac{5 - 820(0.00187)}{220} = \frac{3.47}{220} = 0.0158 = \mathbf{15.8\text{ mA}}$$

Substituting I_2 and I_3 values into the current equation at node A yields

$$\begin{aligned} I_1 - 0.0158 + 0.00187 &= 0 \\ I_1 = 0.0158 - 0.00187 &= 0.0139\text{ A} = \mathbf{13.9\text{ mA}} \end{aligned}$$

Related Problem Determine the branch currents in Figure 8 with the polarity of the 5 V source reversed.

Use Multisim file E09-07 to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 2 CHECKUP

1. What basic circuit laws are used in the branch current method?
2. When assigning branch currents, you should be careful that the assigned directions match the actual directions. (T or F)
3. What is a loop?
4. What is a node?

3 LOOP CURRENT METHOD

In the loop current method (also known as the mesh current method), you will work with loop currents instead of branch currents. An ammeter placed in a given branch will measure the branch current. Unlike branch currents, loop currents are mathematical quantities, rather than actual physical currents, that are used to make circuit analysis somewhat easier than with the branch current method.

After completing this section, you should be able to

- **Use loop analysis to find unknown quantities in a circuit**
  - Assign loop currents
  - Apply Kirchhoff's voltage law around each loop
  - Develop the loop equations
  - Solve the loop equations

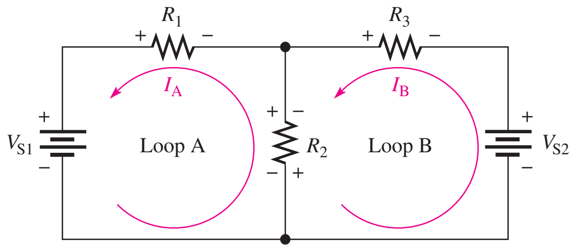
A systematic method of loop analysis is given in the following steps and is illustrated in Figure 9, which is the same circuit configuration used in the branch current analysis. It demonstrates the basic principles well.

Step 1: Although direction of an assigned loop current is arbitrary, we will assign a current in the counterclockwise (CCW) direction around each nonredundant closed loop, for consistency. This may not be the actual current direction, but it does not matter. The number of loop-current assignments must be sufficient to include current through all components in the circuit.

Step 2: Indicate the voltage drop polarities in each loop based on the assigned current directions.

Step 3: Apply Kirchhoff's voltage law around each closed loop. When more than one loop current passes through a component, include its voltage drop. This results in one equation for each loop.

Step 4: Using substitution or determinants, solve the resulting equations for the loop currents.



▲ FIGURE 9

First, the **loop currents** I_A and I_B are assigned in the CCW direction as shown in Figure 9. A loop current could be assigned around the outer perimeter of the circuit, but this would be redundant since I_A and I_B already pass through all of the components.

Second, the polarities of the voltage drops across R_1, R_2, and R_3 are shown based on the loop-current directions. Notice that I_A and I_B are in opposite directions through R_2 because R_2 is common to both loops. Therefore, two voltage polarities are indicated. In reality, the R_2 current cannot be separated into two parts, but remember that the loop currents are basically mathematical quantities used for analysis purposes. The polarities of the voltage sources are fixed and are not affected by the current assignments.

Third, Kirchhoff's voltage law applied to the two loops results in the following two equations:

$$R_1I_A + R_2(I_A - I_B) = V_{S1} \qquad \text{for loop A}$$
$$R_3I_B + R_2(I_B - I_A) = -V_{S2} \qquad \text{for loop B}$$

Notice that I_A is positive in loop A and I_B is positive in loop B.

Fourth, the like terms in the equations are combined and rearranged into standard form for convenient solution so that they have the same position in each equation, that is, the I_A term is first and the I_B term is second. The equations are rearranged into the following form. Once the loop currents are evaluated, all of the branch currents can be determined.

$$(R_1 + R_2)I_A - R_2I_B = V_{S1} \qquad \text{for loop A}$$
$$-R_2I_A + (R_2 + R_3)I_B = -V_{S2} \qquad \text{for loop B}$$

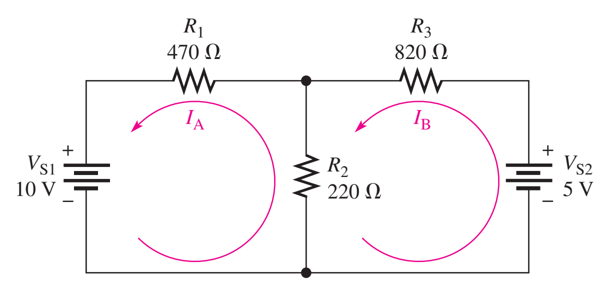
Notice that in the loop current method only two equations are required for the same circuit that required three equations in the branch current method. The last two equations (developed in the fourth step) follow a form to make loop analysis easier. Referring to these last two equations, notice that for loop A, the total resistance in the loop, $R_1 + R_2$, is multiplied by I_A (its loop current). Also in the loop A equation, the resistance common to both loops, R_2, is multiplied by the other loop current, I_B, and subtracted from the first term. The same form is seen in the loop B equation except that the terms have been rearranged. From these observations, a concise rule for applying steps 1 to 4 is as follows:

> **(Sum of resistors in loop) times (loop current) minus (each resistor common to both loops) times (associated adjacent loop current) equals (source voltage in the loop).**

Example 8 illustrates the application of this rule to the loop current analysis of a circuit.

EXAMPLE 8

Using the loop current method, find the branch currents in Figure 10.



▲ FIGURE 10

Solution Assign the loop currents (I_A and I_B) as shown in Figure 10; resistance values are in ohms and voltage values are in volts. Use the rule described to set up the two loop equations.

$$(470 + 220)I_A - 220I_B = 10$$
$$690I_A - 220I_B = 10 \qquad \text{for loop A}$$
$$-220I_A + (220 + 820)I_B = -5$$
$$-220I_A + 1040I_B = -5 \qquad \text{for loop B}$$

Use determinants to find I_A.

$$I_A = \frac{\begin{vmatrix} 10 & -220 \\ -5 & 1040 \end{vmatrix}}{\begin{vmatrix} 690 & -220 \\ -220 & 1040 \end{vmatrix}} = \frac{(10)(1040) - (-5)(-220)}{(690)(1040) - (-220)(-220)} = \frac{104000 - 1100}{717600 - 48400} = \frac{102900}{669200} = 13.9\text{ mA}$$

Solving for I_B yields

$$I_B = \frac{\begin{vmatrix} 690 & 10 \\ -220 & -5 \end{vmatrix}}{669200} = \frac{(690)(-5) - (-220)(10)}{669200} = \frac{-3450 - (-2200)}{669200} = -1.87\text{ mA}$$

The negative sign on I_B means that its assigned direction is opposite to the actual current.

Now find the actual branch currents. Since I_A is the only current through R_1, it is also the branch current I_1.

$$I_1 = I_A = \mathbf{13.9\text{ mA}}$$

Since I_B is the only current through R_3, it is also the branch current I_3.

$$I_3 = I_B = \mathbf{-1.87\text{ mA}}$$

The negative sign indicates opposite direction of that originally assigned to I_B.

As originally assigned, both loop currents I_A and I_B are through R_2 in opposite directions. The branch current I_2 is the difference between I_A and I_B.

$$I_2 = I_A - I_B = 13.9\text{ mA} - (-1.87\text{ mA}) = \mathbf{15.8\text{ mA}}$$

Keep in mind that once you know the branch currents, you can find the voltages by using Ohm's law. Notice that these results are the same as in Example 7 where the branch current method was used.

Related Problem Solve for the two loop currents using your calculator.

Use Multisim file E09-08 to verify the calculated results in this example and to verify your calculations for the related problem.

Circuits with More Than Two Loops

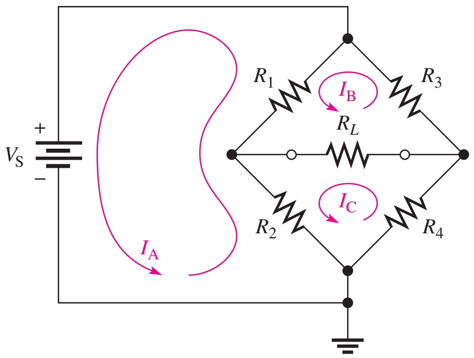
The loop current method can be systematically applied to circuits with any number of loops. Of course, the more loops there are, the more difficult is the solution, but calculators have greatly simplified solving simultaneous equations. Most circuits that you will encounter will not have more than three loops. Keep in mind that the loop currents are not the actual physical currents but are mathematical quantities assigned for analysis purposes.

A widely used circuit that you have already encountered is the Wheatstone bridge. The Wheatstone bridge was originally designed as a stand-alone measuring instrument but has largely been replaced with other instruments. However, the Wheatstone bridge circuit is incorporated in automated measuring instruments, and as explained previously, is widely used in the scale industry and in other measurement applications.

One method for solving the bridge parameters, which directly leads to finding the current in each arm of the bridge and the load current, is to write loop equations for the bridge. Figure 11 shows a Wheatstone bridge with three loops. Example 9 illustrates how to solve for all of the currents in the bridge.

▶ FIGURE 11

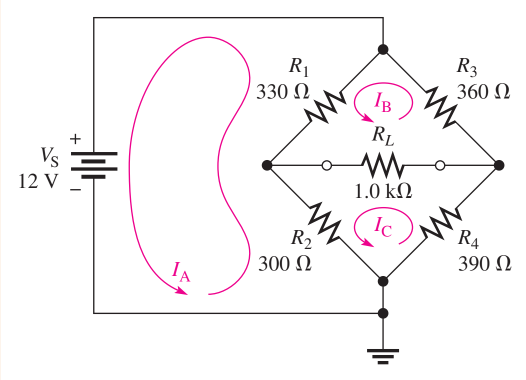
Wheatstone bridge with three loops.



EXAMPLE 9

For the Wheatstone bridge circuit in Figure 12, find the loop currents. Use the loop currents to solve for the current in each resistor (branch current).

▶ FIGURE 12



Solution Assign three counterclockwise loop currents (I_A, I_B, and I_C) as shown in Figure 9-12. Then write the loop equations. The equations for the loops are

$$\begin{aligned}
&\text{Loop A:} && -12 + 330(I_A - I_B) + 300(I_A - I_C) = 0\\
&\text{Loop B:} && 330(I_B - I_A) + 360I_B + 1000(I_B - I_C) = 0\\
&\text{Loop C:} && 300(I_C - I_A) + 1000(I_C - I_B) + 390I_C = 0
\end{aligned}$$

Rearrange the equations into standard form:

$$\begin{aligned}
&\text{Loop A:} && 630I_A - 330I_B - 300I_C = 12\text{ V}\\
&\text{Loop B:} && -330I_A + 1690I_B - 1000I_C = 0\\
&\text{Loop C:} && -300I_A - 1000I_B + 1690I_C = 0
\end{aligned}$$

You can solve these equations with substitution, but this is tedious with three unknowns. The determinant method or directly solving with your calculator are simpler ways. Units are not shown until the end of the problem.

Evaluating the characteristic determinant using the expansion method,

$$\left|\begin{array}{rrr|rr}
630 & -330 & -300 & 630 & -330\\
-330 & 1690 & -1000 & -330 & 1690\\
-300 & -1000 & 1690 & -300 & -1000
\end{array}\right.$$

$$\begin{aligned}
&= [(630)(1690)(1690) + (-330)(-1000)(-300) + (-300)(-330)(-1000)]\\
&\quad -[(-300)(1690)(-300) + (-1000)(-1000)(630) + (1690)(-330)(-330)]\\
&= 635202000
\end{aligned}$$

Solving for I_A:

$$\frac{\begin{vmatrix} 12 & -330 & -300 \\ 0 & 1690 & -1000 \\ 0 & -1000 & 1690 \end{vmatrix}}{635202000} = \frac{(12)(1690)(1690) - (12)(-1000)(-1000)}{635202000} = 0.0351\text{ A} = 35.1\text{ mA}$$

Solving for I_B:

$$\frac{\begin{vmatrix} 630 & 12 & -300 \\ -330 & 0 & -1000 \\ -300 & 0 & 1690 \end{vmatrix}}{635202000} = \frac{(12)(-1000)(-300) - (-330)(12)(1690)}{635202000} = 0.0162\text{ A} = 16.2\text{ mA}$$

Solving for I_C:

$$\frac{\begin{vmatrix} 630 & -330 & 12 \\ -330 & 1690 & 0 \\ -300 & -1000 & 0 \end{vmatrix}}{635202000} = \frac{(12)(-330)(-1000) - (-300)(1690)(12)}{635202000} = 0.0158\text{ A} = 15.8\text{ mA}$$

The current in R_1 is the difference between I_A and I_B:

$$I_1 = (I_A - I_B) = 35.1\text{ mA} - 16.2\text{ mA} = \mathbf{18.9\text{ mA}}$$

The current in R_2 is the difference between I_A and I_C:

$$I_2 = (I_A - I_C) = 35.1\text{ mA} - 15.8\text{ mA} = \mathbf{19.3\text{ mA}}$$

The current in R_3 is I_B:

$$I_3 = I_B = \mathbf{16.2\text{ mA}}$$

The current in R_4 is I_C:

$$I_4 = I_C = \mathbf{15.8\text{ mA}}$$

The current in R_L is the difference between I_B and I_C:

$$I_L = (I_B - I_C) = 16.2\text{ mA} - 15.8\text{ mA} = \mathbf{0.4\text{ mA}}$$

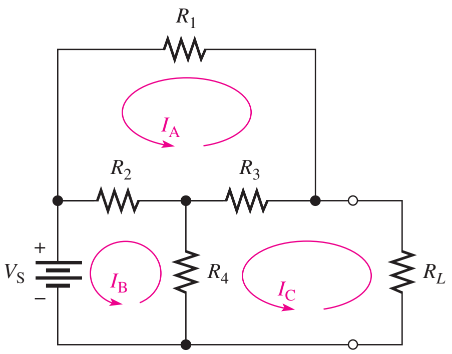
Related Problem Use a calculator to verify the loop currents in this example.

Use Multisim file E09-09 to verify the calculated results in this example and to confirm your calculations for the related problem.

Another useful three-loop circuit is the bridged-T circuit. While the circuit is primarily applied in ac filter circuits using reactive components, it is introduced here to illustrate the three-loop circuit solution. A loaded resistive bridged-T is shown in Figure 13.

Resistors will often be in kΩ (or even MΩ) so the coefficients for simultaneous equations will become quite large if they are shown explicitly in solving equations. To simplify entering and solving equations with kΩ, it is common practice to drop the kΩ in the equations and recognize that the unit for current is the mA if the voltage is volts. The following example of a bridged-T circuit illustrates this idea.

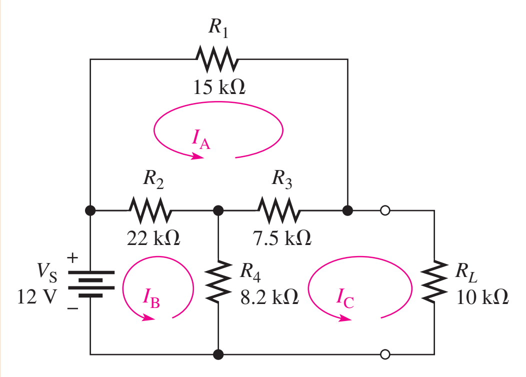
▶ FIGURE 13



EXAMPLE 10

Figure 14 shows a bridged-T circuit with three loops. Set up the standard form equations for the loop currents. Solve the equations with a calculator and find the current in each resistor.

▶ FIGURE 14



Solution Assign three counterclockwise loop currents(I_A, I_B, and I_C) as shown in Figure 14. Write the loop equations, but drop the k prefix from the resistances. Current will be in mA.

$$\begin{aligned}
&\text{Loop A:} \quad 22(I_A - I_B) + 15I_A + 7.5(I_A - I_C) = 0\\
&\text{Loop B:} \quad -12 + 22(I_B - I_A) + 8.2(I_B - I_C) = 0\\
&\text{Loop C:} \quad 8.2(I_C - I_B) + 7.5(I_C - I_A) + 10I_C = 0
\end{aligned}$$

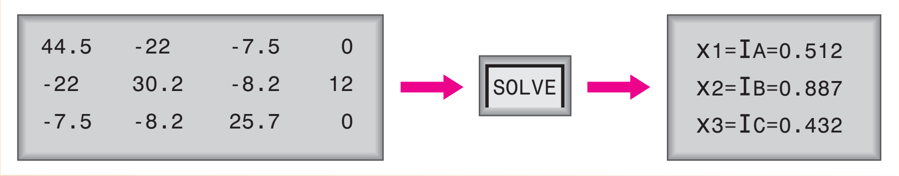
Rearrange the equations into standard form:

$$\begin{aligned}
&\text{Loop A:} \quad 44.5I_A - 22I_B - 7.5I_C = 0\\
&\text{Loop B:} \quad -22I_A + 30.2I_B - 8.2I_C = 12\\
&\text{Loop C:} \quad -7.5I_A - 8.2I_B + 25.7I_C = 0
\end{aligned}$$

Calculator Solution: A calculator solution requires entry of the number of equations (3), the coefficients, and the constants. The calculator SOLVE function produces the results as shown in Figure 15. Because the resistors were in kΩ, the unit for the loop currents is mA. Solve for the current in each resistor. The current in $R_1 = I_A$.

$$I_1 = \mathbf{0.512\ mA}$$

▶ FIGURE 15


```
44.5   -22    -7.5    0
-22    30.2   -8.2   12
-7.5   -8.2   25.7    0
```

```
X1=IA=0.512
X2=IB=0.887
X3=IC=0.432
```

The current in R_2 is the difference between I_A and I_B.

$$I_2 = (I_A - I_B) = 0.512\text{ mA} - 0.887\text{ mA} = \mathbf{-0.375\text{ mA}}$$

The negative sign indicates that current is the opposite direction to I_A; the positive side of the resistor is the right side.

The current in R_3 is $I_A - I_C$.

$$I_3 = 0.512\text{ mA} - 0.432\text{ mA} = \mathbf{0.08\text{ mA}}$$

The current in R_4 is $I_B - I_C$.

$$I_4 = I_B - I_C = 0.887\text{ mA} - 0.432\text{ mA} = \mathbf{0.455\text{ mA}}$$

The current in R_L is I_C.

$$I_L = \mathbf{0.432\text{ mA}}$$

Related Problem Find the voltage across each resistor.

Use Multisim files E09-10A and E09-10B to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 3 CHECKUP

1. Do the loop currents necessarily represent the actual currents in the branches?
2. When you solve for a current using the loop method and get a negative value, what does it mean?
3. What circuit law is used in the loop current method?

4 NODE VOLTAGE METHOD

Another method of analysis of multiple-loop circuits is called the node voltage method. It is based on finding the voltages at each node in the circuit using Kirchhoff's current law. Recall that a node is the junction of two or more components.

After completing this section, you should be able to

- **Use node analysis to find unknown quantities in a circuit**
  - Select the nodes at which the voltage is unknown and assign currents
  - Apply Kirchhoff's current law at each node
  - Develop and solve the node equations

The general steps for the node voltage method of circuit analysis are as follows:

Step 1: Determine the number of nodes.

Step 2: Select one node as a reference. All voltages will be relative to the reference node. Assign voltage designations to each node where the voltage is unknown.

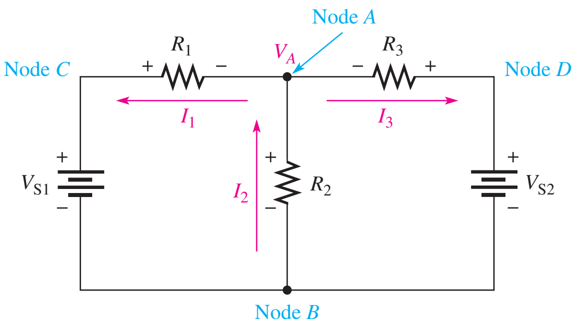
Step 3: Assign currents at each node where the voltage is unknown, except at the reference node. The directions are arbitrary.

Step 4: Apply Kirchhoff's current law to each node where currents are assigned.

Step 5: Express the current equations in terms of voltages, and solve the equations for the unknown node voltages using Ohm's law.

FIGURE 16

Circuit for node voltage analysis.



We will use Figure 16 to illustrate the general approach to node voltage analysis. First, establish the nodes. In this case, there are four nodes, as indicated in the figure. Second, let's use node *B* as the reference. Think of it as the circuit's reference ground. Node voltages *C* and *D* are already known to be the source voltages. The voltage at node *A* is the only unknown; it is designated as V_A. Third, arbitrarily assign the branch currents at node *A* as indicated in the figure. Fourth, the Kirchhoff current equation at node *A* is

$$I_1 - I_2 + I_3 = 0$$

Fifth, express the currents in terms of circuit voltages using Ohm's law.

$$I_1 = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_A}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{S3} - V_A}{R_3}$$

Substituting these terms into the current equation yields

$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0$$

The only unknown is V_A; so solve the single equation by combining and rearranging terms. Once you know the voltage, you can calculate all branch currents. Example 11 illustrates this method further.

EXAMPLE 11

Find the node voltage V_A in Figure 17 and determine the branch currents.

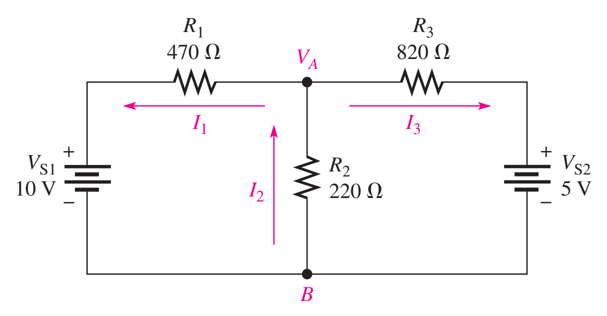


FIGURE 17

Solution The reference node is chosen at B. The unknown node voltage is V_A, as indicated in Figure 17. This is the only unknown voltage. Branch currents are assigned at node A as shown. The current equation is

$$I_1 - I_2 + I_3 = 0$$

Substitution for currents using Ohm's law gives the equation in terms of voltages.

$$\frac{10 - V_A}{470} - \frac{V_A}{220} + \frac{5 - V_A}{820} = 0$$

Rearranging the terms yields

$$\frac{10}{470} - \frac{V_A}{470} - \frac{V_A}{220} + \frac{5}{820} - \frac{V_A}{820} = 0$$

$$-\frac{V_A}{470} - \frac{V_A}{220} - \frac{V_A}{820} = -\frac{10}{470} - \frac{5}{820}$$

To solve for V_A, combine the terms on each side of the equation and find the common denominator.

$$\frac{1804V_A + 3854V_A + 1034V_A}{847880} = \frac{820 + 235}{38540}$$

$$\frac{6692V_A}{847880} = \frac{1055}{38540}$$

$$V_A = \frac{(1055)(847880)}{(6692)(38540)} = \mathbf{3.47\ V}$$

You can now determine the branch currents.

$$I_1 = \frac{10\ \text{V} - 3.47\ \text{V}}{470\ \Omega} = \mathbf{13.9\ mA}$$

$$I_2 = \frac{3.47\ \text{V}}{220\ \Omega} = \mathbf{15.8\ mA}$$

$$I_3 = \frac{5\ \text{V} - 3.47\ \text{V}}{820\ \Omega} = \mathbf{1.87\ mA}$$

These results agree with those for the same circuit in Example 7 and in Example 8 using the branch and loop current methods.

Related Problem Find V_A in Figure 17 if the 5 V source is reversed.

Use Multisim file E09-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

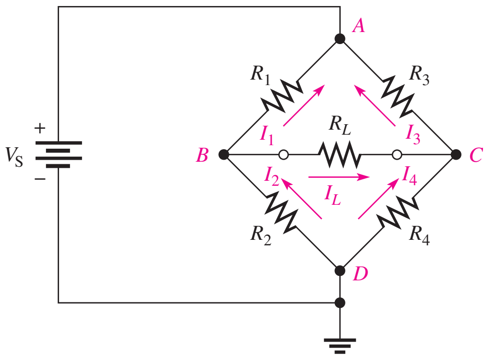
Example 11 illustrated an obvious advantage to the node method. The branch current method required three equations for the three unknown currents. The loop current method reduced the number of simultaneous equations but required the extra step of converting the fictitious loop currents to the actual currents in the resistors. The node method for the circuit in Figure 17 reduced the equations to one, in which all of the currents were written in terms of one unknown node voltage. The node voltage method also has the advantage of finding unknown voltages, which are easier to directly measure than current.

Node Voltage Method for a Wheatstone Bridge

The node voltage method can be applied to a Wheatstone bridge. The Wheatstone bridge is shown with nodes identified in Figure 18 with currents shown. Node D is usually

▶ FIGURE 18

Wheatstone bridge with node assignments.



selected as the reference node, and node A has the same potential as the source voltage. When setting up the equations for the two unknown node voltages (B and C), it is necessary to specify a current direction as described in the general steps. The direction of current in R_L is dependent on the bridge resistances; if the assigned direction is incorrect, it will show up as a negative current in the solution.

Kirchhoff's current law is then written for each of the unknown nodes. Each current is then expressed in terms of node voltages using Ohm's law as follows:

Node B:

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

Node C:

$$I_3 = I_L + I_4$$

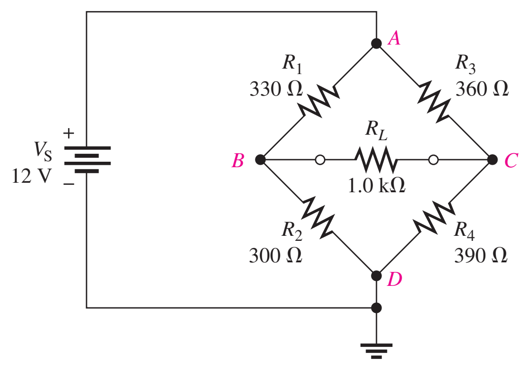
$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

The equations are put in standard form and can be solved with any of the methods you have learned. The following example illustrates this for the Wheatstone bridge that was solved by loop equations in Example 9.

EXAMPLE 12

For the Wheatstone bridge circuit in Figure 19, find the node voltages at node B and node C. Node D is the reference, and node A has the same voltage as the source.

▶ FIGURE 19



Solution Apply Kirchhoff's current law at node B and node C in terms of node voltages. To keep coefficients more manageable, all resistances are shown in kΩ; current will be in mA.

Node B:

$$I_1 + I_L = I_2$$
$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$
$$\frac{12 - V_B}{0.33\,\text{k}\Omega} + \frac{V_C - V_B}{1.0\,\text{k}\Omega} = \frac{V_B}{0.30\,\text{k}\Omega}$$

Node C:

$$I_3 = I_L + I_4$$
$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$
$$\frac{12\text{ V} - V_C}{0.36\,\text{k}\Omega} = \frac{V_C - V_B}{1.0\,\text{k}\Omega} + \frac{V_C}{0.39\,\text{k}\Omega}$$

Rearrange the equations for each node into standard form. Resistance units of kΩ will be omitted for simplicity.

Node B: Multiply each term in the expression for node B by $R_1R_2R_L$ and combine like terms to get the standard form.

$$R_2R_L(V_A - V_B) + R_1R_2(V_C - V_B) = R_1R_LV_B$$
$$(0.30)(1.0)(12 - V_B) + (0.33)(0.30)(V_C - V_B) = (0.33)(1.0)V_B$$
$$0.729V_B - 0.099V_C = 3.6$$

Node C: Multiply each term in the expression for node C by $R_3R_4R_L$ and combine like terms to get the standard form.

$$R_4R_L(V_A - V_C) = R_3R_4(V_C - V_B) + R_3R_LV_C$$
$$(0.39)(1.0)(12 - V_C) = (0.36)(0.39)(V_C - V_B) + (0.36)(1.0)V_C$$
$$0.1404V_B - 0.8904V_C = -4.68$$

You can solve the two simultaneous equations using substitution, determinants, or the calculator. Solving by determinants,

$$0.729V_B - 0.099V_C = 3.6$$
$$0.1404V_B - 0.8904V_C = -4.68$$

$$V_B = \frac{\begin{vmatrix} 3.6 & -0.099 \\ -4.68 & -0.8904 \end{vmatrix}}{\begin{vmatrix} 0.729 & -0.099 \\ 0.1404 & -0.8904 \end{vmatrix}} = \frac{(3.6)(-0.8904) - (-0.099)(-4.68)}{(0.729)(-0.8904) - (0.1404)(-0.099)} = \mathbf{5.78\ V}$$

$$V_C = \frac{\begin{vmatrix} 0.729 & 3.6 \\ 0.1404 & -4.68 \end{vmatrix}}{\begin{vmatrix} 0.729 & -0.099 \\ 0.1404 & -0.8904 \end{vmatrix}} = \frac{(0.729)(-4.68) - (0.1404)(3.6)}{(0.729)(-0.8904) - (0.1404)(-0.099)} = \mathbf{6.17\ V}$$

Related Problem Using Ohm's law, determine the current in each resistor. Compare your results to the loop current method in Example 9.

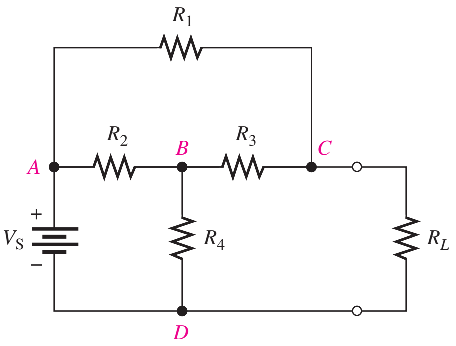
Use Multisim file E09-12 to verify the calculated results in this example and to confirm your calculations for the related problem.

Node Voltage Method for the Bridged-T Circuit

Applying the node voltage method to the bridged-T circuit also results in two equations with two unknowns. As in the case of the Wheatstone bridge, there are four nodes as shown in Figure 20. Node *D* is the reference and node *A* is the source voltage, so the two unknown voltages are at nodes *C* and *D*. The effect of a load resistor on the circuit is usually the most important question, so the voltage at node *C* is the focus. A calculator solution of the simultaneous equations is simplified for analyzing the effect of various loads because only the equation for node *C* is affected when the load changes. Example 13 illustrates this idea.

▶ **FIGURE 20**

The bridged-T circuit with node assignments.



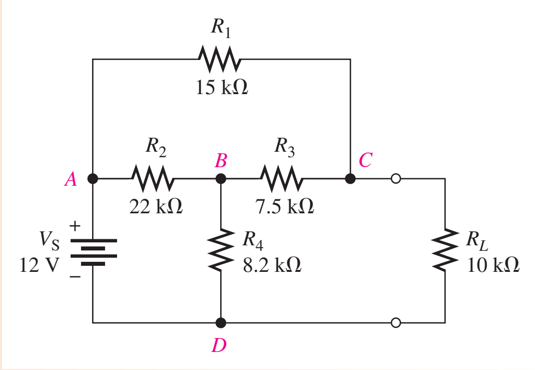
EXAMPLE 13

The circuit in Figure 21 is the same as in Example 10.

(a) Solve for the voltage across R_L using node analysis and a calculator.

(b) Find the effect on the load voltage when the load resistor is changed to 15 kΩ.

▶ **FIGURE 21**



Solution **(a)** Apply Kirchhoff's current law at nodes *B* and *C* in terms of node voltages.

Node *B*:

$$I_2 = I_3 + I_4$$

$$\frac{V_A - V_B}{R_2} = \frac{V_B - V_C}{R_3} + \frac{V_B}{R_4}$$

$$\frac{12 - V_B}{22\text{ k}\Omega} = \frac{V_B - V_C}{7.5\text{ k}\Omega} + \frac{V_B}{8.2\text{ k}\Omega}$$

Node C:

$$I_1 + I_3 = I_L$$

$$\frac{V_A - V_C}{R_1} + \frac{V_B - V_C}{R_3} = \frac{V_C}{R_L}$$

$$\frac{12\text{ V} - V_C}{15\text{ k}\Omega} + \frac{V_B - V_C}{7.5\text{ k}\Omega} = \frac{V_C}{10\text{ k}\Omega}$$

Rearrange the equations for each node into standard form. Resistance units of kΩ will be omitted for simplicity.

Node B: Multiply each term in the equation for node B by $R_2R_3R_4$ to cancel the denominator. Combine like terms to get the standard form.

$$R_3R_4(V_A - V_B) = R_2R_4(V_B - V_C) + R_2R_3V_B$$

$$(7.5)(8.2)(12 - V_B) = (22)(8.2)(V_B - V_C) + (22)(7.5)V_B$$

$$406.9V_B - 180.4V_C = 738$$

Node C: Multiply each term in the equation for node C by $R_1R_3R_L$ and combine like terms to get the standard form.

$$R_3R_L(V_A - V_C) + R_1R_L(V_B - V_C) = R_1R_3V_C$$

$$(7.5)(10)(12 - V_C) + (15)(10)(V_B - V_C) = (15)(7.5)V_C$$

$$150V_B - 337.5V_C = -900$$

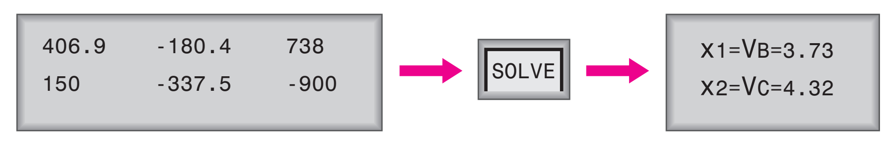
Calculator Solution: The two equations in standard form are

$$406.9V_B - 180.4V_C = 738$$

$$150V_B - 337.5V_C = -900$$

Enter the number of equations (2), the coefficients, and the constants into a calculator to solve for V_B and V_C as shown in Figure 22. As a check, notice that this voltage implies that the load current is 0.432 mA, which is in agreement with the result found by the loop current method in Example 10.

▶ FIGURE 22



(b) To calculate the load voltage with a 15 kΩ load resistor, notice that the equation for node B is unaffected. The node C equation is modified as follows:

$$\frac{12\text{ V} - V_C}{15\text{ k}\Omega} + \frac{V_B - V_C}{7.5\text{ k}\Omega} = \frac{V_C}{15\text{ k}\Omega}$$

$$(7.5)(15)(12 - V_C) + (15)(15)(V_B - V_C) = (15)(7.5)V_C$$

$$225V_B - 450V_C = -1350$$

Change the parameters for the node C equation and press solve. The result is

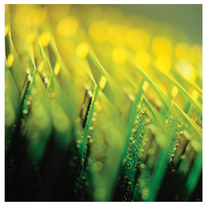
$$V_C = V_L = \mathbf{5.02\ V}$$

Related Problem For the 15 kΩ load, what is the voltage at node B?

Use Multisim files E09-13A and E09-13B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 5 CHECKUP

1. What circuit law is the basis for the node voltage method?
2. What is the reference node?



Application Activity

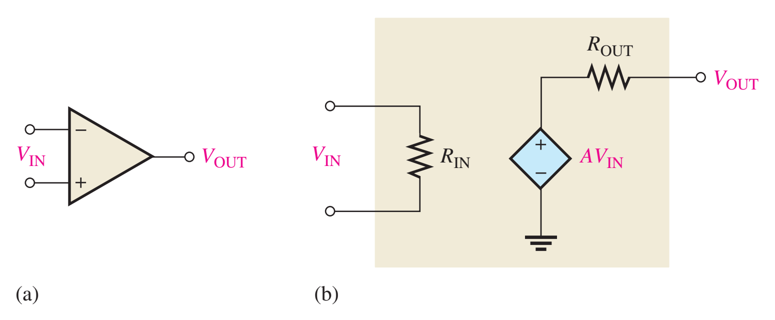
In this circuit application, you will see how a particular type of amplifier can be modeled and analyzed using the methods introduced in this chapter. The point is not to learn how amplifiers work because that is beyond the scope of this text and will be covered in a later course. The focus is on the application of circuit analysis methods to circuit models and not the amplifier itself. The amplifier is simply used as an example to illustrate how you can apply analysis methods to a practical circuit.

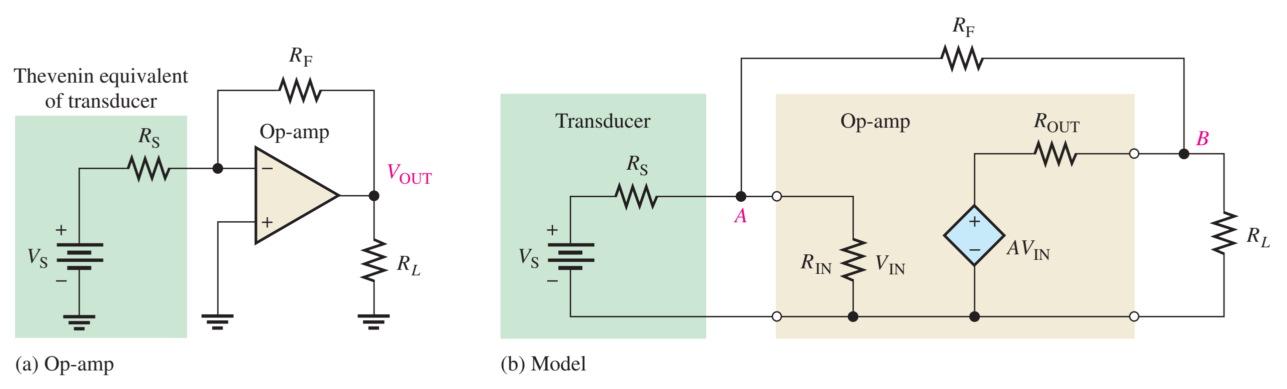
Operational amplifiers are integrated circuit devices that are widely used in analog applications for signal processing. An operational amplifier symbol is shown in Figure 23(a). The equivalent dependent source model is shown in part (b). The gain (*A*) of the dependent source can be positive or negative, depending on how it is configured.

Assume you need to calculate in detail the effect of an operational amplifier circuit on a transducer that serves as an input. Some transducers, such as pH meters, appear as a small source voltage with a high series resistance. The transducer that is shown here is modeled as a small Thevenin dc voltage source in series with a Thevenin resistance of 10 kΩ.

Practical amplifiers are created using an operational amplifier with external components. Figure 24(a) shows one type of amplifier configuration that includes the Thevenin resistance of the source along with two other external resistors. R_S represents the Thevenin source resistance. R_L is connected from the operational amplifier output to ground as a load, and R_F is connected from the output to the input as a feedback resistor. Feedback is used in most operational amplifier circuits and is simply a path from the output back to the input. It has many advantages as you will learn in a later course.

FIGURE 23

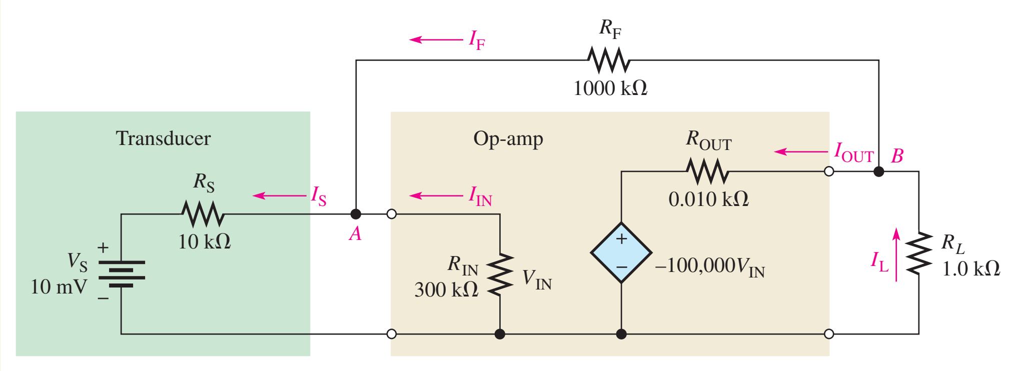




(a) Op-amp

(b) Model

FIGURE 24



▲ FIGURE 25

Figure 24(b) is the equivalent circuit model of the source, the op-amp, and the load that we will use for analysis purposes. The internal gain of the dependent source, shown with the letter *A* on the op-amp block, is negative because it is an inverting amplifier (the output has the opposite sign of the input). This internal gain is generally extremely high. Although it is a very large number, the actual gain of the circuit with the external components is much lower because it is controlled by the external components rather than the internal gain.

The specific values for the circuit in this application are given in Figure 25 along with assigned currents. All values are shown in kΩ to simplify entering the coefficients in the equations that you will use. Although op-amp circuits have excellent simple approximations for determining the output voltage, there are times you may want to know the exact output. You can apply your knowledge from this chapter to the circuit to find the exact output voltage.

The amplifier model in Figure 25 can be analyzed easiest with the node voltage method because there are only two nodes, *A* and *B*, that have unknown voltages. At node *A*, the voltage is designated and is the same as the input to the op-amp (V_{IN}). At node *B*, the voltage is designated and also represents the output (or load voltage), V_L. Current names and directions are assigned as shown.

Analysis

Apply Kirchhoff's current law at each of the unknown nodes to write the node equations.

Node *A*: $I_S = I_F + I_{IN}$

Node *B*: $I_{OUT} + I_F = I_L$

Next, apply Ohm's law and let $V_{IN} = V_A$. The internal source voltage for the op-amp is AV_{IN}, so this is written as AV_A to express the unknowns in terms of V_A and V_B.

$$\text{Node } A\text{:}\quad \frac{V_S - V_A}{R_S} = \frac{V_A}{R_{IN}} + \frac{V_A - V_B}{R_F}$$

$$\text{Node } B\text{:}\quad \frac{AV_A - V_B}{R_{OUT}} + \frac{V_A - V_B}{R_F} = \frac{V_B}{R_L}$$

Expressing the equations in standard form,

$$\text{Node } A\text{:}\ -\left(\frac{1}{R_S} + \frac{1}{R_{IN}} + \frac{1}{R_F}\right)V_A + \left(\frac{1}{R_F}\right)V_B = -\left(\frac{1}{R_S}\right)V_S$$

$$\text{Node } B\text{:}\ -\left(\frac{A}{R_{OUT}} + \frac{1}{R_F}\right)V_A + \left(\frac{1}{R_L} + \frac{1}{R_{OUT}} + \frac{1}{R_F}\right)V_B = 0$$

1. Substitute the values that were given in Figure 25 into the standard form equations. Solve the equations to find V_{IN} and V_L. (Resistance can be entered in kΩ.)
2. Calculate the input current, I_{IN}, and the current in the feedback resistor, I_F.

Review

3. Does the output voltage change if the load resistor R_L is doubled?
4. Does the output voltage change if the feedback resistor R_F is doubled?

SUMMARY

- Simultaneous equations can be solved by substitution, by determinants, or by a graphics calculator.
- The number of equations must be equal to the number of unknowns.
- Second-order determinants are evaluated by adding the signed cross-products.
- Third-order determinants are evaluated by the expansion method.
- The branch current method is based on Kirchhoff's voltage law and Kirchhoff's current law.

- The loop current method is based on Kirchhoff's voltage law.
- A loop current is not necessarily the actual current in a branch.
- The node voltage method is based on Kirchhoff's current law.

KEY TERMS

Branch One current path that connects two nodes.

Determinant The solution of a matrix consisting of an array of coefficients and constants for a set of simultaneous equations.

Loop A closed current path in a circuit.

Matrix An array of numbers.

Node The junction of two or more components.

Simultaneous equations A set of *n* equations containing *n* unknowns, where *n* is a number with a value of 2 or more.

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. An equation with two variables is called a second-order equation.
2. A set of three simultaneous equations is required to solve for two unknowns.
3. Three ways to solve simultaneous equations are substitution, determinants, and calculator.
4. A complete current path in a circuit is a loop.
5. A node and a branch are the same thing.
6. A loaded Wheatstone bridge contains three loops.
7. A loaded bridged-T circuit has only two loops.
8. In the loop current method, an equation is written for each loop.
9. Kirchhoff's current law is applied in the node voltage method.
10. A loaded Wheatstone bridge has four unknown nodes.

SELF-TEST

Answers are at the end of the chapter.

1. Assuming the voltage source values in Figure 6 are known, there is/are
   (a) 3 nonredundant loops
   (b) 1 unknown node
   (c) 2 nonredundant loops
   (d) 2 unknown nodes
   (e) both answers (b) and (c)
2. In assigning the direction of branch currents,
   (a) the directions are critical
   (b) they must all be in the same direction
   (c) they must all point into a node
   (d) the directions are not critical
3. The branch current method uses
   (a) Ohm's law and Kirchhoff's voltage law
   (b) Kirchhoff's voltage and current laws
   (c) the superposition theorem and Kirchhoff's current law
   (d) Thevenin's theorem and Kirchhoff's voltage law

4. A characteristic determinant for two simultaneous equations will have
   (a) 2 rows and 1 column **(b)** 1 row and 2 columns
   (c) 2 rows and 2 columns
5. The first row of a certain determinant has the numbers 2 and 4. The second row has the numbers 6 and 1. The value of this determinant is
   (a) 22 **(b)** 2 **(c)** −22 **(d)** 8
6. The expansion method for evaluating determinants is
   (a) good only for second-order determinants
   (b) good only for both second and third-order determinants
   (c) good for any determinant
   (d) easier than using a calculator
7. The loop current method is based on
   (a) Kirchhoff's current law **(b)** Ohm's law
   (c) the superposition theorem **(d)** Kirchhoff's voltage law
8. The node voltage method is based on
   (a) Kirchhoff's current law **(b)** Ohm's law
   (c) the superposition theorem **(d)** Kirchhoff's voltage law
9. In the node voltage method,
   (a) currents are assigned at each node
   (b) currents are assigned at the reference node
   (c) the current directions are arbitrary
   (d) currents are assigned only at the nodes where the voltage is unknown
   (e) both answers (c) and (d)
10. Generally, the node voltage method results in
   (a) more equations than the loop current method
   (b) fewer equations than the loop current method
   (c) the same number of equations as the loop current method

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 26.

1. If R_2 opens, the current through R_3
   (a) increases **(b)** decreases **(c)** stays the same
2. If the 6 V source shorts out, the voltage at point A with respect to ground
   (a) increases **(b)** decreases **(c)** stays the same
3. If R_2 becomes disconnected from ground, the voltage at point A with respect to ground
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 27.

4. If the current source fails open, the current through R_2
   (a) increases **(b)** decreases **(c)** stays the same
5. If R_2 opens, the current through R_3
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 30.

6. If R_1 opens, the magnitude of the voltage between the A and B terminals
   (a) increases **(b)** decreases **(c)** stays the same

7. If R_3 is replaced by a 10 Ω resistor, V_{AB}

(a) increases **(b)** decreases **(c)** stays the same

8. If point B shorts to the negative side of the source, V_{AB}

(a) increases **(b)** decreases **(c)** stays the same

9. If the negative side of the source is grounded, V_{AB}

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 32.

10. If a voltage source V_{S2} fails open, the voltage at A with respect to ground

(a) increases **(b)** decreases **(c)** stays the same

11. If a short develops from point A to ground, the current through R_3

(a) increases **(b)** decreases **(c)** stays the same

12. If R_2 opens, the voltage across R_3

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the entry.

SECTION 1 Simultaneous Equations in Circuit Analysis

1. Using the substitution method, solve the following set of equations for I_{R1} and I_{R2}.

$$100I_1 + 50I_2 = 30$$
$$75I_1 + 90I_2 = 15$$

2. Evaluate each determinant:

(a) $\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix}$ **(b)** $\begin{vmatrix} 9 & -1 \\ 0 & 5 \end{vmatrix}$ **(c)** $\begin{vmatrix} 12 & 15 \\ -2 & -1 \end{vmatrix}$ **(d)** $\begin{vmatrix} 100 & 50 \\ 30 & -20 \end{vmatrix}$

3. Using determinants, solve the following set of equations for both currents:

$$-I_1 + 2I_2 = 4$$
$$7I_1 + 3I_2 = 6$$

4. Evaluate each of the determinants:

(a) $\begin{vmatrix} 1 & 0 & -2 \\ 5 & 4 & 1 \\ 2 & 10 & 0 \end{vmatrix}$ **(b)** $\begin{vmatrix} 0.5 & 1 & -0.8 \\ 0.1 & 1.2 & 1.5 \\ -0.1 & -0.3 & 5 \end{vmatrix}$

5. Evaluate each of the determinants:

(a) $\begin{vmatrix} 25 & 0 & -20 \\ 10 & 12 & 5 \\ -8 & 30 & -16 \end{vmatrix}$ **(b)** $\begin{vmatrix} 1.08 & 1.75 & 0.55 \\ 0 & 2.12 & -0.98 \\ 1 & 3.49 & -1.05 \end{vmatrix}$

6. Find I_3 in Example 4.

7. Solve for I_1, I_2, I_3 in the following set of equations using determinants:

$$2I_1 - 6I_2 + 10I_3 = 9$$
$$3I_1 + 7I_2 - 8I_3 = 3$$
$$10I_1 + 5I_2 - 12I_3 = 0$$

***8.** Find V_1, V_2, V_3, and V_4 from the following set of equations using your calculator:

$$16V_1 + 10V_2 - 8V_3 - 3V_4 = 15$$
$$2V_1 + 0V_2 + 5V_3 + 2V_4 = 0$$
$$-7V_1 - 12V_2 + 0V_3 + 0V_4 = 9$$
$$-1V_1 + 20V_2 - 18V_3 + 0V_4 = 10$$

9. Solve the two simultaneous equations in Problem 1 using your calculator.

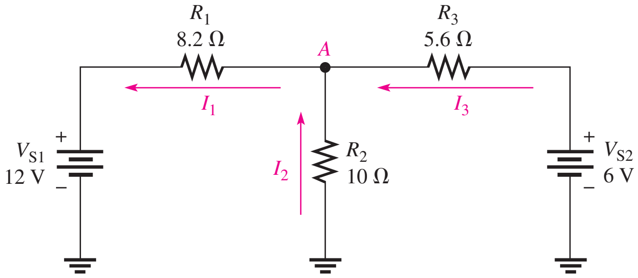
10. Solve the three simultaneous equations in Problem 7 using your calculator.

SECTION 2 Branch Current Method

11. Write the Kirchhoff current equation for the current assignment shown at node *A* in Figure 26.

12. Solve for each of the branch currents in Figure 26.

13. Find the voltage drop across each resistor in Figure 26 and indicate its actual polarity.



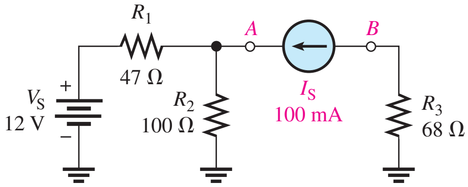
R_1
8.2 Ω
A
R_3
5.6 Ω
I_1
I_3
I_2
V_{S1}
12 V
R_2
10 Ω
V_{S2}
6 V
+
–

▲ FIGURE 26

***14.** Find the current through each resistor in Figure 27.

15. In Figure 27, determine the voltage across the current source (points *A* and *B*).

▶ FIGURE 27



R_1
47 Ω
A
B
V_S
12 V
R_2
100 Ω
I_S
100 mA
R_3
68 Ω
+
–

SECTION 3 Loop Current Method

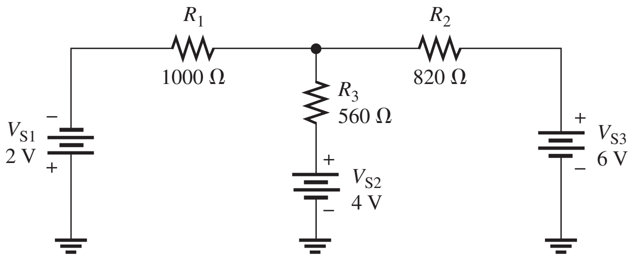
16. Write the characteristic determinant for the equations:

$$
\begin{aligned}
0.045I_A + 0.130I_B + 0.066I_C &= 0\\
0.177I_A + 0.0420I_B + 0.109I_C &= 12\\
0.078I_A + 0.196I_B + 0.029I_C &= 3.0
\end{aligned}
$$

17. Using the loop current method, find the loop currents in Figure 28.

18. Find the branch currents in Figure 28.

19. Determine the voltages and their proper polarities for each resistor in Figure 28.



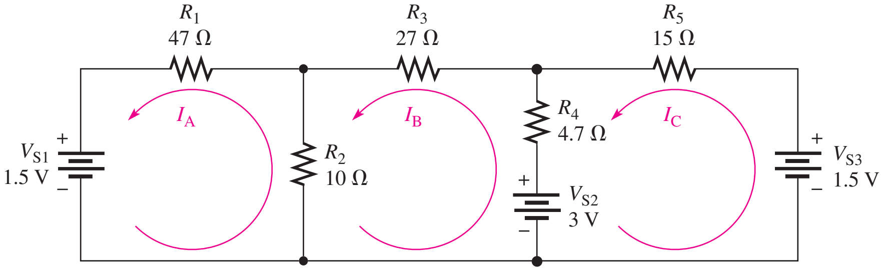
R_1
1000 Ω
R_2
820 Ω
R_3
560 Ω
V_{S1}
2 V
V_{S2}
4 V
V_{S3}
6 V
+
–

▲ FIGURE 28

20. Write the loop equations for the circuit in Figure 29.

21. Solve for the loop currents in Figure 29 using your calculator.

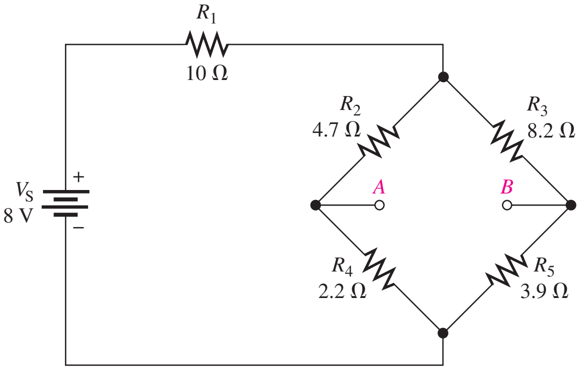
22. Find the current through each resistor in Figure 29.


R_1
47 Ω
R_3
27 Ω
R_5
15 Ω
I_A
I_B
I_C
R_4
4.7 Ω
V_{S1}
1.5 V
+
–
R_2
10 Ω
+
V_{S2}
3 V
–
+
V_{S3}
1.5 V
–

▲ FIGURE 29

23. Determine the voltage across the open bridge terminals, A and B, in Figure 30.

24. When a 10 Ω resistor is connected from terminal A to terminal B in Figure 30, what is the current through it?

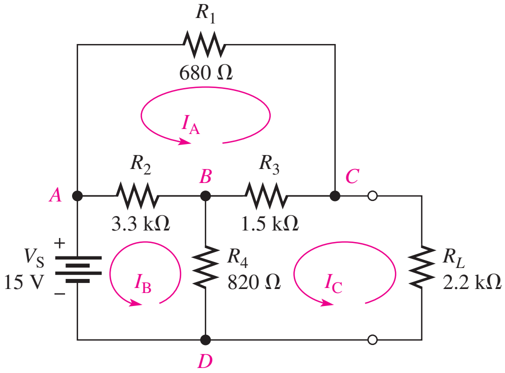

R_1
10 Ω
R_2
4.7 Ω
R_3
8.2 Ω
V_S
8 V
+
–
A
B
R_4
2.2 Ω
R_5
3.9 Ω

▲ FIGURE 30

25. Write the loop equations in standard form for the bridged-T circuit in Figure 31.

26. Find the current through R_L in Figure 31.

***27.** What is the voltage across R_3 in Figure 31?

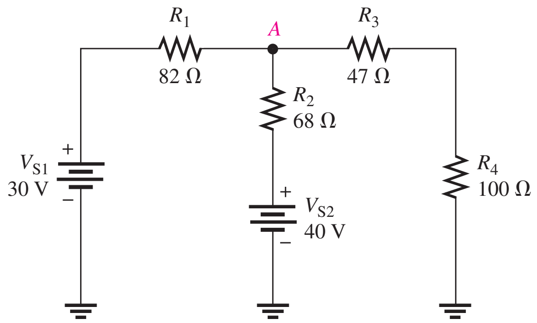

R_1
680 Ω
I_A
R_2
B
R_3
C
A
3.3 kΩ
1.5 kΩ
V_S
15 V
+
–
I_B
R_4
820 Ω
I_C
R_L
2.2 kΩ
D

▲ FIGURE 31

SECTION 4 Node Voltage Method

28. In Figure 9-32, use the node voltage method to find the voltage at point *A* with respect to ground.

29. What are the branch current values in Figure 32? Show the actual direction of current in each branch.

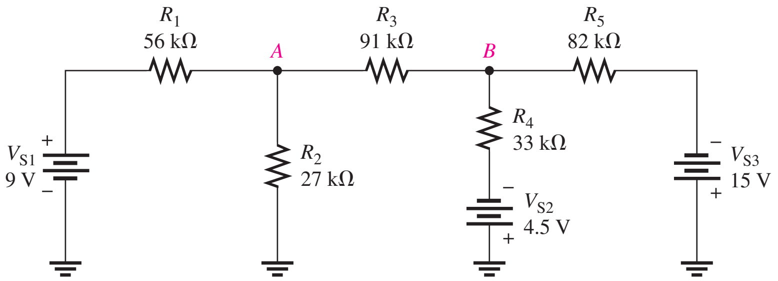


R_1
82 Ω
A
R_3
47 Ω
R_2
68 Ω
V_{S1}
30 V
+
−
+
V_{S2}
40 V
−
R_4
100 Ω

▲ FIGURE 32

30. Write the node voltage equations for Figure 29. Use your calculator to find the node voltages.

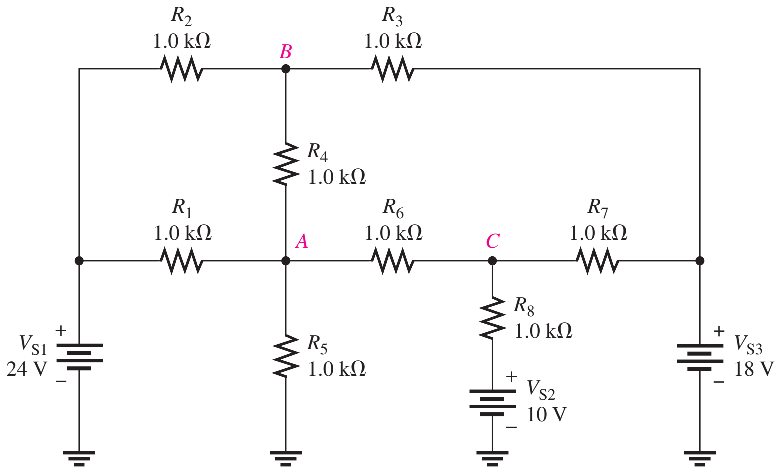
31. Use node analysis to determine the voltage at points *A* and *B* with respect to ground in Figure 33.



R_1
56 kΩ
A
R_3
91 kΩ
B
R_5
82 kΩ
R_4
33 kΩ
V_{S1}
9 V
+
−
R_2
27 kΩ
−
V_{S2}
4.5 V
+
−
V_{S3}
15 V
+

▲ FIGURE 33

***32.** Find the voltage at points *A*, *B*, and *C* in Figure 34.

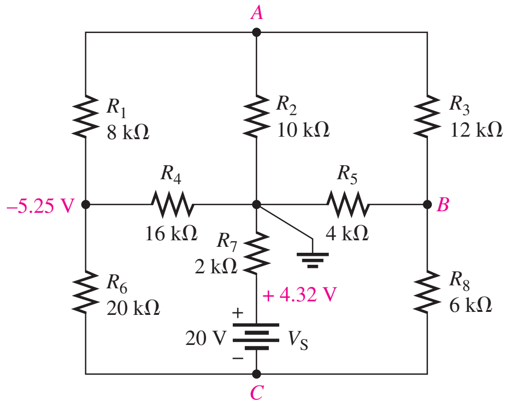


R_2
1.0 kΩ
B
R_3
1.0 kΩ
R_4
1.0 kΩ
R_1
1.0 kΩ
A
R_6
1.0 kΩ
C
R_7
1.0 kΩ
R_8
1.0 kΩ
V_{S1}
24 V
+
−
R_5
1.0 kΩ
+
V_{S2}
10 V
−
+
V_{S3}
18 V
−

▲ FIGURE 34

***33.** Use node analysis, loop analysis, or any other procedure to find all currents and the voltages at each unknown node in Figure 35.

▶ FIGURE 35



Multisim Troubleshooting and Analysis

These problems require Multisim.

34. Open file P09-34 and measure the current through each resistor.

35. Open file P09-35 and measure the current through each resistor.

36. Open file P09-36 and measure the voltages with respect to ground at nodes *A* and *B*.

37. Open file P09-37. Determine if there is a fault and, if so, specify the fault.

38. Open file P09-38 and measure the voltages with respect to ground at output terminals 1 and 2.

39. Open file P09-39 and determine what the fault is.

40. Open file P09-40 and determine what the fault is.

41. Open file P09-41 and determine what the fault is.

ANSWERS

SECTION CHECKUPS

SECTION 1 Simultaneous Equations in Circuit Analysis

1. **(a)** 4 **(b)** 0.415 **(c)** −98

2. $\begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix}$

3. $-0.286\text{ A} = -286\text{ mA}$

4. $I_1 = -.038893513289$
$I_2 = .084110232475$
$I_3 = .041925798204$
$I_4 = -.067156192401$

5. $I_1 = -.056363148617$
$I_2 = .07218287729$
$I_3 = .065684612774$
$I_4 = -.041112571034$

SECTION 2 Branch Current Method

1. Kirchhoff's voltage law and Kirchhoff's current law are used in the branch current method.

2. False, but write the equations so that they are consistent with your assigned directions.

3. A loop is a closed path within a circuit.

4. A node is a junction of two or more components.

SECTION 3 Loop Current Method

1. No, loop currents are not necessarily the same as branch currents.
2. A negative value means the direction should be reversed.
3. Kirchhoff's voltage law is used in loop analysis.

SECTION 4 Node Voltage Method

1. Kirchhoff's current law is the basis for node analysis.
2. A reference node is the junction to which all circuit voltages are referenced.

RELATED PROBLEMS FOR EXAMPLES

1 $20x_1 - 11x_2 = -15$
$18x_1 + 25x_2 = 10$

2 $10V_1 - 21V_2 - 50V_3 = -15$
$18V_1 + 25V_2 - 12V_3 = 10$
$18V_1 - 25V_2 + 12V_3 = 9$

3 3.71 A

4 −298 mA

5 $X_1 = -1.76923076923$; $X_2 = -18.5384615385$; $X_3 = -34.4615384615$

6 Same results as answer for 5

7 $I_1 = 17.2$ mA; $I_2 = 8.74$ mA; $I_3 = -8.44$ mA

8 $I_1 = X_1 = .013897190675$ ($\approx$13.9 mA); $I_2 = X_2 = -.001867901972$ (≈ -1.87 mA)

9 They are correct.

10 $V_1 = 7.68$ V, $V_2 = 8.25$ V, $V_3 = 0.6$ V, $V_4 = 3.73$ V, $V_L = 4.32$ V

11 1.92 V

12 $I_1 = 18.8$ mA, $I_2 = 19.3$ mA, $I_3 = 16.2$ mA, $I_4 = 15.8$ mA, $I_L = 0.4$ mA

13 $V_B = 4.04$ V

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F
6. T **7.** F **8.** T **9.** T **10.** F

SELF-TEST

1. (e) **2.** (d) **3.** (b) **4.** (c) **5.** (c)
6. (b) **7.** (d) **8.** (a) **9.** (e) **10.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (b)
7. (a) **8.** (a) **9.** (c) **10.** (b) **11.** (b) **12.** (b)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. $I_1 = 371\text{ mA}; I_2 = -143\text{ mA}$

3. $I_1 = 0\text{ A}, I_2 = 2\text{ A}$

5. **(a)** $-16{,}470$ **(b)** -1.59

7. $I_1 = 1.24\text{ A}, I_2 = 2.05\text{ A}, I_3 = 1.89\text{ A}$

9. $\text{X1} = .371428571429\ (I_1 = 371\text{ mA})$
$\text{X2} = -.142857142857\ (I_2 = -143\text{ mA})$

11. $I_1 - I_2 - I_3 = 0$

13. $V_1 = 5.66\text{ V}, V_2 = 6.33\text{ V}, V_3 = 325\text{ mV}$

15. -1.84 V

17. $I_1 = -5.11\text{ mA}, I_2 = -3.52\text{ mA}$

19. $V_1 = 5.11\text{ V}, V_3 = 890\text{ mV}, V_2 = 2.89\text{ V}$

21. $I_1 = 15.6\text{ mA}, I_2 = -61.3\text{ mA}, I_3 = 61.5\text{ mA}$

23. -11.2 mV

25. Note: all *Rs* (coefficients) are in kΩ.
Loop A: $5.48I_\text{A} - 3.3I_\text{B} - 1.5I_\text{C} = 0$
Loop B: $-3.3I_\text{A} + 4.12I_\text{B} - 0.82I_\text{C} = 15$
Loop C: $-1.5I_\text{A} - 0.82I_\text{B} + 4.52I_\text{C} = 0$

27. 4.76 V

29. $I_1 = 20.6\text{ mA}, I_3 = 193\text{ mA}, I_2 = -172\text{ mA}$

31. $V_A = 1.5\text{ V}, V_B = -5.65\text{ V}$

33. $I_1 = 193\ \mu\text{A}, I_2 = 370\ \mu\text{A}, I_3 = 179\ \mu\text{A}$,
$I_4 = 328\ \mu\text{A}, I_5 = 1.46\text{ mA}, I_6 = 522\ \mu\text{A}$,
$I_7 = 2.16\text{ mA}, I_8 = 1.64\text{ mA}, V_A = -3.70\text{ V}$,
$V_B = -5.85\text{ V}, V_C = -15.7\text{ V}$

35. No fault

37. R_4 open

39. Lower fuse open

41. R_4 open

MAGNETISM AND ELECTROMAGNETISM

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MAGNETISM AND ELECTROMAGNETISM

CHAPTER OUTLINE

1 The Magnetic Field
2 Electromagnetism
3 Electromagnetic Devices
4 Magnetic Hysteresis
5 Electromagnetic Induction
6 The DC Generator
7 The DC Motor
Application Activity

CHAPTER OBJECTIVES

- Explain the principles of a magnetic field
- Explain the principles of electromagnetism
- Describe the principle of operation for several types of electromagnetic devices
- Explain magnetic hysteresis
- Discuss the principle of electromagnetic induction
- Explain how a dc generator works
- Explain how a dc motor works

KEY TERMS

- Magnetic field
- Lines of force
- Magnetic flux
- Weber (Wb)
- Tesla
- Hall effect
- Electromagnetism
- Electromagnetic field
- Permeability
- Reluctance
- Magnetomotive force (mmf)
- Ampere-turn (At)
- Solenoid
- Relay
- Speaker
- Hysteresis
- Retentivity
- Induced voltage (v_{ind})
- Electromagnetic induction
- Induced current (i_{ind})
- Faraday's law
- Lenz's law

APPLICATION ACTIVITY PREVIEW

In this application activity, you will learn how electromagnetic relays can be used in security alarm systems, and you will develop a procedure to check out a basic alarm system.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

This chapter introduces the concepts of magnetism and electromagnetism. The operation of devices such as the relay, the solenoid, and the speaker is based partially on magnetic or electromagnetic principles. Electromagnetic induction is important in an electrical component called an inductor or coil.

Two types of magnets are the permanent magnet and the electromagnet. The permanent magnet maintains a constant magnetic field between its two poles with no external excitation. The electromagnet produces a magnetic field only when there is current through it. The electromagnet is basically a coil of wire wound around a magnetic core material.

The chapter concludes with an introduction to dc generators and dc motors.

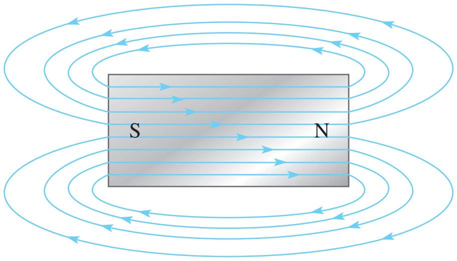
1 THE MAGNETIC FIELD

A permanent magnet has a magnetic field surrounding it. **A magnetic field** is visualized by **lines of force** that radiate from the north pole (N) to the south pole (S) and back to the north pole through the magnetic material.

After completing this section, you should be able to

- **Explain the principles of a magnetic field**
  - Define *magnetic flux*
  - Define *magnetic flux density*
  - Discuss how materials are magnetized
  - Explain how a magnetic switch works

A permanent magnet, such as the bar magnet shown in Figure 1, has a magnetic field surrounding it. All magnetic fields have their origin in moving charge, which in solid materials is caused by moving electrons. In certain materials such as iron, atoms can be aligned so that the electron motion is reinforced, creating a measurable field that extends in three dimensions. Even some electrical insulators can exhibit this behavior; ceramics make excellent magnets but are electrical insulators.



Blue lines represent only a few of the many magnetic lines of force in the magnetic field.

FIGURE 1

Magnetic lines of force around a bar magnet.

To illustrate magnetic fields, "lines of force" or flux lines are used to represent an unseen field. Flux lines are widely used as a description of a field, showing the strength and direction of the field. The flux lines never cross. When lines are close together, the field is more intense; when they are farther apart, the field is weaker. The flux lines are always drawn from the north pole (N) to the south pole (S) of a magnet. Even in small magnets, the number of lines based on the mathematical definition is extremely large, so for clarity, only a few lines are generally shown in drawings of magnetic fields.

When unlike poles of two permanent magnets are placed close together, their magnetic fields produce an attractive force, as indicated in Figure 2(a). When two like poles are brought close together, they repel each other, as shown in part (b).

When a nonmagnetic material such as paper, glass, wood, or plastic is placed in a magnetic field, the lines of force are unaltered, as shown in Figure 3(a). However, when a magnetic material such as iron is placed in the magnetic field, the lines of force tend to change course and pass through the iron rather than through the surrounding air. They do so because the iron provides a magnetic path that is more easily established than that of air.

HISTORY NOTE

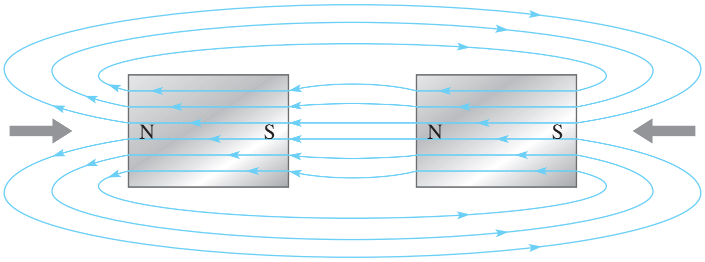
Wilhelm Eduard Weber 1804–1891

Weber was a German physicist who worked closely with Gauss, whose biography appears later. Independently, he established a system of absolute electrical units and also performed work that was crucial to the later development of the electromagnetic theory of light. The unit of magnetic flux is named in his honor. (Photo credit: Courtesy of the Smithsonian Institution. Photo No. 52,604.)

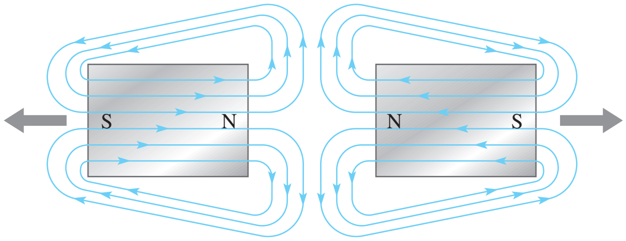
This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

SAFETY NOTE

Many strong magnets are very brittle and can shatter on impact. Eye protection should always be worn when you work with strong magnets. Strong magnets are not toys and should not be given to children. People with pacemakers should stay away from strong magnets.



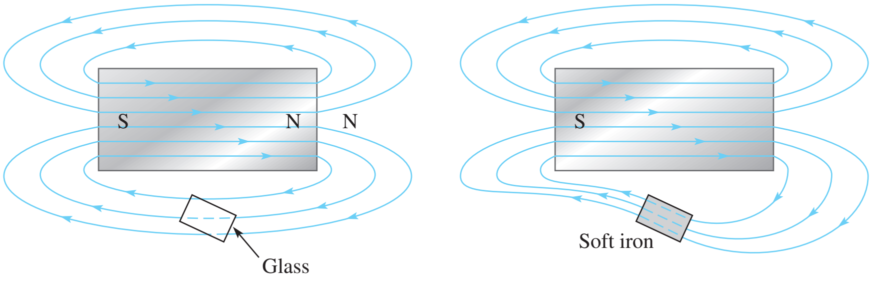
(a) Unlike poles attract.



(b) Like poles repel.

▲ FIGURE 2

Magnetic attraction and repulsion.



(a) (b)

▲ FIGURE 3

Effect of (a) nonmagnetic and (b) magnetic materials on a magnetic field.

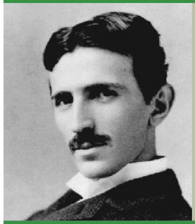
Figure 3(b) illustrates this principle. The fact that magnetic lines of force follow a path through iron or other materials is a consideration in the design of shields that prevent stray magnetic fields from affecting sensitive circuits.

Magnetic Flux (ϕ)

The group of force lines going from the north pole to the south pole of a magnet is called the **magnetic flux**, symbolized by ϕ (the Greek letter phi). A stronger field is represented by more lines of force. Several factors determine the strength of a magnet, including the material and physical geometry as well as the distance from the magnet. Magnetic field lines tend to be more concentrated at the poles.

The unit of magnetic flux is the **weber (Wb)**. One weber equals 10^8 lines. The weber is a very large unit; thus, in most practical situations, the microweber (μWb) is used. One microweber equals 100 lines of magnetic flux.

HISTORY NOTE



Nikola Tesla
1856–1943

Tesla was born in Croatia (then Austria-Hungary). He was an electrical engineer who invented the ac induction motor, polyphase ac systems, the Tesla coil transformer, wireless communications, and fluorescent lights. He worked for Edison when he first came to the U.S. in 1884 and later for Westinghouse. The SI unit of magnetic flux density is named in his honor. (Photo credit: Courtesy of the Nikola Tesla Museum, Belgrade, Yugoslavia.)

Magnetic Flux Density (*B*)

The **magnetic flux density** is the amount of flux per unit area perpendicular to the magnetic field. Its symbol is *B*, and its SI unit is the **tesla (T)**. One tesla equals one weber per square meter (Wb/m^2). The following formula expresses the flux density:

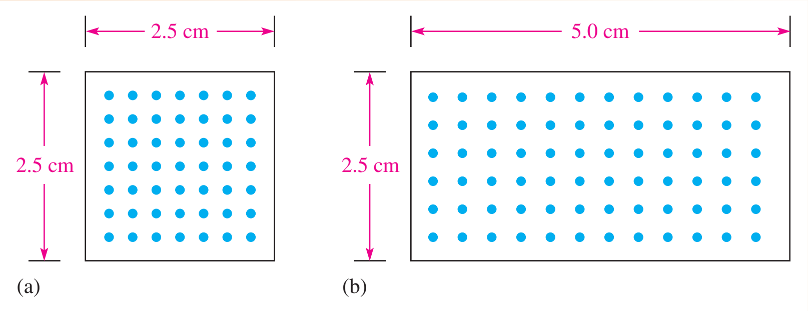
$$B = \frac{\phi}{A} \qquad \text{Equation 1}$$

where ϕ is the flux (Wb) and A is the cross-sectional area in square meters (m^2) of the magnetic field.

EXAMPLE 1

Compare the flux and the flux density in the two magnetic cores shown in Figure 4. The diagram represents the cross section of a magnetized material. Assume that each dot represents 100 lines or 1 μWb.

FIGURE 4



Solution The flux is simply the number of lines. In Figure 4(a) there are 49 dots. Each represents 1 μWb, so the flux is 49 μWb. In Figure 4(b) there are 72 dots, so the flux is 72 μWb.

To calculate the flux density in Figure 4(a), first calculate the area in m^2.

$$A = l \times w = 0.025\text{ m} \times 0.025\text{ m} = 6.25 \times 10^{-4}\text{ m}^2$$

For Figure 4(b) the area is

$$A = l \times w = 0.025\text{ m} \times 0.050\text{ m} = 1.25 \times 10^{-3}\text{ m}^2$$

Use Equation 1 to calculate the flux density. For Figure 4(a) the flux density is

$$B = \frac{\phi}{A} = \frac{49\ \mu\text{Wb}}{6.25 \times 10^{-4}\text{ m}^2} = 78.4 \times 10^{-3}\text{ Wb/m}^2 = 78.4 \times 10^{-3}\text{ T}$$

For Figure 4(b) the flux density is

$$B = \frac{\phi}{A} = \frac{72\ \mu\text{Wb}}{1.25 \times 10^{-3}\text{ m}^2} = 57.6 \times 10^{-3}\text{ Wb/m}^2 = 57.6 \times 10^{-3}\text{ T}$$

The data in Table 1 compares the two cores. Note that the core with the largest flux does not necessarily have the highest flux density.

TABLE 1

	FLUX (Wb)	AREA (m^2)	FLUX DENSITY (T)
Figure 4(a)	49 μWb	$6.25 \times 10^{-4}\text{m}^2$	$78.4 \times 10^{-3}\text{T}$
Figure 4(b)	72 μWb	$1.25 \times 10^{-3}\text{m}^2$	$57.6 \times 10^{-3}\text{T}$

*Related Problem** What happens to the flux density if the same flux shown in Figure 4(a) is in a core that is 5.0 cm × 5.0 cm?

---

*Answers are at the end of the chapter.

EXAMPLE 2

If the flux density in a certain magnetic material is 0.23 T and the area of the material is 0.38 in.^{2}, what is the flux through the material?

Solution First, 0.38 in.^{2} must be converted to square meters. 39.37 in. = 1 m; therefore,

$$A = 0.38\text{ in.}^2[1\text{ m}^2/(39.37\text{ in.})^2] = 245 \times 10^{-6}\text{ m}^2$$

The flux through the material is

$$\phi = BA = (0.23\text{ T})(245 \times 10^{-6}\text{ m}^2) = \mathbf{56.4\ \mu Wb}$$

Related Problem Calculate B if $A = 0.05$ in.^{2} and $\phi = 1000\ \mu$Wb.

HISTORY NOTE

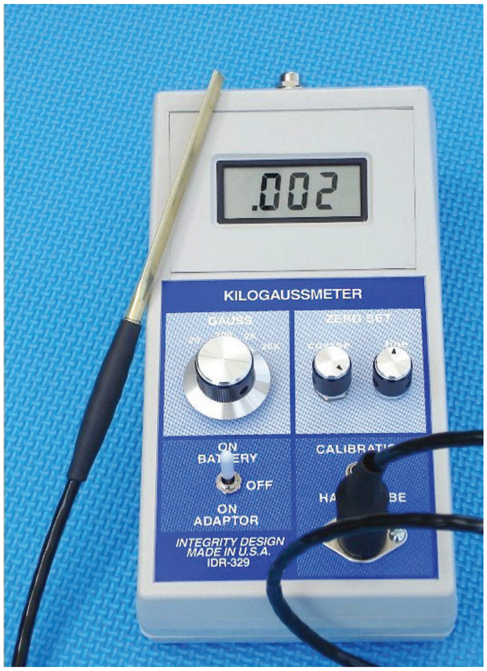
Karl Friedrich Gauss 1777–1855

Gauss, a German mathematician, disproved many 18th century mathematical theories. Later, he worked closely with Weber on a worldwide system of stations for systematic observations of terrestrial magnetism. The most important result of their work in electromagnetism was the later development of telegraphy by others. The CGS unit of magnetic flux density is named in his honor. (Credit: Illustration by Steven S. Nau.)

The Gauss Although the tesla (T) is the SI unit for flux density, another unit called the **gauss**, **G**, from the CGS (centimeter-gram-second) system, is used (10^4 G = 1 T). In fact, the instrument used to measure flux density is the gaussmeter. A typical gaussmeter is shown in Figure 5. This particular gaussmeter is a portable unit with four ranges that can measure magnetic fields as small as the earth's field (about 0.5 G, but changes depending on location) to strong fields such as in a MRI unit (about 10,000 G). The unit *gauss* is still in widespread use, so you should be familiar with it as well as the *tesla*.

▶ **FIGURE 5**

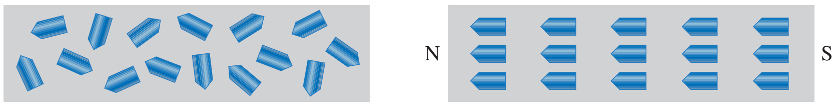
A dc gaussmeter. (Integrity Model IDR-329 distributed by Less EMF Inc.)



How Materials Become Magnetized

Ferromagnetic materials such as iron, nickel, and cobalt become magnetized when placed in the magnetic field of a magnet. We have all seen a permanent magnet pick up things like paper clips, nails, and iron filings. In these cases, the object becomes magnetized (that is, it actually becomes a magnet itself) under the influence of the permanent magnetic field and becomes attracted to the magnet. When removed from the magnetic field, the object tends to lose its magnetism.

Ferromagnetic materials have minute magnetic domains created within their atomic structure. These domains can be viewed as very small bar magnets with north and south



(a) The magnetic domains (N ◀ S) are randomly oriented in the unmagnetized material.

(b) The magnetic domains become aligned when the material is magnetized.

FIGURE 6

Magnetic domains in (a) an unmagnetized and in (b) a magnetized material.

poles. When the material is not exposed to an external magnetic field, the magnetic domains are randomly oriented, as shown in Figure 6(a). When the material is placed in a magnetic field, the domains align themselves as shown in part (b). Thus, the object itself effectively becomes a magnet.

Effects of the Magnetic Material The magnetic material affects not only the magnetic flux density at the poles but also how the magnetic flux density falls off as distance from the poles increases. The physical size also affects the flux density. For example, two disk magnets (both made from sintered Alnico) have very similar densities near the pole, but the larger magnet has much higher flux density, as you move away from the pole, as shown in Figure 7. Notice that the flux density falls off rapidly as you move away from the pole. This type of plot can illustrate if a given magnet is effective for a specific application that depends on the distance in which the magnet must work.

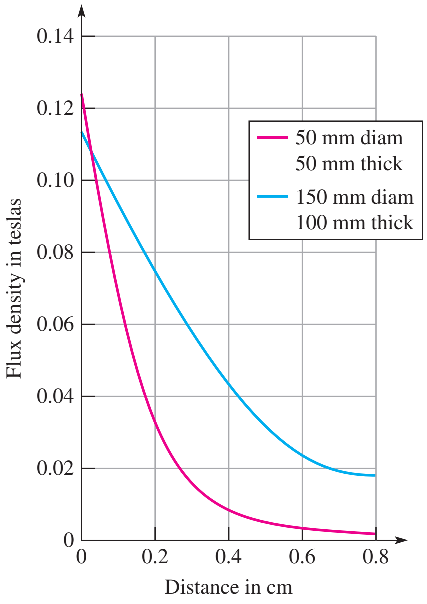


FIGURE 7

Example of magnetic flux density for two disk magnets as a function of distance. The blue curve represents the larger magnet.

The type of material is an important parameter for the actual flux density of magnets. Table 2 lists the flux densities of typical magnetic fields in teslas. For permanent magnets, the numbers given are based on the flux density of the field that is typical if measured close to the pole. As previously discussed, these values can drop significantly as distance from the poles is increased. The strongest field most people will ever experience is about 1 T (10,000 G) if they have an MRI exam. The strongest commercially available permanent magnets are neodymium-iron-boron composites (NdFeB). To find the flux density in gauss, multiply the values in teslas by 10^4 (10,000).

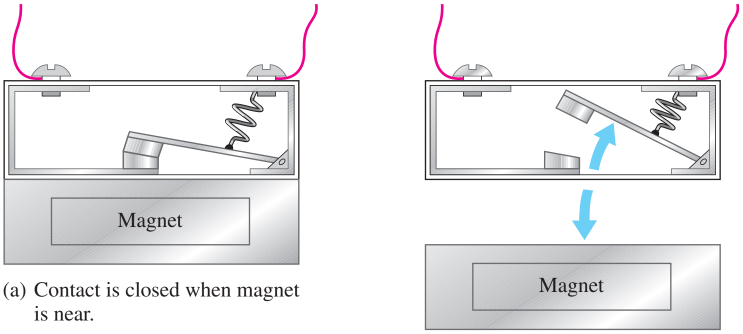
▶ TABLE 2

Flux density of various magnetic fields.

SOURCE	TYPICAL FLUX DENSITY IN TESLAS (T)
Earth's magnetic field	4×10^{-5} (varies with location)
Small "refrigerator" magnets	0.08 to 0.1
Ceramic magnets	0.2 to 0.3
Alnico 5 reed switch magnet	0.1 to 0.2
Neodymium magnets	0.3 to 0.52
Magnetic resonance imaging (MRI)	1
The strongest steady magnetic field ever achieved in a laboratory	45

Applications

Permanent magnets are widely used in brushless motors (discussed in Section 7), magnetic separators, speakers, microphones, automobiles, and magnetic resonance-imaging devices. They are also commonly used in switches, such as the normally closed switch illustrated in Figure 8. When the magnet is near the switch mechanism, as in Figure 8(a), the switch is closed. When the magnet is moved away, as in part (b), the spring pulls the arm open. Magnetic switches are widely used in security systems.



(a) Contact is closed when magnet is near.

(b) Contact opens when magnet is moved away.

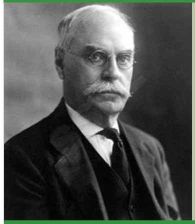
▲ FIGURE 8

Operation of a magnetic switch.

Another important application of permanent magnets is in sensors that take advantage of an effect known as the Hall effect. The **Hall effect** is the occurrence of a small voltage (a few μV) that is generated on opposite sides of a thin current-carrying conductor or semiconductor (the Hall element) in a magnetic field. A voltage called the Hall voltage appears across the Hall element, as illustrated in Figure 9. The Hall voltage is due to the forces exerted on the electrons as they traverse the magnetic field, causing an excess of charge on one side of the Hall element. Although the effect was first noticed in a conductor, it is more pronounced in semiconductors, which are normally used in Hall-effect sensors. Notice that the magnetic field, the electric current, and the Hall voltage are all at right angles to each other. This voltage is amplified and can be used to detect the presence of the magnetic field. The detection of a magnetic field is useful in sensor applications.

Hall-effect sensors are widely used because they are small, inexpensive, and have no moving parts. In addition, they are noncontacting sensors, so they can last for billions of repeated operations, a clear advantage over contacting sensors that can wear out. Hall-effect sensors can detect the nearby presence of a magnet by sensing its magnetic

HISTORY NOTE



Edwin Herbert Hall (1855–1938)

The Hall effect was discovered by Hall in 1879 while he worked on his doctoral thesis in physics at Johns Hopkins University. Hall's experiments consisted of exposing thin gold leaf on a glass plate to a magnetic field and tapping off the gold leaf at points down its length. After applying a current through the gold leaf, he observed a tiny voltage across the tap points. (Photo credit: Voltiana, Como, Italy-September 10, 1927 issue, courtesy AIP Emilio Segre Visual Archives.)

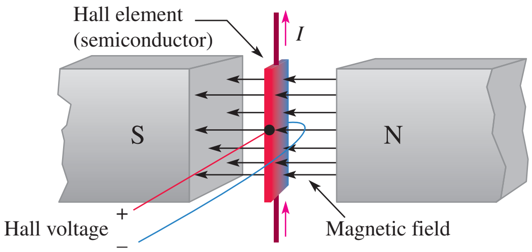


FIGURE 9

The Hall effect. The Hall voltage is induced across the Hall element. For illustration, the positive side is shown in red and the negative side is shown in blue.

field. For this reason, they can be used for position measurements or for sensing motion. They are used in conjunction with other sensor elements to measure current, temperature, or pressure.

There are many applications for Hall-effect sensors. In automobiles, Hall-effect sensors are used to measure various parameters such as throttle angle, crankshaft and camshaft positions, distributor position, tachometer, power seat and rear-view mirror positions. A few other applications for Hall-effect sensors include measuring parameters for rotating devices such as drills, fans, vanes in flow meters, and disk speed detection. They are also used in dc motors as will be seen in Section 7.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. When the north poles of two magnets are placed close together, do they repel or attract each other?
2. What is the difference between magnetic flux and magnetic flux density?
3. What are two units for measuring magnetic flux density?
4. What is the flux density when $\phi = 4.5\ \mu\text{Wb}$ and $A = 5 \times 10^{-3}\ \text{m}^2$?

2 ELECTROMAGNETISM

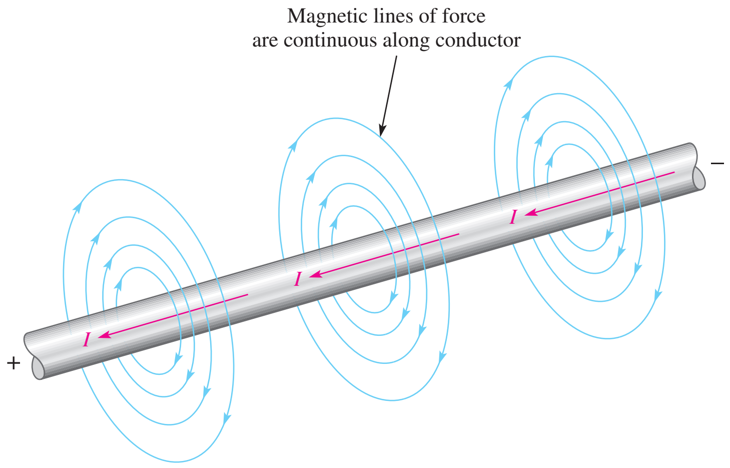
Electromagnetism is the production of a magnetic field by current in a conductor.

After completing this section, you should be able to

- **Explain the principles of electromagnetism**
    - Determine the direction of the magnetic lines of force
    - Define *permeability*
    - Define *reluctance*
    - Define *magnetomotive force*
    - Describe a basic electromagnet

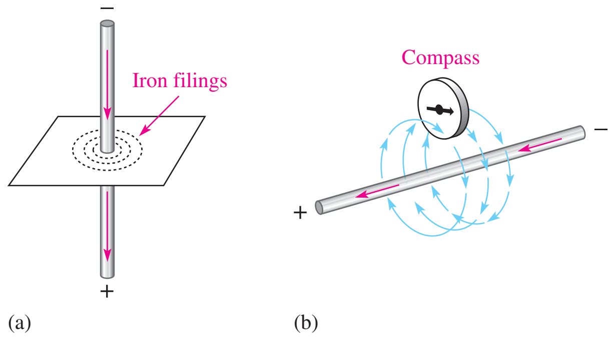
Current produces a magnetic field, called an **electromagnetic field**, around a conductor, as illustrated in Figure 10. The invisible lines of force of the magnetic field form a concentric circular pattern around the conductor and are continuous along its length. Unlike the bar magnet, the magnetic field surrounding a wire does not have a north or south pole. The direction of the lines of force surrounding the conductor shown in the figure is for electron flow. The lines are in a clockwise direction. When current is reversed, the magnetic field lines are in a counterclockwise direction.

Although the magnetic field cannot be seen, it is capable of producing visible effects. For example, if a current-carrying wire is inserted through a sheet of paper in a perpendicular



▲ **FIGURE 10**

Magnetic field around a current-carrying conductor. The red arrows indicate the direction of electron flow current (− to +).



(a) (b)

▲ **FIGURE 11**

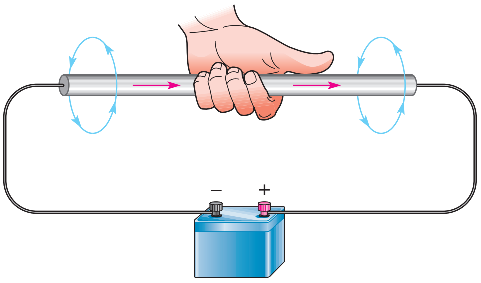
Visible effects of an electromagnetic field.

direction, iron filings placed on the surface of the paper arrange themselves along the magnetic lines of force in concentric rings, as illustrated in Figure 11(a). Part (b) of the figure illustrates that the north pole of a compass placed in the electromagnetic field will point in the direction of the lines of force. The field is stronger closer to the conductor and becomes weaker with increasing distance from the conductor.

Left-Hand Rule An aid to remembering the direction of the lines of force is illustrated in Figure 12. Imagine that you are grasping the conductor with your left hand, with your thumb pointing in the direction of current. Your fingers point in the direction of the magnetic lines of force.

▶ **FIGURE 12**

Illustration of left-hand rule. The left-hand rule is used for electron flow current (− to +).



Electromagnetic Properties

Several important properties are related to electromagnetic fields.

Permeability (μ) The ease with which a magnetic field can be established in a given material is measured by the **permeability** of that material. The higher the permeability, the more easily a magnetic field can be established.

The symbol of permeability is μ (the Greek letter mu), and its value varies depending on the type of material. The permeability of a vacuum (μ_0) is $4\pi \times 10^{-7}$ Wb/At · m (webers/ampere-turn · meter) and is used as a reference. Ferromagnetic materials typically have permeabilities hundreds of times larger than that of a vacuum, indicating that a magnetic field can be set up with relative ease in these materials. Ferromagnetic materials include iron, steel, nickel, cobalt, and their alloys.

The *relative permeability* (μ_r) of a material is the ratio of its absolute permeability to the permeability of a vacuum (μ_0).

$$\mu_r = \frac{\mu}{\mu_0}$$

Equation 2

Because it is a ratio of permeabilities, μ_r is dimensionless. Typical magnetic materials, such as iron, have a relative permeability of a few hundred. Highly permeable materials can have a relative permeability as high as 100,000.

Reluctance ($\mathscr{R}$) The opposition to the establishment of a magnetic field in a material is called **reluctance**. The value of reluctance is directly proportional to the length (l) of the magnetic path and inversely proportional to the permeability (μ) and to the cross-sectional area (A) of the material, as expressed by the following equation:

$$\mathscr{R} = \frac{l}{\mu A}$$

Equation 3

Reluctance in magnetic circuits is analogous to resistance in electric circuits. The unit of reluctance can be derived using l in meters, A (area) in square meters, and μ in Wb/At · m as follows:

$$\mathscr{R} = \frac{l}{\mu A} = \frac{\cancel{\text{m}}}{(\text{Wb/At}\cdot\cancel{\text{m}})(\cancel{\text{m}}^{\cancel{2}})} = \frac{\text{At}}{\text{Wb}}$$

At/Wb is ampere-turns/weber.

Equation 3 is similar to the equation for determining wire resistance:

$$R = \frac{\rho l}{A}$$

The reciprocal of resistivity (ρ) is conductivity (σ). By substituting $1/\sigma$ for ρ, the equation can be written as

$$R = \frac{l}{\sigma A}$$

Compare this last equation for wire resistance with Equation 3. The length (l) and the area (A) have the same meaning in both equations. The conductivity (σ) in electrical circuits is analogous to permeability (μ) in magnetic circuits. Also, resistance (R) in electric circuits is analogous to reluctance ($\mathscr{R}$) in magnetic circuits; both are oppositions. Typically, the reluctance of a magnetic circuit is 50,000 At/Wb or more, depending on the size and type of material.

EXAMPLE 3

Calculate the reluctance of a torus (a doughnut-shaped core) made of low-carbon steel. The inner radius of the torus is 1.75 cm and the outer radius of the torus is 2.25 cm. Assume the permeability of low-carbon steel is 2×10^{-4} Wb/At · m.

Solution You must convert centimeters to meters before you calculate the area and length. From the dimensions given, the thickness (diameter) is 0.5 cm = 0.005 m. Thus, the cross-sectional area is

$$A = \pi r^2 = \pi(0.0025)^2 = 1.96 \times 10^{-5}\ \text{m}^2$$

The length is equal to the circumference of the torus measured at the average radius of 2.0 cm or 0.020 m.

$$l = C = 2\pi r = 2\pi(0.020\ \text{m}) = 0.125\ \text{m}$$

Substituting values into Equation 3,

$$\mathcal{R} = \frac{l}{\mu A} = \frac{0.125\ \text{m}}{(2 \times 10^{-4}\ \text{Wb/At}\cdot\text{m})(1.96 \times 10^{-5}\ \text{m}^2)} = \mathbf{31.9 \times 10^6\ At/Wb}$$

Related Problem What happens to the reluctance if cast steel with a permeability of 5×10^{-4} Wb/At · m is substituted for the cast iron core?

EXAMPLE 4

Mild steel has a relative permeability of 800. Calculate the reluctance of a mild steel core that has a length of 10 cm and has a cross section of 1.0 cm × 1.2 cm.

Solution First, determine the permeability of mild steel.

$$\mu = \mu_0\mu_r = (4\pi \times 10^{-7}\ \text{Wb/At}\cdot\text{m})(800) = 1.00 \times 10^{-3}\ \text{Wb/At}\cdot\text{m}$$

Next, convert the length to meters and the area to square meters.

$$l = 10\ \text{cm} = 0.10\ \text{m}$$
$$A = 0.010\ \text{m} \times 0.012\ \text{m} = 1.2 \times 10^{-4}\ \text{m}^2$$

Substituting values into Equation 3,

$$\mathcal{R} = \frac{l}{\mu A} = \frac{0.10\ \text{m}}{(1.00 \times 10^{-3}\ \text{Wb/At}\cdot\text{m})(1.2 \times 10^{-4}\ \text{m}^2)} = \mathbf{8.33 \times 10^5\ At/Wb}$$

Related Problem What happens to the reluctance if the core is made from 78 Permalloy with a relative permeability of 4000?

Magnetomotive Force (mmf) As you have learned, current in a conductor produces a magnetic field. The cause of a magnetic field is called the **magnetomotive force (mmf)**. Magnetomotive force is something of a misnomer because in a physics sense mmf is not really a force, but rather it is a direct result of the movement of charge (current). The unit of mmf, the **ampere-turn (At)**, is established on the basis of the current in a single loop (turn) of wire. The formula for mmf is

Equation 4

$$F_m = NI$$

where F_m is the magnetomotive force, N is the number of turns of wire, and I is the current in amperes.

Figure 13 illustrates that a number of turns of wire carrying a current around a magnetic material creates a force that sets up flux lines through the magnetic path. The amount

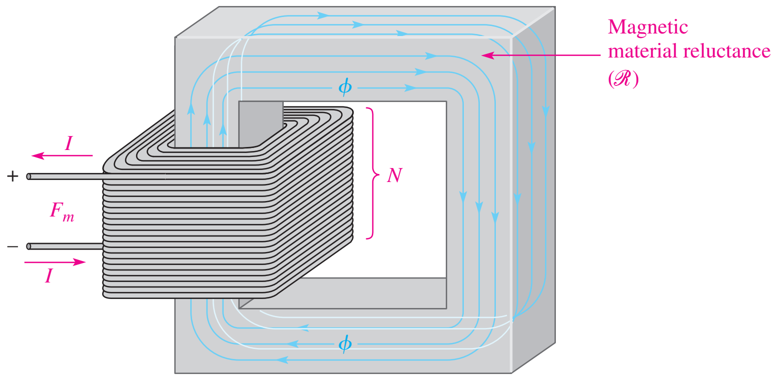


FIGURE 13

A basic magnetic circuit.

of flux depends on the magnitude of the mmf and on the reluctance of the material, as expressed by the following equation:

Equation 5

$$\phi = \frac{F_m}{\mathcal{R}}$$

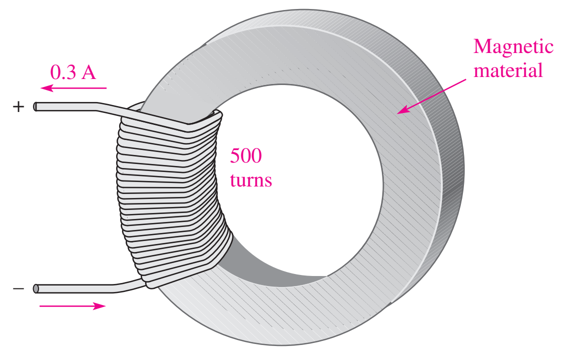
Equation 5 is known as the *Ohm's law for magnetic circuits* because the flux (ϕ) is analogous to current, the mmf (F_m) is analogous to voltage, and the reluctance ($\mathcal{R}$) is analogous to resistance. Like other phenomena in science, the flux is an effect, the mmf is a cause, and the reluctance is an opposition.

One important difference between an electric circuit and a magnetic circuit is that in magnetic circuits, Equation 5 is only valid up to a certain point before the magnetic material saturates (flux becomes a maximum). You will see this when you look at magnetization curves in Section 4. Another difference that was already noted is that flux does occur in permanent magnets with no source of mmf. In a permanent magnet, the flux is due to internal electron motion rather than an external current. No equivalent effect occurs in electric circuits.

EXAMPLE 5

How much flux is established in the magnetic path of Figure 14 if the reluctance of the material is 2.8×10^5 At/Wb?

FIGURE 14



Solution

$$\phi = \frac{F_m}{\mathcal{R}} = \frac{NI}{\mathcal{R}} = \frac{(500\text{ t})(0.300\text{ A})}{2.8 \times 10^5\text{ At/Wb}} = \mathbf{536\ \mu Wb}$$

Related Problem How much flux is established in the magnetic path of Figure 14 if the reluctance is 7.5×10^3 At/Wb, the number of turns is 300, and the current is 0.18 A?

EXAMPLE 6

There is 0.1 A of current through a coil with 400 turns.

(a) What is the mmf?

(b) What is the reluctance of the circuit if the flux is 250 μWb?

Solution

(a) $N = 400$ and $I = 0.1\text{ A}$

$$F_m = NI = (400\text{ t})(0.1\text{ A}) = \mathbf{40\ At}$$

(b) $$\mathcal{R} = \frac{F_m}{\phi} = \frac{40\text{ At}}{250\ \mu\text{Wb}} = \mathbf{1.60 \times 10^5\ At/Wb}$$

Related Problem

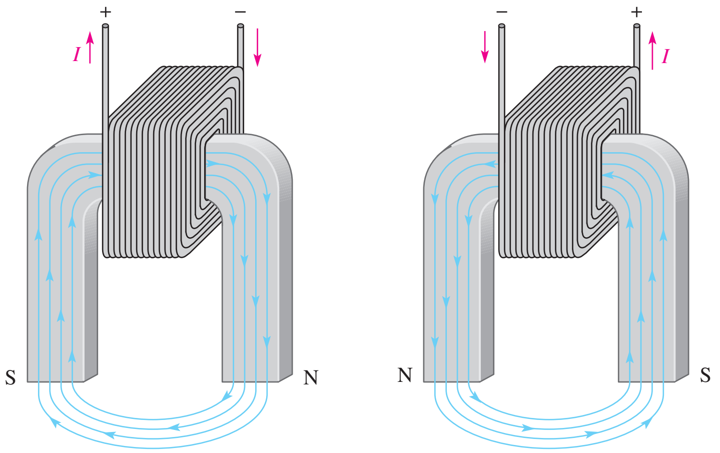
Rework the example for $I = 85$ mA and $N = 500$. The flux is 500 μWb.

In many magnetic circuits, the core is not continuous. For example, if an air gap is cut into the core, it will increase the reluctance of the magnetic circuit. This means that more current is required to establish the same flux as before because an air gap represents a significant opposition to establishing flux. The situation is analogous to a series electrical circuit; the total reluctance of the magnetic circuit is the sum of the reluctance of the core and the reluctance of the air gap.

The Electromagnet

An electromagnet is based on the properties that you have just learned. A basic electromagnet is simply a coil of wire wound around a core material that can be easily magnetized.

The shape of an electromagnet can be designed for various applications. For example, Figure 15 shows a U-shaped magnetic core. When the coil of wire is connected to a battery and there is current, as shown in part (a), a magnetic field is established as indicated. If the current is reversed, as shown in part (b), the direction of the magnetic field is also reversed. The closer the north and south poles are brought together, the smaller the air gap between them becomes, and the easier it becomes to establish a magnetic field because the reluctance is lessened.



(a)

(b)

▲ **FIGURE 15**

Reversing the current in the coil causes the electromagnetic field to reverse.

SECTION 2 CHECKUP

1. Explain the difference between magnetism and electromagnetism.
2. What happens to the magnetic field in an electromagnet when the current through the coil is reversed?
3. State Ohm's law for a magnetic circuit.
4. Compare each quantity in Question 3 to its electrical counterpart.

3 ELECTROMAGNETIC DEVICES

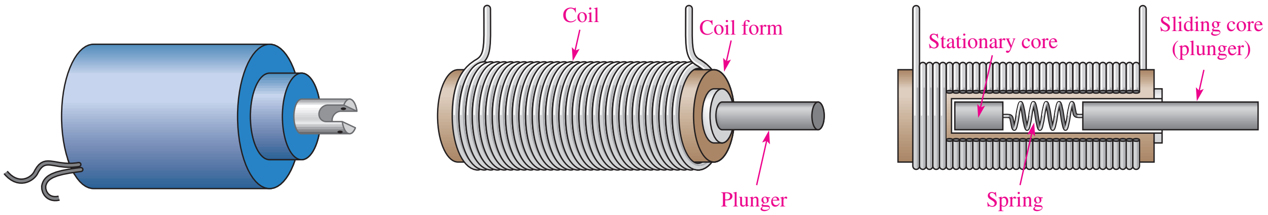
Many types of useful devices such as tape recorders, electric motors, speakers, solenoids, and relays are based on electromagnetism.

After completing this section, you should be able to

- **Describe the principle of operation for several types of electromagnetic devices**
  - Discuss how a solenoid and a solenoid valve work
  - Discuss how a relay works
  - Discuss how a speaker works
  - Discuss the basic analog meter movement
  - Explain a magnetic disk and tape Read/Write operation
  - Explain the concept of the magneto-optical disk

The Solenoid

The **solenoid** is a type of electromagnetic device that has a movable iron core called a *plunger.* The movement of this iron core depends on both an electromagnetic field and a mechanical spring force. The basic structure of a solenoid is shown in Figure 16. It consists of a cylindrical coil of wire wound around a nonmagnetic hollow form. A stationary iron core is fixed in position at the end of the shaft and a sliding iron core (plunger) is attached to the stationary core with a spring.

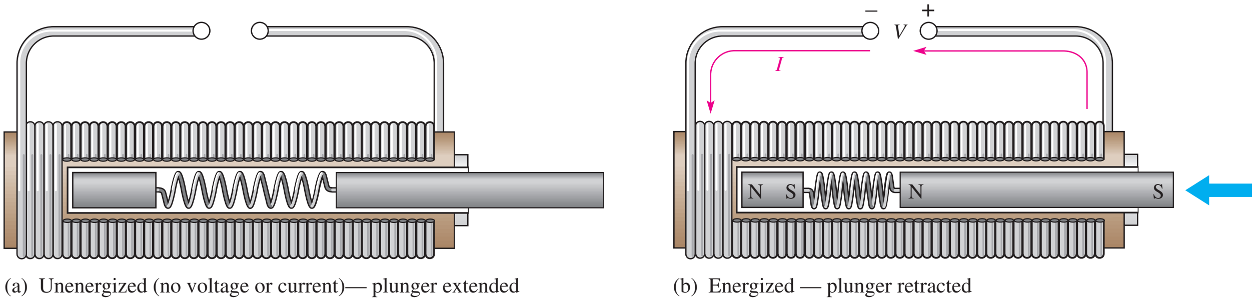


(a) Solenoid (b) Basic construction (c) Cutaway view

▲ FIGURE 16

Basic solenoid structure.

The basic solenoid operation is illustrated in Figure 17 for the unenergized and the energized conditions. In the at-rest (or unenergized) state, the plunger is extended. The solenoid is energized by current through the coil, which sets up an electromagnetic field that magnetizes both iron cores. The south pole of the stationary core attracts the north pole of



(a) Unenergized (no voltage or current)— plunger extended

(b) Energized — plunger retracted

▲ FIGURE 17

Basic solenoid operation.

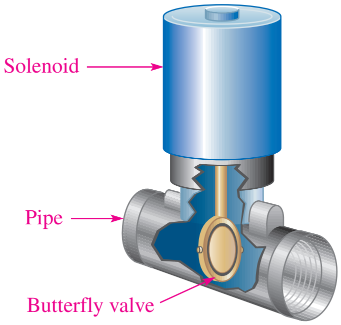
the movable core causing it to slide inward, thus retracting the plunger and compressing the spring. As long as there is coil current, the plunger remains retracted by the attractive force of the magnetic fields. When the current is cut off, the magnetic fields collapse and the force of the compressed spring pushes the plunger back out. The solenoid is used for applications such as opening and closing valves and automobile door locks.

The Solenoid Valve In industrial controls, **solenoid valves** are widely used to control the flow of air, water, steam, oils, refrigerants, and other fluids. Solenoid valves are used in both pneumatic (air) and hydraulic (oil) systems, common in machine controls. Solenoid valves are also common in the aerospace and medical fields. Solenoid valves can either move a plunger to open or close a port or can rotate a blocking flap a fixed amount.

A solenoid valve consists of two functional units: a solenoid coil that provides the magnetic field to provide the required movement to open or close the valve and a valve body, which is isolated from the coil assembly via a leakproof seal and includes a pipe and butterfly valve. Figure 18 shows a cutaway of one type of solenoid valve. When the solenoid is energized, the butterfly valve is turned to open a normally closed (NC) valve or to close a normally open (NO) valve.

▶ FIGURE 18

A basic solenoid valve structure.



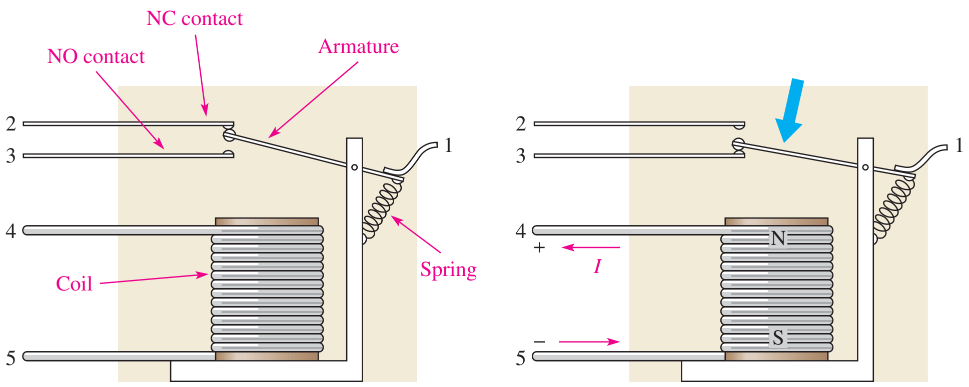
Solenoid valves are available with a wide variety of configurations including normally open or normally closed valves. They are rated for different types of fluids (for example, gas or water), pressures, number of pathways, sizes, and more. The same valve may control more than one line and may have more than one solenoid to move.

The Relay

The **relay** differs from the solenoid in that the electromagnetic action is used to open or close electrical contacts rather than to provide mechanical movement. Figure 19 shows the basic operation of an *armature-type relay* with one normally open (NO) contact and one normally closed (NC) contact (single pole–double throw). When there is no coil current, the armature is held against the upper contact by the spring, thus providing continuity from

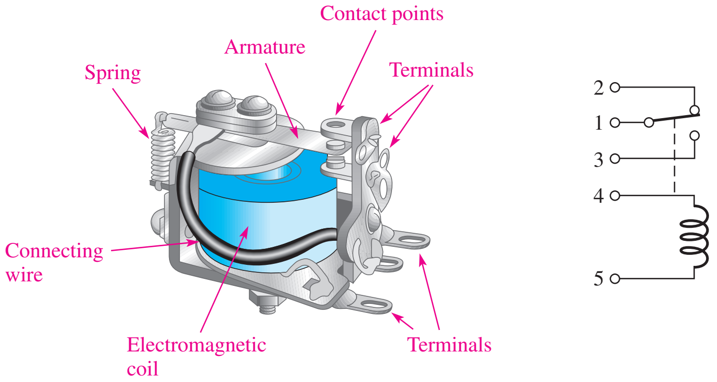
◀ **FIGURE 19**

Basic structure of a single-pole–double-throw armature relay.


NC contact
NO contact
Armature
2
3
1
4
Coil
Spring
5
2
3
1
4
+
I
N
S
–
5

(a) Unenergized: continuity from terminal 1 to terminal 2

(b) Energized: continuity from terminal 1 to terminal 3

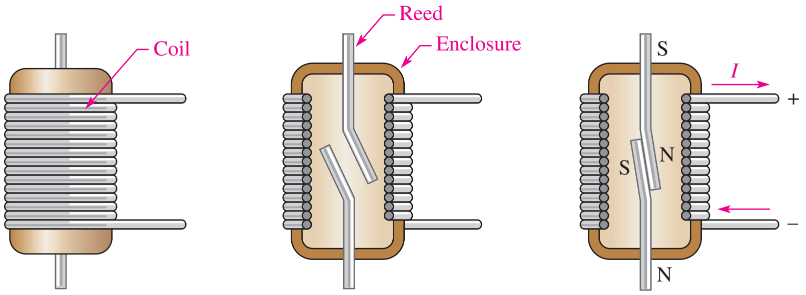

Contact points
Armature
Spring
Terminals
Connecting wire
Electromagnetic coil
Terminals
2
1
3
4
5

(c) Typical relay structure

(d) Symbol

terminal 1 to terminal 2, as shown in part (a) of the figure. When energized with coil current, the armature is pulled down by the attractive force of the electromagnetic field and makes connection with the lower contact to provide continuity from terminal 1 to terminal 3, as shown in Figure 19(b). A typical armature relay is shown in part (c), and the schematic symbol is shown in part (d).

Another widely used type of relay is the *reed relay,* which is shown in Figure 20. The reed relay, like the armature relay, uses an electromagnetic coil. The contacts are thin reeds of magnetic material and are usually located inside the coil. When there is no coil current, the reeds are in the open position as shown in part (b). When there is current through the coil, the reeds make contact because they are magnetized and attract each other as shown in part (c).


Coil
Reed
Enclosure
S
I
+
S
N
–
N

(a)

(b) Unenergized

(c) Energized

▲ **FIGURE 20**

Basic structure of a reed relay.

Reed relays are superior to armature relays in that they are faster, more reliable, and produce less contact arcing. However, they have less current-handling capability than armature relays and are more susceptible to mechanical shock.

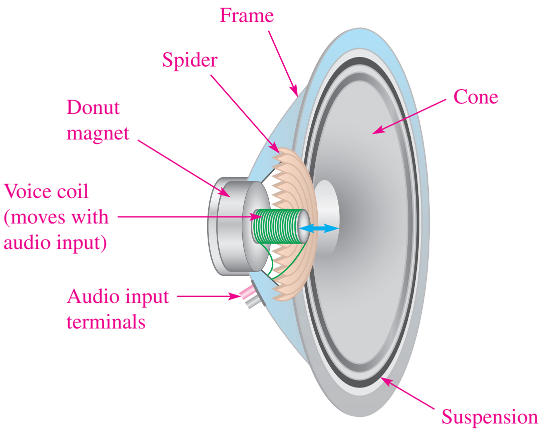
The Speaker

A **speaker** is an electromagnetic device that converts electrical signals into sound. Essentially, it is a linear motor that alternately attracts and repels an electromagnet to and from a permanent magnet called a donut magnet. Figure 21 shows the key parts of a speaker. The audio signal is connected using very flexible wires to a cylindrical coil called the voice coil. The voice coil and its movable core form an electromagnet, which is suspended in an accordion-like structure called the spider. The spider acts like an accordion spring, keeping the voice coil in the center and restoring it to the rest position when there is no input signal.

Current from the audio input alternates back and forth and powers the electromagnet; when there is more current, the attraction or repulsion is greater. When the input current reverses direction, the polarity of the electromagnet reverses direction also, faithfully following the input signal. The voice coil and its moving magnet are firmly attached to the cone. The cone is a flexible diaphragm that vibrates to produce sound.

▶ FIGURE 21

Key parts of a speaker (cutaway view).

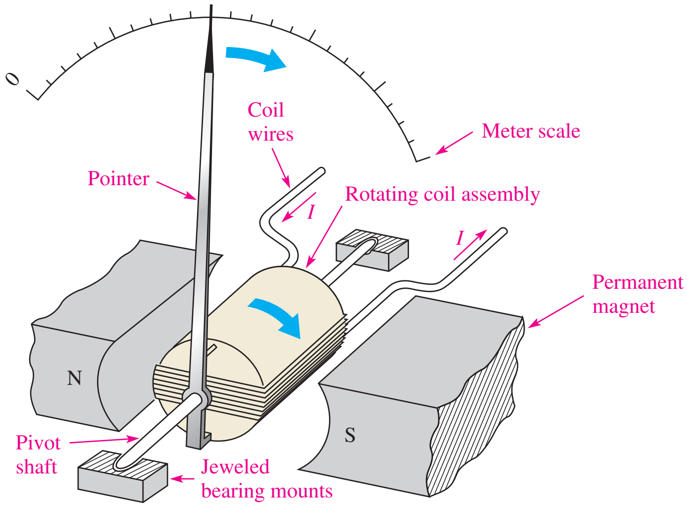

Frame
Spider
Cone
Donut magnet
Voice coil (moves with audio input)
Audio input terminals
Suspension

Meter Movement

The d'Arsonval meter movement is the most common type used in analog multimeters. In this type of meter movement, the pointer is deflected in proportion to the amount of current through a coil. Figure 22 shows a basic d'Arsonval meter movement. It consists of a

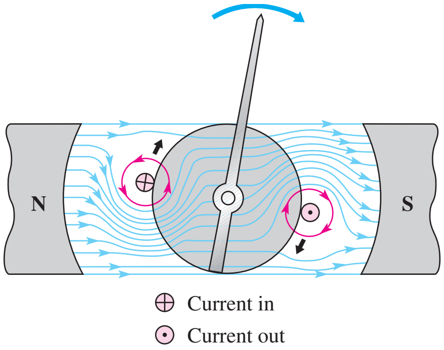
▶ FIGURE 22

The basic d'Arsonval meter movement.


0
Coil wires
Meter scale
Pointer
Rotating coil assembly
I
I
Permanent magnet
N
S
Pivot shaft
Jeweled bearing mounts

coil of wire wound on a bearing-mounted assembly that is placed between the poles of a permanent magnet. A pointer is attached to the moving assembly. With no current through the coil, a spring mechanism keeps the pointer at its left-most (zero) position. When there is current through the coil, electromagnetic forces act on the coil, causing a rotation to the right. The amount of rotation depends on the amount of current.

Figure 23 illustrates how the interaction of magnetic fields produces rotation of the coil assembly. Current is inward at the "cross" and outward at the "dot" in the single winding shown. The outward current produces a clockwise electromagnetic field that reinforces the permanent magnetic field above it. The result is a downward force on the right side of the coil as shown. The inward current produces a counterclockwise electromagnetic field that reinforces the permanent magnetic field below it. The result is an upward force on the left side of the coil as shown. These forces produce a clockwise rotation of the coil assembly and are opposed by a spring mechanism. The indicated forces and the spring force are balanced at the value of the current. When current is removed, the spring force returns the pointer to its zero position.

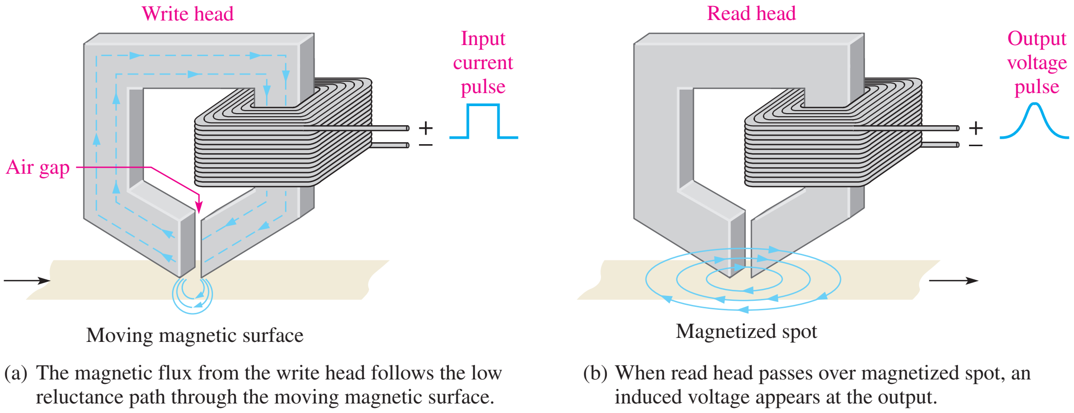


◀ **FIGURE 23**

When the electromagnetic field interacts with the permanent magnetic field, forces are exerted on the rotating coil assembly, causing it to move clockwise and thus deflecting the pointer.

Magnetic Disk and Tape Read/Write Head

A simplified diagram of a magnetic disk or tape surface read/write operation is shown in Figure 24. A data bit (1 or 0) is written on the magnetic surface by the magnetization of a small segment of the surface as it moves by the write head. The direction of the magnetic flux lines is controlled by the direction of the current pulse in the winding, as shown in Figure 24(a). At the air gap in the write head, the magnetic flux takes a path through the surface of the storage device. This magnetizes a small spot on the surface in the direction of the field. A magnetized spot of one polarity represents a binary 1, and one of the opposite polarity represents a binary 0. Once a spot on the surface is magnetized, it remains until written over with an opposite magnetic field.



(a) The magnetic flux from the write head follows the low reluctance path through the moving magnetic surface.

(b) When read head passes over magnetized spot, an induced voltage appears at the output.

▲ **FIGURE 24**

Read/write function on a magnetic surface.

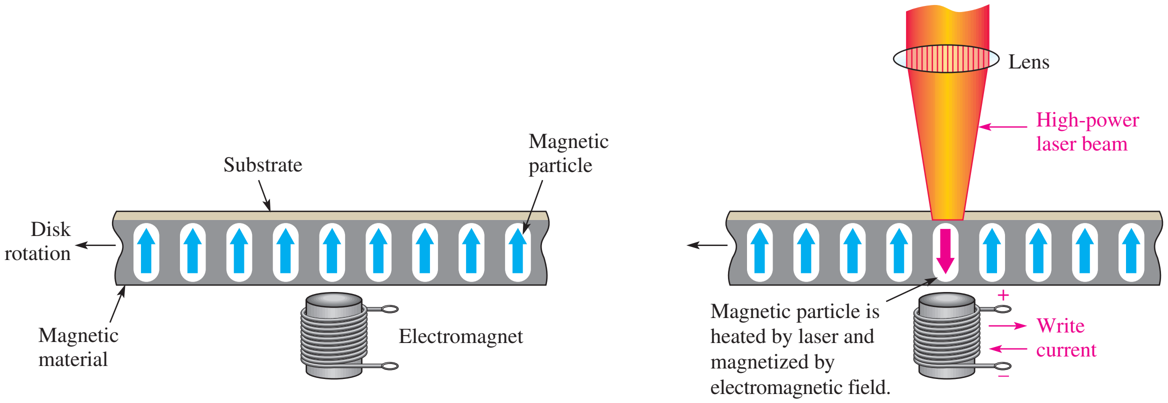
When the magnetic surface passes a read head, the magnetized spots produce magnetic fields in the read head, which induce voltage pulses in the winding. The polarity of these pulses depends on the direction of the magnetized spot and indicates whether the stored bit is a 1 or a 0. This process is illustrated in Figure 24(b). Often the read and write heads are combined into a single unit.

The Magneto-Optical Disk

The magneto-optical disk uses an electromagnet and laser beams to read and write (record) data on a magnetic surface. Magneto-optical disks are formatted in tracks and sectors similar to magnetic floppy disks and hard disks. However, because of the ability of a laser beam to be precisely directed to an extremely small spot, magneto-optical disks are capable of storing much more data than standard magnetic hard disks.

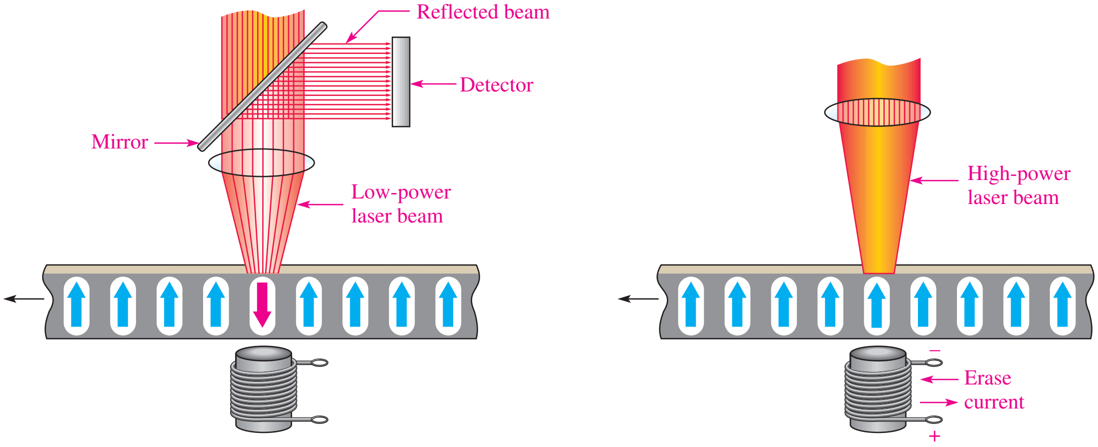
Figure 25(a) illustrates a small cross-sectional area of a disk before recording, with an electromagnet positioned below it. Tiny magnetic particles, represented by the arrows, are all magnetized in the same direction.

Writing (recording) on the disk is accomplished by applying an external magnetic field opposite to the direction of the magnetic particles, as indicated in Figure 25(b), and


Lens
High-power laser beam
Magnetic particle
Substrate
Disk rotation
Magnetic material
Electromagnet
Magnetic particle is heated by laser and magnetized by electromagnetic field.
+
Write current
−

(a) Small cross-section of unrecorded disk

(b) Writing: A high-power laser beam heats the spot, causing the magnetic particle to align with the electromagnetic field.


Reflected beam
Detector
Mirror
Low-power laser beam
High-power laser beam
−
Erase current
+

(c) Reading: A low-power laser beam reflects off of the reversed-polarity magnetic particle and its polarization shifts. If the particle is not reversed, the polarization of the reflected beam is unchanged.

(d) Erasing: The electromagnetic field is reversed as the high-power laser beam heats the spot, causing the magnetic particle to be restored to the original polarity.

▲ FIGURE 25

Basic concept of the magneto-optical disk.

then directing a high-power laser beam to heat the disk at a precise point where a binary 1 is to be stored. The disk material, a magneto-optic alloy, is highly resistant to magnetization at room temperature; but at the spot where the laser beam heats the material, the inherent direction of magnetism is reversed by the external magnetic field produced by the electromagnet. At points where binary 0s are to be stored, the laser beam is not applied and the inherent upward direction of the magnetic particle remains.

As illustrated in Figure 25(c), reading data from the disk is accomplished by turning off the external magnetic field and directing a low-power laser beam at a spot where a bit is to be read. Basically, if a binary 1 is stored at the spot (reversed magnetization), the laser beam is reflected and its polarization is shifted; but if a binary 0 is stored, the polarization of the reflected laser beam is unchanged. A detector senses the difference in the polarity of the reflected laser beam to determine if the bit being read is a 1 or a 0.

Figure 25(d) shows that the disk is erased by restoring the original magnetic direction of each particle by reversing the external magnetic field and applying the high power laser beam.

SECTION 3
CHECKUP

1. Explain the difference between a solenoid and a relay.
2. What is the movable part of a solenoid called?
3. What is the movable part of a relay called?
4. Upon what basic principle is the d'Arsonval meter movement based?
5. What is the function of the spider in a speaker?

4 MAGNETIC HYSTERESIS

When a magnetizing force is applied to a material, the flux density in the material changes in a certain way.

After completing this section, you should be able to

- **Explain magnetic hysteresis**
  - State the formula for magnetic field intensity
  - Discuss a hysteresis curve
  - Define *retentivity*

Magnetic Field Intensity (*H*)

The **magnetic field intensity** (also called *magnetizing force*) in a material is defined to be the magnetomotive force (F_m) per unit length (l) of the material, as expressed by the following equation. The unit of magnetic field intensity (H) is ampere-turns per meter (At/m).

$$H = \frac{F_m}{l}$$

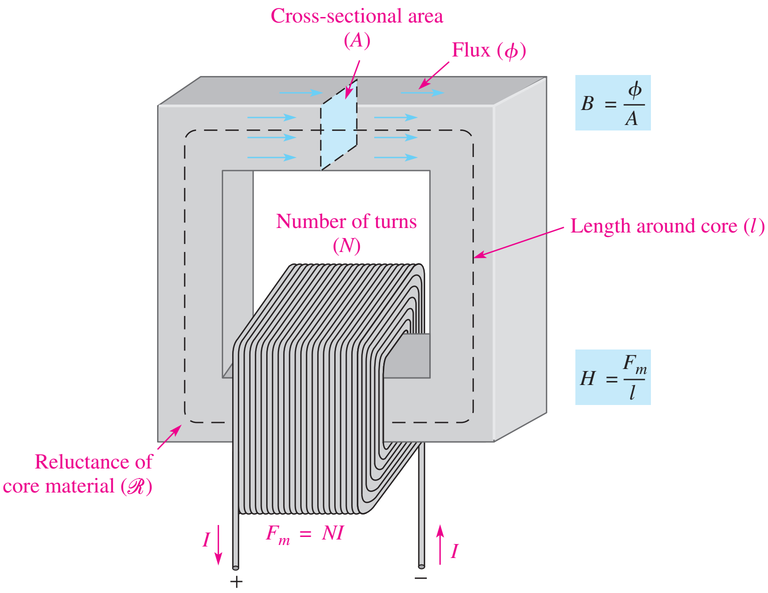
Equation 6

where $F_m = NI$. Note that the magnetic field intensity depends on the number of turns (N) of the coil of wire, the current (I) through the coil, and the length (l) of the material. It does not depend on the type of material.

Since $\phi = F_m/\mathcal{R}$, as F_m increases, the flux increases. Also, the magnetic field intensity (H) increases. Recall that the flux density (B) is the flux per unit cross-sectional area ($B = \phi/A$), so B is also proportional to H. The curve showing how these two quantities (B and H) are related is called the B-H curve or the hysteresis curve. The parameters that influence both B and H are illustrated in Figure 26.

▶ **FIGURE 26**

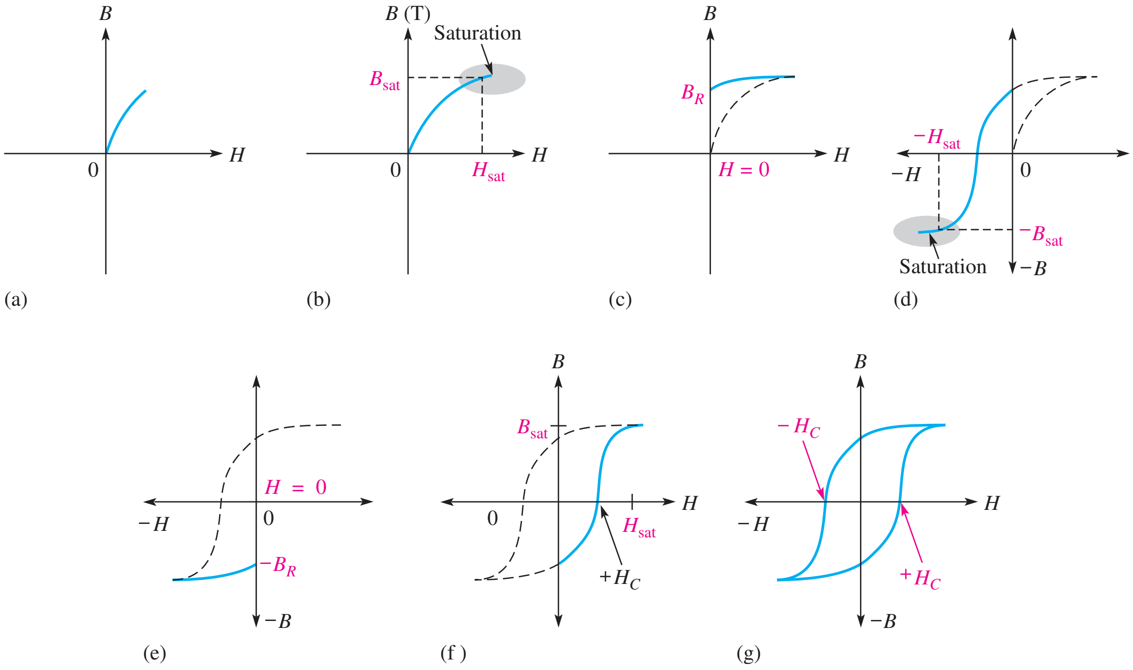
Parameters that determine the magnetic field intensity (H) and the flux density (B).


Cross-sectional area
(A)
Flux (ϕ)
B = ϕ/A
Number of turns
(N)
Length around core (l)
H = F_m/l
Reluctance of
core material (ℛ)
I
F_m = NI
I
+
−

The Hysteresis Curve and Retentivity

Hysteresis is a characteristic of a magnetic material whereby a change in magnetization lags the application of the magnetic field intensity. The magnetic field intensity (H) can be readily increased or decreased by varying the current through the coil of wire, and it can be reversed by reversing the voltage polarity across the coil.

Figure 27 illustrates the development of the hysteresis curve. Let's start by assuming a magnetic core is unmagnetized so that $B = 0$. As the magnetic field intensity (H) is increased from zero, the flux density (B) increases proportionally, as indicated by the curve



▲ **FIGURE 27**

Development of a magnetic hysteresis curve.

in Figure 27(a). When H reaches a certain value, B begins to level off. As H continues to increase, B reaches a saturation value (B_{sat}) when H reaches a value (H_{sat}), as illustrated in Figure 27(b). Once saturation is reached, a further increase in H will not increase B.

Now, if H is decreased to zero, B will fall back along a different path to a residual value (B_R), as shown in Figure 27(c). This indicates that the material continues to be magnetized even when the magnetic field intensity is zero ($H = 0$). The ability of a material to maintain a magnetized state without magnetic field intensity is called **retentivity**. The retentivity of a material represents the maximum flux that can be retained after the material has been magnetized to saturation and is indicated by the ratio of B_R to B_{sat}.

Reversal of the magnetic field intensity is represented by negative values of H on the curve and is achieved by reversing the current in the coil of wire. An increase in H in the negative direction causes saturation to occur at a value ($-H_{\text{sat}}$) where the flux density is at its maximum negative value, as indicated in Figure 27(d).

When the magnetic field intensity is removed ($H = 0$), the flux density goes to its negative residual value ($-B_R$), as shown in Figure 27(e). From the $-B_R$ value, the flux density follows the curve indicated in part (f) back to its maximum positive value when the magnetic field intensity equals H_{sat} in the positive direction.

The complete B-H curve is shown in Figure 27(g) and is called the *hysteresis curve.* The magnetic field intensity required to make the flux density zero is called the *coercive force,* H_C.

Materials with a low retentivity do not retain a magnetic field very well, while those with high retentivities exhibit values of B_R very close to the saturation value of B. Depending on the application, retentivity in a magnetic material can be an advantage or a disadvantage. In permanent magnets and magnetic tape, for example, high retentivity is required; while in tape recorder read/write heads, low retentivity is necessary. In ac motors, retentivity is undesirable because the residual magnetic field must be overcome each time the current reverses, thus wasting energy.

SECTION 4 CHECKUP

1. **For a given wirewound core, how does an increase in current through the coil affect the flux density?**
2. **Define *retentivity*.**
3. **Why is low retentivity required for tape recorder read/write heads but high retentivity is required for magnetic tape?**

5 ELECTROMAGNETIC INDUCTION

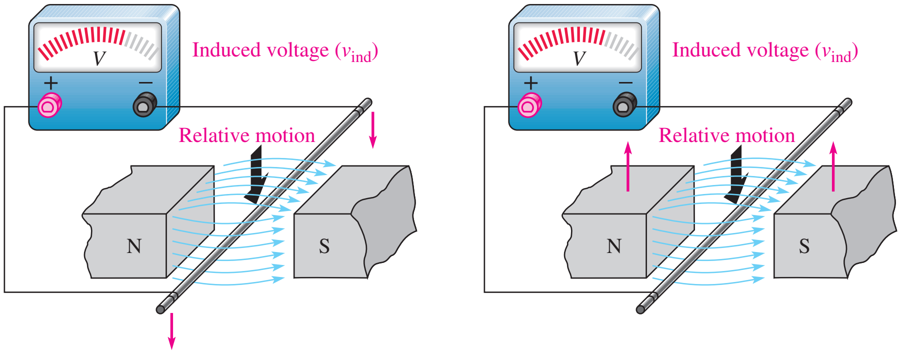
In this section, electromagnetic induction is introduced. The principle of electromagnetic induction is what makes transformers, electrical generators, and many other devices possible.

After completing this section, you should be able to

- **Discuss the principle of electromagnetic induction**
    - Explain how voltage is induced in a conductor in a magnetic field
    - Determine polarity of an induced voltage
    - Discuss forces on a conductor in a magnetic field
    - State Faraday's law
    - State Lenz's law
    - Explain how a crankshaft position sensor works

FIGURE 28

Relative motion between a straight conductor and a magnetic field.



(a) Conductor moving downward

(b) Magnetic field moving upward

Relative Motion

When a straight conductor is moved perpendicular to a magnetic field, there is a relative motion between the conductor and the magnetic field. Likewise, when a magnetic field is moved past a stationary conductor, there is also relative motion. In either case, this relative motion results in an **induced voltage (v_{ind})** in the conductor, as Figure 28 indicates. This principle is known as **electromagnetic induction**. The lowercase v stands for instantaneous voltage. Voltage is only induced when the conductor "cuts" magnetic lines as shown. The amount of the induced voltage (v_{ind}) depends on the flux density (B), the length of the conductor (l) that is exposed to the magnetic field, and the rate at which the conductor and the magnetic field move with respect to each other. The faster the relative speed, the greater the induced voltage. The equation for induced voltage in a straight conductor is

Equation 7

$$v_{ind} = B_{\perp} l v$$

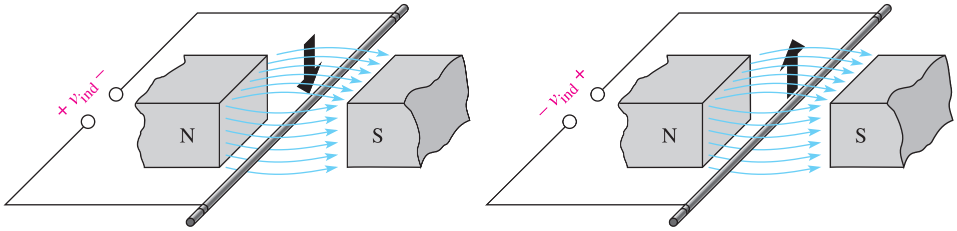
where v_{ind} is the induced voltage, $B_{\perp}$ is the flux density in teslas, l is the length of the conductor exposed to the magnetic field in meters, and v is the relative velocity in meters per second.

Polarity of the Induced Voltage

If the conductor in Figure 28 is moved first one way and then another in the magnetic field, a reversal of the polarity of the induced voltage will be observed. As the conductor is moved downward, a voltage is induced with the polarity indicated in Figure 29(a). As the conductor is moved upward, the polarity is as indicated in part (b) of the figure.

FIGURE 29

Polarity of induced voltage depends on direction of motion.



(a) Downward relative motion

(b) Upward relative motion

EXAMPLE 7

Assume the conductor in Figure 29 is 10 cm long and the pole face of the magnet is 5.0 cm wide. The flux density is 0.5 T, and the conductor is moved upward at a velocity of 0.8 m/s. What voltage is induced in the conductor?

Solution Although the conductor is 10 cm, only 5.0 cm (0.05 m) is in the magnetic field because of the size of the pole faces. Therefore,

$$v_{\text{ind}} = B_{\perp} lv = (0.5\text{ T})(0.05\text{ m})(0.8\text{ m/s}) = \mathbf{20\text{ mV}}$$

Related Problem What is the induced voltage if the velocity is doubled?

Induced Current

When a load resistor is connected to the conductor in Figure 29, the voltage induced by the relative motion in the magnetic field will cause a current in the load, as shown in Figure 30. This current is called the **induced current (i_{ind})**. The lowercase i stands for instantaneous current.

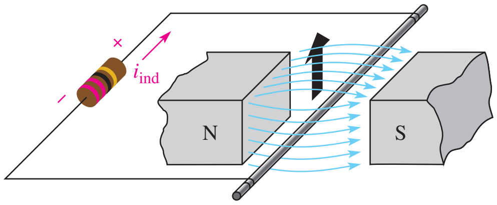


FIGURE 30

Induced current in a load as the conductor moves through the magnetic field.

The action of producing a voltage and a resulting current in a load by moving a conductor across a magnetic field is the basis for electrical generators. A single conductor will have a small induced current, so practical generators use coils with many turns. The concept of a conductor in a moving magnetic field is the basis for inductance in an electric circuit.

Faraday's law

Michael Faraday discovered the principle of electromagnetic induction in 1831. The key idea behind Faraday's law is that a changing magnetic field can induce a voltage in a conductor. Sometimes Faraday's law is stated as Faraday's law of induction. Faraday experimented with coils, and his law is an extension of the principle of electromagnetic induction for straight conductors discussed previously.

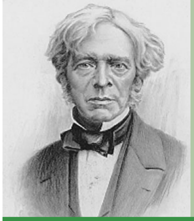
When a conductor is coiled into multiple turns, more conductors can be exposed to the magnetic field, increasing the induced voltage. When the magnetic flux is caused to change by any means, an induced voltage will result. The change in the magnetic field can be caused by relative motion between the magnetic field and the coil. Faraday's observations can be stated as follows:

1. The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil ($d\phi/dt$).
2. The amount of voltage induced in a coil is directly proportional to the number of turns of wire in the coil (N).

Faraday's first observation is demonstrated in Figure 31, where a bar magnet is moved through a coil, thus creating a changing magnetic field. In part (a) of the figure, the magnet is moved at a certain rate, and a certain induced voltage is produced as indicated. In part (b), the magnet is moved at a faster rate through the coil, creating a greater induced voltage.

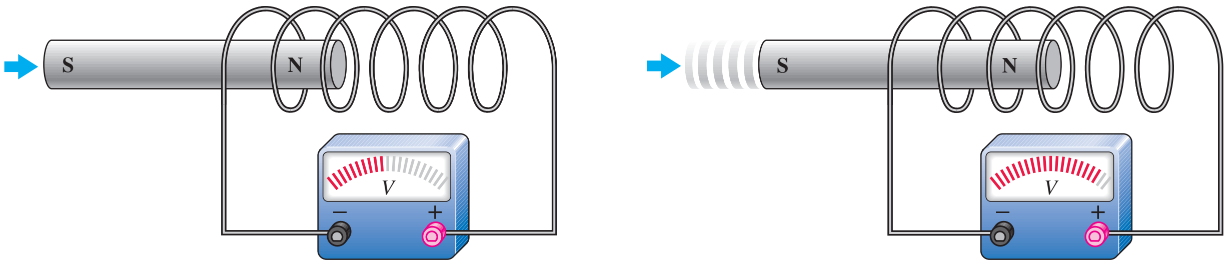
Faraday's second observation is demonstrated in Figure 32. In part (a), the magnet is moved through the coil and a voltage is induced as shown. In part (b), the magnet is moved at the same speed through a coil that has a greater number of turns. The greater number of turns creates a greater induced voltage.

HISTORY NOTE



Michael Faraday 1791–1867

Faraday was an English physicist and chemist who is best remembered for his contribution to the understanding of electromagnetism. He discovered that electricity could be produced by moving a magnet inside a coil of wire, and he was able to build the first electric motor. He later built the first electromagnetic generator and transformer. The statement of the principles of electromagnetic induction is known today as Faraday's law. Also, the unit of capacitance, the *farad*, is named after him. (Photo credit: Library of Congress.)

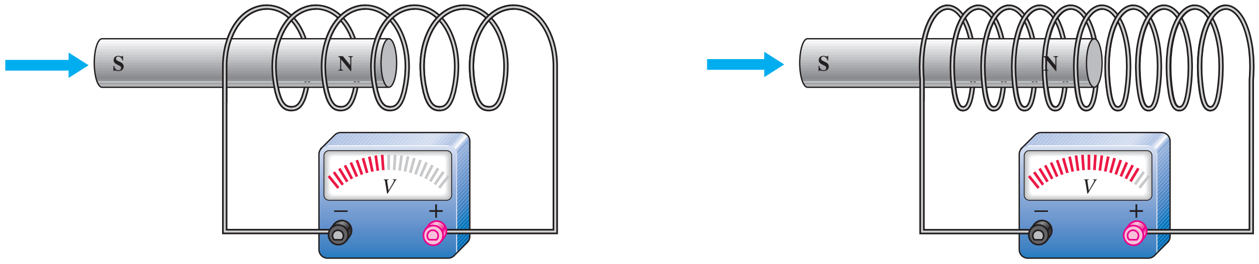


(a) As the magnet moves slowly to the right, its magnetic field is changing with respect to the coil, and a voltage is induced.

(b) As the magnet moves more rapidly to the right, its magnetic field is changing more rapidly with respect to the coil, and a greater voltage is induced.

▲ **FIGURE 31**

A demonstration of Faraday's first observation: The amount of induced voltage is directly proportional to the rate of change of the magnetic field with respect to the coil.



(a) Magnet moves through a coil and induces a voltage.

(b) Magnet moves at same rate through a coil with more turns (loops) and induces a greater voltage.

▲ **FIGURE 32**

A demonstration of Faraday's second observation: The amount of induced voltage is directly proportional to the number of turns in the coil.

Faraday's law is stated as follows:

The voltage induced across a coil equals the number of turns in the coil times the rate of change of the magnetic flux.

Faraday's law is expressed in equation form as

Equation 8

$$v_{\text{ind}} = N\left(\frac{d\phi}{dt}\right)$$

where v_{ind} is the induced voltage, N is the number of turns in the coil, and $d\phi/dt$ is the rate of change of the magnetic flux.

EXAMPLE 8

Apply Faraday's law to find the induced voltage across a coil with 500 turns that is located in a magnetic field that is changing at a rate of 8000 μWb/s.

Solution

$$v_{\text{ind}} = N\left(\frac{d\phi}{dt}\right) = (500\ \text{t})(8000\ \mu\text{Wb/s}) = \mathbf{4.0\ V}$$

Related Problem

Find the induced voltage across a 250 turn coil in a magnetic field that is changing at 50 μWb/s.

Any *relative* motion between the magnetic field and the coil will produce the changing magnetic field that will induce a voltage in the coil. The changing magnetic field can even be induced by ac applied to an electromagnet, just as if there were motion. This type of changing magnetic field is the basis for transformer action in ac circuits.

Lenz's Law

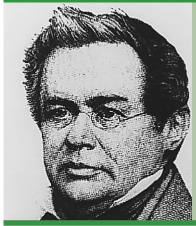
Faraday's law states that a changing magnetic field induces a voltage in a coil that is directly proportional to the rate of change of the magnetic field and the number of turns in the coil. **Lenz's law** defines the polarity or direction of the induced voltage.

> **When the current through a coil changes, an induced voltage is created as a result of the changing electromagnetic field and the polarity of the induced voltage is such that it always opposes the change in current.**

An Application of Electromagnetic Induction

In automobiles, it is necessary to know the position of the crankshaft to control ignition timing and sometimes to adjust the fuel mixture. As mentioned previously, a Hall-effect sensor is one method to determine the position of the crankshaft (or camshaft). Another widely used method is to detect a change in the magnetic field as a metallic tab moves though the air gap in a magnetic assembly. A basic concept is shown in Figure 33. A steel disk with protruding tabs is connected to the end of the crankshaft. As the crankshaft turns, the tabs move through the magnetic field. Steel has a much lower reluctance than does air, so the magnetic flux increases when a tab is in the air gap. This change in the magnetic flux causes an induced voltage to appear across the coil, indicating the position of the crankshaft.

HISTORY NOTE



Heinrich F. E. Lenz 1804–1865

Lenz was born in Estonia (then Russia) and became a professor at the University of St. Petersburg. He carried out many experiments following Faraday's lead and formulated the principle of electromagnetism, which defines the polarity of induced voltage in a coil. The statement of this principle is named in his honor. (Photo credit: AIP Emilio Segré Visual Archives, E. Scott Barr Collection.)

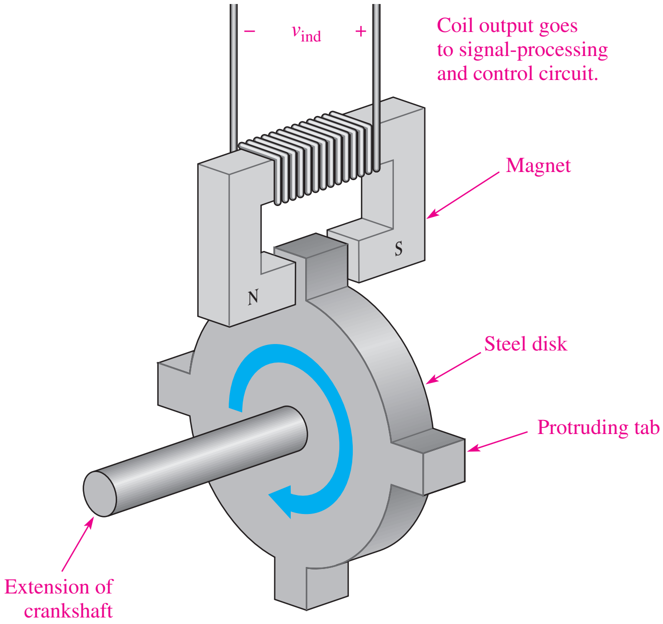


FIGURE 33

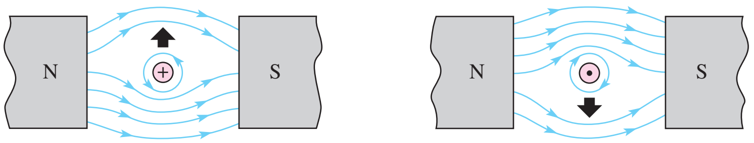
A crankshaft position sensor that produces a voltage when a tab passes through the air gap of the magnet.

Forces on a Current-Carrying Conductor in a Magnetic Field (Motor Action)

Figure 34(a) shows current inward through a wire in a magnetic field. The electromagnetic field set up by the current interacts with the permanent magnetic field; as a result, the permanent lines of force above the wire tend to be deflected down under the wire because they are opposite in direction to the electromagnetic lines of force. Therefore, the

FIGURE 34

Forces on a current-carrying conductor in a magnetic field (motor action).



(a) Upward force: weak field above, strong field below.

(b) Downward force: strong field above, weak field below

⊙ Current out
⊕ Current in

flux density above is reduced, and the magnetic field is weakened. The flux density below the conductor is increased, and the magnetic field is strengthened. An upward force on the conductor results, and the conductor tends to move toward the weaker magnetic field.

Figure 34(b) shows the current outward, resulting in a force on the conductor in the downward direction. This force is the basis for electrical motors. The force on a current-carrying conductor is given by the equation

Equation 9

$$F = BIl$$

where F is the force in newtons, B is the magnetic flux density in teslas, I is the current in amperes, and l is the length of the conductor exposed to the magnetic field in meters.

EXAMPLE 9

Assume a magnetic pole face is a square that is 3.0 cm on a side. Find the force on a conductor that has a current of 2 A if the conductor is perpendicular to the field and the flux density is 0.35 T.

Solution

The length of conductor exposed to the magnetic flux is 3.0 cm (0.030 m). Therefore,

$$F = BIl = (0.35\text{ T})(2.0\text{ A})(0.03\text{ m}) = \mathbf{0.21\text{ N}}$$

Related Problem

What is the direction of the force if the field is directed up (along the y axis) and the current is directed inward (along the z axis)?

SECTION 5 CHECKUP

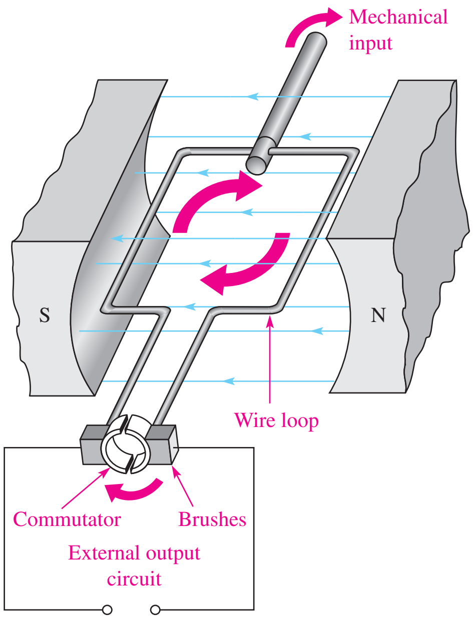
1. What is the induced voltage across a stationary conductor in a stationary magnetic field?
2. When the speed at which a conductor is moved through a magnetic field is increased, does the induced voltage increase, decrease, or remain the same?
3. When there is current through a conductor in a magnetic field, what happens?
4. If the steel disk in the crankshaft position sensor has stopped with a tab in the magnet's air gap, what is the induced voltage?

6 THE DC GENERATOR

DC generators produce a voltage that is proportional to the magnetic flux and the rotational speed of the armature.

After completing this section, you should be able to

- **Explain how a dc generator works**
  - Draw an equivalent circuit for a self-excited shunt-type dc generator
  - Discuss the parts of a dc generator

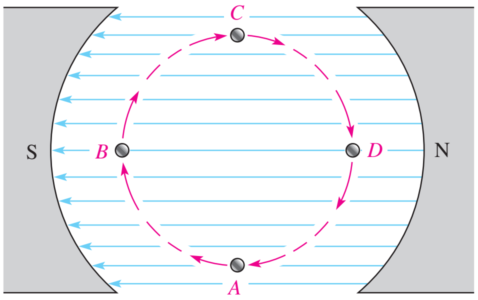


◀ FIGURE 35

A simplified dc generator.

Figure 35 shows a simplified dc generator consisting of a single loop of wire in a permanent magnetic field. Notice that each end of the wire loop is connected to a split-ring arrangement. This conductive metal ring is called a *commutator.* As the wire loop rotates in the magnetic field, the split commutator ring also rotates. Each half of the split ring rubs against the fixed contacts, called *brushes,* and connects the wire loop to an external circuit.

As the wire loop rotates through the magnetic field, it cuts through the flux lines at varying angles, as illustrated in Figure 36. At position *A* in its rotation, the loop of wire is effectively moving parallel with the magnetic field. Therefore, at this instant, the rate at which it is cutting through the magnetic flux lines is zero. As the loop moves from position *A* to position *B*, it cuts through the flux lines at an increasing rate. At position *B*, it is moving effectively perpendicular to the magnetic field and thus is cutting through a maximum number of lines. As the loop rotates from position *B* to position *C*, the rate at which it cuts the flux lines decreases to minimum (zero) at *C*. From position *C* to position *D*, the rate at which the loop cuts the flux lines increases to a maximum at *D* and then back to a minimum again at *A*.

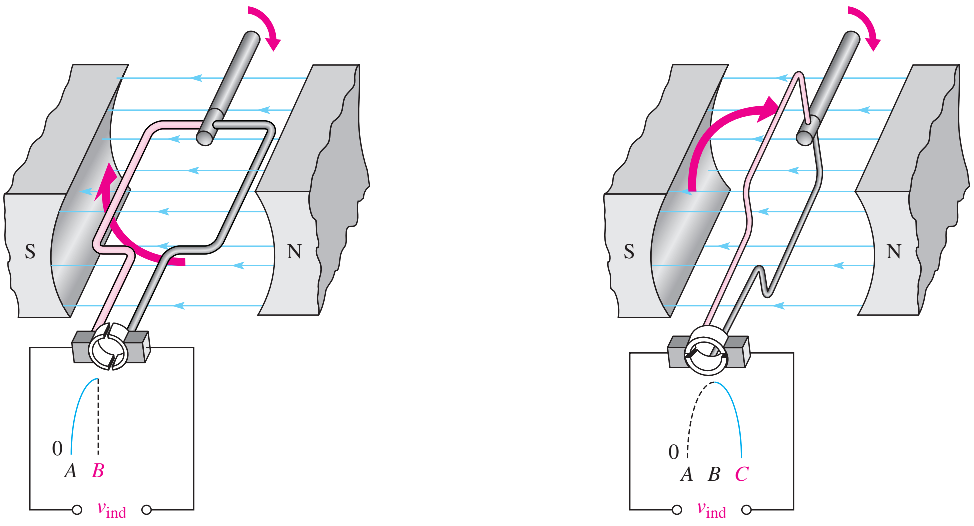


◀ FIGURE 36

End view of wire loop cutting through the magnetic field.

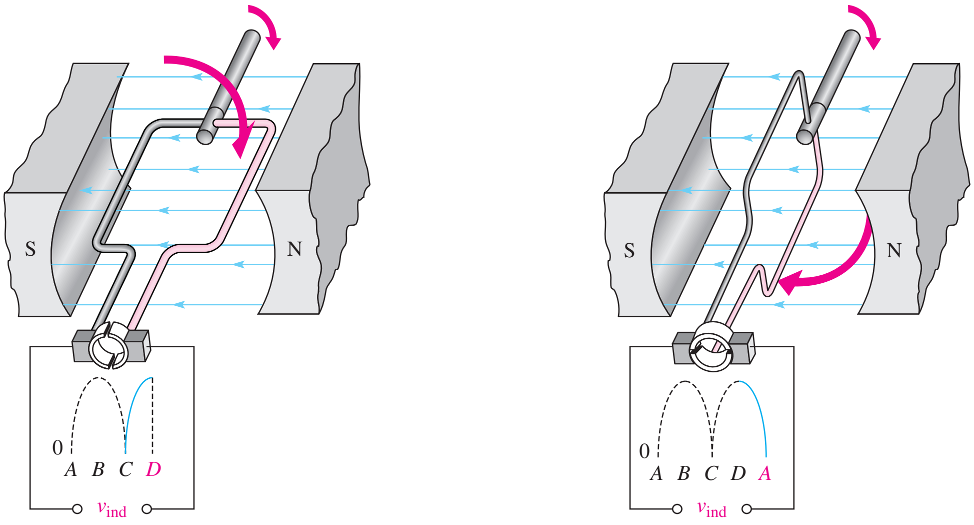
As you have learned, when a wire moves through a magnetic field, a voltage is induced, and by Faraday's law, the amount of induced voltage is proportional to the number of loops (turns) in the wire and the rate at which it is moving with respect to the magnetic field. The angle at which the wire moves with respect to the magnetic flux lines determines the amount of induced voltage because the rate at which the wire cuts through the flux lines depends on the angle of motion.

Figure 37 illustrates how a voltage is induced in the external circuit as the single loop rotates in the magnetic field. Assume that the loop is in its instantaneous horizontal position,



Position *B*: Loop is moving perpendicular to flux lines, and voltage is maximum.

Position *C*: Loop is moving parallel with flux lines, and voltage is zero.



Position *D*: Loop is moving perpendicular to flux lines, and voltage is maximum.

Position *A*: Loop is moving parallel with flux lines, and voltage is zero.

▲ **FIGURE 37**

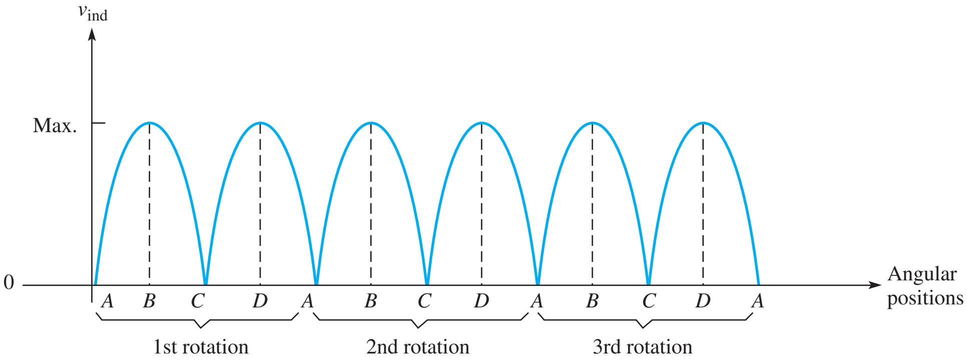
Operation of a basic dc generator.

so the induced voltage is zero. As the loop continues in its rotation, the induced voltage builds up to a maximum at position *B*, as shown in part (a) of the figure. Then, as the loop continues from *B* to *C*, the voltage decreases to zero at position *C*, as shown in part (b).

During the second half of the revolution, shown in Figure 37(c) and (d), the brushes switch to opposite commutator sections, so the polarity of the voltage remains the same across the output. Thus, as the loop rotates from position *C* to position *D* and then back to position *A*, the voltage increases from zero at *C* to a maximum at *D* and back to zero at *A*.

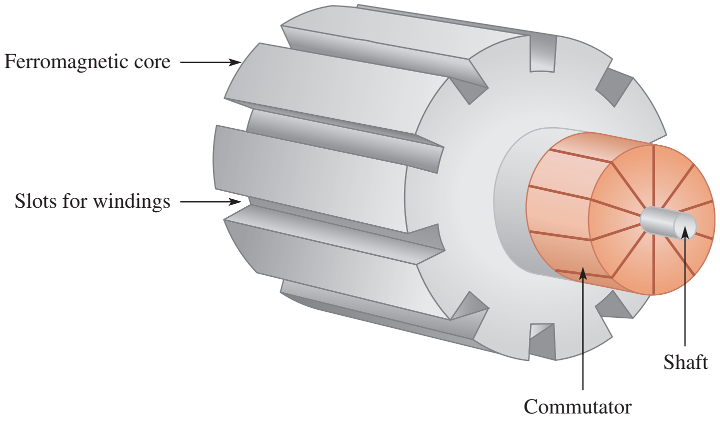
Figure 38 shows how the induced voltage varies as the wire loop in the dc generator goes through several rotations (three in this case). This voltage is a dc voltage because its polarities do not change. However, the voltage is pulsating between zero and its maximum value.

In practical generators, multiple coils are pressed into slots in a ferromagnetic-core assembly. The entire assembly, called the **rotor**, is connected to bearings and rotates in the magnetic field. Figure 39 is a diagram of the rotor core with no wire loops (coils)



▲ **FIGURE 38**

Induced voltage over three rotations of the wire loop in the dc generator.



◀ **FIGURE 39**

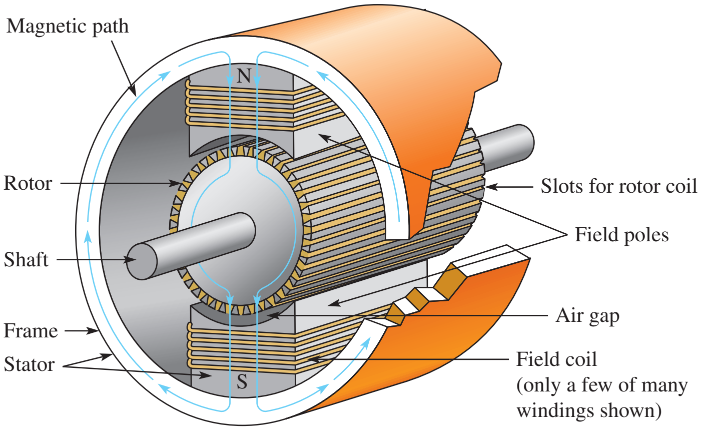
A simplified rotor core. The coils are pressed into the slots and connected to the commutator.

shown. The commutator is divided into segments, with each pair of segments connected to the end of a coil. With more coils, the voltages from several coils are combined because the brushes can contact more than one of the commutator segments at once. The loops do not reach maximum voltage at the same time, so the pulsating output voltage is much smoother than is the case with only one coil or loop shown previously. The variations can be further smoothed by filters to produce a nearly constant dc output.

Instead of permanent magnets, most generators use electromagnets to provide the required magnetic field. One advantage to this is that flux density can be controlled, thus controlling the output voltage of the generator. The windings for the electromagnets, called field windings, require current to produce the magnetic field.

The current for the field windings can be provided from a separate voltage source, but this is a disadvantage. A better method is to use the generator itself to provide the current for the electromagnets; this is called a **self-excited generator**. The generator starts because there is normally enough residual magnetism in the field magnets due to hysteresis that causes a small initial field and allows the generator to start producing a voltage. In cases where a generator has not been used for a long time, it may be necessary to provide an external source to the field windings to start it.

The stationary part of a generator (or motor) includes all of the nonmoving parts and is called the **stator**. Figure 40 illustrates a simplified two-pole dc generator showing the magnetic paths (end caps, bearings, and commutator are not shown). Notice that the frame is part of the magnetic path for the field magnets. To make a generator have high efficiency, the air gap is kept as small as possible. The **armature** is the power-producing component and can be either on the rotor or the stator. In the dc generator described previously, the armature is the rotor because power is produced in the moving conductors and taken from the rotor via the commutator.


Magnetic path
Rotor
Shaft
Frame
Stator
N
S
Slots for rotor coil
Field poles
Air gap
Field coil
(only a few of many
windings shown)

▲ FIGURE 40

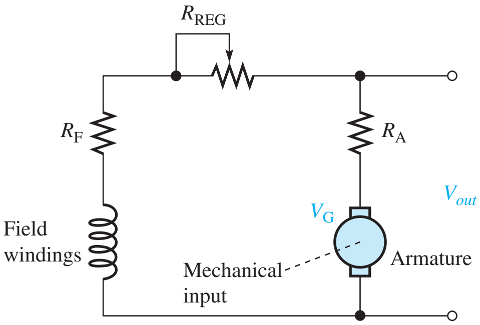
The magnetic structure of a generator or motor. In this case, the rotor is also the armature because it produces the power.

Equivalent Circuit for a DC Generator

A self-excited generator can be represented by a basic dc circuit with a coil to produce the magnetic field and a mechanically driven generator, as shown in Figure 41. There are other configurations of dc generators, but this represents a common one. In the case shown, the field windings are in parallel with the source; this configuration is called a *shunt-wound generator*. The resistance of the field windings is shown as R_F. For the equivalent circuit, this resistance is shown in series with the field windings. The armature is driven by a mechanical input, causing it to spin; it looks like a voltage source of V_G. The armature resistance is shown as the series resistance, R_A. The rheostat, R_{REG}, is in series with the field winding resistance and regulates the output voltage by controlling the current to the field windings and thus the flux density.

▶ FIGURE 41

Equivalent circuit for a self-excited shunt dc generator.


R_{REG}
R_F
R_A
V_{out}
V_G
Field
windings
Mechanical
input
Armature

When a load is connected to the output, current in the armature is shared between the load and the field windings. The efficiency of the generator can be calculated as the ratio of the power delivered to the load (P_L) to the total power (P_T), which includes the losses in the armature and the resistance in the field circuit.

SECTION 6 CHECKUP

1. What is the moving part of a generator called?
2. What is the purpose of a commutator?
3. How does greater resistance in the field windings of a generator affect the output voltage?
4. What is meant by a self-excited generator?

7 THE DC MOTOR

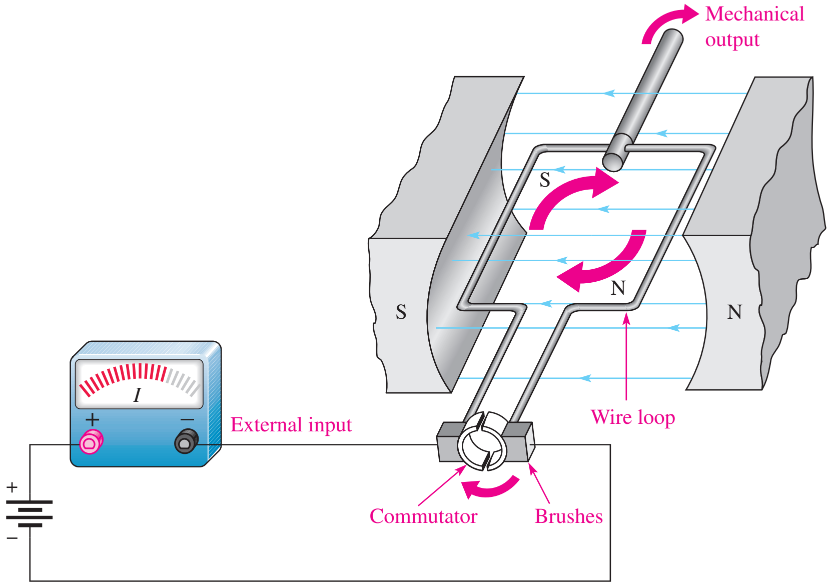
Motors convert electrical energy to mechanical motion by taking advantage of the force produced when a current-carrying conductor is in a magnetic field. A dc motor operates from a dc source and can use either an electromagnet or a permanent magnet to supply the field.

After completing this section, you should be able to

- **Explain how a dc motor works**
  - Draw an equivalent circuit for a series and a shunt-type dc motor
  - Discuss back emf and how it reduces armature current
  - Discuss power rating of motors

Basic Operation

As in the case of generators, motor action is the result of the interaction of magnetic fields. In a dc motor, the rotor field interacts with the magnetic field set up by current in the stator windings. The rotor in all dc motors contains the armature winding, which sets up a magnetic field. The rotor moves because of the attractive force between opposite poles and the repulsive force between like poles, as illustrated in the simplified diagram of Figure 42. The rotor moves because of the attraction of its north pole with the south pole of the stator (and vice-versa). As the two poles near each other, the polarity of the rotor current is suddenly switched by the commutator, reversing the magnetic poles of the rotor. The commutator serves as a mechanical switch to reverse the current in the armature just as the unlike poles are near each other, continuing the rotation of the rotor.



▲ FIGURE 42

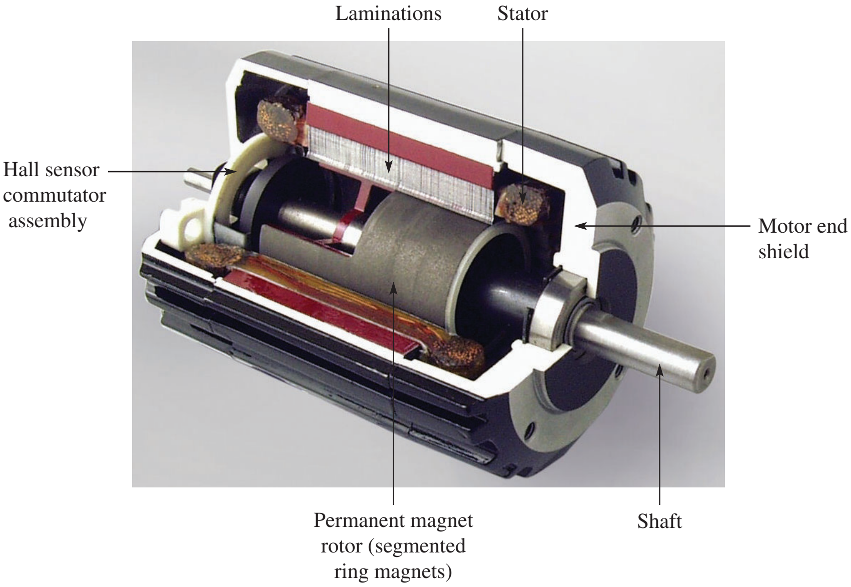
Simplified dc motor.

Brushless DC Motors

Many dc motors do not use a commutator to reverse the polarity of the current. Instead of supplying current to a moving armature, the magnetic field is rotated in the stator windings using an electronic controller. The direction of current in the field coils is periodically reversed by the controller by producing an ac waveform (or modified ac waveform) from the dc input. This causes the stator field to rotate, and the permanent magnet rotor moves in the same direction to keep up with the rotating field. A common way to sense the position of the rotating magnet is to use a Hall-effect sensor, which provides the controller with position information. Brushless motors have higher reliability than traditional brushed motors because they do not need to have periodic brush replacement, but have the added complexity of the electronic controller. Figure 43 shows a cutaway view of a brushless dc motor.

▶ FIGURE 43

Cutaway view of a brushless dc motor. (Courtesy of Bodine Electric Company)


Laminations
Stator
Hall sensor commutator assembly
Motor end shield
Permanent magnet rotor (segmented ring magnets)
Shaft

Back EMF

When a dc motor is first started, a magnetic field is present from the field windings. Armature current develops another magnetic field that interacts with the one from the field windings and starts the motor turning. The armature windings are now spinning in the presence of the magnetic field, so generator action occurs. In effect, the spinning armature has a voltage developed across it that opposes the original applied voltage in accordance with Lenz's law. This self-generated voltage is called **back emf** (electromotive force). The term *emf* was once common for voltage but is not favored because voltage is not a "force" in the physics sense, but back emf is still applied to the self-generated voltage in motors. Back emf, also called *counter emf*, serves to significantly reduce the armature current when the motor is turning at constant speed.

Motor Ratings

Some motors are rated by the torque they can provide, others are rated by the power they produce. Torque and power are important parameters for any motor. Although torque and power are different physical parameters, if one is known, the other can be obtained.

Torque tends to rotate an object. In a dc motor, the torque is proportional to the amount of flux and to the armature current. Torque, T, in a dc motor can be calculated from the equation

Equation 10

$$T = K\phi I_A$$

where T is torque in newton-meters (N-m), K is a constant that depends on physical parameters of the motor, ϕ is magnetic flux in webers (Wb), and I_A is armature current in amperes (A).

Recall that power is defined as the rate of doing work. To calculate power from torque, you must know the speed of the motor in rpm for the torque that you measured. The equation to determine the power, given the torque at a certain speed, is

$$P = 0.105Ts \qquad \text{Equation 11}$$

where P is power in W, T is torque in N-m, and s is speed of motor in rpm.

EXAMPLE 10 What is the power developed by a motor that turns at 350 rpm when the torque is 3.6 N-m?

Solution Substitute into Equation 11.

$$P = 0.105Ts = 0.105(3.6\text{ N-m})(350\text{ rpm}) = \mathbf{132\ W}$$

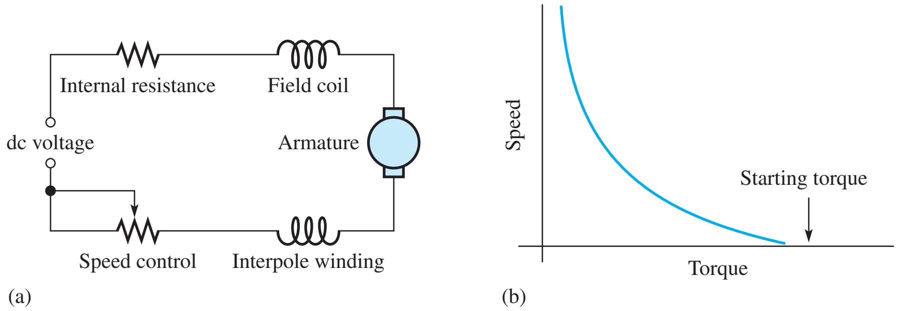
Related Problem Determine the power produced by a motor if $T = 5$ N-m and $s = 1000$ rpm.

The Series DC Motor

The series dc motor has the field coil windings and the armature coil windings in series. A schematic of this arrangement is shown in Figure 44(a). The internal resistance is generally small and consists of field coil resistance, armature winding resistance, and brush resistance. As in the case of generators, dc motors may also contain an interpole winding, as shown, and current limiting for speed control. In a series dc motor, the armature current, field current, and line current are all the same.

TECH NOTE

One characteristic of dc motors is that if they are allowed to run without a load, the torque can cause the motor to "run away" to a speed beyond the manufacturer's rating. Therefore, dc motors should always be operated with a load to prevent self-destruction.



▲ **FIGURE 44**

Simplified schematic and torque-speed characteristic for a series dc motor.

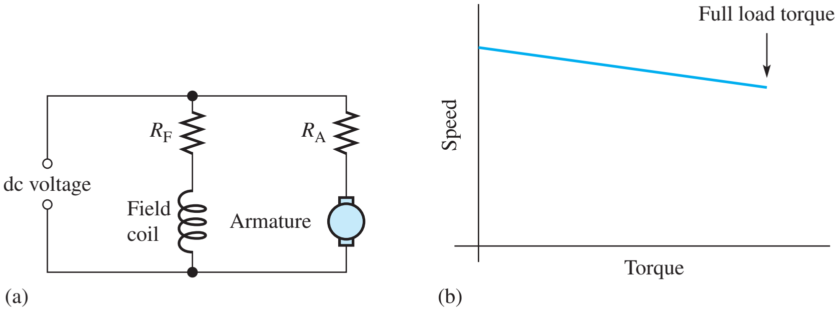
As you know, magnetic flux is proportional to the current in a coil. The magnetic flux created by the field windings is proportional to armature current because of the series connection. Thus, when the motor is loaded, armature current rises, and the magnetic flux rises too. Recall that Equation 10 showed that the torque in a dc motor is proportional to both the armature current and the magnetic flux. Thus, the series-wound motor will have a very high starting torque when the current is high because flux and armature current are high. For this reason, series dc motors are used when high starting torques are required (such as a starter motor in a car).

The plot of the torque and motor speed for a series dc motor is shown in Figure 44(b). The starting torque is at its maximum value. At low speeds, the torque is still very high, but drops off dramatically as the speed increases. As you can see, the speed can be very high if the torque is low; for this reason, the series-wound dc motor always is operated with a load.

The Shunt DC Motor

A shunt dc motor has the field coil in parallel with the armature, as shown in the equivalent circuit in Figure 45(a). In the shunt motor, the field coil is supplied by a constant voltage source, so the magnetic field set up by the field coils is constant. The armature resistance and the back emf produced by generator action in the armature determine the armature current.

The torque-speed characteristic for a shunt dc motor is quite different than for a series dc motor. When a load is applied, the shunt motor will slow down, causing the back emf to be reduced and the armature current to increase. The increase in armature current tends to compensate for the added load by increasing the torque of the motor. Although the motor has slowed because of the additional load, the torque-speed characteristic is nearly a straight line for the shunt dc motor as shown in Figure 45(b). At full load, the shunt dc motor still has high torque.

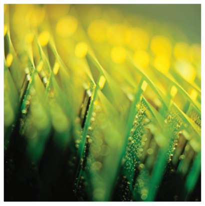


▲ FIGURE 45

Simplified schematic and torque-speed characteristic for a shunt dc motor.

SECTION 7 CHECKUP

1. What creates back emf?
2. How does back emf affect armature current as the motor comes up to speed?
3. What type of dc motor has the highest starting torque?
4. What is the major advantage of a brushless motor over a brushed motor?

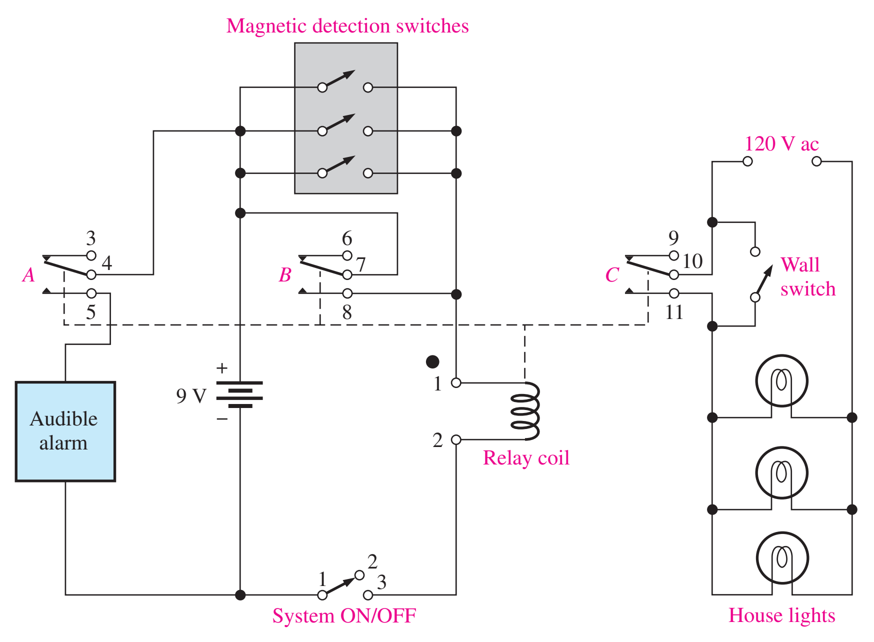


Application Activity

The relay is a common type of electromagnetic device that is used in many types of control applications. With a relay, a lower voltage, such as from a battery, can be used to switch a much higher voltage, such as the 120 V from an ac outlet. You will see how a relay can be used in a basic security alarm system.

The schematic in Figure 46 shows a simplified intrusion alarm system that uses a relay to turn on an audible alarm (siren) and lights. The system operates from a 9 V battery so that even if power to the house is off, the audible alarm will still work.

The detection switches are normally open (NO) magnetic switches that are parallel connected and located in the windows and doors. The relay is a triple-pole–double-throw device that operates with a coil voltage of 9 V dc and draws approximately 50 mA. When an intrusion occurs, one of the switches closes and allows current from the battery to the relay coil, which energizes the relay and causes the three sets of normally open contacts to close. Closure of contact *A* turns on the alarm, which


Magnetic detection switches
120 V ac
3
4
5
A
6
7
8
B
9
10
11
C
Wall switch
Audible alarm
9 V
+
–
1
2
Relay coil
1
2
3
System ON/OFF
House lights

▲ FIGURE 46

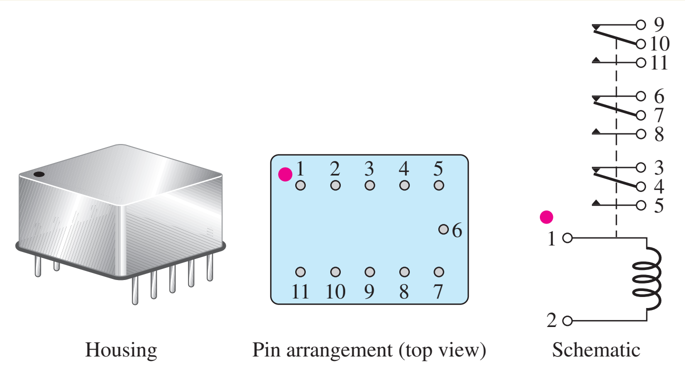
Simplified burglar alarm system.

draws 2 A from the battery. Closure of contact *C* turns on a light circuit in the house. Closure of contact *B* latches the relay and keeps it energized even if the intruder closes the door or window through which entry was made. If not for contact *B* in parallel with the detection switches, the alarm and lights would go off as soon as the window or door was shut behind the intruder.

The relay contacts are not physically remote in relation to the coil as the schematic indicates. The schematic is drawn this way for functional clarity. The entire relay is housed in the package shown in Figure 47. Also shown are the pin diagram and internal schematic for the relay.

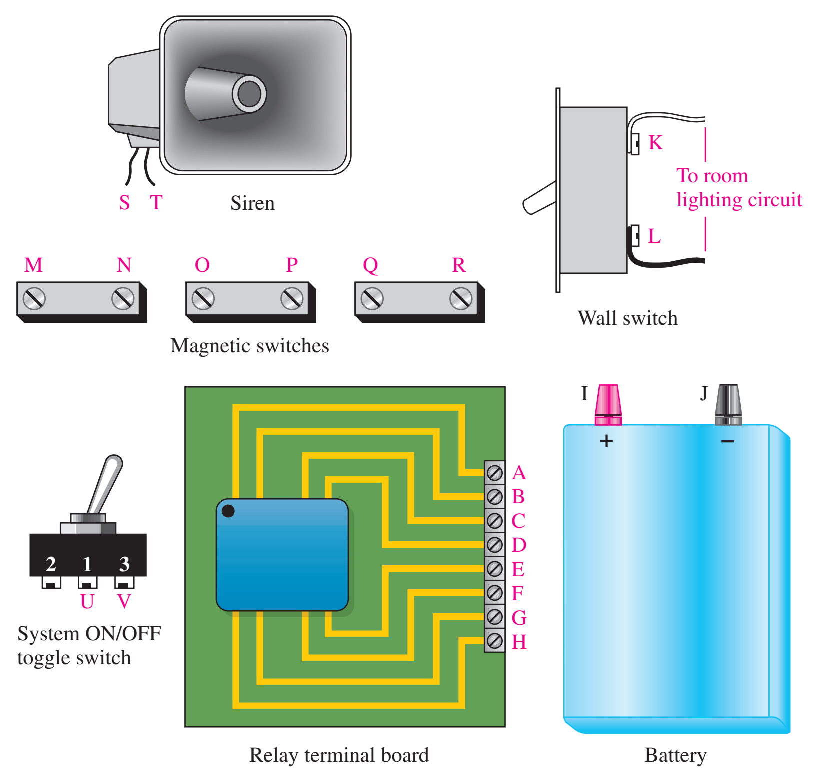
System Interconnections

1. Develop a connection block diagram and point-to-point wire list for interconnecting the components in Figure 48 to create the alarm system shown in the schematic of Figure 46. The connection points on the components are indicated by letters.


1
2
3
4
5
6
11
10
9
8
7
9
10
11
6
7
8
3
4
5
1
2
Housing
Pin arrangement (top view)
Schematic

▲ FIGURE 47

Triple-pole–double-throw relay.


S
T
Siren
K
To room lighting circuit
L
Wall switch
M
N
O
P
Q
R
Magnetic switches
I
J
+
−
2
1
3
U
V
System ON/OFF toggle switch
A
B
C
D
E
F
G
H
Relay terminal board
Battery

▲ FIGURE 48

Array of security alarm components.

A Test Procedure

2. Develop a detailed step-by-step procedure to check out the completely wired security alarm system.

Review

3. What is the purpose of the detection switches?
4. What is the purpose of contact *B* in the relay in Figure 46?

SUMMARY

- Unlike magnetic poles attract each other, and like poles repel each other.
- Materials that can be magnetized are called *ferromagnetic*.
- When there is current through a conductor, it produces an electromagnetic field around the conductor.
- You can use the left-hand rule to establish the direction of the electromagnetic lines of force around a conductor.
- An electromagnet is basically a coil of wire around a magnetic core.
- When a conductor moves within a magnetic field, or when a magnetic field moves relative to a conductor, a voltage is induced across the conductor.
- The faster the relative motion between a conductor and a magnetic field, the greater the induced voltage.
- Table 3 summarizes the quantities and units used in this chapter.
- Hall-effect sensors use current to sense the presence of a magnetic field.
- DC generators convert mechanical power to dc electrical power.

TABLE 3

SYMBOL	QUANTITY	SI UNIT
B	Magnetic flux density	Tesla (T)
ϕ	Magnetic flux	Weber (Wb)
μ	Permeability	Webers/ampere-turn · meter (Wb/At · m)
$\mathcal{R}$	Reluctance	Ampere-turns/weber (At/Wb)
F_m	Magnetomotive force (mmf)	Ampere-turn (At)
H	Magnetic field intensity	Ampere-turns/meter (At/m)
F	Force	Newton (N)
T	Torque	Newton-meter (N-m)

- The moving part of a generator or motor is called the *rotor*; the stationary part is called the *stator*.
- DC motors convert electrical power to mechanical power.
- Brushless DC motors use a permanent magnet as the rotor, and the stator is the armature.

KEY TERMS

Ampere-turn (At) The current in a single loop (turn) of wire.

Electromagnetic field A formation of a group of magnetic lines of force surrounding a conductor created by electrical current in the conductor.

Electromagnetic induction The phenomenon or process by which a voltage is produced in a conductor when there is relative motion between the conductor and a magnetic or electromagnetic field.

Electromagnetism The production of a magnetic field by current in a conductor.

Faraday's law A law stating that the voltage induced across a coil equals the number of turns in the coil times the rate of change of the magnetic flux.

Hall-effect A change in current across a conductor or semiconductor when current in the material is perpendicular to a magnetic field. The change in current produces a small transverse voltage in the material called the *Hall voltage*.

Hysteresis A characteristic of a magnetic material whereby a change in magnetization lags the application of the magnetic field intensity.

Induced current (i_{ind}) A current induced in a conductor when the conductor moves through a magnetic field.

Induced voltage (v_{ind}) Voltage produced as a result of a changing magnetic field.

Lenz's law A law that states when the current through a coil changes, the polarity of the induced voltage created by the changing magnetic field is such that it always opposes the change in current that caused it. The current cannot change instantaneously.

Lines of force Magnetic flux lines in a magnetic field radiating from the north pole to the south pole.

Magnetic field A force field radiating from the north pole to the south pole of a magnet.

Magnetic flux The lines of force between the north and south poles of a permanent magnet or an electromagnet.

Magnetomotive force (mmf) The cause of a magnetic field, measured in ampere-turns.

Permeability The measure of ease with which a magnetic field can be established in a material.

Relay An electromagnetically controlled mechanical device in which electrical contacts are opened or closed by a magnetizing current.

Reluctance The opposition to the establishment of a magnetic field in a material.

Retentivity The ability of a material, once magnetized, to maintain a magnetized state without the presence of a magnetizing force.

Solenoid An electromagnetically controlled device in which the mechanical movement of a shaft or plunger is activated by a magnetizing current.

Speaker An electromagnetic device that converts electrical signals to sound waves.

Tesla (T) The SI unit for magnetic flux density.

Weber (Wb) The SI unit of magnetic flux, which represents 10^8 lines.

FORMULAS

1	$B = \frac{\phi}{A}$	Magnetic flux density
2	$\mu_r = \frac{\mu}{\mu_0}$	Relative permeability
3	$\mathcal{R} = \frac{l}{\mu A}$	Reluctance
4	$F_m = NI$	Magnetomotive force
5	$\phi = \frac{F_m}{R}$	Magnetic flux
6	$H = \frac{F_m}{l}$	Magnetic field intensity
7	$v_{\text{ind}} = B_{\perp} lv$	Induced voltage across a moving conductor perpendicular to a magnetic field
8	$v_{\text{ind}} = N\left(\frac{d\phi}{dt}\right)$	Faraday's law
9	$F = BIl$	Force on a current-carrying conductor
10	$T = K\phi I_{\text{A}}$	Torque in a dc motor
11	$P = 0.105Ts$	Power from torque

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The tesla (T) and the gauss (G) are both units for magnetic flux density.
2. The unit for measuring magnetomotive force (mmf) is the volt.
3. Ohm's law for a magnetic circuit gives the relationship between flux density, magnetomotive force, and reluctance.
4. A solenoid is a form of electromagnetic switch that opens and closes mechanical contacts.
5. A hysteresis curve is a plot of flux density (B) as a function of field intensity (H).
6. To produce an induced voltage in a coil, the magnetic field surrounding it can be changed.
7. The speed of a generator can be controlled with a rheostat in the field windings.
8. A self-excited dc generator will normally have enough residual magnetism in the field magnets to start the generator producing voltage at the output when it is first turned on.
9. The power developed by a motor is proportional to its torque.
10. In a brushless motor, the magnetic field is supplied by permanent magnets.

SELF-TEST

Answers are at the end of the chapter.

1. When the south poles of two bar magnets are brought close together, there will be
   (a) a force of attraction **(b)** a force of repulsion
   (c) an upward force **(d)** no force
2. A magnetic field is made up of
   (a) positive and negative charges **(b)** magnetic domains
   (c) flux lines **(d)** magnetic poles

3. The direction of a magnetic field is from
   (a) north pole to south pole **(b)** south pole to north pole
   (c) inside to outside the magnet **(d)** front to back
4. Reluctance in a magnetic circuit is analogous to
   (a) voltage in an electric circuit **(b)** current in an electric circuit
   (c) power in an electric circuit **(d)** resistance in an electric circuit
5. The unit of magnetic flux is the
   (a) tesla **(b)** weber **(c)** ampere-turn **(d)** ampere-turns/weber
6. The unit of magnetomotive force is the
   (a) tesla **(b)** weber **(c)** ampere-turn **(d)** ampere-turns/weber
7. The unit of flux density is the
   (a) tesla **(b)** weber **(c)** ampere-turn **(d)** electron-volt
8. The electromagnetic activation of a movable shaft is the basis for
   (a) relays **(b)** circuit breakers **(c)** magnetic switches **(d)** solenoids
9. When there is current through a wire placed in a magnetic field,
   (a) the wire will overheat **(b)** the wire will become magnetized
   (c) a force is exerted on the wire **(d)** the magnetic field will be cancelled
10. A coil of wire is placed in a changing magnetic field. If the number of turns in the coil is increased, the voltage induced across the coil will
    (a) remain unchanged **(b)** decrease **(c)** increase **(d)** be excessive
11. If a conductor is moved back and forth at a constant rate in a constant magnetic field, the voltage induced in the conductor will
    (a) remain constant **(b)** reverse polarity **(c)** be reduced **(d)** be increased
12. In the crankshaft position sensor in Figure 33, the induced voltage across the coil is caused by
    (a) current in the coil
    (b) rotation of the steel disk
    (c) a tab passing through the magnetic field
    (d) acceleration of the steel disk's rotational speed
13. The purpose of the commutator in a generator or motor is to
    (a) change the direction of the current to the rotor as it spins
    (b) change the direction of the current to the stator windings
    (c) support the motor or generator shaft
    (d) provide the magnetic field for the motor or generator
14. In a motor, back emf serves to
    (a) increase the power from the motor **(b)** decrease the flux
    (c) increase the current in the field windings **(d)** decrease the current in the armature
15. The torque of a motor is proportional to the
    (a) amount of flux **(b)** armature current
    (c) both of the above **(d)** none of the above

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Magnetic Field

1. The cross-sectional area of a magnetic field is increased, but the flux remains the same. Does the flux density increase or decrease?
2. In a certain magnetic field the cross-sectional area is 0.5 m^2 and the flux is $1500\ \mu\text{Wb}$. What is the flux density?

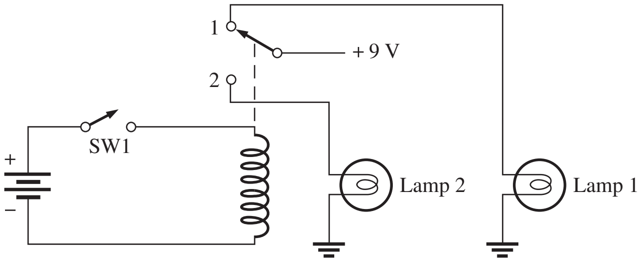
3. What is the flux in a magnetic material when the flux density is 2500×10^{-6} T and the cross-sectional area is 150 cm^2?
4. At a given location, assume the earth's magnetic field is 0.6 G. Express this flux density in tesla.
5. A very strong permanent magnet has a magnetic field of 100,000 μT. Express this flux density in gauss.

SECTION 2 Electromagnetism

6. What happens to the compass needle in Figure 11 when the current through the conductor is reversed?
7. What is the relative permeability of a ferromagnetic material whose absolute permeability is 750×10^{-6} Wb/At·m?
8. Determine the reluctance of a material with a length of 0.28 m and a cross-sectional area of 0.08 m^2 if the absolute permeability is 150×10^{-7} Wb/At·m.
9. What is the magnetomotive force in a 50 turn coil of wire when there are 3 A of current through it?

SECTION 3 Electromagnetic Devices

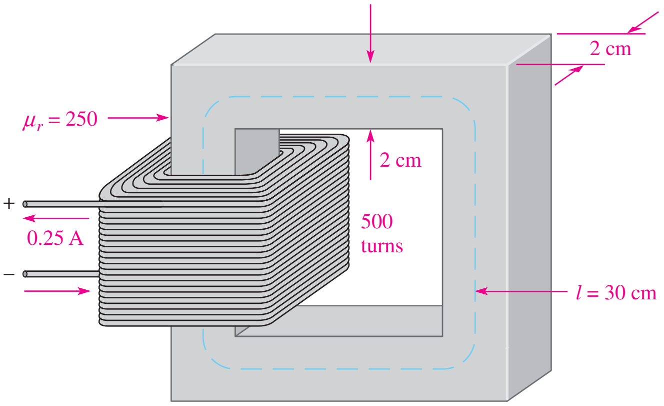
10. Typically, when a solenoid is activated, is the plunger extended or retracted?
11. **(a)** What force moves the plunger when a solenoid is activated?
    (b) What force causes the plunger to return to its at-rest position?
12. Explain the sequence of events in the circuit of Figure 49 starting when switch 1 (SW1) is closed.
13. What causes the pointer in a d'Arsonval movement to deflect when there is current through the coil?



▲ FIGURE 49

SECTION 4 Magnetic Hysteresis

14. What is the magnetizing force in Problem 9 if the length of the core is 0.2 m?
15. How can the flux density in Figure 50 be changed without altering the physical characteristics of the core?

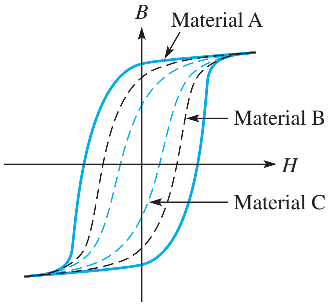


▲ FIGURE 50

16. In Figure 50, there are 500 turns. Determine

(a) H **(b)** ϕ **(c)** B

17. Determine from the hysteresis curves in Figure 51 which material has the most retentivity.



▲ FIGURE 51

SECTION 5 Electromagnetic Induction

18. According to Faraday's law, what happens to the induced voltage across a given coil if the rate of change of magnetic flux doubles?

19. What are three factors that determine the voltage in a conductor that is moving perpendicular to a magnetic field?

20. A magnetic field is changing at a rate of 3500×10^{-3} Wb/s. How much voltage is induced across a 50 turn coil that is placed in the magnetic field?

21. How does Lenz's law complement Faraday's law?

22. In Figure 33, why is there no induced voltage when the steel disk is not rotating?

SECTION 6 The DC Generator

23. Explain the purpose of the commutator and brushes in Figure 35.

***24.** A basic one-loop dc generator is rotated at 60 rev/s. How many times each second does the dc output voltage peak (reach a maximum)?

***25.** Assume that another loop, 90 degrees from the first loop, is added to the dc generator in Problem 24. Make a graph of voltage versus time to show how the output voltage appears. Let the maximum voltage be 10 V.

26. Assume the self-excited shunt dc generator in Figure 41 has a load connected to it that draws 12 A. If the field windings draw 1.0 A, what is the armature current?

27. **(a)** If the output voltage in Problem 26 is 14 V, what power is supplied to the load?

(b) What power is dissipated in the field resistance?

SECTION 7 The DC Motor

28. **(a)** What power is developed by a motor that turns at 1200 rpm and has a torque of 3.0 N-m?

(b) What is the horsepower rating of the motor? (746 W = 1 hp)

29. Assume a motor dissipates 12 W internally when it delivers 50 W to a load. What is the efficiency?

ANSWERS

SECTION CHECKUPS

SECTION 1 The Magnetic Field

1. North poles repel.
2. Magnetic flux is the group of lines of force that makes up a magnetic field. Magnetic flux density is a measure of the concentration of magnetic flux.
3. Gauss and tesla
4. $B = \phi/A = 900\ \mu\text{T}$

SECTION 2 Electromagnetism

1. Electromagnetism is produced by current through a conductor. An electromagnetic field exists only when there is current. A magnetic field exists independently of current.
2. When current reverses, the direction of the magnetic field also reverses.
3. Flux (ϕ) equals magnetomotive force (F_m) divided by reluctance ($\mathcal{R}$).
4. Flux: current, mmf: voltage, reluctance: resistance.

SECTION 3 Electromagnetic Devices

1. A solenoid produces a movement only. A relay provides an electrical contact closure.
2. The movable part of a solenoid is the plunger.
3. The movable part of a relay is the armature.
4. The d'Arsonval movement is based on the interaction of magnetic fields.
5. The spider acts as a spring to return and support the coil in its rest position.

SECTION 4 Magnetic Hysteresis

1. An increase in current increases the flux density.
2. Retentivity is the ability of a material to remain magnetized after removal of the magnetizing force.
3. Heads should not remain magnetized after magnetic force is removed, but tape should.

SECTION 5 Electromagnetic Induction

1. Zero voltage is induced.
2. Induced voltage increases.
3. A force is exerted on the conductor when there is current.
4. The induced voltage is zero.

SECTION 6 The DC Generator

1. The rotor
2. The commutator reverses the current in the rotating coil.
3. Greater resistance will reduce the magnetic flux, causing the output voltage to drop.
4. A generator in which the field windings derive their current from the output

SECTION 7 The DC Motor

1. Back emf is a voltage developed in a motor because of generator action as the rotor turns. It opposes the original supply voltage.
2. Back emf reduces the armature current.
3. A series-wound motor
4. Higher reliability because there are no brushes to wear out

RELATED PROBLEMS FOR EXAMPLES

1 Flux density will decrease.

2 31.0 T

3 The reluctance is reduced to 12.8×10^6 At/Wb.

4 1.66×10^5 At/Wb

5 7.2 mWb

6 **(a)** $F_m = 42.5$ At

(b) $\mathscr{R} = 85 \times 10^3$ At/Wb

7 40 mV

8 12.5 mV

9 The direction is along the negative x axis.

10 525 W

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** F **5.** T

6. T **7.** F **8.** T **9.** T **10.** F

SELF-TEST

1. (b) **2.** (c) **3.** (a) **4.** (d) **5.** (b) **6.** (c) **7.** (a) **8.** (d)

9. (c) **10.** (c) **11.** (b) **12.** (c) **13.** (a) **14.** (d) **15.** (c)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. Decreases

3. 37.5 μWb

5. 1000 G

7. 597

9. 150 At

11. **(a)** Electromagnetic field **(b)** Spring

13. Forces produced by the interaction of the electromagnetic field and the permanent magnetic field

15. Change the current.

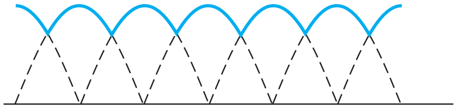
17. Material A

19. The strength of the magnetic field, the length of the conductor exposed to the field, and the rotational rate of the conductor

21. Lenz's law defines the polarity of the induced voltage.

23. The commutator and brush arrangement electrically connect the loop to the external circuit.

25. Figure 52.



▲ FIGURE 52

27. **(a)** 168 W **(b)** 14 W

29. 81%

INTRODUCTION TO ALTERNATING CURRENT AND VOLTAGE

From Chapter 11 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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INTRODUCTION TO ALTERNATING CURRENT AND VOLTAGE

CHAPTER OUTLINE

1 The Sinusoidal Waveform
2 Sinusoidal Voltage and Current Values
3 Angular Measurement of a Sine Wave
4 The Sine Wave Formula
5 Introduction to Phasors
6 Analysis of AC Circuits
7 The Alternator (AC Generator)
8 The AC Motor
9 Nonsinusoidal Waveforms
10 The Oscilloscope
Application Activity

CHAPTER OBJECTIVES

- Identify a sinusoidal waveform and measure its characteristics
- Determine the various voltage and current values of a sine wave
- Describe angular relationships of sine waves
- Mathematically analyze a sinusoidal waveform
- Use a phasor to represent a sine wave
- Apply the basic circuit laws to resistive ac circuits
- Describe how an alternator generates electricity
- Explain how ac motors convert electrical energy into rotational motion
- Identify the characteristics of basic nonsinusoidal waveforms
- Use an oscilloscope to measure waveforms

KEY TERMS

- Waveform
- Sine wave
- Cycle
- Period (T)
- Frequency (f)
- Hertz (Hz)
- Oscillator
- Function generator
- Instantaneous value
- Peak value
- Amplitude
- Peak-to-peak value
- rms value
- Average value
- Degree
- Radian
- Phase
- Phasor
- Angular velocity
- Induction motor
- Synchronous motor
- Squirrel cage
- Slip
- Pulse
- Rise time (t_r)
- Fall time (t_f)
- Pulse width (t_W)
- Periodic
- Duty cycle
- Ramp
- Fundamental frequency
- Harmonics
- Oscilloscope

APPLICATION ACTIVITY PREVIEW

In this application activity, you will learn to measure voltage signals in an AM receiver using an oscilloscope.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

This chapter provides an introduction to ac circuit analysis in which time-varying electrical signals, particularly the sine wave, are studied. An electrical signal is a voltage or current that changes in some consistent manner with time. In other words, the voltage or current fluctuates according to a certain pattern called a waveform.

An alternating voltage is one that changes polarity at a certain rate, and an alternating current is one that changes direction at a certain rate. The sinusoidal waveform (sine wave) is the most common and fundamental type because all other types of repetitive waveforms can be broken down into composite sine waves. The sine wave is a periodic type of waveform that repeats at fixed intervals. The use of phasors to represent sine waves is discussed.

Special emphasis is given to the sinusoidal waveform (sine wave) because of its fundamental importance in ac circuit analysis. Alternators, which generate sine waves, and ac motors are introduced. Other types of waveforms are also introduced, including pulse, triangular, and sawtooth. The use of the oscilloscope for displaying and measuring waveforms is introduced.

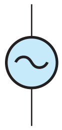
1 THE SINUSOIDAL WAVEFORM

The sinusoidal waveform or *sine wave* is the fundamental type of alternating current (ac) and alternating voltage. It is also referred to as a sinusoidal wave or, simply, sinusoid. The electrical service provided by the power company is in the form of sinusoidal voltage and current. In addition, other types of repetitive **waveforms** are composites of many individual sine waves called harmonics.

After completing this section, you should be able to

- **Identify a sinusoidal waveform and measure its characteristics**
  - Determine the period
  - Determine the frequency
  - Relate the period and the frequency
  - Describe two types of electronic signal generators.

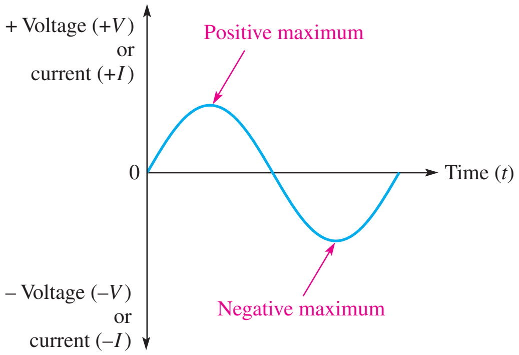
Sinusoidal voltages are produced by two types of sources: rotating electrical machines (ac generators) or electronic oscillator circuits, which are used in instruments commonly known as electronic signal generators. Figure 1 shows the symbol used to represent either source of sinusoidal voltage. AC generators are covered in Section 7, and electronic signal generators are discussed at the end of this section.



▲ FIGURE 1

Symbol for a sinusoidal voltage source.

Figure 2 is a graph showing the general shape of a **sine wave**, which can be either an alternating current or an alternating voltage. Voltage (or current) is displayed on the vertical axis and time (t) is displayed on the horizontal axis. Notice how the voltage (or current) varies with time. Starting at zero, the voltage (or current) increases to a positive maximum (peak), returns to zero, and then increases to a negative maximum (peak) before returning again to zero, thus completing one full cycle.



◀ FIGURE 2

Graph of one cycle of a sine wave.

Polarity of a Sine Wave

As mentioned, a sine wave changes polarity at its zero value; that is, it alternates between positive and negative values. When a sinusoidal voltage source (V_s) is applied to a resistive circuit, as in Figure 3, an alternating sinusoidal current results. When the voltage changes polarity, the current correspondingly changes direction as indicated.

During the positive alternation of the applied voltage V_s, the current is in the direction shown in Figure 3(a). During a negative alternation of the applied voltage, the current

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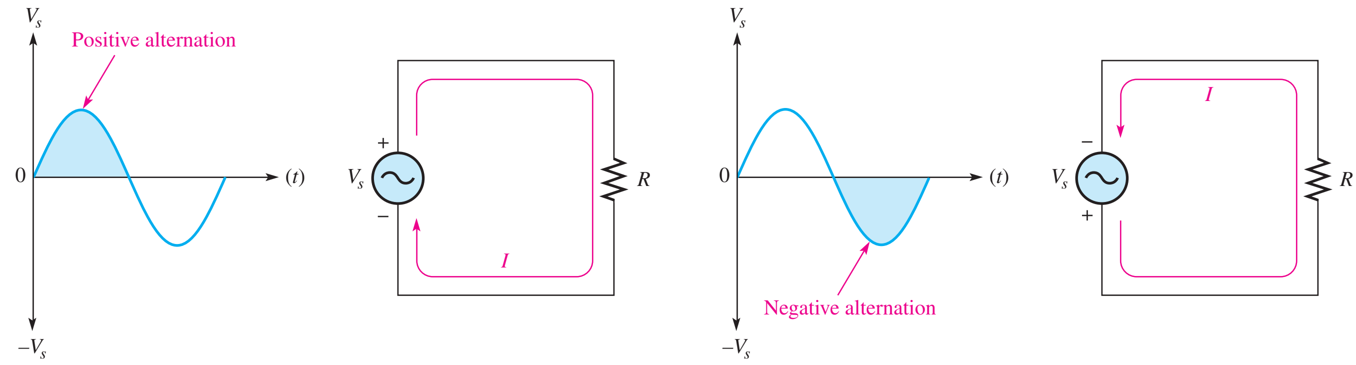
This icon indicates selected websites for further information on topics in this section. See the Companion Websites provided with this text.



(a) During a positive alternation of voltage, current is in the direction shown.

(b) During a negative alternation of voltage, current reverses direction, as shown.

▲ **FIGURE 3**

Alternating current and voltage.

is in the opposite direction, as shown in Figure 3(b). The combined positive and negative alternations make up one **cycle** of a sine wave.

Period of a Sine Wave

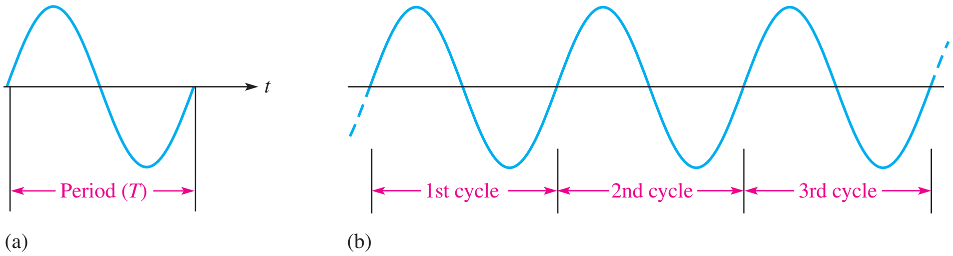
A sine wave varies with time (t) in a definable manner.

The time required for a sine wave to complete one full cycle is called the period (T).

Figure 4(a) illustrates the period of a sine wave. Typically, a sine wave continues to repeat itself in identical cycles, as shown in Figure 4(b). Since all cycles of a repetitive sine wave are the same, the period is always a fixed value for a given sine wave. The period of a sine wave can be measured from a zero crossing to the next corresponding zero crossing, as indicated in Figure 4(a). The period can also be measured from any peak in a given cycle to the corresponding peak in the next cycle.

▶ **FIGURE 4**

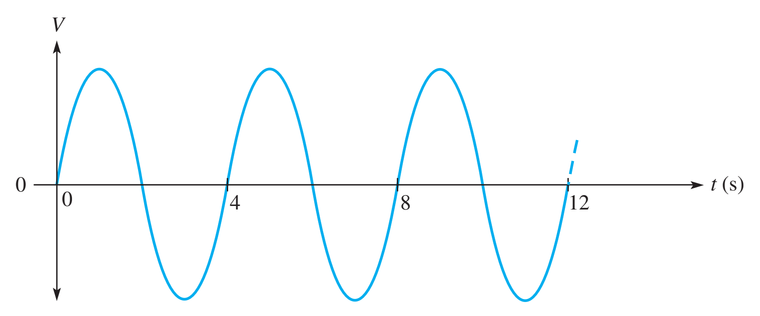
The period of a sine wave is the same for each cycle.



EXAMPLE 1

What is the period of the sine wave in Figure 5?

▶ **FIGURE 5**



Solution As shown in Figure 5, it takes four seconds (4 s) to complete each cycle. Therefore, the period is 4 s.

$$T = \mathbf{4\ s}$$

*Related Problem** What is the period if the sine wave goes through five cycles in 12 s?

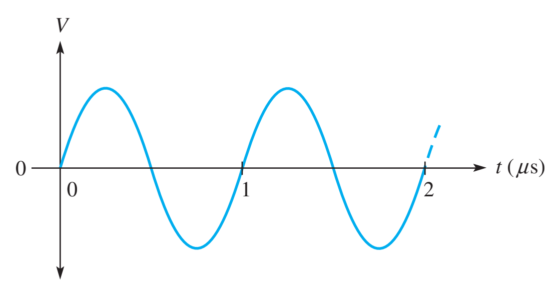
---

*Answers are at the end of the chapter.

EXAMPLE 2

Show three possible ways to measure the period of the sine wave in Figure 6. How many cycles are shown?

▶ FIGURE 6



Solution **Method 1:** The period can be measured from one zero crossing to the corresponding zero crossing in the next cycle (the slope must be the same at the corresponding zero crossings).

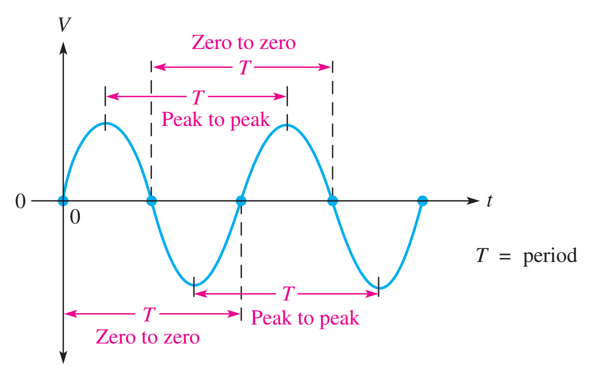
Method 2: The period can be measured from the positive peak in one cycle to the positive peak in the next cycle.

Method 3: The period can be measured from the negative peak in one cycle to the negative peak in the next cycle.

These measurements are indicated in Figure 7, where **two cycles of the sine wave** are shown. Keep in mind that you obtain the same value for the period no matter which corresponding points on the waveform you use.

▶ FIGURE 7

Measurement of the period of a sine wave.



Related Problem If a positive peak occurs at 1 ms and the next positive peak occurs at 2.5 ms, what is the period?

Frequency of a Sine Wave

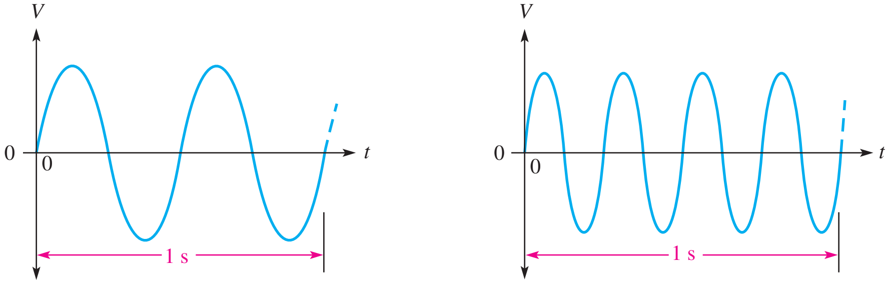
Frequency (f) is the number of cycles that a sine wave completes in one second.

The more cycles completed in one second, the higher the frequency. Frequency (f) is measured in units of hertz. One **hertz (Hz)** is equivalent to one cycle per second; 60 Hz is 60 cycles per second, for example. Figure 8 shows two sine waves. The sine wave in part (a) completes two full cycles in one second. The one in part (b) completes four cycles in one second. Therefore, the sine wave in part (b) has twice the frequency of the one in part (a).

HISTORY NOTE

Heinrich Rudolf Hertz 1857–1894

Hertz, a German physicist, was the first to broadcast and receive electromagnetic (radio) waves. He produced electromagnetic waves in the labratory and measured their parameters. Hertz also proved that the nature of the reflection and refraction of electromagnetic waves was the same as that of light. The unit of frequency is named in his honor. (Photo credit: Deutsches Museum, courtesy AIP Emilio Segrè Visual Archives.)



(a) Lower frequency: fewer cycles per second (b) Higher frequency: more cycles per second

FIGURE 8

Illustration of frequency.

Relationship of Frequency and Period

The formulas for the relationship between frequency (f) and period (T) are as follows:

Equation 1

$$f = \frac{1}{T}$$

Equation 2

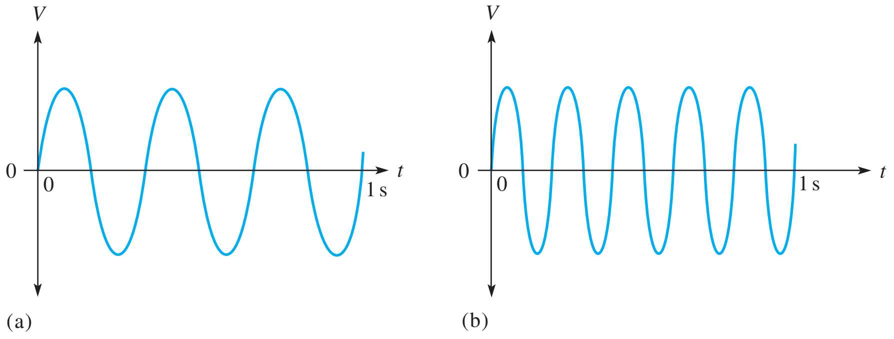
$$T = \frac{1}{f}$$

There is a reciprocal relationship between f and T. Knowing one, you can calculate the other with the x^{-1} or $1/x$ key on your calculator. This inverse relationship makes sense because a sine wave with a longer period goes through fewer cycles in one second than one with a shorter period.

EXAMPLE 3

Which sine wave in Figure 9 has a higher frequency? Determine the frequency and the period of both waveforms.

FIGURE 9



Solution The sine wave in Figure 9(b) has the higher frequency because it completes more cycles in 1 s than does the sine wave in part (a).

In Figure 9(a), three cycles are completed in 1 s; therefore,

$$f = \mathbf{3\ Hz}$$

One cycle takes 0.333 s (one-third second), so the period is

$$T = 0.333\ \text{s} = \mathbf{333\ ms}$$

In Figure 9(b), five cycles are completed in 1 s; therefore,

$$f = \mathbf{5\ Hz}$$

One cycle takes 0.2 s (one-fifth second), so the period is

$$T = 0.2\ \text{s} = \mathbf{200\ ms}$$

Related Problem If the time between negative peaks of a given sine wave is 50 μs, what is the frequency?

EXAMPLE 4 The period of a certain sine wave is 10 ms. What is the frequency?

Solution Use Equation 1.

$$f = \frac{1}{T} = \frac{1}{10\ \text{ms}} = \frac{1}{10 \times 10^{-3}\ \text{s}} = \mathbf{100\ Hz}$$

Related Problem A certain sine wave goes through four cycles in 20 ms. What is the frequency?

EXAMPLE 5 The frequency of a sine wave is 60 Hz. What is the period?

Solution Use Equation 2.

$$T = \frac{1}{f} = \frac{1}{60\ \text{Hz}} = \mathbf{16.7\ ms}$$

Related Problem If $T = 15\ \mu$s, what is f?

Electronic Signal Generators

The signal generator is an instrument that electronically produces sine waves for use in testing or controlling electronic circuits and systems. There are a variety of signal generators, ranging from special-purpose instruments that produce only one type of waveform in a limited frequency range, to programmable instruments that produce a wide range of frequencies and a variety of waveforms. All signal generators consist basically of an **oscillator**, which is an electronic circuit that produces repetitive waves. All generators have controls for adjusting the amplitude and frequency.

Function Generators and Arbitrary Waveform Generators A **function generator** is an instrument that produces more than one type of waveform. It provides pulse waveforms as well as sine waves and triangular waves. Typical function generators are shown in Figure 10(a).

(a) Examples of function generators

(b) A typical arbitrary waveform generator

FIGURE 10

Typical signal generators. Copyright © Tektronix, Inc. Reproduced by permission.

An arbitrary waveform generator can be used to generate standard signals like sine waves, triangular waves, and pulses as well as signals with various shapes and characteristics. Waveforms can be defined by mathematical or graphical input. A typical arbitrary waveform generator is shown in Figure 10(b).

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Describe one cycle of a sine wave.
2. At what point does a sine wave change polarity?
3. How many maximum points does a sine wave have during one cycle?
4. How is the period of a sine wave measured?
5. Define *frequency,* and state its unit.
6. Determine f when $T = 5\ \mu s$.
7. Determine T when $f = 120$ Hz.

2 SINUSOIDAL VOLTAGE AND CURRENT VALUES

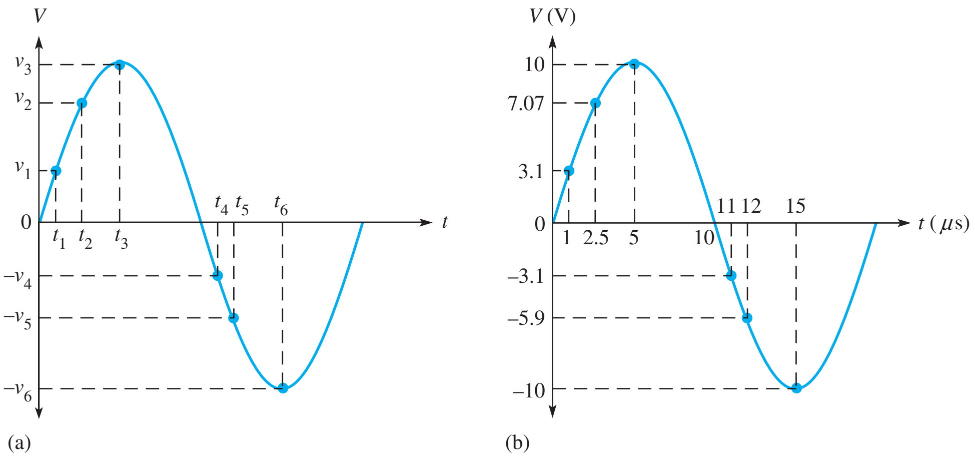
Five ways to express the value of a sine wave in terms of its voltage or its current magnitude are instantaneous, peak, peak-to-peak, rms, and average values.

After completing this section, you should be able to

- **Determine the various voltage and current values of a sine wave**
  - Find the instantaneous value at any point
  - Find the peak value
  - Find the peak-to-peak value
  - Define *rms*
  - Explain why the average value is always zero over a complete cycle
  - Find the half-cycle average value

Instantaneous Value

Figure 11 illustrates that at any point in time on a sine wave, the voltage (or current) has an **instantaneous value**. This instantaneous value is different at different points along the



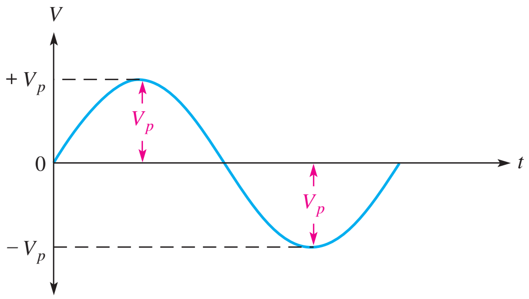
◄ **FIGURE 11**

Instantaneous values.

curve. Instantaneous values are positive during the positive alternation and negative during the negative alternation. Instantaneous values of voltage and current are symbolized by lowercase v and i, respectively. The curve in part (a) shows voltage only, but it applies equally for current when the v's are replaced with i's. An example of instantaneous values is shown in part (b) where the instantaneous voltage is 3.1 V at 1 μs, 7.07 V at 2.5 μs, 10 V at 5 μs, 0 V at 10 μs, −3.1 V at 11 μs, and so on.

Peak Value

The **peak value** of a sine wave is the value of voltage (or current) at the positive or the negative maximum (peak) with respect to zero. Since the positive and negative peak values are equal in **magnitude**, a sine wave is characterized by a single peak value. This is illustrated in Figure 12. For a given sine wave, the peak value is constant and is represented by V_p or I_p. The peak value is also called the **amplitude**.

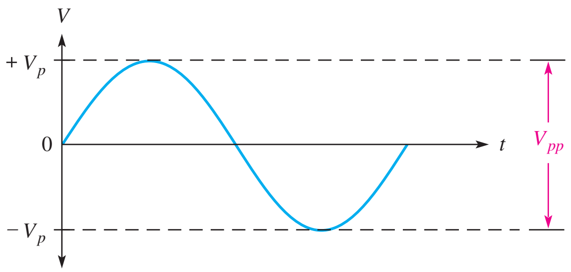


◄ **FIGURE 12**

Peak values.

Peak-to-Peak Value

The **peak-to-peak value** of a sine wave, as shown in Figure 13, is the voltage or current from the positive peak to the negative peak. It is always twice the peak value as



◄ **FIGURE 13**

Peak-to-peak value.

expressed in the following equations. Peak-to-peak voltage or current values are represented by V_{pp} or I_{pp}.

Equation 3

$$V_{pp} = 2V_p$$

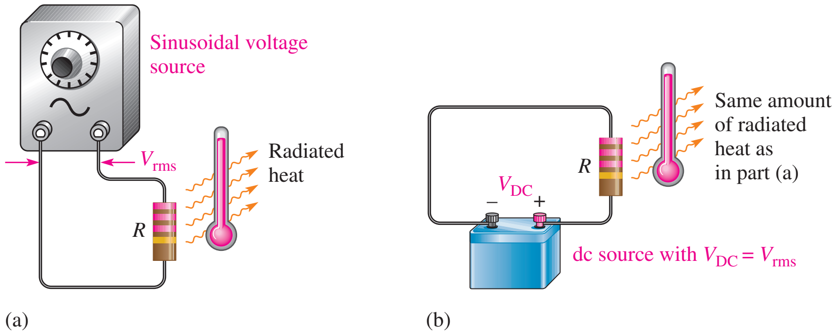
Equation 4

$$I_{pp} = 2I_p$$

RMS Value

The term *rms* stands for *root mean square.* Most ac voltmeters display rms voltage. The 120 V at your wall outlet is an rms value. The **rms value**, also referred to as the **effective value**, of a sinusoidal voltage is actually a measure of the heating effect of the sine wave. For example, when a resistor is connected across an ac (sinusoidal) voltage source, as shown in Figure 14(a), a certain amount of heat is generated by the power in the resistor. Figure 14(b) shows the same resistor connected across a dc voltage source. The value of the dc voltage can be adjusted so that the resistor gives off the same amount of heat as it does when connected to the ac source.

The rms value of a sinusoidal voltage is equal to the dc voltage that produces the same amount of heat in a resistance as does the sinusoidal voltage.


Sinusoidal voltage source
V_{rms}
R
Radiated heat
(a)
V_{DC}
−
+
R
Same amount of radiated heat as in part (a)
dc source with $V_{DC} = V_{rms}$
(b)

▲ FIGURE 14

When the same amount of heat is produced in both setups, the sinusoidal voltage has an rms value equal to the dc voltage.

The peak value of a sine wave can be converted to the corresponding rms value using the following relationships for either voltage or current:

Equation 5

$$V_{rms} = 0.707V_p$$

Equation 6

$$I_{rms} = 0.707I_p$$

Using these formulas, you can also determine the peak value if you know the rms value.

$$V_p = \frac{V_{rms}}{0.707}$$

Equation 7

$$V_p = 1.414V_{rms}$$

Similarly,

Equation 8

$$I_p = 1.414I_{rms}$$

To get the peak-to-peak value, simply double the peak value.

Equation 9

$$V_{pp} = 2.828V_{rms}$$

and

Equation 10

$$I_{pp} = 2.828I_{rms}$$

Average Value

The average value of a sine wave taken over one complete cycle is always zero because the positive values (above the zero crossing) offset the negative values (below the zero crossing).

To be useful for certain purposes such as measuring types of voltages found in power supplies, the average value of a sine wave is defined over a half-cycle rather than over a full cycle. The **average value** is the total area under the half-cycle curve divided by the distance in radians of the curve along the horizontal axis. The result is derived in Appendix B and is expressed in terms of the peak value as follows for both voltage and current sine waves:

$$V_{avg} = \left(\frac{2}{\pi}\right)V_p$$

$$V_{avg} = 0.637V_p$$

Equation 11

$$I_{avg} = \left(\frac{2}{\pi}\right)I_p$$

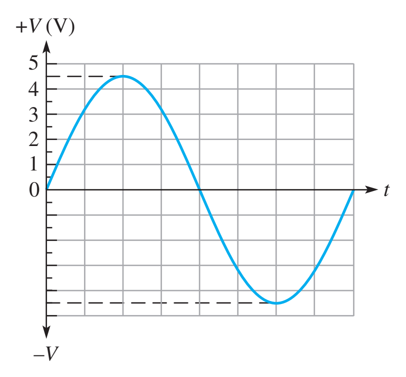
$$I_{avg} = 0.637I_p$$

Equation 12

EXAMPLE 6

Determine V_p, V_{pp}, V_{rms}, and the half-cycle V_{avg} for the sine wave in Figure 15.

▶ **FIGURE 15**



Solution V_p = **4.5 V** is read directly from the graph. From this, calculate the other values.

$$V_{pp} = 2V_p = 2(4.5\text{ V}) = \mathbf{9\text{ V}}$$

$$V_{rms} = 0.707V_p = 0.707(4.5\text{ V}) = \mathbf{3.18\text{ V}}$$

$$V_{avg} = 0.637V_p = 0.637(4.5\text{ V}) = \mathbf{2.87\text{ V}}$$

Related Problem If $V_p = 25$ V, determine V_{pp}, V_{rms}, and V_{avg} for a voltage sine wave.

SECTION 2 CHECKUP

1. Determine V_{pp} in each case when
   (a) $V_p = 1$ V **(b)** $V_{rms} = 1.414$ V **(c)** $V_{avg} = 3$ V
2. Determine V_{rms} in each case when
   (a) $V_p = 2.5$ V **(b)** $V_{pp} = 10$ V **(c)** $V_{avg} = 1.5$ V
3. Determine the half-cycle V_{avg} in each case when
   (a) $V_p = 10$ V **(b)** $V_{rms} = 2.3$ V **(c)** $V_{pp} = 60$ V

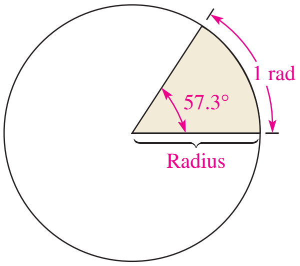
3 ANGULAR MEASUREMENT OF A SINE WAVE

As you have seen, sine waves can be measured along the horizontal axis on a time basis; however, since the time for completion of one full cycle or any portion of a cycle is frequency-dependent, it is often useful to specify points on the sine wave in terms of an angular measurement expressed in degrees or radians.

After completing this section, you should be able to

- **Describe angular relationships of sine waves**
  - Show how to measure a sine wave in terms of angles
  - Define *radian*
  - Convert radians to degrees
  - Determine the phase angle of a sine wave

A sinusoidal voltage can be produced by an alternator, which is an ac generator. There is a direct relationship between the rotation of the rotor in an alternator and the sine wave output. Thus, the angular measurement of the rotor's position is directly related to the angle assigned to the sine wave.



▲ FIGURE 16

Angular measurement showing relationship of radian (rad) to degrees (°).

Angular Measurement

A **degree** is an angular measurement corresponding to 1/360 of a circle or a complete revolution. A **radian** is the angular measurement along the circumference of a circle that is equal to the radius of the circle. One radian (rad) is equivalent to 57.3°, as illustrated in Figure 16. In a 360° revolution, there are 2π radians.

The Greek letter π (pi) represents the ratio of the circumference of any circle to its diameter and has a constant value of approximately 3.1416.

Scientific calculators have a π function so that the actual numerical value does not have to be entered.

Table 1 lists several values of degrees and the corresponding radian values. These angular measurements are illustrated in Figure 17.

▶ TABLE 1

DEGREES (°)	RADIANS (RAD)
0	0
45	$\pi/4$
90	$\pi/2$
135	$3\pi/4$
180	π
225	$5\pi/4$
270	$3\pi/2$
315	$7\pi/4$
360	2π

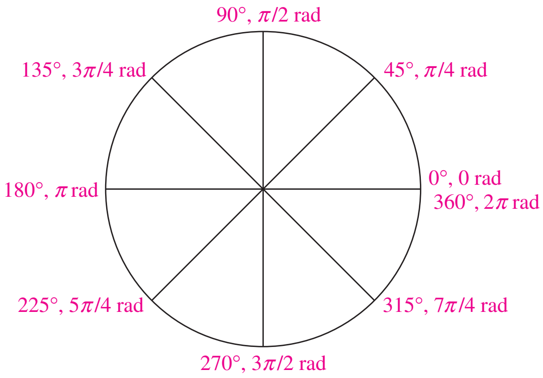

90°, π/2 rad
135°, 3π/4 rad
45°, π/4 rad
180°, π rad
0°, 0 rad
360°, 2π rad
225°, 5π/4 rad
315°, 7π/4 rad
270°, 3π/2 rad

FIGURE 17

Angular measurements starting at 0° and going counterclockwise.

Radian/Degree Conversion

Degrees can be converted to radians.

$$\text{rad} = \left(\frac{\pi\ \text{rad}}{180^\circ}\right) \times \text{degrees}$$

Equation 13

Similarly, radians can be converted to degrees.

$$\text{degrees} = \left(\frac{180^\circ}{\pi\ \text{rad}}\right) \times \text{rad}$$

Equation 14

EXAMPLE 7

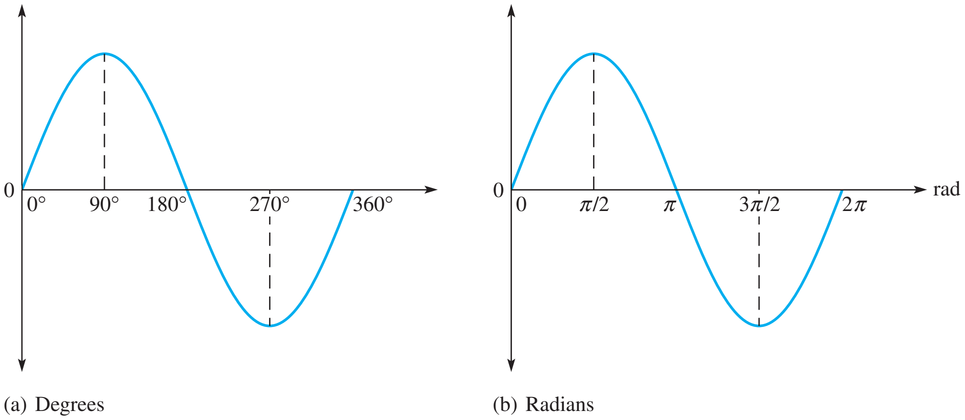
(a) Convert 60° to radians. **(b)** Convert π/6 rad to degrees.

Solution **(a)** $\text{Rad} = \left(\frac{\pi\ \text{rad}}{180^\circ}\right)60^\circ = \boldsymbol{\frac{\pi}{3}}\ \textbf{rad}$ **(b)** $\text{Degrees} = \left(\frac{180^\circ}{\pi\ \text{rad}}\right)\left(\frac{\pi}{6}\ \text{rad}\right) = \mathbf{30^\circ}$

Related Problem **(a)** Convert 15° to radians. **(b)** Convert 5π/8 rad to degrees.

Sine Wave Angles

The angular measurement of a sine wave is based on 360° or 2π rad for a complete cycle. A half-cycle is 180° or π rad; a quarter-cycle is 90° or π/2 rad; and so on. Figure 18(a) shows angles in degrees for a full cycle of a sine wave; part (b) shows the same points in radians.


0
0°
90°
180°
270°
360°
0
0
π/2
π
3π/2
2π
rad

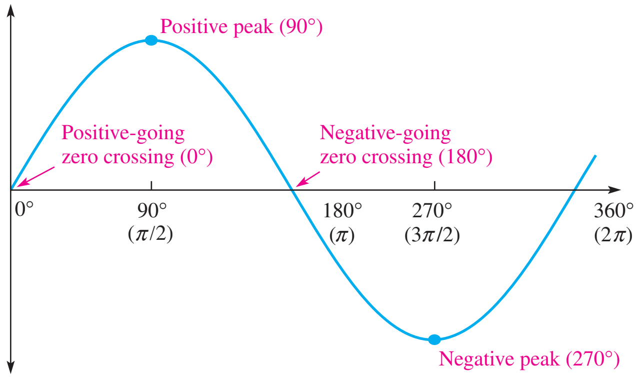
(a) Degrees (b) Radians

FIGURE 18

Sine wave angles.

Phase of a Sine Wave

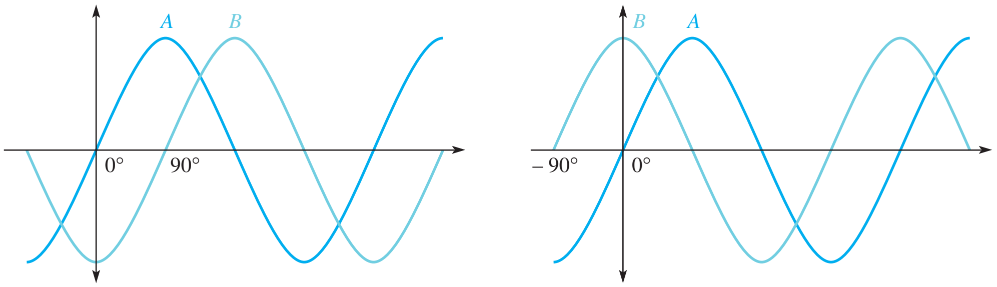
The **phase** of a sine wave is an angular measurement that specifies the position of that sine wave relative to a reference. Figure 19 shows one cycle of a sine wave to be used as the reference. Note that the first positive-going crossing of the horizontal axis (zero crossing) is at 0° (0 rad), and the positive peak is at 90° ($\pi/2$ rad). The negative-going zero crossing is at 180° (π rad), and the negative peak is at 270° ($3\pi/2$ rad). The cycle is completed at 360° (2π rad). When the sine wave is shifted left or right with respect to this reference, there is a phase shift.



▲ FIGURE 19

Phase reference.

Figure 20 illustrates phase shifts of a sine wave. In part (a), sine wave *B* is shifted to the right by 90° ($\pi/2$ rad) with respect to sine wave *A*. Thus, there is a phase angle of 90° between sine wave *A* and sine wave *B*. In terms of time, the positive peak of sine wave *B* occurs later than the positive peak of sine wave *A* because time increases to the right along the horizontal axis. In this case, sine wave *B* is said to **lag** sine wave *A* by 90° or $\pi/2$ radians. Stated another way, sine wave *A* leads sine wave *B* by 90°.



(a) *A* leads *B* by 90°, or *B* lags *A* by 90°.

(b) *B* leads *A* by 90°, or *A* lags *B* by 90°.

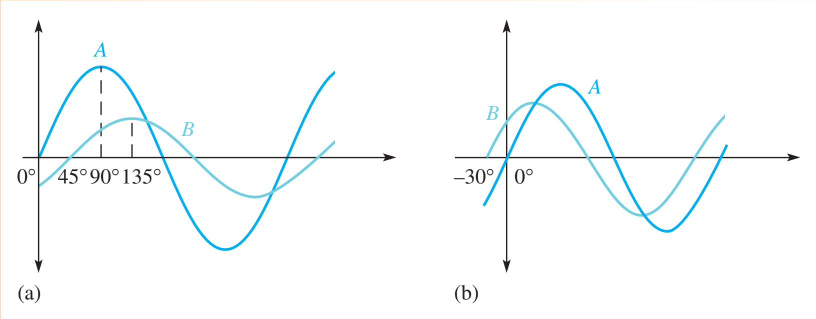
▲ FIGURE 20

Illustration of a phase shift.

In Figure 20(b), sine wave *B* is shown shifted left by 90° with respect to sine wave *A*. Thus, again there is a phase angle of 90° between sine wave *A* and sine wave *B*. In this case, the positive peak of sine wave *B* occurs earlier in time than that of sine wave *A*; therefore, sine wave *B* is said to **lead** sine wave *A* by 90°.

EXAMPLE 8

What are the phase angles between the two sine waves in parts (a) and (b) of Figure 21?



▲ **FIGURE 21**

Solution In Figure 21(a) the zero crossing of sine wave A is at 0°, and the corresponding zero crossing of sine wave B is at 45°. There is a 45° phase angle between the two waveforms with sine wave B lagging sine wave A.

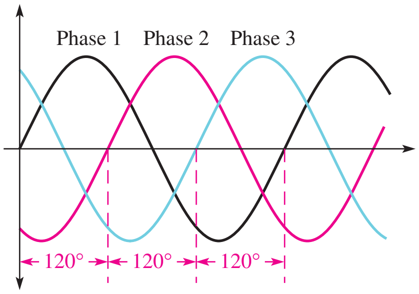
In Figure 21(b) the zero crossing of sine wave B is at $-30°$, and the corresponding zero crossing of sine wave A is at 0°. There is a 30° phase angle between the two waveforms with sine wave B leading sine wave A.

Related Problem If the positive-going zero crossing of one sine wave is at 15° and that of the second sine wave is at 23°, what is the phase angle between them?

As a practical matter, when you measure the phase shift between two waveforms on an oscilloscope, you should make them appear to have the same amplitude. This is done by taking one of the oscilloscope channels out of vertical calibration and adjusting the corresponding waveform until its apparent amplitude equals that of the other waveform. This procedure eliminates the error caused if both waveforms are not measured at their exact center.

Polyphase Power

One important application of phase-shifted sine waves is in electrical power systems. Electrical utilities generate ac with three phases that are separated by 120°, as shown in Figure 22. The reference is called neutral. Normally, three-phase power is delivered to the user with four lines (three hot lines and neutral). There are important advantages to three-phase power for ac motors. Three-phase motors are more efficient and simpler than an equivalent single-phase motor. Motors are discussed further in Section 8.



◀ **FIGURE 22**

Three-phase power waveforms.

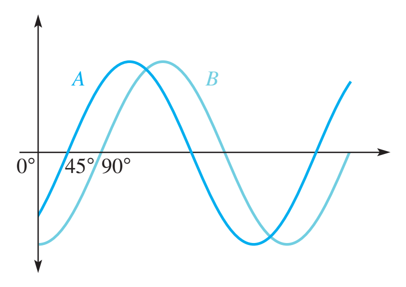
The three phases can be split up by the utility company to supply three separate single-phase systems. If only one of the three phases plus neutral is supplied, the result is standard

120 V, which is single-phase power. Single-phase power is distributed to residential and small commercial buildings; it consists of two 120 V hot lines that are 180° out of phase with each other and a neutral, which is grounded at the service entrance. The two hot lines allow for connecting to 240 V for high-power appliances (dryers, air conditioners).

SECTION 3 CHECKUP

1. When the positive-going zero crossing of a sine wave occurs at 0°, at what angle does each of the following points occur?
   - **(a)** Positive peak
   - **(b)** Negative-going zero crossing
   - **(c)** Negative peak
   - **(d)** End of first complete cycle
2. A half-cycle is completed in ________ degrees or ________ radians.
3. A full cycle is completed in ________ degrees or ________ radians.
4. Determine the phase angle between the two sine waves in Figure 23.

FIGURE 23



4 THE SINE WAVE FORMULA

A sine wave can be graphically represented by voltage or current values on the vertical axis and by angular measurement (degrees or radians) along the horizontal axis. This graph can be expressed mathematically, as you will see.

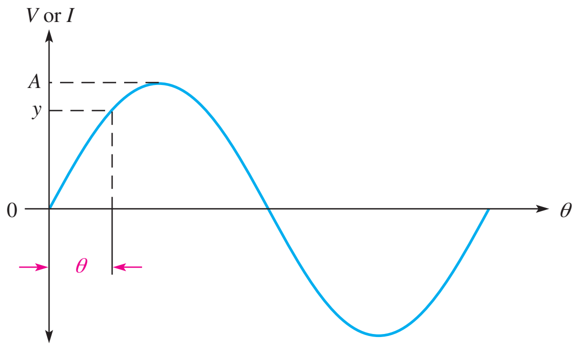
After completing this section, you should be able to

- **Mathematically analyze a sinusoidal waveform**
  - State the sine wave formula
  - Find instantaneous values using the sine wave formula

A generalized graph of one cycle of a sine wave is shown in Figure 24. The sine wave amplitude (A) is the maximum value of the voltage or current on the vertical axis; angular values run along the horizontal axis. The variable y is an instantaneous value that

FIGURE 24

One cycle of a generic sine wave showing amplitude and phase.



represents either voltage or current at a given angle, θ. The symbol θ is the Greek letter *theta.*

All electrical sine waves follow a specific mathematical formula. The general expression for the sine wave curve in Figure 24 is

$$y = A \sin\theta$$

Equation 15

This formula states that any point on the sine wave, represented by an instantaneous value (y), is equal to the maximum value A times the sine (sin) of the angle θ at that point. For example, a certain voltage sine wave has a peak value of 10 V. You can calculate the instantaneous voltage at a point 60° along the horizontal axis as follows, where $y = v$ and $A = V_p$:

$$v = V_p \sin\theta = (10\text{ V})\sin 60° = (10\text{ V})(0.866) = 8.66\text{ V}$$

Figure 25 shows this particular instantaneous value of the curve. You can find the sine of any angle on most calculators by first entering the value of the angle and then pressing the SIN key. Verify that your calculator is in the degree mode.

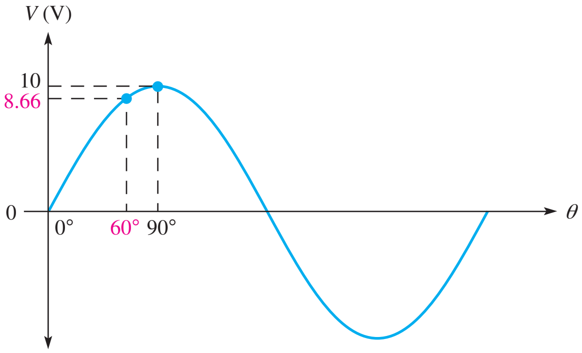


FIGURE 25

Illustration of the instantaneous value of a voltage sine wave at $\theta = 60°$.

Expressions for Phase-Shifted Sine Waves

When a sine wave is shifted to the right of the reference (lagging) by a certain angle, ϕ (Greek letter phi), as illustrated in Figure 26(a) where the reference is the vertical axis, the general expression is

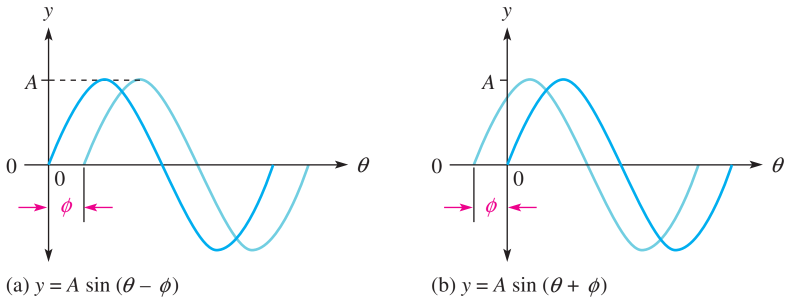
$$y = A \sin(\theta - \phi)$$

Equation 16

where y represents instantaneous voltage or current, and A represents the peak value (amplitude). When a sine wave is shifted to the left of the reference (leading) by a certain angle, ϕ, as shown in Figure 26(b), the general expression is

$$y = A\sin(\theta + \phi)$$

Equation 17



(a) $y = A \sin(\theta - \phi)$

(b) $y = A \sin(\theta + \phi)$

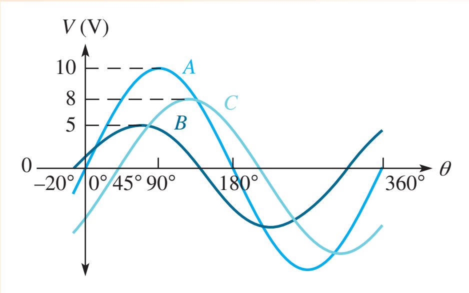
FIGURE 26

Shifted sine waves.

EXAMPLE 9

Determine the instantaneous value at the 90° reference point on the horizontal axis for each voltage sine wave in Figure 27.

▶ **FIGURE 27**



Solution Sine wave A is the reference. Sine wave B is shifted left by 20° with respect to A, so it leads. Sine wave C is shifted right by 45° with respect to A, so it lags.

$$\begin{aligned} v_A &= V_p \sin\theta \\ &= (10\text{ V})\sin(90°) = (10\text{ V})(1) = \mathbf{10\text{ V}} \\ v_B &= V_p \sin(\theta + \phi_B) \\ &= (5\text{ V})\sin(90° + 20°) = (5\text{ V})\sin(110°) = (5\text{ V})(0.9397) = \mathbf{4.70\text{ V}} \\ v_C &= V_p \sin(\theta - \phi_C) \\ &= (8\text{ V})\sin(90° - 45°) = (8\text{ V})\sin(45°) = (8\text{ V})(0.7071) = \mathbf{5.66\text{ V}} \end{aligned}$$

Related Problem A voltage sine wave has a peak value of 20 V. What is its instantaneous value at 65° from its zero crossing?

SECTION 4 CHECKUP

1. Calculate the instantaneous value at 120° for the voltage sine wave in Figure 25.
2. Determine the instantaneous value at 45° of a voltage sine wave that leads the reference by 10° ($V_p = 10$ V).
3. Find the instantaneous value of 90° of a voltage sine wave that leads the reference by 25° ($V_p = 5$ V).

5 INTRODUCTION TO PHASORS

Phasors provide a graphic means for representing quantities that have both magnitude and direction (angular position). Phasors are especially useful for representing sine waves in terms of their magnitude and phase angle and also for the analysis of reactive circuits.

After completing this section, you should be able to

- **Use a phasor to represent a sine wave**
    - Define *phasor*
    - Explain how phasors are related to the sine wave formula
    - Draw a phasor diagram
    - Discuss angular velocity

You may already be familiar with vectors. In math and science, a vector is any quantity with both magnitude and direction. Examples of vectors are force, velocity, and acceleration. The simplest way to describe a vector is to assign a magnitude and an angle to a quantity.

In electronics, a **phasor** is a type of vector but the term generally refers to quantities that vary with time, such as sine waves. Examples of phasors are shown in Figure 28. The length of the phasor "arrow" represents the magnitude of a quantity. The angle, θ (relative to 0°), represents the angular position, as shown in part (a) for a positive angle. The specific phasor example in part (b) has a magnitude of 2 and a phase angle of 45°. The phasor in part (c) has a magnitude of 3 and a phase angle of 180°. The phasor in part (d) has a magnitude of 1 and a phase angle of $-45°$ (or $+315°$). Notice that positive angles are measured counterclockwise (CCW) from the reference (0°) and negative angles are measured clockwise (CW) from the reference.

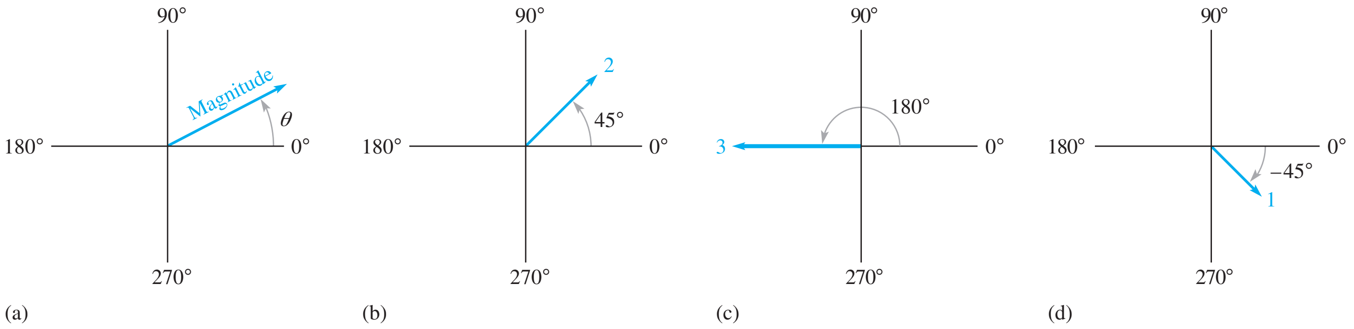


FIGURE 28

Examples of phasors.

Phasor Representation of a Sine Wave

A full cycle of a sine wave can be represented by rotation of a phasor through 360 degrees.

> **The instantaneous value of the sine wave at any point is equal to the vertical distance from the tip of the phasor to the horizontal axis.**

Figure 29 shows how the phasor traces out the sine wave as it goes from 0° to 360°. You can relate this concept to the rotation in an ac generator. Notice that the length of the phasor is equal to the peak value of the sine wave (observe the 90° and the 270° points). The angle of the phasor measured from 0° is the corresponding angular point on the sine wave.

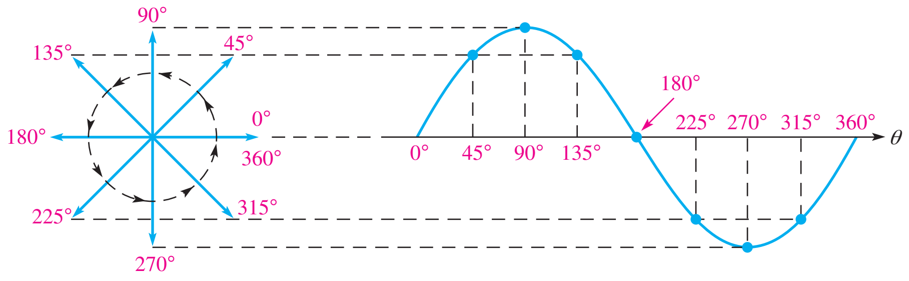
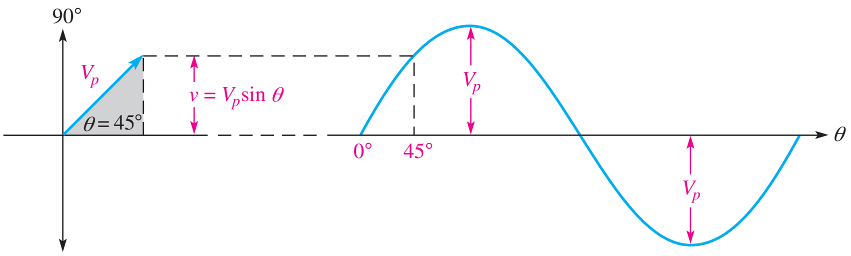


FIGURE 29

Sine wave represented by rotational phasor motion.

Phasors and the Sine Wave Formula

Let's examine a phasor representation at one specific angle. Figure 30 shows a voltage phasor at an angular position of 45° and the corresponding point on the sine wave. The instantaneous value of the sine wave at this point is related to both the position and the length of the



▲ FIGURE 30

Right triangle derivation of sine wave formula.

phasor. As previously mentioned, the vertical distance from the phasor tip down to the horizontal axis represents the instantaneous value of the sine wave at that point.

Notice that when a vertical line is drawn from the phasor tip down to the horizontal axis, a right triangle is formed, as shown shaded in Figure 30. The length of the phasor is the hypotenuse of the triangle, and the vertical projection is the opposite side. From trigonometry,

The opposite side of a right triangle is equal to the hypotenuse times the sine of the angle θ.

The length of the phasor is the peak value of the sinusoidal voltage, V_p. Thus, the opposite side of the triangle, which is the instantaneous value, can be expressed as

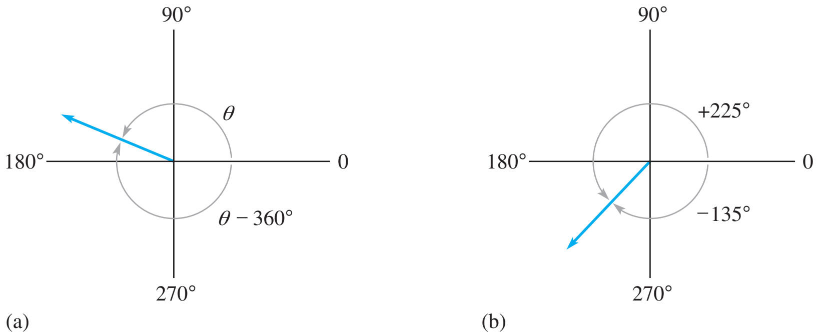
$$v = V_p \sin \theta$$

Recall that this formula is the one stated earlier for calculating instantaneous sinusoidal voltage. A similar formula applies to a sinusoidal current.

$$i = I_p \sin \theta$$

Positive and Negative Phasor Angles

The position of a phasor at any instant can be expressed as a positive angle, as you have seen, or as an equivalent negative angle. Positive angles are measured counterclockwise from 0°. Negative angles are measured clockwise from 0°. For a given positive angle θ, the corresponding negative angle is $\theta - 360°$, as illustrated in Figure 31(a). In part (b), a specific example is shown. The angle of the phasor in this case can be expressed as $+225°$ or $-135°$.

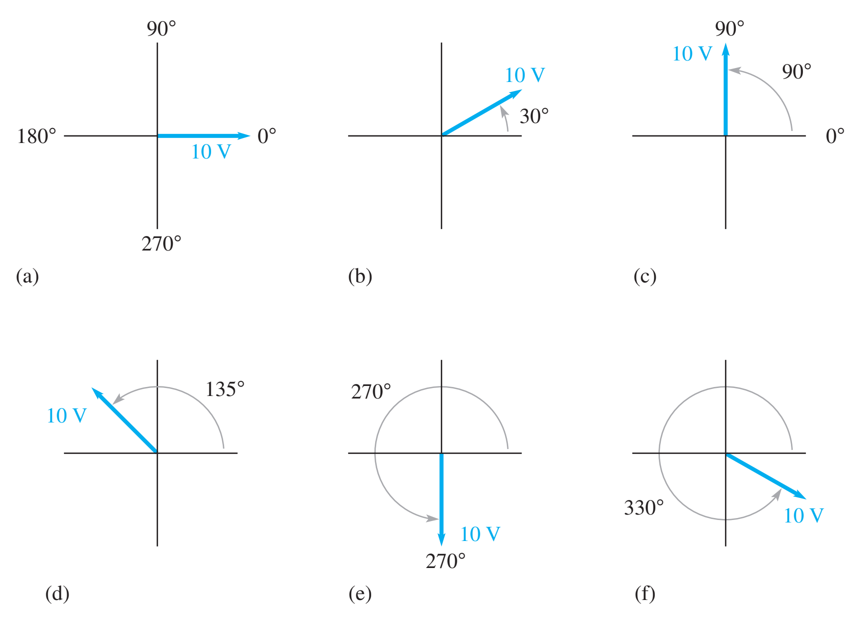


▲ FIGURE 31

Positive and negative phasor angles.

EXAMPLE 10

For the phasor in each part of Figure 32, determine the instantaneous voltage value. Also express each positive angle shown as an equivalent negative angle. The length of each phasor represents the peak value of the sinusoidal voltage.



▲ FIGURE 32

Solution

(a) $v = (10\text{ V})\sin 0° = (10\text{ V})(0) = \mathbf{0\text{ V}}$
$0° - 360° = \mathbf{-360°}$

(b) $v = (10\text{ V})\sin 30° = (10\text{ V})(0.5) = \mathbf{5\text{ V}}$
$30° - 360° = \mathbf{-330°}$

(c) $v = (10\text{ V})\sin 90° = (10\text{ V})(1) = \mathbf{10\text{ V}}$
$90° - 360° = \mathbf{-270°}$

(d) $v = (10\text{ V})\sin 135° = (10\text{ V})(0.707) = \mathbf{7.07\text{ V}}$
$135° - 360° = \mathbf{-225°}$

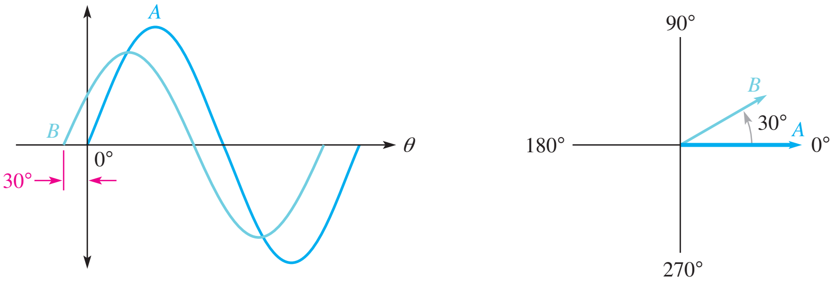
(e) $v = (10\text{ V})\sin 270° = (10\text{ V})(-1) = \mathbf{-10\text{ V}}$
$270° - 360° = \mathbf{-90°}$

(f) $v = (10\text{ V})\sin 330° = (10\text{ V})(-0.5) = \mathbf{-5\text{ V}}$
$330° - 360° = \mathbf{-30°}$

Related Problem If a phasor is at 45° and its length represents 15 V, what is the instantaneous sine wave value?

Phasor Diagrams

A phasor diagram can be used to show the relative relationship of two or more sine waves of the same frequency. A phasor in a *fixed position* is used to represent a complete sine wave because once the phase angle between two or more sine waves of the same frequency or between the sine wave and a reference is established, the phase angle remains constant throughout the cycles. For example, the two sine waves in Figure 33(a) can be represented by a phasor diagram, as shown in part (b). As you can see, sine wave *B* leads sine wave *A* by 30° and has less amplitude than sine wave *A*, as indicated by the lengths of the phasors.



(a) (b)

FIGURE 33

Example of a phasor diagram representing sinusoidal waveforms.

EXAMPLE 11 Use a phasor diagram to represent the sine waves in Figure 34.

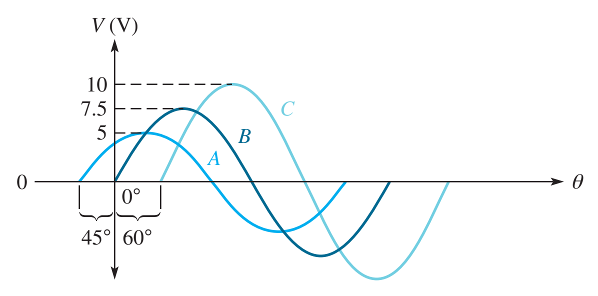
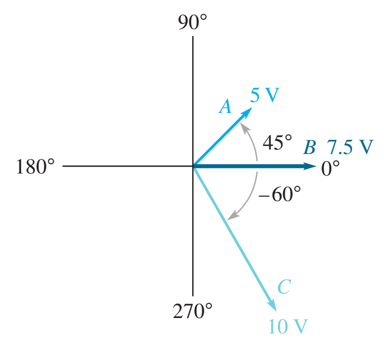


FIGURE 34

Solution The phasor diagram representing the sine waves is shown in Figure 35. The length of each phasor represents the peak value of the sine wave.

FIGURE 35



Related Problem Describe a phasor to represent a 5 V peak sine wave that lags sine wave *C* in Figure 34 by 25°.

Angular Velocity of a Phasor

As you have seen, one cycle of a sine wave is traced out when a phasor is rotated through 360 degrees or 2π radians. The faster it is rotated, the faster the sine wave cycle is traced out. Thus, the period and frequency are related to the velocity of rotation of the phasor. The velocity of rotation is called the **angular velocity** and is designated ω (the small Greek letter omega).

When a phasor rotates through 2π radians, one complete cycle is traced out. Therefore, the time required for the phasor to go through 2π radians is the period of the sine wave. Because the phasor rotates through 2π radians in a time equal to the period, T, the angular velocity can be expressed as

$$\omega = \frac{2\pi}{T}$$

Since $f = 1/T$,

$$\omega = 2\pi f$$

Equation 18

When a phasor is rotated at an angular velocity ω, then ωt is the angle through which the phasor has passed at any instant. Therefore, the following relationship can be stated:

$$\theta = \omega t$$

Equation 19

Substituting $2\pi f$ for ω results in $\theta = 2\pi ft$. With this relationship between angle and time, the equation for the instantaneous value of a sinusoidal voltage, $v = V_p \sin\theta$, can be written as

$$v = V_p \sin 2\pi ft$$

Equation 20

You can calculate the instantaneous value at any point in time along the sine wave curve if you know the frequency and peak value. The unit of $2\pi ft$ is the radian so your calculator must be in the radian mode.

EXAMPLE 12

What is the value of a sinusoidal voltage at 3 μs from the positive-going zero crossing when $V_p = 10$ V and $f = 50$ kHz?

Solution

$$\begin{aligned} v &= V_p \sin 2\pi ft \\ &= (10\text{ V}) \sin\left[2\pi(50\text{ kHz})(3 \times 10^{-6}\text{ s})\right] = \mathbf{8.09\ V} \end{aligned}$$

Related Problem

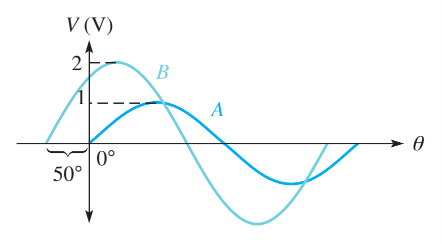
What is the value of a sinusoidal voltage at 12 μs from the positive-going zero crossing when $V_p = 50$ V and $f = 10$ kHz?

SECTION 5 CHECKUP

1. What is a phasor?
2. What is the angular velocity of a phasor representing a sine wave with a frequency of 1500 Hz?
3. A certain phasor has an angular velocity of 628 rad/s. To what frequency does this correspond?

4. Draw a phasor diagram to represent the two sine waves in Figure 36. Use peak values.

▶ FIGURE 36



6 ANALYSIS OF AC CIRCUITS

When a time-varying ac voltage such as a sinusoidal voltage is applied to a circuit, the circuit laws and power formulas that you studied earlier still apply. Ohm's law, Kirchhoff's laws, and the power formulas apply to ac circuits in the same way that they apply to dc circuits.

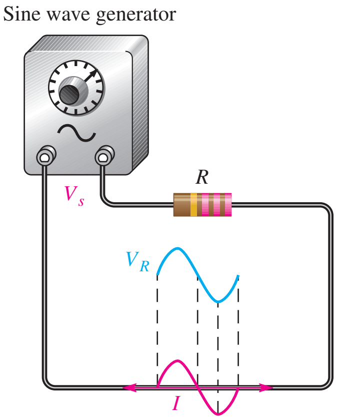
After completing this section, you should be able to

- **Apply the basic circuit laws to resistive ac circuits**
  - Apply Ohm's law to resistive circuits with ac sources
  - Apply Kirchhoff's voltage law and current law to resistive circuits with ac sources
  - Determine power in resistive ac circuits
  - Determine total voltages that have both ac and dc components

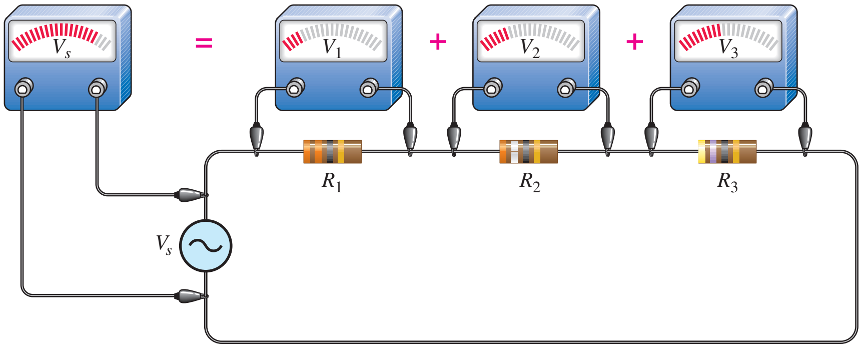
If a sinusoidal voltage is applied across a resistor as shown in Figure 37, there is a sinusoidal current. The current is zero when the voltage is zero and is maximum when the voltage is maximum. When the voltage changes polarity, the current reverses direction. As a result, the voltage and current are said to be in phase with each other.

▶ FIGURE 37

A sinusoidal voltage produces a sinusoidal current.



When you use Ohm's law in ac circuits, remember that both the voltage and the current must be expressed consistently, that is, both as peak values, both as rms values, both as average values, and so on. Kirchhoff's voltage and current laws apply to ac circuits as well as to dc circuits. Figure 38 illustrates Kirchhoff's voltage law in a resistive circuit that has a sinusoidal voltage source. The source voltage is the sum of all the voltage drops across the resistors, just as in a dc circuit.



▲ FIGURE 38

Illustration of Kirchhoff's voltage law in an ac circuit.

Power in resistive ac circuits is determined the same as for dc circuits except that you must use rms values of current and voltage. Recall that the rms value of a sine wave voltage is equivalent to a dc voltage of the same value in terms of its heating effect. The general power formulas are restated for a resistive ac circuit as

$$P = V_{\text{rms}}I_{\text{rms}}$$

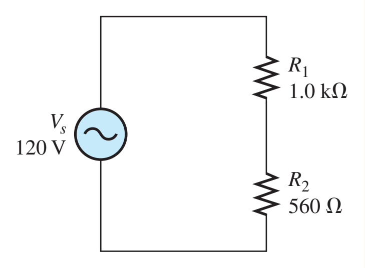
$$P = \frac{V^2_{\text{rms}}}{R}$$

$$P = I^2_{\text{rms}}R$$

EXAMPLE 13

Determine the rms voltage across each resistor and the rms current in Figure 39. The source voltage is given as an rms value. Also determine the total power.

▶ FIGURE 39



Solution The total resistance of the circuit is

$$R_{tot} = R_1 + R_2 = 1.0\,\text{k}\Omega + 560\,\Omega = 1.56\,\text{k}\Omega$$

Use Ohm's law to find the rms current.

$$I_{\text{rms}} = \frac{V_{s(\text{rms})}}{R_{tot}} = \frac{120\text{ V}}{1.56\,\text{k}\Omega} = \mathbf{76.9\ mA}$$

The rms voltage drop across each resistor is

$$V_{1(\text{rms})} = I_{\text{rms}}R_1 = (76.9\text{ mA})(1.0\text{ k}\Omega) = \mathbf{76.9\text{ V}}$$
$$V_{2(\text{rms})} = I_{\text{rms}}R_2 = (76.9\text{ mA})(560\ \Omega) = \mathbf{43.1\text{ V}}$$

The total power is

$$P_{tot} = I^2_{\text{rms}}R_{tot} = (76.9\text{ mA})^2(1.56\text{ k}\Omega) = 9.23\text{ W}$$

Related Problem Repeat this example for a source voltage of 10 V peak.

Use Multisim files E11-13A and E11-13B to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 14

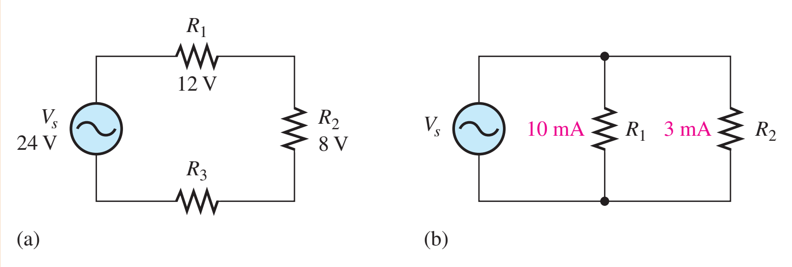
All values in Figure 40 are given in rms.

(a) Find the unknown peak voltage drop in Figure 40(a).

(b) Find the total rms current in Figure 40(b).

(c) Find the total power in Figure 40(b) if $V_{\text{rms}} = 24$ V.

► FIGURE 40



Solution **(a)** Use Kirchhoff's voltage law to find V_3.

$$V_s = V_1 + V_2 + V_3$$
$$V_{3(\text{rms})} = V_{s(\text{rms})} - V_{1(\text{rms})} - V_{2(\text{rms})} = 24\text{ V} - 12\text{ V} - 8\text{ V} = 4\text{ V}$$

Convert rms to peak.

$$V_{3(p)} = 1.414V_{3(\text{rms})} = 1.414(4\text{ V}) = \mathbf{5.66\text{ V}}$$

(b) Use Kirchhoff's current law to find I_{tot}.

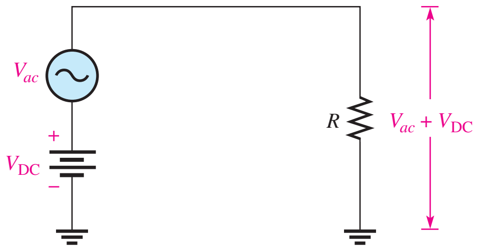
$$I_{tot(\text{rms})} = I_{1(\text{rms})} + I_{2(\text{rms})} = 10\text{ mA} + 3\text{ mA} = \mathbf{13\text{ mA}}$$

(c) $P_{tot} = V_{\text{rms}}I_{\text{rms}} = (24\text{ V})(13\text{ mA}) = \mathbf{312\text{ mW}}$

Related Problem A series circuit has the following voltage drops: $V_{1(\text{rms})} = 3.50$ V, $V_{2(p)} = 4.25$ V, $V_{3(\text{avg})} = 1.70$ V. Determine the peak-to-peak source voltage.

Superimposed DC and AC Voltages

In many practical circuits, you will find both dc and ac voltages combined. An example of this is in amplifier circuits where ac signal voltages are superimposed on dc operating voltages. This is a common application of the superposition theorem.

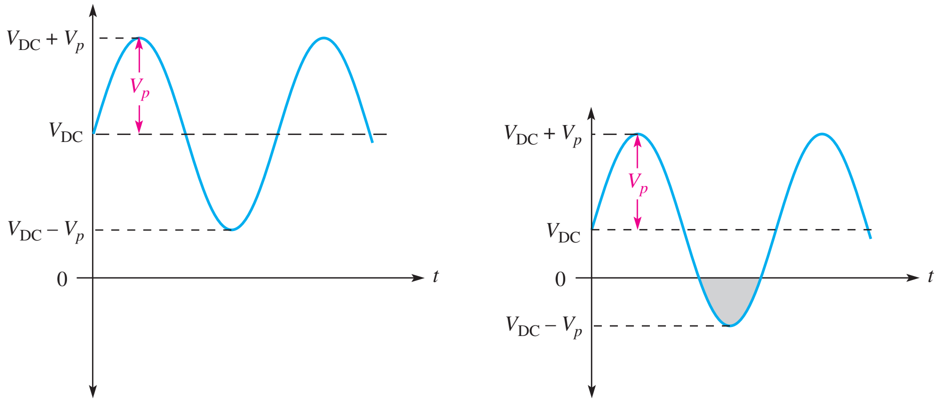


◀ **FIGURE 41**

Superimposed dc and ac voltages.

Figure 41 shows a dc source and an ac source in series. These two voltages will add algebraically to produce an ac voltage "riding" on a dc level, as measured across the resistor.

If V_{DC} is greater than the peak value of the sinusoidal voltage, the combined voltage is a sine wave that never reverses polarity and is therefore nonalternating. That is, the sine wave is riding on a dc level, as shown in Figure 42(a). If V_{DC} is less than the peak value of the sine wave, the sine wave will be negative during a portion of its lower half-cycle, as illustrated in Figure 42(b), and is therefore alternating. In either case, the sine wave will reach a maximum voltage equal to $V_{DC} + V_p$, and it will reach a minimum voltage equal to $V_{DC} - V_p$.



(a) $V_{DC} > V_p$. The sine wave never goes negative.

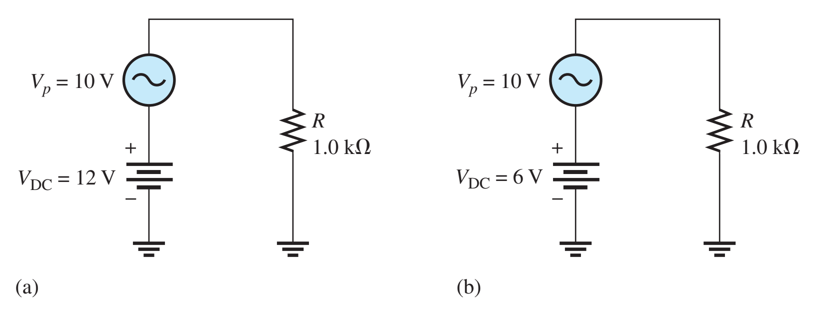
(b) $V_{DC} < V_p$. The sine wave reverses polarity during a portion of its cycle, as indicated by the gray area.

▲ **FIGURE 42**

Sine waves with dc levels.

EXAMPLE 15

Determine the maximum and minimum voltage across the resistor in each circuit of Figure 43.



▲ **FIGURE 43**

Solution In Figure 43(a), the maximum voltage across *R* is

$$V_{max} = V_{\text{DC}} + V_p = 12\text{ V} + 10\text{ V} = \mathbf{22\text{ V}}$$

The minimum voltage across *R* is

$$V_{min} = V_{\text{DC}} - V_p = 12\text{ V} - 10\text{ V} = \mathbf{2\text{ V}}$$

Therefore, $V_{R(tot)}$ is a nonalternating sine wave that varies from +22 V to +2 V, as shown in Figure 44(a).

In Figure 43(b), the maximum voltage across *R* is

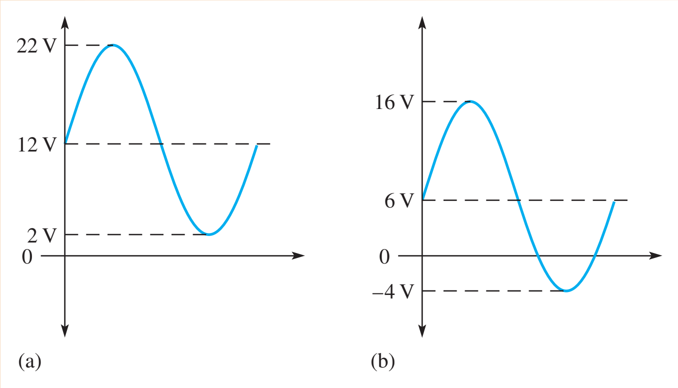
$$V_{max} = V_{\text{DC}} + V_p = 6\text{ V} + 10\text{ V} = \mathbf{16\text{ V}}$$

The minimum voltage across *R* is

$$V_{min} - = V_{\text{DC}} - V_p = \mathbf{-4}\text{ V}$$

Therefore, $V_{R(tot)}$ is an alternating sine wave that varies from +16 V to −4 V, as shown in Figure 44(b).

▶ FIGURE 44



(a) (b)

Related Problem Explain why the waveform in Figure 44(a) is nonalternating but the waveform in part (b) is considered to be alternating.

Use Multisim files E11-15A and E11-15B to verify the calculated results in this example.

SECTION 6 CHECKUP

1. A sinusoidal voltage with a half-cycle average value of 12.5 V is applied to a circuit with a resistance of 330 Ω. What is the peak current in the circuit?
2. The peak voltage drops in a series resistive circuit are 6.2 V, 11.3 V, and 7.8 V. What is the rms value of the source voltage?
3. What is the maximum positive value of the resulting total voltage when a sine wave with $V_p = 5$ V is added to a dc voltage of +2.5 V?
4. Will the resulting voltage in Question 3 alternate polarity?
5. If the dc voltage in Question 3 is −2.5 V, what is the maximum positive value of the resulting total voltage?

7 THE ALTERNATOR (AC GENERATOR)

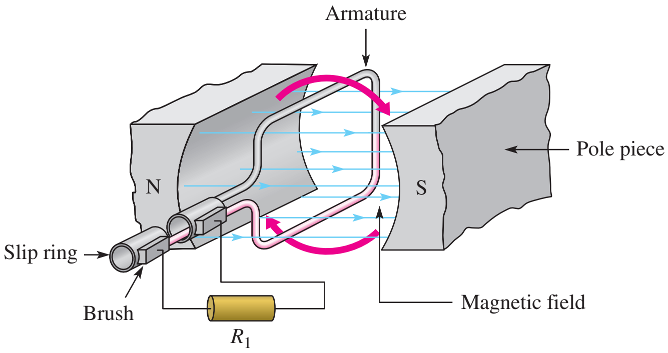
An **alternator** is an ac generator that converts energy of motion into electrical energy. Although it is similar to a dc generator, the alternator is more efficient than the dc generator. Alternators are widely used in vehicles, boats, and other applications even when dc is the final output.

After completing this section, you should be able to

- **Describe how an alternator generates electricity**
  - Identify the main parts of an alternator, including the rotor, stator, and slip rings
  - Explain why the output of a rotating-field alternator is taken from the stator
  - Describe the purpose of the slip rings
  - Explain how an alternator can be used to produce dc

Simplified Alternator

Both the dc generator and the alternator, which generates ac voltage, are based on the principle of electromagnetic induction that produces a voltage when there is relative motion between a magnetic field and a conductor. For a simplified alternator, a single rotating loop passes permanent magnetic poles. The natural voltage that is generated by a rotating loop is ac. In an alternator, instead of the split rings used in a dc generator, solid rings called slip rings are used to connect to the rotor, and the output is ac. The simplest form of an alternator has the same appearance as a dc generator except for the slip rings, as shown in Figure 45.



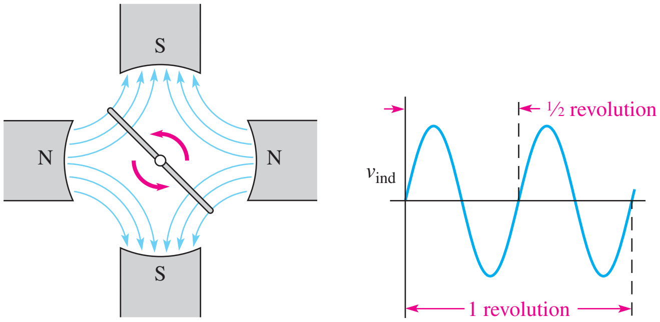
▲ FIGURE 45

A simplified alternator.

Frequency

In the simplified alternator in Figure 45, each revolution of the loop produces one cycle of a sine wave. The positive and negative peaks occur when the loop cuts the maximum number of flux lines. The rate the loop spins determines the time for one complete cycle and the frequency. If it takes 1/60 of a second to make a revolution, the period of the sine wave is 1/60 of a second and the frequency is 60 Hz. Thus, the faster the loop rotates, the higher the frequency.

Another way of achieving a higher frequency is to use more magnetic poles. When four poles are used instead of two, as shown in Figure 46, the conductor passes under a north and a south pole during one-half a revolution, which doubles the frequency. Alternators can



▲ FIGURE 46

Four poles produce twice the frequency of two poles for the same rotational speed.

have many more poles, depending on the requirements, some have as many as 100. The number of poles and the speed of the rotor determine the frequency in accordance with the following equation:

Equation 21

$$f = \frac{Ns}{120}$$

where f is the frequency in hertz, N is the number of poles, and s is the rotational speed in revolutions per minute.

EXAMPLE 16 Assume a large alternator is turned by a turbine at 300 rpm and has 24 poles. What is the output frequency?

Solution

$$f = \frac{Ns}{120} = \frac{(24)(300\text{ rpm})}{120} = \mathbf{60\text{ Hz}}$$

Related Problem At what speed must the rotor move to produce a 50 Hz output?

Practical Alternators

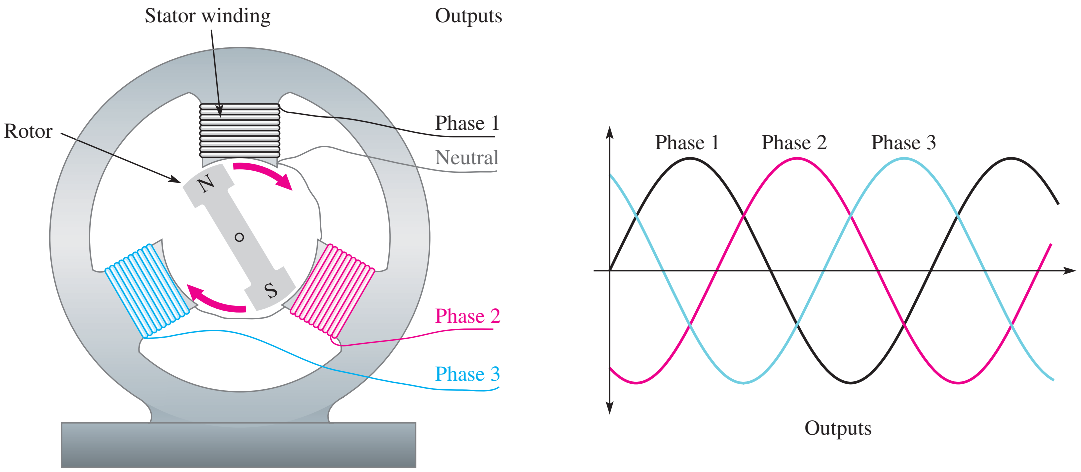
The single loop used in our simplified alternator produces only a tiny voltage. In a practical alternator, hundreds of loops are wound on a magnetic core, which forms the rotor. Practical alternators usually have fixed windings surrounding the rotor instead of permanent magnets. Depending on the type of alternator, these fixed windings can either provide the magnetic field (in which case they are called field windings) or act as the fixed conductors that produce the output (in which case they are the armature windings).

Rotating-Armature Alternators In a rotating-armature alternator, the magnetic field is stationary and is supplied by permanent magnets or electromagnets operated from dc. With electromagnets, field windings are used instead of permanent magnets and provide a fixed magnetic field that interacts with the rotor coils. Power is generated in the rotating assembly and supplied to the load through the slip rings.

In a rotating-armature alternator, the rotor is the component from which power is taken. In addition to hundreds of windings, the practical rotating-armature alternator usually has many pole pairs in the stator that alternate as north and south poles, which serve to increase the output frequency.

Rotating-Field Alternator The rotating-armature alternator is generally limited to low-power applications because all output current must pass through the slip rings and brushes. To avoid this problem, rotating-field alternators take the output from the stator coils and use a rotating magnet, hence the name. Small alternators may have a permanent magnet for a rotor, but most use an electromagnet formed by a wound rotor. A relatively small amount of dc is supplied to the rotor (through the slip rings) to power the electromagnet. As the rotating magnetic field sweeps by the stator windings, power is generated in the stator. The stator is therefore the armature in this case.

Figure 47 shows how a rotating-field alternator can generate three-phase sine waves. (For simplicity, a permanent magnet is shown for the rotor.) AC is generated in each winding as the north pole and the south pole of the rotor alternately sweep by a stator winding. If the north pole generates the positive portion of the sine wave, the south pole will generate the negative portion; thus, one rotation produces a complete sine wave. Each winding has a sine wave output; but because the windings are separated by 120°, the three sine waves are also shifted by 120°. This produces the three-phase output as shown. Most alternators generate three-phase voltage because it is more efficient to produce and is widely used in industry. If the final output is dc, three-phase is easier to convert to dc.



▲ **FIGURE 47**

The rotor shown is a permanent magnet that produces a strong magnetic field. As it sweeps by each stator winding, a sine wave is produced across that winding. The neutral is the reference.

Rotor Current

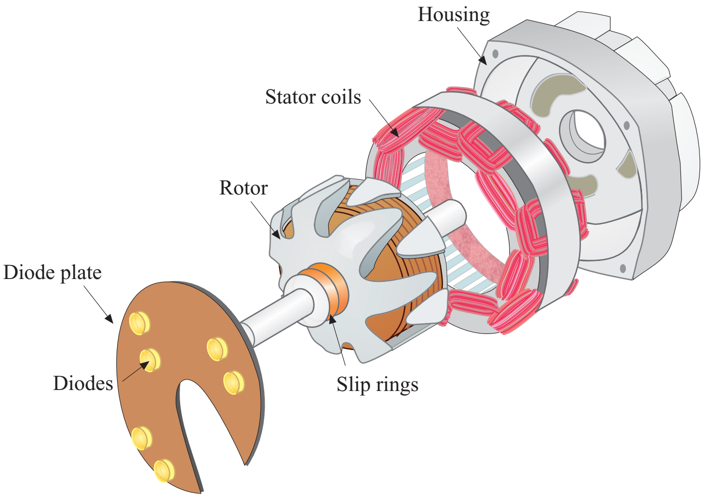
A wound rotor offers important control advantages to alternators. A wound rotor enables control over the strength of the magnetic field by controlling the rotor current and hence the output voltage. For wound rotors, dc must be supplied to the rotor. This current is usually supplied through brushes and slip rings, which are made of a continuous ring of material, (unlike a commutator, which is segmented). Because the brushes need to pass only the magnetizing rotor current, they last longer and are smaller than the brushes in an equivalent dc generator, which pass all of the output current.

In wound-rotor alternators, the only current through the brushes and slip rings is the dc that is used to maintain its magnetic field. The dc is usually derived from a small portion of the output current, which is taken from the stator and converted to dc. Large alternators, such as in power stations, may have a separate dc generator, called an **exciter**, to supply current to the field coils. An exciter can respond very fast to changes in output voltage to keep the alternator's output constant, an important consideration in high-power alternators. Some exciters are set up using a stationary field with the armature on the rotating main shaft. The result is a brushless system because the exciter output is on the rotating shaft. A brushless system eliminates the primary maintenance issue with large alternators in cleaning, repairing, and replacing brushes.

An Application

Alternators are used in nearly all modern automobiles, trucks, tractors, and other vehicles. In vehicles, the output is usually three-phase ac taken from stator windings and then converted to dc with diodes that are housed inside the alternator case. (Diodes are solid-state devices that allow current in only one direction.) Current to the rotor is controlled by a voltage regulator, which is also internal to the alternator. The voltage regulator keeps the output voltage relatively constant for engine speed changes or changing loads. Alternators have replaced dc generators in automobiles and most other applications because they are more efficient and more reliable.

Important parts of a small alternator, such as you might find in an automobile are shown in Figure 48. Like the self-excited generator, the rotor has a small residual magnetism to begin with, so an ac voltage is generated in the stators as soon as the rotor starts spinning. This ac is converted to dc by a set of diodes. A portion of the dc is used to provide current to the rotor; the rest is available for the loads. The amount of current required by the rotor is much less than the total current from the alternator, so it can easily provide the required current to the load.



▲ **FIGURE 48**

Expanded and simplified view of a rotor, stator, and diode plate for a small alternator that produces dc.

In addition to the fact that three-phase voltage is more efficient to produce, it can produce a stable dc output easily by using two diodes in each winding. Since vehicles require dc for the charging system and loads, the output of the alternator is converted to dc internally using a diode array mounted on a diode plate. Thus, a standard three-phase automotive alternator will normally have six diodes inside to convert the output to dc. (Some alternators have six independent stator coils and twelve diodes.)

SECTION 7 CHECKUP

1. What two factors affect the frequency of an alternator?
2. What is the advantage of taking the output from the stator in a rotating-field alternator?
3. What is an exciter?
4. What is the purpose of the diodes in an automobile alternator?

8 THE AC MOTOR

Motors are electromagnetic devices that represent the most common loads for ac in power applications. AC motors are used to operate household appliances such as heat pumps, refrigerators, washers, dryers, and vacuums. In industry, ac motors are used in many applications to move and process materials as well as refrigeration and heating units, machining operations, pumps, and much more. In this section, the two major types of ac motors, induction motors and synchronous motors, are introduced.

After completing this section, you should be able to

- **Explain how ac motors convert electrical energy into rotational motion**
  - Cite the main differences between induction and synchronous motors
  - Explain how the magnetic field rotates in an ac motor
  - Explain how an induction motor develops torque

AC Motor Classification

The two major classifications of ac motors are induction motors and synchronous motors. Several considerations determine which of these types are best for any given application. These considerations include the speed and power requirements, voltage rating, load characteristics (such as starting torque required), efficiency requirements, maintenance requirements, and operating environment (such as underwater operation or temperature).

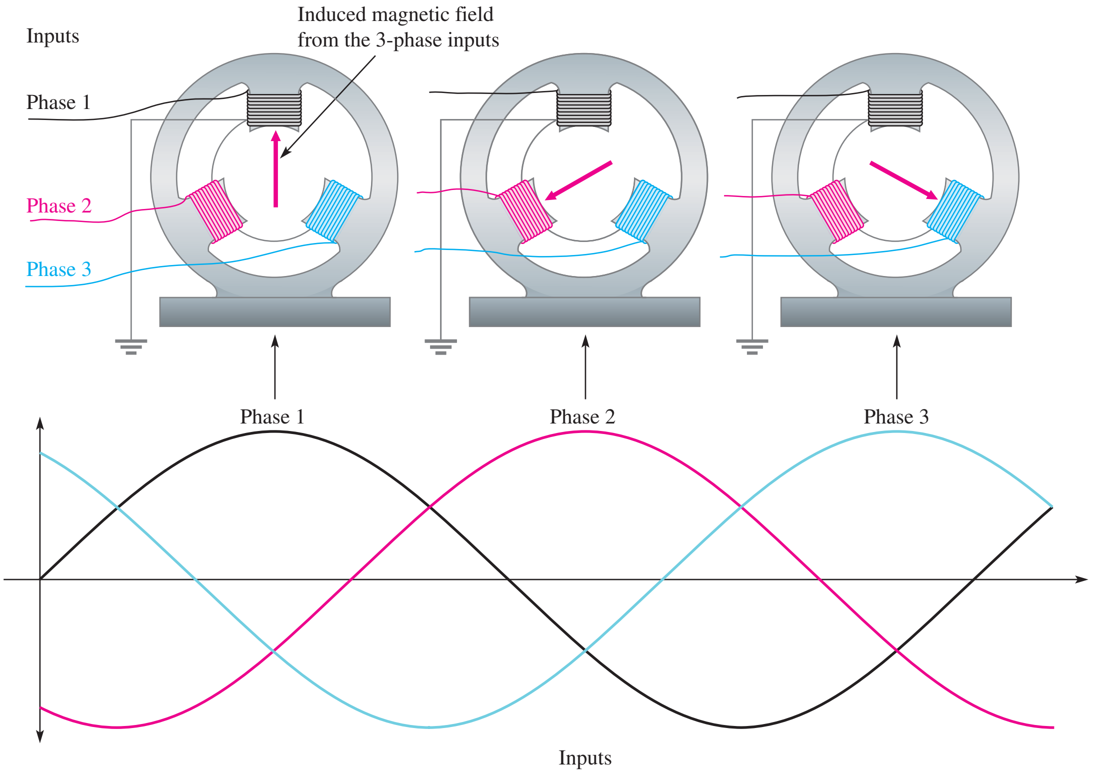
An **induction motor** is so named because a magnetic field induces current in the rotor, creating a magnetic field that interacts with the stator field. Normally, there is no electrical connection to the rotor, so there is no need for slip rings or brushes, which tend to wear out. The rotor current is caused by electromagnetic induction, which also occurs in transformers, so induction motors are said to work by transformer action.

In a **synchronous motor**, the rotor moves in sync (at the same rate) as the rotating field of the stator. Synchronous motors are used in applications where maintaining constant speed is important. Synchronous motors are not self-starting and must receive starting torque from an external source or from built-in starting windings. Like alternators, synchronous motors use slip rings and brushes to provide current to the rotor.

Rotating Stator Field

Both synchronous and induction ac motors have a similar arrangement for the stator windings, which allow the magnetic field of the stator to rotate. The rotating stator field is equivalent to moving a magnet in a circle except that the rotating field is produced electrically, with no moving parts.

How can the magnetic field in the stator rotate if the stator itself does not move? The rotating field is created by the changing ac itself. Let’s look at a rotating field with a three-phase stator, as shown in Figure 49. Notice that one of the three phases “dominates” at different times. When phase 1 is at 90°, the current in the phase 1 winding is at a maximum and current in the other windings is smaller. Therefore, the stator magnetic field will be oriented toward the phase-1 stator winding. As the phase-1 current declines, the phase-2 current increases, and the field rotates toward the phase-2 winding. The magnetic field will be oriented toward the phase-2 winding when current in it is a maximum. As the phase-2 current declines, the phase-3 current increases, and the field rotates toward the phase-3 winding. The process repeats as the field returns to the phase-1 winding. Thus, the field rotates at a rate



▲ **FIGURE 49**

The application of three phases to the stator produces a net magnetic field as shown by the red arrow. The rotor (not shown) moves in response to this field.

determined by the frequency of the applied voltage. With a more detailed analysis, it can be shown that the magnitude of the field is unchanged; only the direction of the field changes.

As the stator field moves, the rotor moves in sync with it in a synchronous motor but lags behind in an induction motor. The rate the stator field moves is called the *synchronous speed* of the motor.

Induction Motors

The theory of operation is essentially the same for both single-phase and three-phase induction motors. Both types use the rotating field described previously, but the single-phase motor requires starting windings or other method to produce torque for starting the motor, whereas the three-phase motor is self-starting. When starting windings are employed in a single-phase motor, they are removed from the circuit by a mechanical centrifugal switch as the motor speeds up.

The core of the induction motor's rotor consists of an aluminum frame that forms the conductors for the circulating current in the rotor. (Some larger induction motors use copper bars.) The aluminum frame is similar in appearance to the exercise wheel for pet squirrels (common in the early 20th century), so it is aptly called a **squirrel cage**, illustrated in Figure 50. The aluminum squirrel cage itself is the *electrical* path; it is embedded within a ferromagnetic material to provide a low reluctance *magnetic* path through the rotor. In addition, the rotor has cooling fins that may be molded into the same piece of aluminum as the squirrel cage. The entire assembly must be balanced so that it spins easily and without vibrating.

Operation of an Induction Motor When the magnetic field from the stator moves across the squirrel cage of the inductor, a current is generated in the squirrel cage. This current creates a magnetic field that reacts with the moving field of the stator, causing the

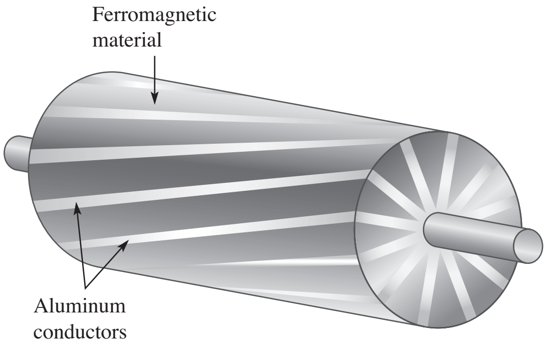


FIGURE 50

Diagram of a squirrel-cage rotor.

rotor to start turning. The rotor will try to "catch-up" with the moving field, but cannot, in a condition known as slip. **Slip** is defined as the difference between the synchronous speed of the stator and the rotor speed. The rotor can never reach the synchronous speed of the stator field because, if it did, it would not cut any field lines and the torque would drop to zero. Without torque, the rotor could not turn itself.

Initially, before the rotor starts moving, there is no back emf, so the stator current is high. As the rotor speeds up, it generates a back emf that opposes the stator current. As the motor speeds up, the torque produced balances the load and the current is just enough to keep the rotor turning. The running current is significantly lower than the initial start-up current because of the back emf. If the load on the motor is then increased, the motor will slow down and generate less back emf. This increases the current to the motor and increases the torque it can apply to the load. Thus, an induction motor can operate over a range of speeds and torque. Maximum torque occurs when the rotor is spinning at about 75% of the synchronous speed.

Synchronous Motors

Recall that an induction motor develops no torque if it runs at the synchronous speed, so it must run slower than the synchronous speed, depending on the load. The synchronous motor will run at the synchronous speed and still develop the required torque for different loads. The only way to change the speed of a synchronous motor is to change the frequency.

The fact that synchronous motors maintain a constant speed for all load conditions is a major advantage in certain industrial operations and in applications where clock or timing requirements are involved (such as a telescope drive motor or a chart recorder). In fact, the first application of synchronous motors was in electric clocks (in 1917).

Another important advantage to large synchronous motors is their efficiency. Although their original cost is higher than a comparable induction motor, the savings in power will often pay for the cost difference in a few years.

Operation of a Synchronous Motor Essentially, the rotating stator field of the synchronous motor is identical to that of an induction motor. The primary difference in the two motors is in the rotor. The induction motor has a rotor that is electrically isolated from a supply and the synchronous motor uses a magnet to follow the rotating stator field. Small synchronous motors use a permanent magnet for the rotor; larger motors use an electromagnet. When an electromagnet is used, dc is supplied from an external source via slip rings as in the case of the alternator.

SECTION 8 CHECKUP

1. What is the main difference between an induction motor and a synchronous motor?
2. What happens to the magnitude of the rotating stator field as it moves?
3. What is the purpose of a squirrel cage?
4. With reference to motors, what does the term *slip* mean?

9 NONSINUSOIDAL WAVEFORMS

Sine waves are important in electronics, but they are by no means the only type of ac or time-varying waveform. Two other major types of waveforms are the pulse waveform and the triangular waveform.

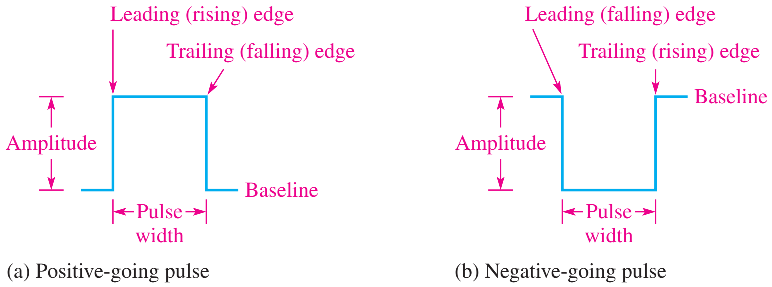
After completing this section, you should be able to

- **Identify the characteristics of basic nonsinusoidal waveforms**
  - Discuss the properties of a pulse waveform
  - Define *duty cycle*
  - Discuss the properties of triangular and sawtooth waveforms
  - Discuss the harmonic content of a waveform

Pulse Waveforms

Basically, a **pulse** can be described as a very rapid transition **(leading edge)** from one voltage or current level **(baseline)** to an amplitude level, and then, after an interval of time, a very rapid transition **(trailing edge)** back to the original baseline level. The transitions in level are also called *steps.* An ideal pulse consists of two opposite-going steps of equal amplitude. When the leading or trailing edge is positive-going, it is called a **rising edge**. When the leading or trailing edge is negative-going, it is called a **falling edge**.

Figure 51(a) shows an ideal positive-going pulse consisting of two equal but opposite instantaneous steps separated by an interval of time called the *pulse width*. Part (b) of Figure 51 shows an ideal negative-going pulse. The height of the pulse measured from the baseline is its voltage (or current) amplitude.



(a) Positive-going pulse (b) Negative-going pulse

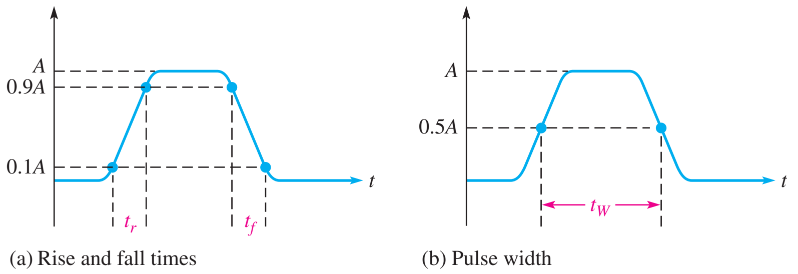
▲ FIGURE 51

Ideal pulses.

In many applications, analysis is simplified by treating all pulses as ideal (composed of instantaneous steps and perfectly rectangular in shape). Actual pulses, however, are never ideal. All pulses possess certain characteristics that cause them to be different from the ideal.

In practice, pulses cannot change from one level to another instantaneously. Time is always required for a transition (step), as illustrated in Figure 52(a). As you can see, there is an interval of time during the rising edge in which the pulse is going from its lower value to its higher value. This interval is called the *rise time,* t_r.

> **Rise time is the time required for the pulse to go from 10% of its amplitude to 90% of its amplitude.**



▲ **FIGURE 52**

Nonideal pulse.

The interval of time during the falling edge in which the pulse is going from its higher value to its lower value is called the *fall time*, t_f.

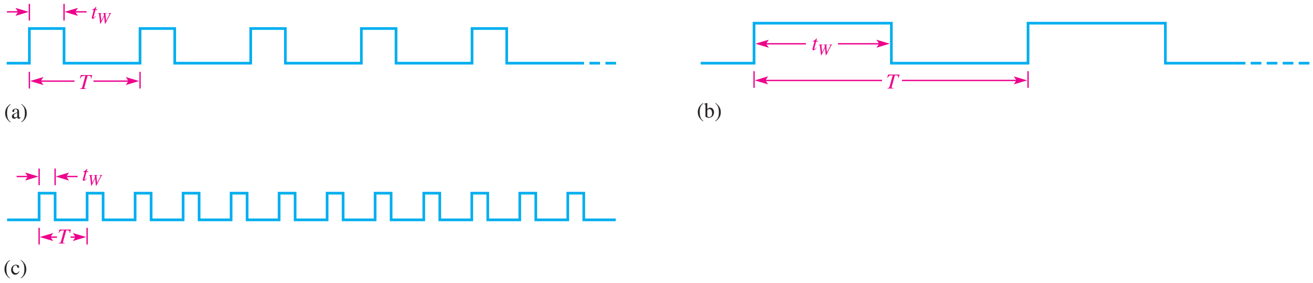
Fall time is the time required for the pulse to go from 90% of its amplitude to 10% of its amplitude.

Pulse width, t_W, also requires a precise definition for the nonideal pulse because the rising and falling edges are not vertical.

Pulse width is the time between the point on the rising edge, where the value is 50% of amplitude, to the point on the falling edge, where the value is 50% of amplitude.

Pulse width is shown in Figure 52(b).

Repetitive Pulses Any waveform that repeats itself at fixed intervals is **periodic**. Some examples of periodic pulse waveforms are shown in Figure 53. Notice that, in each case, the pulses repeat at regular intervals. The rate at which the pulses repeat is the **pulse repetition frequency**, which is the fundamental frequency of the waveform. The frequency can be expressed in hertz or in pulses per second. The time from one pulse to the corresponding point on the next pulse is the period, T. The relationship between frequency and period is the same as with the sine wave, $f = 1/T$.



▲ **FIGURE 53**

Repetitive pulse waveforms.

An important characteristic of repetitive pulse waveforms is the duty cycle.

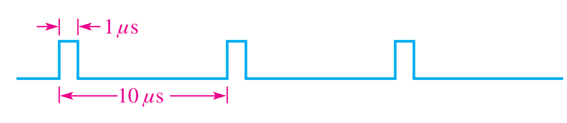
The duty cycle is the ratio of the pulse width (t_W) to the period (T) and is usually expressed as a percentage.

$$\text{percent duty cycle} = \left(\frac{t_W}{T}\right)100\% \qquad \text{Equation 22}$$

EXAMPLE 17

Determine the period, frequency, and duty cycle for the pulse waveform in Figure 54.

▶ **FIGURE 54**



Solution

$$T = \mathbf{10\ \mu s}$$

$$f = \frac{1}{T} = \frac{1}{10\ \mu\text{s}} = \mathbf{100\ kHz}$$

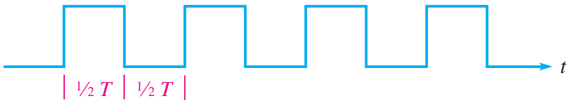
$$\text{percent duty cycle} = \left(\frac{1\ \mu\text{s}}{10\ \mu\text{s}}\right)100\% = \mathbf{10\%}$$

Related Problem A certain pulse waveform has a frequency of 200 kHz and a pulse width of 0.25 μs. Determine the duty cycle.

Square Waves A square wave is a pulse waveform with a duty cycle of 50%. Thus, the pulse width is equal to one-half of the period. A square wave is shown in Figure 55.

▶ **FIGURE 55**

Square wave.



The Average Value of a Pulse Waveform The average value (V_{avg}) of a pulse waveform is equal to its baseline value plus its duty cycle times its amplitude. The lower level of a positive-going waveform or the upper level of a negative-going waveform is taken as the baseline. The formula is as follows:

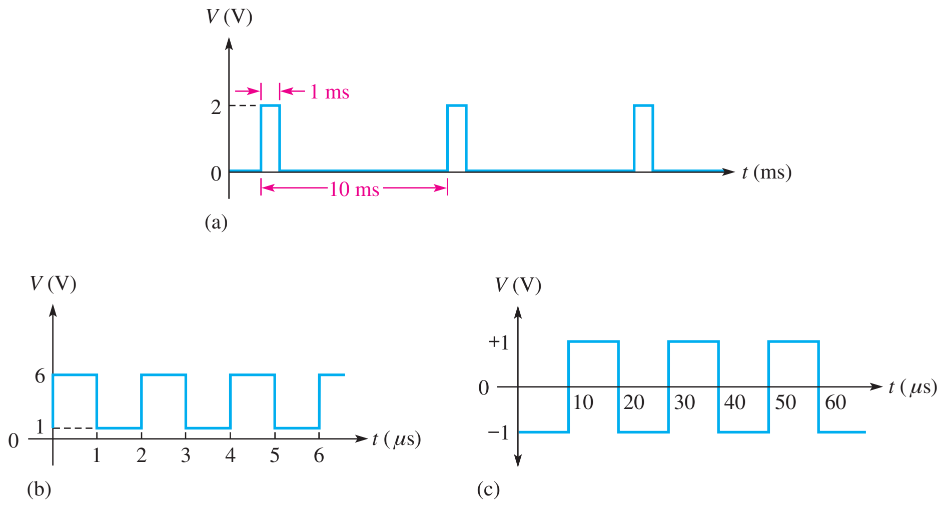
Equation 23

$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$

The following example illustrates the calculation of the average value.

EXAMPLE 18

Determine the average value of each of the waveforms in Figure 56.



▲ **FIGURE 56**

Solution In Figure 56(a), the baseline is at 0 V, the amplitude is 2 V, and the duty cycle is 10%. The average value is

$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$
$$= 0\text{ V} + (0.1)(2\text{ V}) = \mathbf{0.2\text{ V}}$$

The waveform in Figure 56(b) has a baseline of +1 V, an amplitude of 5 V, and a duty cycle of 50%. The average value is

$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$
$$= 1\text{ V} + (0.5)(5\text{ V}) = 1\text{ V} + 2.5\text{ V} = \mathbf{3.5\text{ V}}$$

Figure 56(c) shows a square wave with a baseline of −1 V and an amplitude of 2 V. The average value is

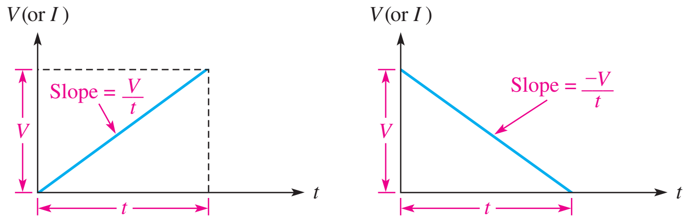
$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$
$$= -1\text{ V} + (0.5)(2\text{ V}) = -1\text{ V} + 1\text{ V} = \mathbf{0\text{ V}}$$

This is an alternating square wave, and, as with an alternating sine wave, it has an average of zero.

Related Problem If the baseline of the waveform in Figure 56(a) is shifted to 1 V, what is the average value?

Triangular and Sawtooth Waveforms

Triangular and sawtooth waveforms are formed by voltage or current ramps. A **ramp** is a linear increase or decrease in the voltage or current. Figure 57 shows both positive- and negative-going ramps. In part (a), the ramp has a positive slope; in part (b), the ramp has a negative slope. The slope of a voltage ramp is $\pm V/t$ and is expressed in units of V/s. The slope of a current ramp is $\pm I/t$ and is expressed in units of A/s.



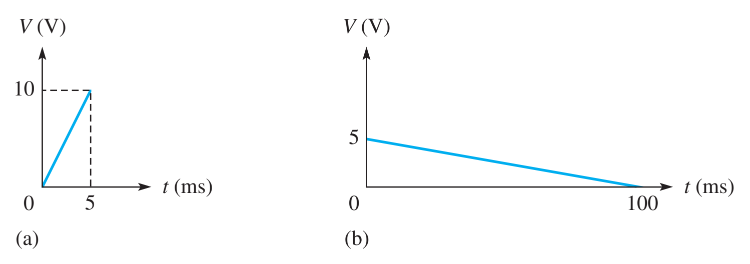
(a) Positive ramp (b) Negative ramp

▲ **FIGURE 57**

Voltage ramps.

EXAMPLE 19 What are the slopes of the voltage ramps in Figure 58?

▶ **FIGURE 58**



Solution In Figure 58(a), the voltage increases from 0 V to +10 V in 5 ms. Thus, $V = 10$ V and $t = 5$ ms. The slope is

$$\frac{V}{t} = \frac{10\text{ V}}{5\text{ ms}} = \mathbf{2\ V/ms}$$

In Figure 58(b), the voltage decreases from +5 V to 0 V in 100 ms. Thus, $V = -5$ V and $t = 100$ ms. The slope is

$$\frac{V}{t} = \frac{-5\text{ V}}{100\text{ ms}} = \mathbf{-0.05\ V/ms}$$

Related Problem A certain voltage ramp has a slope of $+12$ V/μs. If the ramp starts at zero, what is the voltage at 0.01 ms?

Triangular Waveforms Figure 59 shows that a **triangular waveform** is composed of positive-going and negative-going ramps having equal slopes. The period of this waveform is measured from one peak to the next corresponding peak, as illustrated. This particular triangular waveform is alternating and has an average value of zero.

FIGURE 59

Alternating triangular waveform with a zero average value.

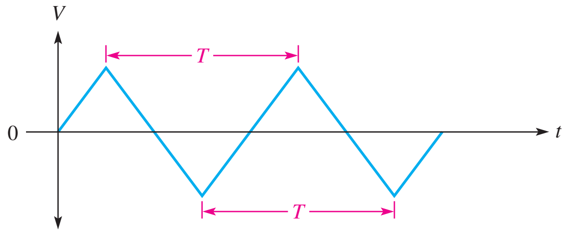
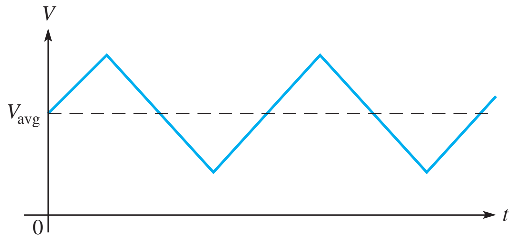


Figure 60 depicts a triangular waveform with a nonzero average value. The frequency for triangular waves is determined in the same way as for sine waves, that is, $f = 1/T$.

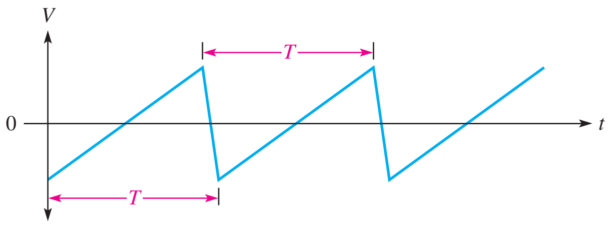
FIGURE 60

Nonalternating triangular waveform with a nonzero average value.



Sawtooth Waveforms The **sawtooth waveform** is actually a special case of the triangular wave consisting of two ramps, one of much longer duration than the other. Sawtooth waveforms are used in many electronic systems. For example, the electron beam that sweeps across the screen of a CRT TV receiver, creating the picture, is controlled by sawtooth voltages and currents. One sawtooth wave produces the horizontal beam movement, and the other produces the vertical beam movement. A sawtooth voltage is sometimes called a *sweep voltage*.

Figure 61 is an example of a sawtooth wave. Notice that it consists of a positive-going ramp of relatively long duration, followed by a negative-going ramp of relatively short duration.



▲ FIGURE 61

Alternating sawtooth waveform.

Harmonics

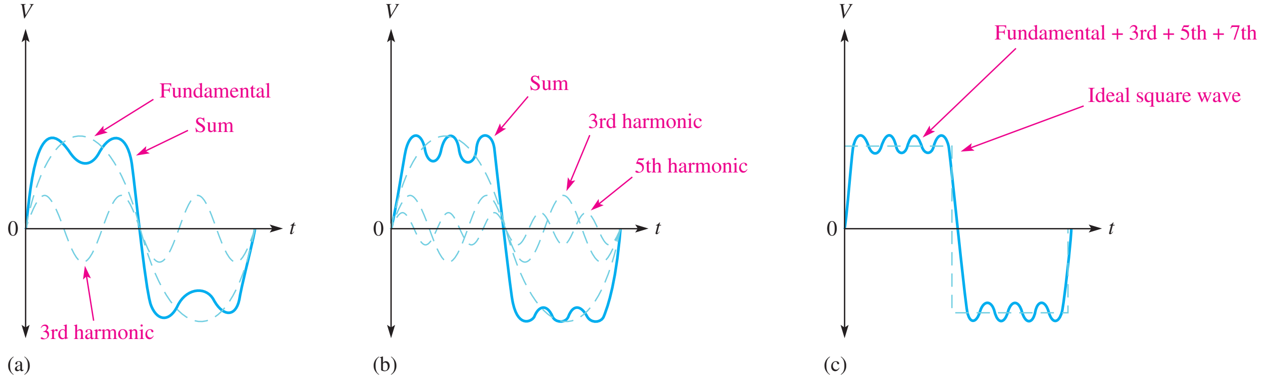
A repetitive nonsinusoidal waveform is composed of a fundamental frequency and harmonic frequencies. The **fundamental frequency** is the repetition rate of the waveform, and the **harmonics** are higher frequency sine waves that are multiples of the fundamental.

Odd Harmonics *Odd harmonics* are frequencies that are odd multiples of the fundamental frequency of a waveform. For example, a 1 kHz square wave consists of a fundamental of 1 kHz and odd harmonics of 3 kHz, 5 kHz, 7 kHz, and so on. The 3 kHz frequency in this case is called the third harmonic, the 5 kHz frequency is the fifth harmonic, and so on.

Even Harmonics *Even harmonics* are frequencies that are even multiples of the fundamental frequency. For example, if a certain wave has a fundamental of 200 Hz, the second harmonic is 400 Hz, the fourth harmonic is 800 Hz, the sixth harmonic is 1200 Hz, and so on. These are even harmonics.

Composite Waveform Any variation from a pure sine wave produces harmonics. A nonsinusoidal wave is a composite of the fundamental and the harmonics. Some types of waveforms have only odd harmonics, some have only even harmonics, and some contain both. The shape of the wave is determined by its harmonic content. Generally, only the fundamental and the first few harmonics are of significant importance in determining the wave shape.

A square wave is an example of a waveform that consists of a fundamental and only odd harmonics. When the instantaneous values of the fundamental and each odd harmonic are added algebraically at each point, the resulting curve will have the shape of a square wave, as illustrated in Figure 62. In part (a) of the figure, the fundamental and the third harmonic produce a wave shape that begins to resemble a square wave. In part (b), the fundamental, third, and fifth harmonics produce a closer resemblance. When the seventh harmonic is included, as in part (c), the resulting wave shape becomes even more like a square wave. As more harmonics are included, a periodic square wave is approached.

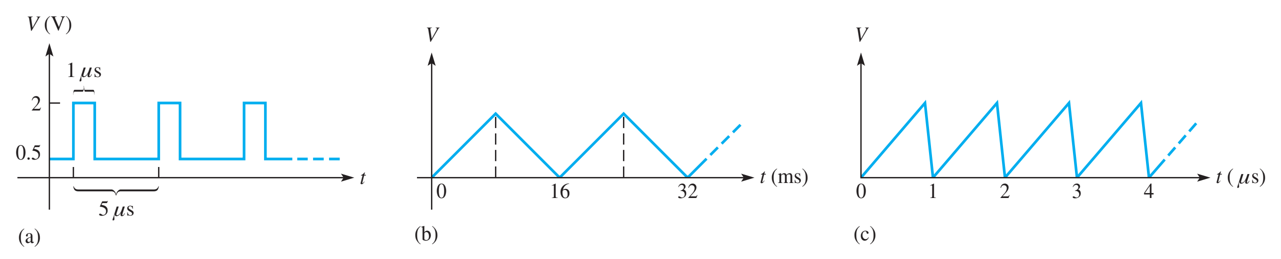


▲ FIGURE 62

Odd harmonics produce a square wave.

SECTION 9 CHECKUP

1. Define the following parameters:
   (a) rise time (b) fall time (c) pulse width
2. In a certain repetitive pulse waveform, the pulses occur once every millisecond. What is the frequency of this waveform?
3. Determine the duty cycle, amplitude, and average value of the waveform in Figure 63(a).
4. What is the period of the triangular wave in Figure 63(b)?
5. What is the frequency of the sawtooth wave in Figure 63(c)?



▲ FIGURE 63

6. Define *fundamental frequency.*
7. What is the second harmonic of a fundamental frequency of 1 kHz?
8. What is the fundamental frequency of a square wave having a period of 10 μs?

10 THE OSCILLOSCOPE

The oscilloscope (scope for short) is a widely used and versatile test instrument for observing and measuring waveforms.

After completing this section, you should be able to

- **Use an oscilloscope to measure waveforms**
    - Recognize common oscilloscope controls
    - Measure the amplitude of a waveform
    - Measure the period and frequency of a waveform

The **oscilloscope** is basically a graph-displaying device that traces a graph of a measured electrical signal on its screen. In most applications, the graph shows how signals change over time. The vertical axis of the display screen represents voltage, and the horizontal axis represents time. You can measure amplitude, period, and frequency of a signal using an oscilloscope. Also, you can determine the pulse width, duty cycle, rise time, and fall time of a pulse waveform. Most scopes can display at least two signals on the screen at one time, enabling you to observe their time relationship. Typical oscilloscopes are shown in Figure 64.

Two basic types of oscilloscopes, digital, and analog can be used to view digital waveforms. The digital scope converts the measured waveform to digital information by a sampling process in an analog-to-digital converter (ADC). The digital information is then used to reconstruct the waveform on the screen. The analog scope works by applying the measured waveform directly to control the up and down motion of the electron beam in the cathode-ray

▲ FIGURE 64

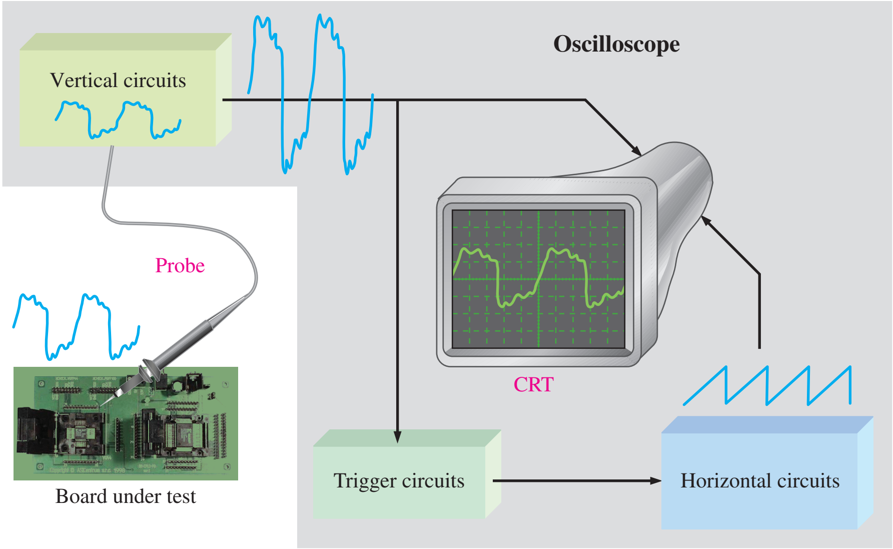
Typical oscilloscopes. Copyright © Tektronix. Reproduced by permission.

tube (CRT) as it sweeps across the screen. As a result, the beam traces out the waveform pattern on the screen.

The digital scope is more widely used than the analog scope. However, either type can be used in many applications; each has characteristics that make it more suitable for certain situations. An analog scope displays waveforms as they occur in "real time." Digital scopes are useful for measuring transient pulses that may occur randomly or only once. Also, because information about the measured waveform can be stored in a digital scope, it may be viewed at some later time, printed out, or thoroughly analyzed by a computer or other means.

Basic Operation of Analog Oscilloscopes

To measure a voltage, a probe must be connected from the scope to the point in a circuit at which the voltage is present. Generally, a ×10 probe is used that reduces (attenuates) the signal amplitude by ten. The signal goes through the probe into the vertical circuits where it is either further attenuated or amplified, depending on the actual amplitude and on where you set the vertical control of the scope. The vertical circuits then drive the vertical deflection plates of the CRT. Also, the signal goes to the trigger circuits that trigger the horizontal circuits to initiate repetitive horizontal sweeps of the electron beam across the screen using a sawtooth waveform. There are many sweeps per second so that the beam appears to form a solid line across the screen in the shape of the waveform. This basic operation is illustrated in Figure 65.

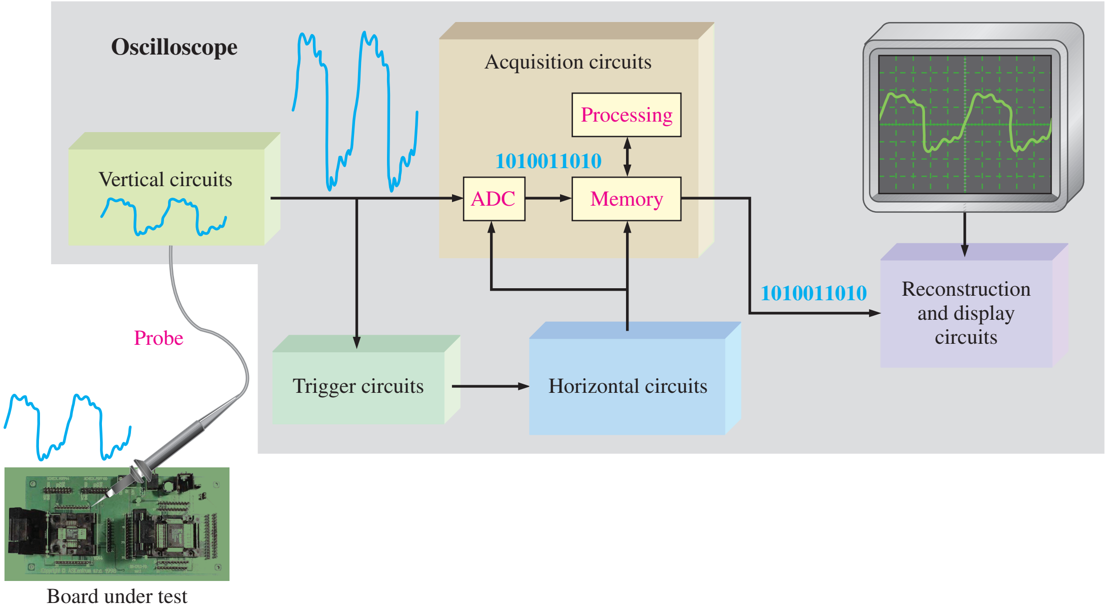


▲ FIGURE 65

Block diagram of an analog oscilloscope.

Basic Operation of Digital Oscilloscopes

Some parts of a digital scope are similar to the analog scope. However, the digital scope is more complex than an analog scope and typically has an LCD screen rather than a CRT. Rather than displaying a waveform as it occurs, the digital scope first acquires the measured analog waveform and converts it to a digital format using an analog-to-digital converter (ADC). The digital data is stored and processed. The data then goes to the reconstruction and display circuits for display in its original analog form. Figure 66 show a basic block diagram for a digital oscilloscope.



▲ **FIGURE 66**

Block diagram of a digital oscilloscope.

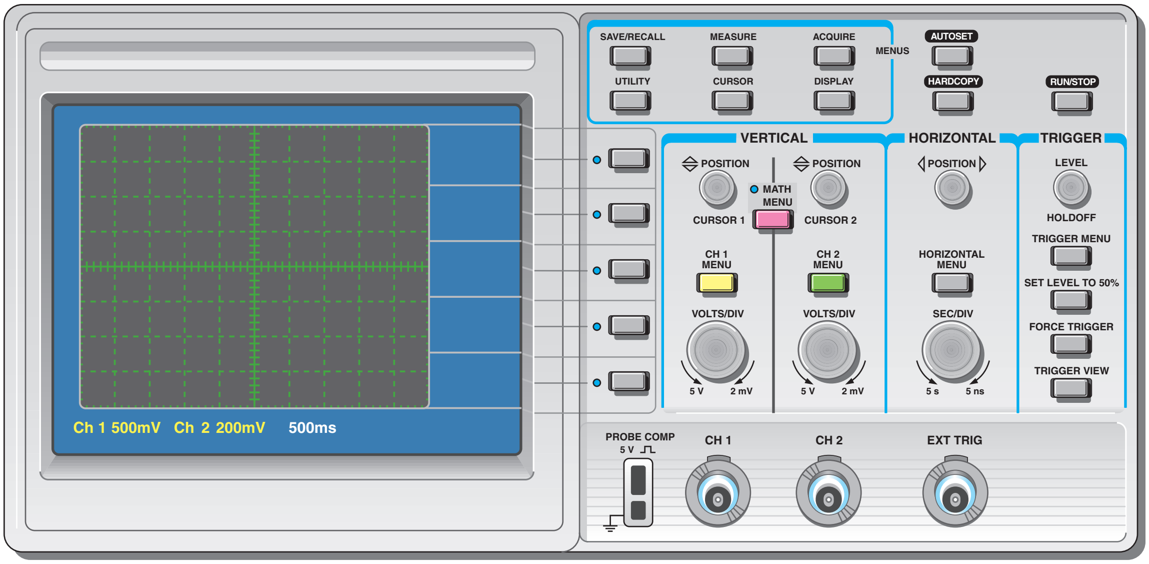
Oscilloscope Controls

A front panel view of a typical dual-channel oscilloscope is shown in Figure 67. Instruments vary depending on model and manufacturer, but most have certain common features. For example, the two vertical sections contain a Position control, a channel menu button, and a Volts/Div control. The horizontal section contains a Sec/Div control.

Some of the main controls are now discussed. Refer to the user manual for complete details of your particular scope.

Vertical Controls In the vertical section of the scope in Figure 67, there are identical controls for each of the two channels (CH1 and CH2). The Position control lets you move a displayed waveform up or down vertically on the screen. The buttons immediately right of the screen provide for the selection of several items that appear on the screen, such as the coupling modes (ac, dc, or ground), coarse or fine adjustment for the Volts/Div, signal inversion, and other parameters. The Volts/Div control adjusts the number of volts represented by each vertical division on the screen. The Volts/Div setting for each channel is displayed on the bottom of the screen.

Horizontal Controls In the horizontal section, the controls apply to both channels. The Position control lets you move a displayed waveform left or right horizontally on the screen. The Horizontal Menu button provides for the selection of several items that appear on the screen such as the main time base, expanded view of a portion of a waveform, and


SAVE/RECALL
MEASURE
ACQUIRE
UTILITY
CURSOR
DISPLAY
MENUS
AUTOSET
HARDCOPY
RUN/STOP
VERTICAL
POSITION
MATH MENU
CURSOR 1
CURSOR 2
CH 1 MENU
CH 2 MENU
VOLTS/DIV
5 V
2 mV
HORIZONTAL
HORIZONTAL MENU
SEC/DIV
5 s
5 ns
TRIGGER
LEVEL
HOLDOFF
TRIGGER MENU
SET LEVEL TO 50%
FORCE TRIGGER
TRIGGER VIEW
PROBE COMP 5 V
CH 1
CH 2
EXT TRIG
Ch 1 500mV Ch 2 200mV 500ms

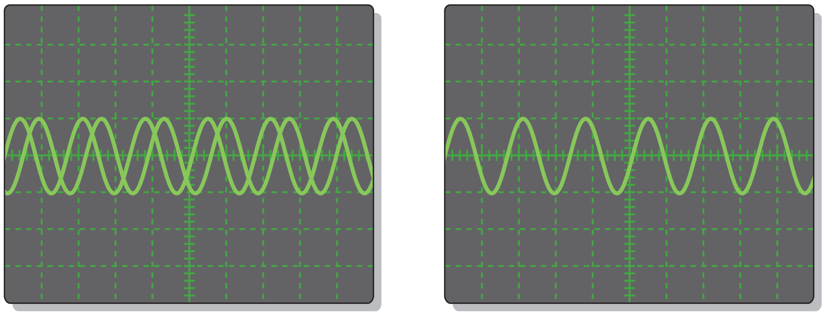
▲ FIGURE 67

A typical dual-channel oscilloscope. Numbers below screen indicate the values for each division on the vertical (voltage) and horizontal (time) scales and can be varied using the vertical and horizontal controls on the scope.

other parameters. The Sec/Div control adjusts the time represented by each horizontal division or main time base. The Sec/Div setting is displayed at the bottom of the screen.

Trigger Controls In the Trigger section, the Level control determines the point on the triggering waveform where triggering occurs to initiate the sweep to display input waveforms. The Trigger Menu button provides for the selection of several items that appear on the screen including edge or slope triggering, trigger source, trigger mode, and other parameters. There is also an input for an external trigger signal.

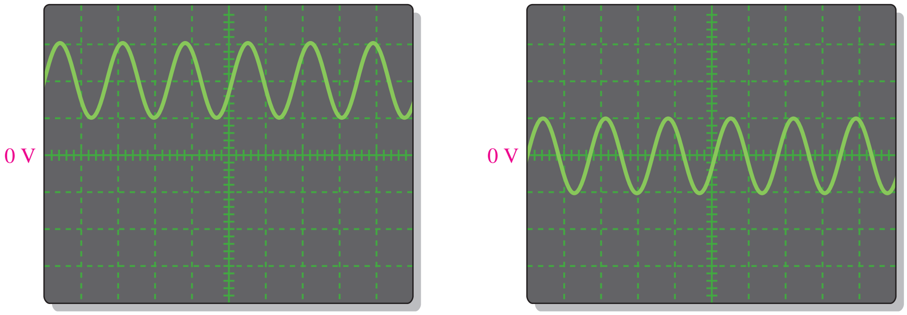
Triggering stabilizes a waveform on the screen and properly triggers on a pulse that occurs only one time or randomly. Also, it allows you to observe time delays between two waveforms. Figure 68 compares a triggered to an untriggered signal. The untriggered signal tends to drift across the screen producing what appears to be multiple waveforms.


(a) Untriggered waveform display (b) Triggered waveform display

▲ FIGURE 68

Comparison of an untriggered and a triggered waveform on an oscilloscope.

Coupling a Signal into the Scope Coupling is the method used to connect a signal voltage to be measured into the oscilloscope. The DC and AC coupling modes are selected from the Vertical menu. DC coupling allows a waveform including its dc component to be displayed. AC coupling blocks the dc component of a signal so that you see the waveform centered at 0 V. The Ground mode allows you to connect the channel input to ground to see where the 0 V reference is on the screen. Figure 69 illustrates the result of DC and AC coupling using a sinusoidal waveform that has a dc component.



(a) DC coupled waveform

(b) AC coupled waveform

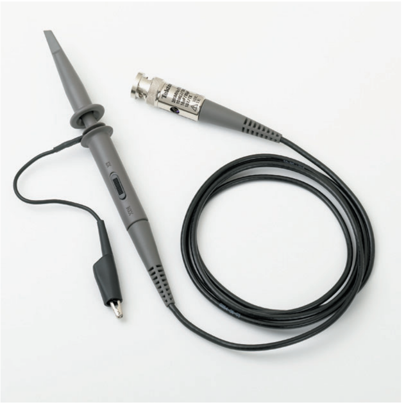
▲ FIGURE 69

Displays of the same waveform having a dc component.

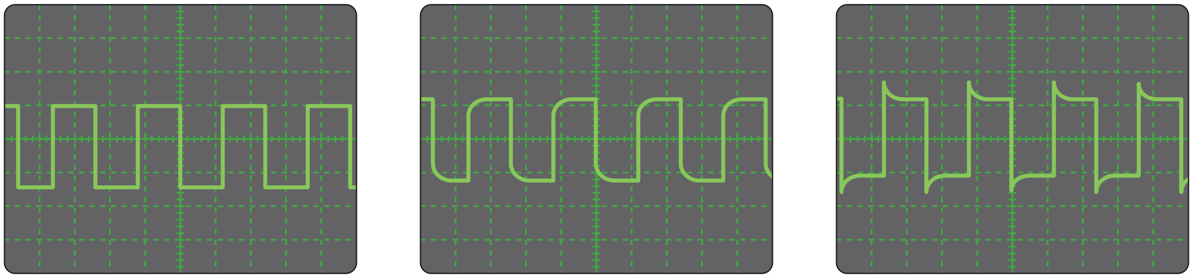
The voltage probe, shown in Figure 70, is used for connecting a signal to the scope. Since all instruments tend to affect the circuit being measured due to loading, most scope probes provide a high series resistance to minimize loading effects. Probes that have a series resistance ten times larger than the input resistance of the scope are called ×10 (times ten) probes. Probes with no series resistance are called ×1 (times one) probes. The oscilloscope adjusts its calibration for the attenuation of the type of probe being used. For most measurements, the ×10 probe should be used. However, if you are measuring very small signals, a ×1 may be the best choice.

▶ FIGURE 70

An oscilloscope voltage probe. Copyright © Tektronix, Inc. Reproduced by permission.



The probe has an adjustment that allows you to compensate for the input capacitance of the scope. Most scopes have a probe compensation output that provides a calibrated square wave for probe compensation. Before making a measurement, you should make sure that the probe is properly compensated to eliminate any distortion introduced. Typically, there is a screw or other means of adjusting compensation on a probe. Figure 71 shows scope waveforms for three probe conditions: properly compensated, undercompensated, and overcompensated. If the waveform appears either over- or under-compensated, adjust the probe until the properly compensated square wave is achieved.



Properly compensated

Undercompensated

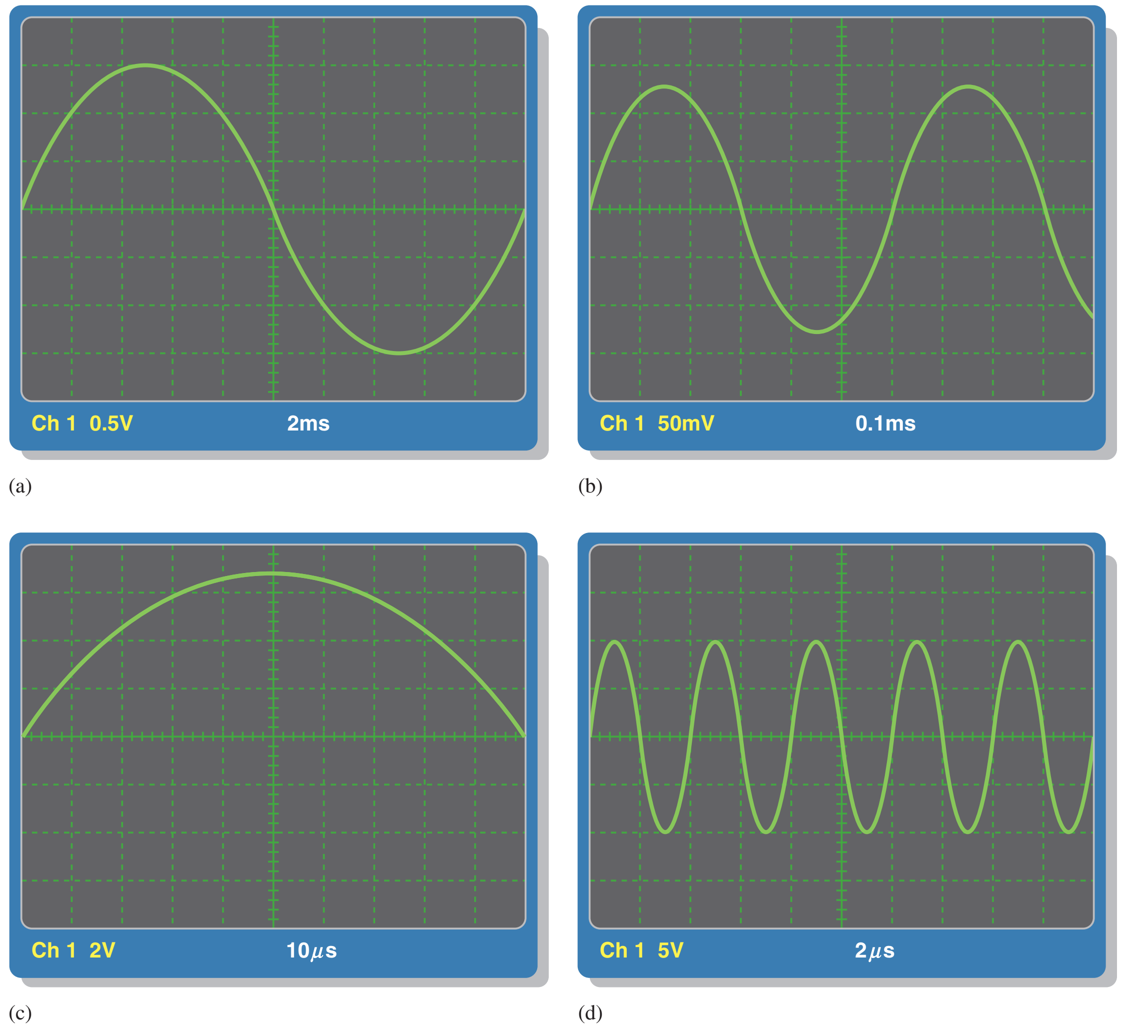
Overcompensated

▲ FIGURE 71

Probe compensation conditions.

EXAMPLE 20

Determine the peak-to-peak value and period of each sine wave in Figure 72 from the digital scope screen displays and the settings for Volts/Div and Sec/Div, which are indicated under the screens. Sine waves are centered vertically on the screens.


Ch 1 0.5V
2ms
(a)
Ch 1 50mV
0.1ms
(b)
Ch 1 2V
10μs
(c)
Ch 1 5V
2μs
(d)

▲ FIGURE 72

Solution Looking at the vertical scale in Figure 72(a),

$$V_{pp} = 6 \text{ divisions} \times 0.5 \text{ V/division} = \mathbf{3.0\ V}$$

From the horizontal scale (one cycle covers ten divisions),

$$T = 10 \text{ divisions} \times 2 \text{ ms/division} = \mathbf{20\ ms}$$

Looking at the vertical scale in Figure 72(b),

$$V_{pp} = 5 \text{ divisions} \times 50 \text{ mV/division} = \mathbf{250\ mV}$$

From the horizontal scale (one cycle covers six divisions),

$$T = 6 \text{ divisions} \times 0.1 \text{ ms/division} = 0.6 \text{ ms} = \mathbf{600\ \mu s}$$

Looking at the vertical scale in Figure 72(c),

$$V_{pp} = 6.8 \text{ divisions} \times 2 \text{ V/division} = \mathbf{13.6\ V}$$

From the horizontal scale (one-half cycle covers ten divisions),

$$T = 20 \text{ divisions} \times 10 \ \mu\text{s/division} = \mathbf{200\ \mu s}$$

Looking at the vertical scale in Figure 72(d),

$$V_{pp} = 4 \text{ divisions} \times 5 \text{ V/division} = \mathbf{20\ V}$$

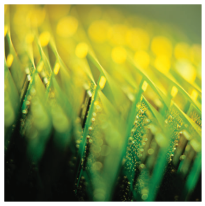
From the horizontal scale (one cycle covers two divisions),

$$T = 2 \text{ divisions} \times 2 \ \mu\text{s/division} = \mathbf{4\ \mu s}$$

Related Problem Determine the rms value and the frequency for each waveform displayed in Figure 72.

SECTION 10 CHECKUP

1. What is the main difference between a digital and an analog oscilloscope?
2. Is voltage read horizontally or vertically on a scope screen?
3. What does the Volts/Div control on an oscilloscope do?
4. What does the Sec/Div control on an oscilloscope do?
5. When should you use a ×10 probe for making a voltage measurement?

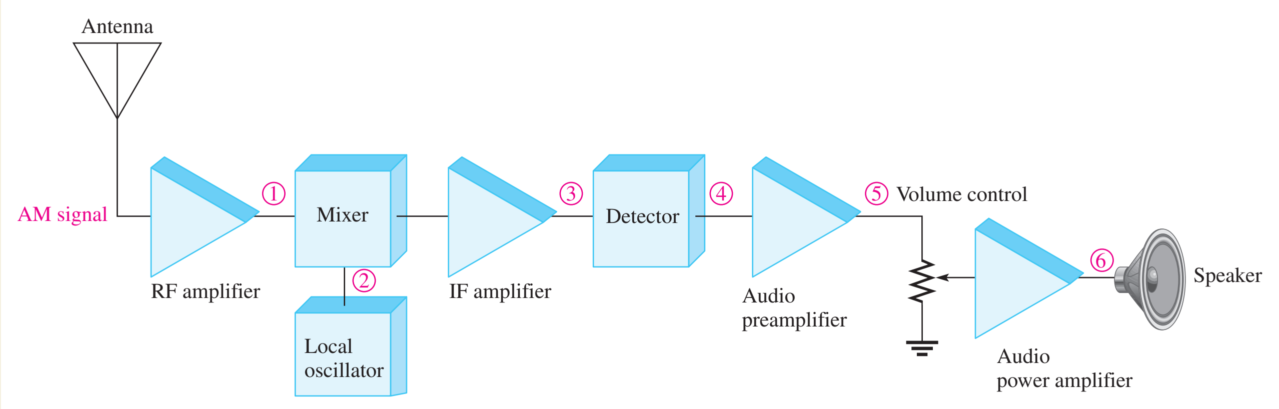


Application Activity

As you learned in this chapter, nonsinusoidal waveforms contain a combination of various harmonic frequencies. Each of these harmonics is a sinusoidal waveform with a certain frequency. Certain sinusoidal frequencies are audible; that is, they can be heard by the human ear. A single audible frequency, or pure sine wave, is called a tone and generally falls in the frequency range from about 300 Hz to about 15 kHz. When you hear a tone reproduced through a speaker, its loudness, or volume, depends on its voltage amplitude. You will use your knowledge of sine wave characteristics and the operation of an oscilloscope to measure the frequency and amplitude of signals at various points in a basic radio receiver.

Actual voice or music signals that are picked up by a radio receiver contain many harmonic frequencies with different voltage values. A voice or music signal is continuously changing, so its harmonic content is also changing. However, if a single sinusoidal frequency is transmitted and picked up by the receiver, you will hear a constant tone from the speaker.

Although, at this point you do not have the background to study amplifiers and receiver systems in detail, you can observe



▲ FIGURE 73

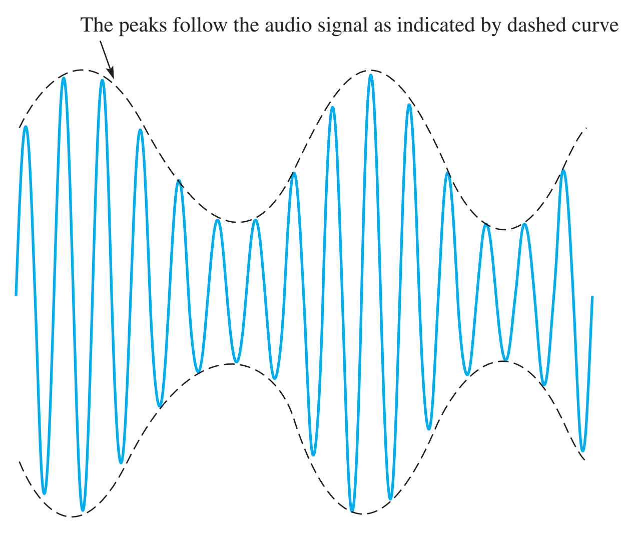
Simplified block diagram of a basic radio receiver. Circled numbers represent test points.

the signals at various points in the receiver. A block diagram of a typical AM receiver is shown in Figure 73. AM stands for amplitude modulation, a topic that will be covered in another course. Figure 74 shows what a basic AM signal looks like, and for now that's all you need to know. As you can see, the amplitude of the sinusoidal waveform is changing. The higher radio frequency (RF) signal is called the *carrier;* and its amplitude is varied or modulated by a lower frequency signal, which is the audio (a tone in this case). Normally, however, the audio signal is a complex voice or music waveform.

Oscilloscope Measurements

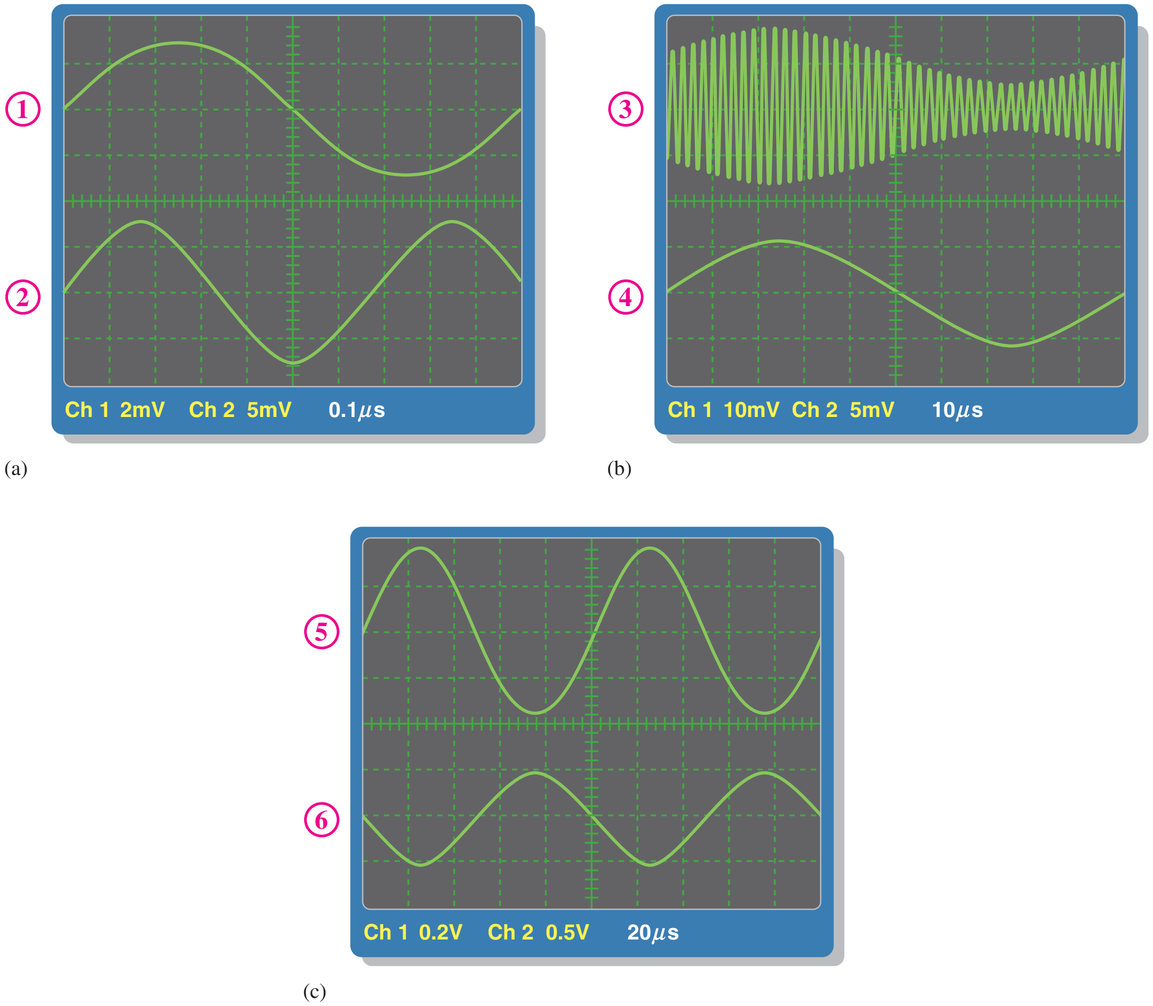
Signals that are indicated by circled numbers at several test points on the receiver block diagram in Figure 73 are displayed on the oscilloscope screen in Figure 75 as indicated by the same corresponding circled numbers. In all cases, the upper waveform on the screen is channel 1 and the lower waveform is channel 2. The readings on the bottom of the screen show readings for both channels.

The signal at point 1 is an AM signal, but you can't see the amplitude variation because of the short time base. The waveform is spread out too much to see the modulating audio signal, which causes amplitude variations; so what you see is just one cycle of the carrier. At point 3, the higher carrier frequency is difficult to determine because the time base was selected to allow viewing of one full cycle of the modulating signal. In an AM receiver, this intermediate frequency is 455 kHz. In actual practice, the modulated carrier signal at point 3 cannot easily be viewed on the scope because it contains two frequencies that make it difficult to synchronize in order to obtain a stable



◀ FIGURE 74

Example of an amplitude modulated (AM) signal.



▲ FIGURE 75

Circled numbers correspond to the numbered test points in Figure 73.

pattern. Sometimes external triggering using the modulating signal or TV field is used to obtain a stable display. A stable pattern is shown in this case to illustrate what the modulated waveform looks like.

1. For each waveform in Figure 75, except point 3, determine the frequency and rms value. The signal at point 4 is the modulating tone extracted by the detector from the higher intermediate frequency (455 kHz).

Amplifier Analysis

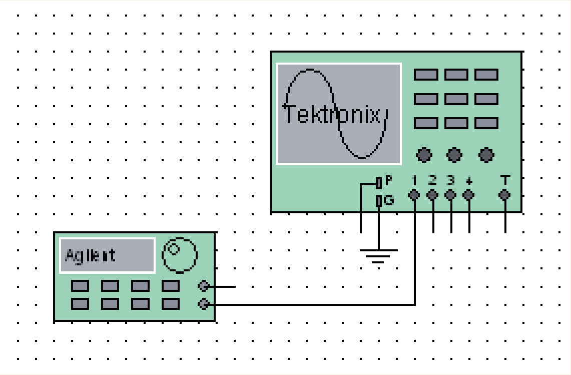
2. All voltage amplifiers have a characteristic known as voltage gain. The voltage gain is the amount by which the amplitude of the output signal is greater than the amplitude of the input signal. Using this definition and the appropriate scope measurements, determine the gain of the audio preamplifier in this particular receiver.
3. When an electrical signal is converted to sound by a speaker, the loudness of the sound depends on the amplitude of the signal applied to the speaker. Based on this, explain how the volume control potentiometer is used to adjust the loudness (volume) of the sound and determine the rms amplitude at the speaker.

Review

4. What does RF stand for?
5. What does IF stand for?
6. Which frequency is higher, the carrier or the audio?
7. What is the variable in a given AM signal?

Multisim Analysis

Open your Multisim software. Place an oscilloscope and a function generator on the screen connected as shown in Figure 76. Double-click each instrument to see the detailed controls in an expanded view. Select the sine wave function; set the amplitude to 100 mV_{pp} and the frequency to 1 kHz. Verify the values by oscilloscope measurement. Repeat for 1 V and 50 kHz and for 10 V and 1 MHz.



▲ FIGURE 76

SUMMARY

- The sine wave is a time-varying, periodic waveform.
- Alternating current changes direction in response to changes in the polarity of the source voltage.
- One cycle of an alternating sine wave consists of a positive alternation and a negative alternation.
- Two common sources of sine waves are the electromagnetic ac generator and the electronic oscillator circuit.
- A full cycle of a sine wave is 360°, or 2π radians. A half-cycle is 180°, or π radian. A quarter-cycle is 90°, or $\pi/2$ radians.
- A sinusoidal voltage can be generated by a conductor rotating in a magnetic field.
- Phase angle is the difference in degrees or radians between a given sine wave and a reference sine wave.
- The angular position of a phasor represents the angle of the sine wave with respect to a 0° reference, and the length or magnitude of a phasor represents the amplitude.
- Alternators (ac generators) produce power when there is relative motion between a magnetic field and a conductor.
- Most alternators take the output from the stator. The rotor provides a moving magnetic field.
- Two major types of ac motors are induction motors and synchronous motors.
- Induction motors have a rotor that turns in response to a rotating field from the stator.
- Synchronous motors move at a constant speed in sync with the field of the stator.
- A pulse consists of a transition from a baseline level to an amplitude level, followed by a transition back to the baseline level.
- A triangle or sawtooth wave consists of positive-going and negative-going ramps.
- Harmonic frequencies are odd or even multiples of the repetition rate of a nonsinusoidal waveform.
- Conversions of sine wave values are summarized in Table 2.

TABLE 2

TO CHANGE FROM	TO	MULTIPLY BY
Peak	rms	0.707
Peak	Peak-to-peak	2
Peak	Average	0.637
rms	Peak	1.414
Peak-to-peak	Peak	0.5
Average	Peak	1.57

KEY TERMS

Amplitude (A) The maximum value of a voltage or current.

Angular velocity The rotational rate of a phasor that is related to the frequency of the sine wave that it represents.

Average value The average of a sine wave over one half-cycle. It is 0.637 times the peak value.

Cycle One repetition of a periodic waveform.

Degree The unit of angular measure corresponding to 1/360 of a complete revolution.

Duty cycle A characteristic of a pulse waveform that indicates the percentage of time that a pulse is present during a cycle; the ratio of pulse width to period expressed as either a fraction or as a percentage.

Fall time (t_f) The time interval required for a pulse to change from 90% to 10% of its amplitude.

Frequency (f) A measure of the rate of change of a periodic function; the number of cycles completed in 1 s. The unit of frequency is the hertz.

Function generator An instrument that produces more than one type of waveform.

Fundamental frequency The repetition rate of a waveform.

Harmonics The frequencies contained in a composite waveform, which are integer multiples of the pulse repetition frequency (fundamental).

Hertz (Hz) The unit of frequency. One hertz equals one cycle per second.

Induction motor An ac motor that achieves excitation to the rotor by transformer action.

Instantaneous value The voltage or current value of a waveform at a given instant in time.

Oscillator An electronic circuit that produces a time-varying signal without an external input signal using positive feedback.

Oscilloscope A measurement instrument that displays signal waveforms on a screen.

Peak-to-peak value The voltage or current value of a waveform measured from its minimum to its maximum points.

Peak value The voltage or current value of a waveform at its maximum positive or negative points.

Period (T) The time interval of one complete cycle of a periodic waveform.

Periodic Characterized by a repetition at fixed-time intervals.

Phase The relative angular displacement of a time-varying waveform in terms of its occurrence with respect to a reference.

Phasor A representation of a sine wave in terms of its magnitude (amplitude) and direction (phase angle).

Pulse A type of waveform that consists of two equal and opposite steps in voltage or current separated by a time interval.

Pulse width (t_W) For a nonideal pulse, the time between the 50% points on the leading and trailing edges; the time interval between the opposite steps of an ideal pulse.

Radian A unit of angular measurement. There are 2π radians in one complete 360° revolution. One radian equals 57.3°.

Ramp A type of waveform characterized by a linear increase or decrease in voltage or current.

Rise time (t_r) The time interval required for a pulse to change from 10% to 90% of its amplitude.

rms value The value of a sinusoidal voltage that indicates its heating effect, also known as the effective value. It is equal to 0.707 times the peak value. *rms* stands for root mean square.

Sine wave A type of waveform that follows a cyclic sinusoidal pattern defined by the formula $y = A \sin \theta$.

Slip The difference between the synchronous speed of the stator field and the rotor speed in an induction motor.

Squirrel cage An aluminum frame within the rotor of an induction motor that forms the electrical conductors for a rotating current.

Synchronous motor An ac motor in which the rotor moves at the same rate as the rotating magnetic field of the stator.

Waveform The pattern of variations of a voltage or current showing how the quantity changes with time.

FORMULAS

1	$f = \frac{1}{T}$	Frequency
2	$T = \frac{1}{f}$	Period
3	$V_{pp} = 2V_p$	Peak-to-peak voltage (sine wave)
4	$I_{pp} = 2I_p$	Peak-to-peak current (sine wave)
5	$V_{\text{rms}} = 0.707V_p$	Root-mean-square voltage (sine wave)
6	$I_{\text{rms}} = 0.707I_p$	Root-mean-square current (sine wave)
7	$V_p = 1.414V_{\text{rms}}$	Peak voltage (sine wave)
8	$I_p = 1.414I_{\text{rms}}$	Peak current (sine wave)
9	$V_{pp} = 2.828V_{\text{rms}}$	Peak-to-peak voltage (sine wave)
10	$I_{pp} = 2.828I_{\text{rms}}$	Peak to peak current (sine wave)
11	$V_{\text{avg}} = 0.637V_p$	Half-cycle average voltage (sine wave)
12	$I_{\text{avg}} = 0.637I_p$	Half-cycle average current (sine wave)
13	$\text{rad} = \left(\frac{\pi\ \text{rad}}{180^\circ}\right) \times \text{degrees}$	Degrees to radian conversion
14	$\text{degrees} = \left(\frac{180^\circ}{\pi\ \text{rad}}\right) \times \text{rad}$	Radian to degrees conversion
15	$y = A \sin \theta$	General formula for a sine wave
16	$y = A \sin(\theta - \phi)$	Sine wave lagging the reference
17	$y = A \sin(\theta + \phi)$	Sine wave leading the reference
18	$\omega = 2\pi f$	Angular velocity
19	$\theta = \omega t$	Phase angle
20	$v = V_p \sin 2\pi f t$	Sine wave voltage
21	$f = \frac{Ns}{120}$	Frequency of an alternator
22	$\text{percent duty cycle} = \left(\frac{t_W}{T}\right)100\%$	Duty cycle
23	$V_{\text{avg}} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$	Average value of a pulse waveform

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The period of a 60 Hz sine wave is 16.7 ms.
2. The rms and average value of a sine wave are the same.
3. A sine wave with a peak value of 10 V has the same heating effect as a 10 V dc source.

4. The peak value of a sine wave is the same as its amplitude.
5. The number of radians in 360° is 2π.
6. In a three-phase electrical system, the phases are separated by 60°.
7. The purpose of an exciter is to supply dc rotor current to an alternator.
8. In an automotive alternator, the output current is taken from the rotor through slip rings.
9. A maintenance issue with induction motors is brush replacement.
10. A synchronous motor can be used when constant speed is required.

SELF-TEST

Answers are at the end of the chapter.

1. The difference between alternating current (ac) and direct current (dc) is
   (a) ac changes value and dc does not **(b)** ac changes direction and dc does not
   (c) both answers (a) and (b) **(d)** neither answer (a) nor (b)
2. During each cycle, a sine wave reaches a peak value
   (a) one time **(b)** two times
   (c) four times **(d)** a number of times depending on the frequency
3. A sine wave with a frequency of 12 kHz is changing at a faster rate than a sine wave with a frequency of
   (a) 20 kHz **(b)** 15,000 Hz **(c)** 10,000 Hz **(d)** 1.25 MHz
4. A sine wave with a period of 2 ms is changing at a faster rate than a sine wave with a period of
   (a) 1 ms **(b)** 0.0025 s **(c)** 1.5 ms **(d)** 1200 ms
5. When a sine wave has a frequency of 60 Hz, in 10 s it goes through
   (a) 6 cycles **(b)** 10 cycles **(c)** 1/16 cycle **(d)** 600 cycles
6. If the peak value of a sine wave is 10 V, the peak-to-peak value is
   (a) 20 V **(b)** 5 V **(c)** 100 V **(d)** none of these
7. If the peak value of a sine wave is 20 V, the rms value is
   (a) 14.14 V **(b)** 6.37 V **(c)** 7.07 V **(d)** 0.707 V
8. The average value of a 10 V peak sine wave over one complete cycle is
   (a) 0 V **(b)** 6.37 V **(c)** 7.07 V **(d)** 5 V
9. The average half-cycle value of a sine wave with a 20 V peak is
   (a) 0 V **(b)** 6.37 V **(c)** 12.74 V **(d)** 14.14 V
10. One sine wave has a positive-going zero crossing at 10° and another sine wave has a positive-going zero crossing at 45°. The phase angle between the two waveforms is
    (a) 55° **(b)** 35° **(c)** 0° **(d)** none of these
11. The instantaneous value of a 15 A peak sine wave at a point 32° from its positive-going zero crossing is
    (a) 7.95 A **(b)** 7.5 A **(c)** 2.13 A **(d)** 7.95 V
12. A phasor represents
    (a) the magnitude of a quantity **(b)** the magnitude and direction of a quantity
    (c) the phase angle **(d)** the length of a quantity
13. If the rms current through a 10 kΩ resistor is 5 mA, the rms voltage drop across the resistor is
    (a) 70.7 V **(b)** 7.07 V **(c)** 5 V **(d)** 50 V
14. Two series resistors are connected to an ac source. If there are 6.5 V rms across one resistor and 3.2 V rms across the other, the peak source voltage is
    (a) 9.7 V **(b)** 9.19 V **(c)** 13.72 V **(d)** 4.53 V
15. An advantage of a three-phase induction motor is that it
    (a) maintains constant speed for any load **(b)** does not require starting windings
    (c) has a wound rotor **(d)** all of the above

16. The difference in the synchronous speed of the stator field and the rotor speed of a motor is called

(a) differential speed **(b)** loading **(c)** lag **(d)** slip

17. A 10 kHz pulse waveform consists of pulses that are 10 μs wide. Its duty cycle is

(a) 100% **(b)** 10% **(c)** 1% **(d)** not determinable

18. The duty cycle of a square wave

(a) varies with the frequency **(b)** varies with the pulse width

(c) both answers (a) and (b) **(d)** is 50%

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 81.

1. If the source voltage increases, the voltage across R_3

(a) increases **(b)** decreases **(c)** stays the same

2. If R_4 opens, the voltage across R_3

(a) increases **(b)** decreases **(c)** stays the same

3. If the half-cycle average value of the source voltage is decreased, the rms voltage across R_2

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 83.

4. If the dc voltage is reduced, the average current through R_L

(a) increases **(b)** decreases **(c)** stays the same

5. If the dc voltage source is reversed, the rms current through R_L

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 90.

6. If the resistor in the upper left of the protoboard has a color code of blue, gray, brown, gold instead of the color bands shown, the CH2 voltage measured by the oscilloscope

(a) increases **(b)** decreases **(c)** stays the same

7. If the CH2 probe shown connected to the right side of the resistor is moved to the left side of the resistor, the amplitude of the measured voltage

(a) increases **(b)** decreases **(c)** stays the same

8. If the bottom lead of the right-most resistor becomes disconnected, the CH2 voltage

(a) increases **(b)** decreases **(c)** stays the same

9. If the wire connecting the two upper resistors becomes disconnected, altering the loading effect on the input signal source, the CH1 voltage

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 91.

10. If the right-most resistor has a third band that is orange instead of red, the CH1 voltage

(a) increases **(b)** decreases **(c)** stays the same

11. If the resistor at the upper left opens, the CH1 voltage

(a) increases **(b)** decreases **(c)** stays the same

12. If the resistor at the lower left opens, the CH1 voltage

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Sinusoidal Waveform

1. Calculate the frequency for each of the following values of period:

(a) 1 s **(b)** 0.2 s **(c)** 50 ms **(d)** 1 ms **(e)** 500 μs **(f)** 10 μs

2. Calculate the period of each of the following values of frequency:

(a) 1 Hz **(b)** 60 Hz **(c)** 500 Hz **(d)** 1 kHz **(e)** 200 kHz **(f)** 5 MHz

3. A sine wave goes through 5 cycles in 10 μs. What is its period?

4. A sine wave has a frequency of 50 kHz. How many cycles does it complete in 10 ms?

5. How long does it take a 10 kHz sine wave to complete 100 cycles?

SECTION 2 Sinusoidal Voltage and Current Values

6. A sine wave has a peak value of 12 V. Determine the following values:

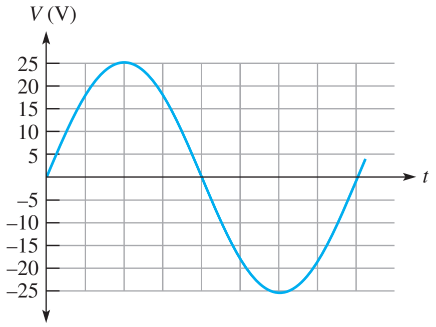
(a) rms **(b)** peak-to-peak **(c)** average

7. A sinusoidal current has an rms value of 5 mA. Determine the following values:

(a) peak **(b)** average **(c)** peak-to-peak

8. For the sine wave in Figure 77, determine the peak, peak-to-peak, rms, and average values.

► FIGURE 77



SECTION 3 Angular Measurement of a Sine Wave

9. Convert the following angular values from degrees to radians:

(a) 30° **(b)** 45° **(c)** 78° **(d)** 135° **(e)** 200° **(f)** 300°

10. Convert the following angular values from radians to degrees:

(a) $\pi/8$ rad **(b)** $\pi/3$ rad **(c)** $\pi/2$ rad **(d)** $3\pi/5$ rad **(e)** $6\pi/5$ rad **(f)** 1.8π rad

11. Sine wave *A* has a positive-going zero crossing at 30°. Sine wave *B* has a positive-going zero crossing at 45°. Determine the phase angle between the two signals. Which signal leads?

12. One sine wave has a positive peak at 75°, and another has a positive peak at 100°. How much is each sine wave shifted in phase from the 0° reference? What is the phase angle between them?

13. Make a sketch of two sine waves as follows: Sine wave *A* is the reference, and sine wave *B* lags *A* by 90°. Both have equal amplitudes.

SECTION 4 The Sine Wave Formula

14. A certain sine wave has a positive-going zero crossing at 0° and an rms value of 20 V. Calculate its instantaneous value at each of the following angles:

(a) 15° **(b)** 33° **(c)** 50° **(d)** 110°
(e) 70° **(f)** 145° **(g)** 250° **(h)** 325°

15. For a particular 0° reference sinusoidal current, the peak value is 100 mA. Determine the instantaneous value at each of the following points:

(a) 35° **(b)** 95° **(c)** 190° **(d)** 215° **(e)** 275° **(f)** 360°

16. For a 0° reference sine wave with an rms value of 6.37 V, determine its instantaneous value at each of the following points:

(a) $\pi/8$ rad **(b)** $\pi/4$ rad **(c)** $\pi/2$ rad **(d)** $3\pi/4$ rad

(e) π rad **(f)** $3\pi/2$ rad **(g)** 2π rad

17. Sine wave A lags sine wave B by 30°. Both have peak values of 15 V. Sine wave A is the reference with a positive-going crossing at 0°. Determine the instantaneous value of sine wave B at 30°, 45°, 90°, 180°, 200°, and 300°.

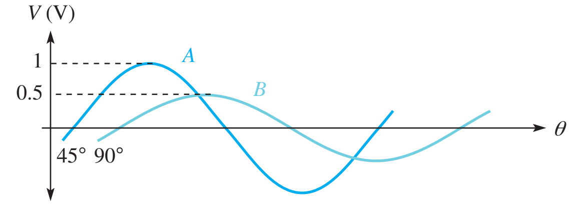
18. Repeat Problem 17 for the case when sine wave A leads sine wave B by 30°.

***19.** A certain sine wave has a frequency of 2.2 kHz and an rms value of 25 V. Assuming a given cycle begins (zero crossing) at $t = 0$ s, what is the change in voltage from 0.12 ms to 0.2 ms?

SECTION 5 Introduction to Phasors

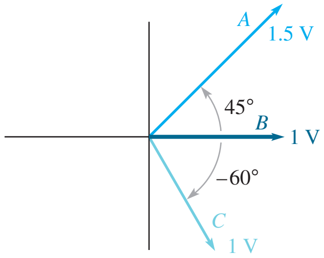
20. Draw a phasor diagram to represent the sine waves in Figure 78 with respect to a 0° reference.

▶ FIGURE 78



21. Draw the sine waves represented by the phasor diagram in Figure 79. The phasor lengths represent peak values.

▶ FIGURE 79



22. Determine the frequency for each angular velocity:

(a) 60 rad/s **(b)** 360 rad/s **(c)** 2 rad/s **(d)** 1256 rad/s

23. Determine the value of sine wave A in Figure 78 at each of the following times, measured from the positive-going zero crossing. Assume the frequency is 5 kHz.

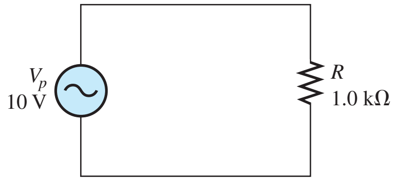
(a) 30 μs **(b)** 75 μs **(c)** 125 μs

SECTION 6 Analysis of AC Circuits

24. A sinusoidal voltage is applied to the resistive circuit in Figure 80. Determine the following:

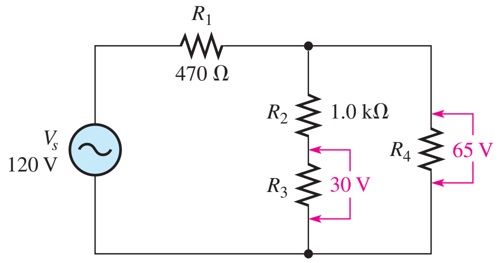
(a) I_{rms} **(b)** I_{avg} **(c)** I_p **(d)** I_{pp} **(e)** i at the positive peak

▶ FIGURE 80



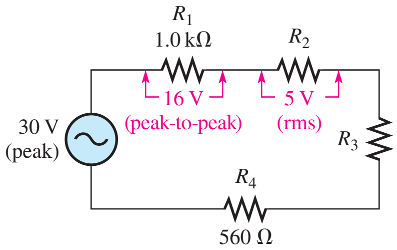
25. Find the half-cycle average values of the voltages across R_1 and R_2 in Figure 81. All values shown are rms.

▶ **FIGURE 81**



26. Determine the rms voltage across R_3 in Figure 82.

▶ **FIGURE 82**



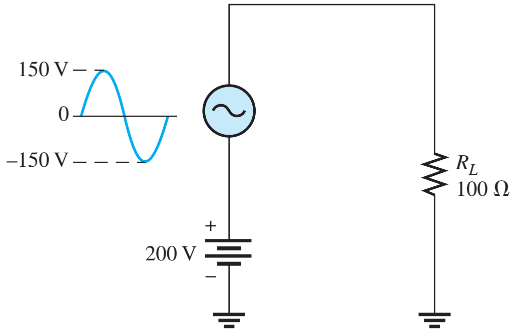
27. A sine wave with an rms value of 10.6 V is riding on a dc level of 24 V. What are the maximum and minimum values of the resulting waveform?

28. How much dc voltage must be added to a 3 V rms sine wave in order to make the resulting voltage nonalternating (no negative values)?

29. A 6 V peak sine wave is riding on a dc voltage of 8 V. If the dc voltage is lowered to 5 V, how far negative will the sine wave go?

***30.** Figure 83 shows a sinusoidal voltage source in series with a dc source. Effectively, the two voltages are superimposed. Determine the power dissipation in the load resistor.

▶ **FIGURE 83**



SECTION 7 The Alternator (AC Generator)

31. The conductive loop on the rotor of a simple two-pole, single-phase generator rotates at a rate of 250 rps. What is the frequency of the induced output voltage?

32. A certain four-pole generator has a speed of rotation of 3600 rpm. What is the frequency of the voltage produced by this generator?

33. At what speed of rotation must a four-pole generator be operated to produce a 400 Hz sinusoidal voltage?

34. A common frequency for alternators on aircraft is 400 Hz. How many poles does a 400 Hz alternator have if the speed of rotation is 3000 rpm?

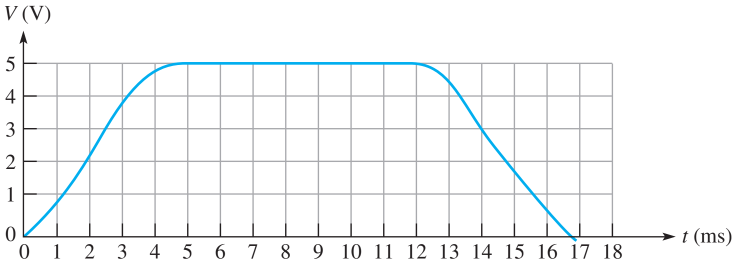
SECTION 8 The AC Motor

35. What is the main difference between a one-phase induction motor and a three-phase induction motor?

36. Explain how the field in a three-phase motor rotates if there are no moving parts to the field coils.

SECTION 9 Nonsinusoidal Waveforms

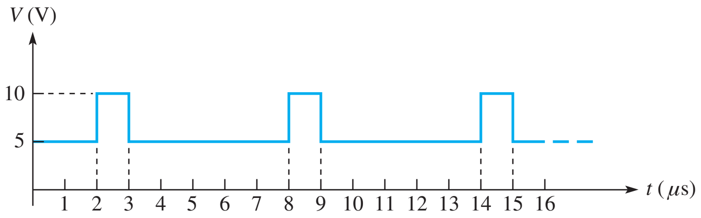
37. From the graph in Figure 84, determine the approximate values of t_r, t_f, t_W, and amplitude.



▲ FIGURE 84

38. The repetition frequency of a pulse waveform is 2 kHz, and the pulse width is 1 μs. What is the percent duty cycle?

39. Calculate the average value of the pulse waveform in Figure 85.

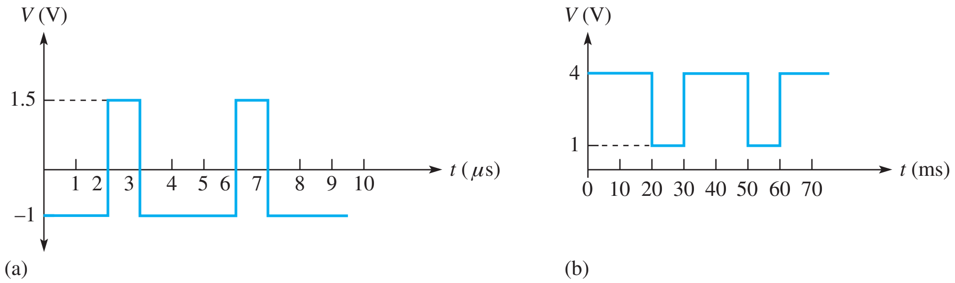


▲ FIGURE 85

40. Determine the duty cycle for each waveform in Figure 86.

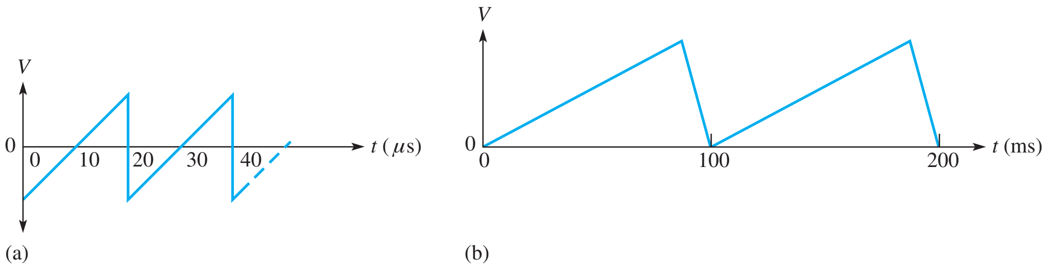
41. Find the average value of each pulse waveform in Figure 86.

42. What is the frequency of each waveform in Figure 86?



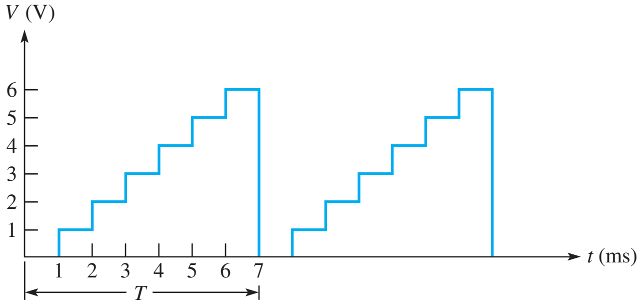
▲ FIGURE 86

43. What is the frequency of each sawtooth waveform in Figure 87?



▲ FIGURE 87

***44.** A nonsinusoidal waveform called a *stairstep* is shown in Figure 88. Determine its average value.



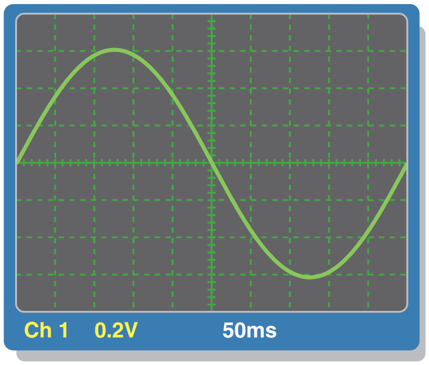
▲ FIGURE 88

45. A square wave has a period of 40 μs. List the first six odd harmonics.

46. What is the fundamental frequency of the square wave mentioned in Problem 45?

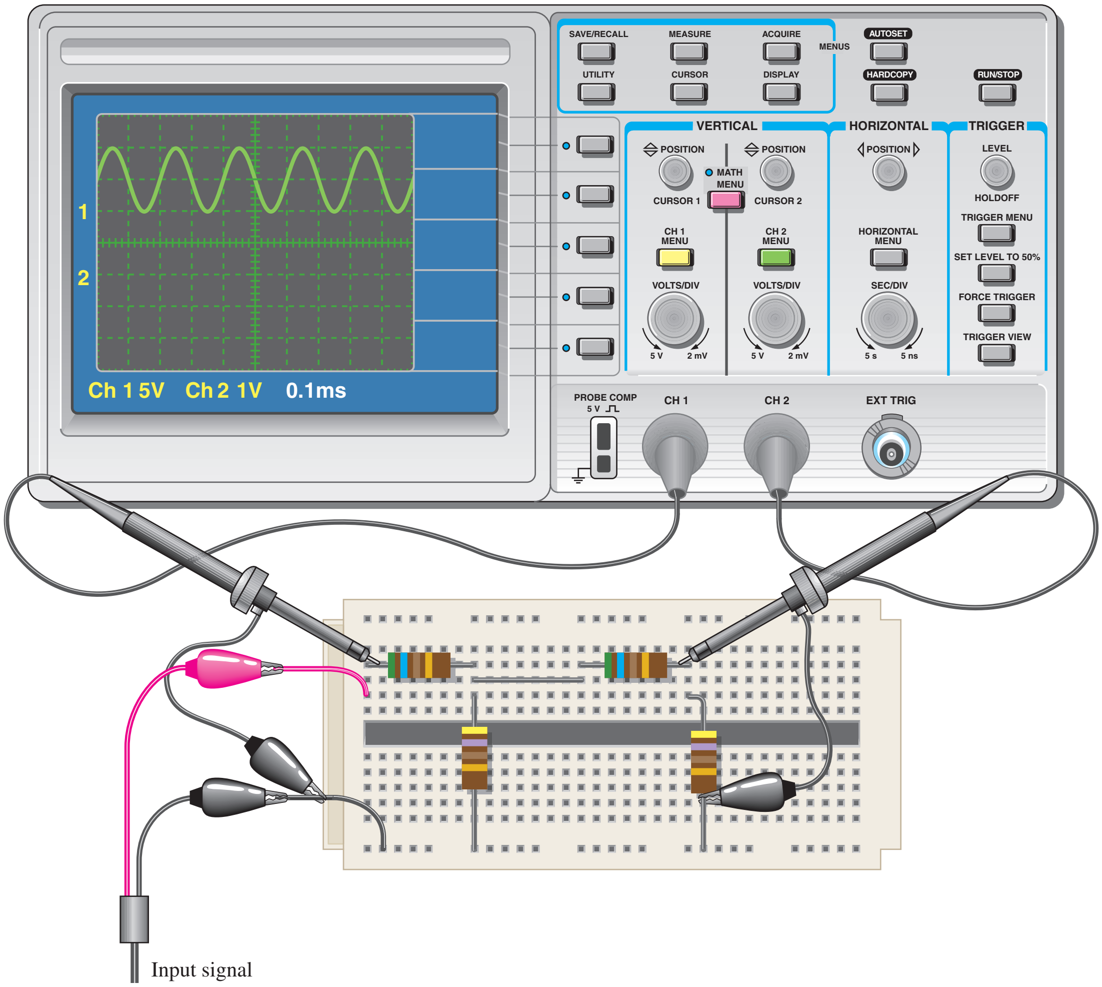
SECTION 10 The Oscilloscope

47. Determine the peak value and the period of the sine wave displayed on the scope screen in Figure 89.



▲ FIGURE 89

***48.** Based on the instrument settings and an examination of the scope display and the protoboard in Figure 90, determine the frequency and peak value of the input signal and output signal. The waveform shown is channel 1. Draw the channel 2 waveform as it would appear on the scope with the indicated settings.


SAVE/RECALL
MEASURE
ACQUIRE
MENUS
AUTOSET
UTILITY
CURSOR
DISPLAY
HARDCOPY
RUN/STOP
1
2
Ch 1 5V
Ch 2 1V
0.1ms
VERTICAL
HORIZONTAL
TRIGGER
POSITION
POSITION
POSITION
LEVEL
MATH MENU
CURSOR 1
CURSOR 2
HOLDOFF
TRIGGER MENU
CH 1 MENU
CH 2 MENU
HORIZONTAL MENU
SET LEVEL TO 50%
VOLTS/DIV
VOLTS/DIV
SEC/DIV
FORCE TRIGGER
TRIGGER VIEW
5 V
2 mV
5 V
2 mV
5 s
5 ns
PROBE COMP
5 V
CH 1
CH 2
EXT TRIG
Input signal

▲ FIGURE 90

***49.** Examine the protoboard and the oscilloscope display in Figure 91 and determine the peak value and the frequency of the unknown input signal.

Multisim Troubleshooting and Analysis

These problems require Multisim.

50. Open file P11-50 and measure the peak and rms voltage across each of the resistors.

51. Open file P11-51 and measure the peak and rms voltage across each of the resistors.

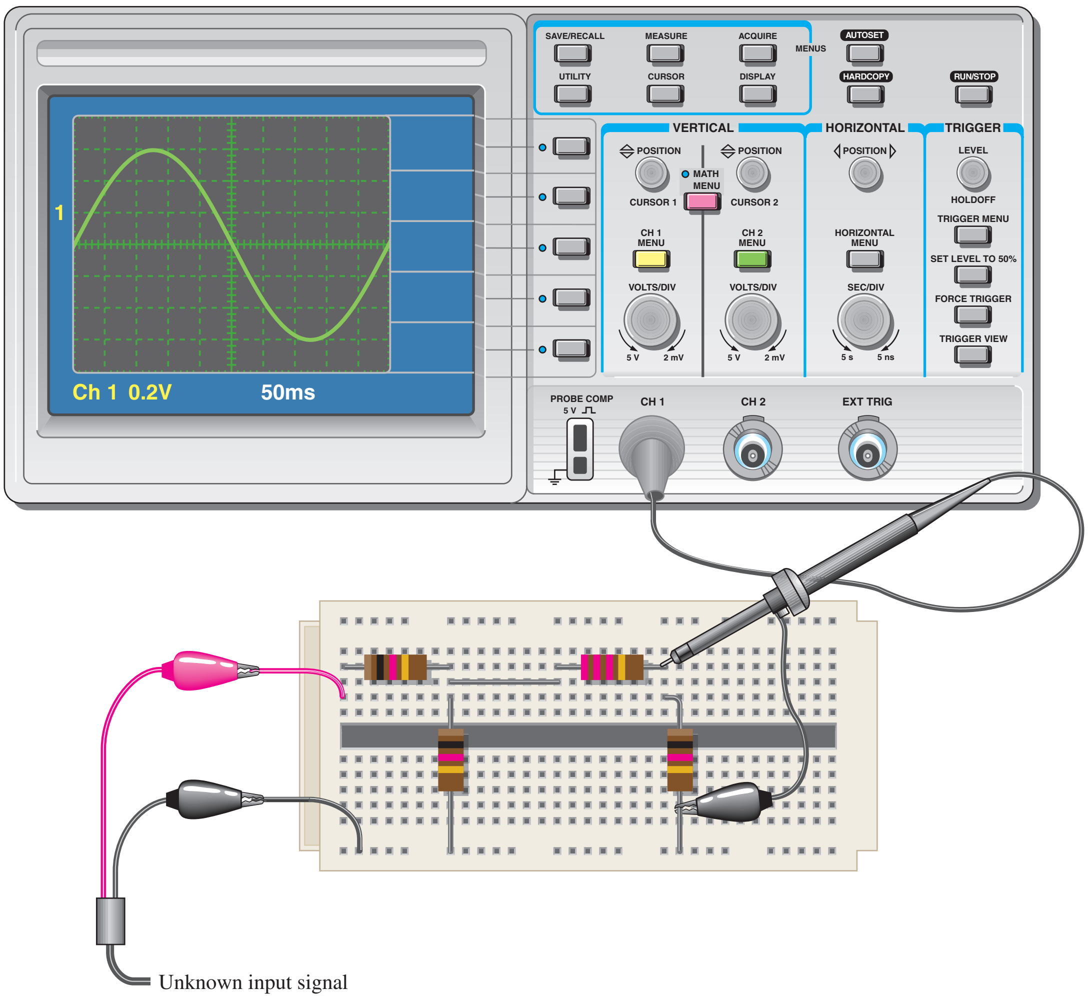
52. Open file P11-52. Determine if there is a fault and, if so, identify the fault.

53. Open file P11-53 and measure the rms current in each branch of the circuit.

54. Open file P11-54. Determine if there is a fault and, if so, identify the fault.

55. Open file P11-55 and measure the total voltage across the resistor using the oscilloscope.

56. Open file P11-56 and measure the total voltage across the resistor using the oscilloscope.


SAVE/RECALL
MEASURE
ACQUIRE
MENUS
AUTOSET
UTILITY
CURSOR
DISPLAY
HARDCOPY
RUN/STOP
VERTICAL
HORIZONTAL
TRIGGER
POSITION
POSITION
POSITION
LEVEL
MATH MENU
CURSOR 1
CURSOR 2
HOLDOFF
TRIGGER MENU
CH 1 MENU
CH 2 MENU
HORIZONTAL MENU
SET LEVEL TO 50%
VOLTS/DIV
VOLTS/DIV
SEC/DIV
FORCE TRIGGER
5 V
2 mV
5 V
2 mV
5 s
5 ns
TRIGGER VIEW
PROBE COMP
5 V
CH 1
CH 2
EXT TRIG
1
Ch 1 0.2V
50ms
Unknown input signal

▲ FIGURE 91

ANSWERS

SECTION CHECKUPS

SECTION 1 The Sinusoidal Waveform

1. One cycle of a sine wave is from the zero crossing through a positive peak, then through zero to a negative peak and back to the zero crossing.
2. A sine wave changes polarity at the zero crossings.
3. A sine wave has two maximum points (peaks) per cycle.
4. The period is from one zero crossing to the next corresponding zero crossing, or from one peak to the next corresponding peak.
5. Frequency is the number of cycles completed in one second; the unit of frequency is the hertz.
6. $f = 1/T = 200\text{ kHz}$
7. $T = 1/f = 8.33\text{ ms}$

SECTION 2 Sinusoidal Voltage and Current Values

1. **(a)** $V_{pp} = 2(1\text{ V}) = 2\text{ V}$ **(b)** $V_{pp} = 2(1.414)(1.414\text{ V}) = 4\text{ V}$
   (c) $V_{pp} = 2(1.57)(3\text{ V}) = 9.42\text{ V}$

2. **(a)** $V_{\text{rms}} = (0.707)(2.5\text{ V}) = 1.77\text{ V}$ **(b)** $V_{\text{rms}} = (0.5)(0.707)(10\text{ V}) = 3.54\text{ V}$
   (c) $V_{\text{rms}} = (0.707)(1.57)(1.5\text{ V}) = 1.66\text{ V}$
3. **(a)** $V_{\text{avg}} = (0.637)(10\text{ V}) = 6.37\text{ V}$ **(b)** $V_{\text{avg}} = (0.637)(1.414)(2.3\text{ V}) = 2.07\text{ V}$
   (c) $V_{\text{avg}} = (0.637)(0.5)(60\text{ V}) = 19.1\text{ V}$

SECTION 3 Angular Measurement of a Sine Wave

1. **(a)** Positive peak at 90° **(b)** Negative-going zero crossing at 180°
   (c) Negative peak at 270° **(d)** End of cycle at 360°
2. Half-cycle: 180°; π
3. Full cycle: 360°; 2π
4. $90^\circ - 45^\circ = 45^\circ$

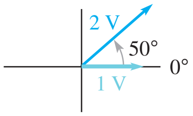
SECTION 4 The Sine Wave Formula

1. $v = (10\text{ V})\sin(120^\circ) = 8.66\text{ V}$
2. $v = (10\text{ V})\sin(45^\circ + 10^\circ) = 8.19\text{ V}$
3. $v = (5\text{ V})\sin(90^\circ - 25^\circ) = 4.53\text{ V}$

SECTION 5 Introduction to Phasors

1. A graphic representation of the magnitude and angular position of a time-varying quantity
2. 9425 rad/s
3. 100 Hz
4. See Figure 92.

► FIGURE 92



SECTION 6 Analysis of AC Circuits

1. $I_p = V_p/R = (1.57)(12.5\text{ V})/330\ \Omega = 59.5\text{ mA}$
2. $V_{s(\text{rms})} = (0.707)(25.3\text{ V}) = 17.9\text{ V}$
3. $+V_{max} = 5\text{ V} + 2.5\text{ V} = 7.5\text{ V}$
4. Yes, it will alternate.
5. $+V_{max} = 5\text{ V} - 2.5\text{ V} = 2.5\text{ V}$

SECTION 7 The Alternator (AC Generator)

1. The number of poles and the rotor speed
2. The brushes do not have to handle the output current.
3. A dc generator that supplies rotor current to larger alternators
4. The diodes convert the ac from the stator to dc for the final output.

SECTION 8 The AC Motor

1. The difference is the rotors. In an induction motor, the rotor obtains current by transformer action; in a synchronous motor, the rotor is a permanent magnet or an electromagnet that is supplied current from an external source through slip rings and brushes.
2. The magnitude is constant.
3. The squirrel cage is composed of the electrical conductors that generate current in the rotor.
4. Slip is the difference between the synchronous speed of the stator field and the rotor speed.

SECTION 9 Nonsinusoidal Waveforms

1. **(a)** Rise time is the time interval from 10% to 90% of the rising pulse edge;
   (b) Fall time is the time interval from 90% to 10% of the falling pulse edge;
   (c) Pulse width is the time interval from 50% of the leading pulse edge to 50% of the trailing pulse edge.
2. $f = 1/1\text{ ms} = 1\text{ kHz}$
3. d.c. $= (1/5)100\% = 20\%$; Ampl. 1.5 V; $V_{\text{avg}} = 0.5\text{ V} + 0.2(1.5\text{ V}) = 0.8\text{ V}$
4. $T = 16\text{ ms}$
5. $f = 1/T = 1/1\ \mu\text{s} = 1\text{ MHz}$
6. Fundamental frequency is the repetition rate of the waveform.
7. 2nd harm.: 2 kHz
8. $f = 1/10\ \mu\text{s} = 100\text{ kHz}$

SECTION 10 The Oscilloscope

1. Analog : Signal drives display directly.
   Digital : Signal is converted to digital for processing and then reconstructed for display.
2. Voltage is measured vertically; time is measured horizontally.
3. The Volts/Div control adjusts the voltage scale.
4. The Sec/Div control adjusts the time scale.
5. Always, unless you are trying to measure a very small, low-frequency signal.

RELATED PROBLEMS FOR EXAMPLES

1 2.4 s
2 1.5 ms
3 20 kHz
4 200 Hz
5 66.7 kHz
6 $V_{pp} = 50\text{ V}$; $V_{\text{rms}} = 17.7\text{ V}$; $V_{\text{avg}} = 15.9\text{ V}$
7 **(a)** $\pi/12$ rad **(b)** 112.5°
8 8°
9 18.1 V
10 10.6 V
11 5 V at −85°
12 34.2 V
13 $I_{\text{rms}} = 4.53\text{ mA}$; $V_{1(\text{rms})} = 4.53\text{ V}$; $V_{2(\text{rms})} = 2.54\text{ V}$; $P_{tot} = 32.0\text{ mW}$
14 23.7 V
15 The waveform in part (a) never goes negative. The waveform in part (b) goes negative for a portion of its cycle.
16 250 rpm
17 5%
18 1.2 V
19 120 V
20 Part (a) 1.06 V, 50 Hz;
part (b) 88.4 mV, 1.67 kHz;
part (c) 4.81 V, 5 kHz;
part (d) 7.07 V, 250 kHz

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** T
6. F **7.** T **8.** F **9.** F **10.** T

SELF-TEST

1. (b) **2.** (b) **3.** (c) **4.** (b) **5.** (d) **6.** (a) **7.** (a)
8. (a) **9.** (c) **10.** (b) **11.** (a) **12.** (b) **13.** (d) **14.** (c)
15. (b) **16.** (d) **17.** (b) **18.** (d)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (a) **3.** (b) **4.** (b) **5.** (c) **6.** (b)
7. (a) **8.** (a) **9.** (a) **10.** (a) **11.** (b) **12.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. **(a)** 1 Hz **(b)** 5 Hz **(c)** 20 Hz
(d) 1 kHz **(e)** 2 kHz **(f)** 100 kHz

3. 2 μs

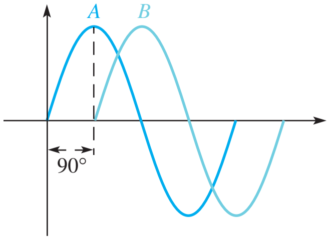
5. 10 ms

7. **(a)** 7.07 mA **(b)** 0 A (full cycle), 4.5 mA (half-cycle)
(c) 14.14 mA

9. **(a)** 0.524 or $\pi/6$ rad **(b)** 0.785 or $\pi/4$ rad
(c) 1.361 or $39\pi/90$ rad **(d)** 2.356 or $3\pi/4$ rad
(e) 3.491 or $10\pi/9$ rad **(f)** 5.236 or $5\pi/3$ rad

11. 15°, A leading

13. See Figure 93.



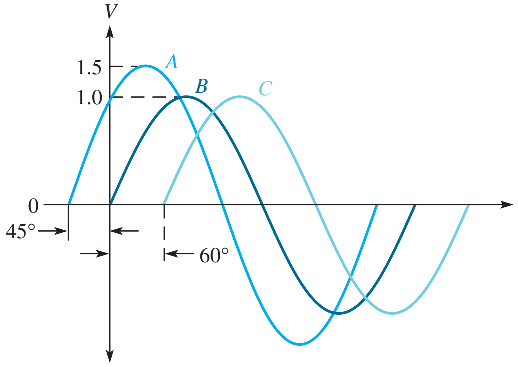
▲ FIGURE 93

15. **(a)** 57.4 mA **(b)** 99.6 mA
(c) −17.4 mA **(d)** −57.4 mA
(e) −99.6 mA **(f)** 0 mA

17. 30°: 13.0 V
45°: 14.5 V
90°: 13.0 V
180°: −7.5 V
200°: −11.5 V
300°: −7.5 V

19. 22.1 V

21. See Figure 94.



▲ FIGURE 94

23. **(a)** 156 mV **(b)** 1 V **(c)** 0 V

25. $V_{1(\text{avg})} = 49.6$ V, $V_{2(\text{avg})} = 31.5$ V

27. $V_{max} = 39$ V, $V_{min} = 9$ V

29. −1 V

31. 250 Hz

33. 200 rps

35. The single-phase motor requires a starting winding; the three-phase does not.

37. $t_r \cong 3.0$ ms, $t_f \cong 3.0$ ms, $t_W \cong 12.0$ ms, Ampl. $\cong 5$ V

39. 5.84 V

41. **(a)** −0.375 V **(b)** 3.01 V

43. **(a)** 50 kHz **(b)** 10 Hz

45. 75 kHz, 125 kHz, 175 kHz, 225 kHz, 275 kHz, 325 kHz

47. $V_p = 600$ mV, $T = 500$ ms

49. $V_{p(in)} = 4.44$ V, $f_{in} = 2$ Hz

51. $V_1 = 16.717\ \text{V}_{\text{pp}}$; $V_1 = 5.911\ \text{V}_{\text{rms}}$;
$V_2 = 36.766\ \text{V}_{\text{pp}}$; $V_2 = 13.005\ \text{V}_{\text{rms}}$;
$V_3 = 14.378\ \text{V}_{\text{pp}}$; $V_3 = 5.084\ \text{V}_{\text{rms}}$

53. No fault

55. $V_{\text{min}} = 2.000\ \text{V}_{\text{p}}$; $V_{\text{max}} = 22.000\ \text{V}_{\text{p}}$

CAPACITORS

From Chapter 12 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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CAPACITORS

CHAPTER OUTLINE

1 The Basic Capacitor
2 Types of Capacitors
3 Series Capacitors
4 Parallel Capacitors
5 Capacitors in DC Circuits
6 Capacitors in AC Circuits
7 Capacitor Applications
8 Switched-Capacitor Circuits
Application Activity

CHAPTER OBJECTIVES

- Describe the basic construction and characteristics of a capacitor
- Discuss various types of capacitors
- Analyze series capacitors
- Analyze parallel capacitors
- Analyze capacitive dc switching circuits
- Analyze capacitive ac circuits
- Discuss some capacitor applications
- Describe the operation of switched-capacitor circuits

KEY TERMS

- Capacitor
- Dielectric
- Farad (F)
- *RC* time constant
- Capacitive reactance
- Instantaneous power
- True power
- Reactive power
- VAR (volt-ampere reactive)
- Ripple voltage

APPLICATION ACTIVITY PREVIEW

In this application activity, you will see how a capacitor is used to couple signal voltages to and from an amplifier. You will also troubleshoot the circuit using oscilloscope waveforms.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, you will learn about the capacitor and its characteristics. The physical construction and electrical properties are examined, and the effects of connecting capacitors in series and in parallel are analyzed. How a capacitor works in both dc and ac circuits is an important part of this coverage and forms the basis for the study of reactive circuits in terms of both frequency response and time response.

The *capacitor* is an electrical device that can store electrical charge, thereby creating an electric field that, in turn, stores energy. The measure of the energy-storing ability of a capacitor is its *capacitance*. When a sinusoidal signal is applied to a capacitor, it reacts in a certain way and produces an opposition to current, which depends on the frequency of the applied signal. This opposition to current is called *capacitive reactance*.

1 THE BASIC CAPACITOR

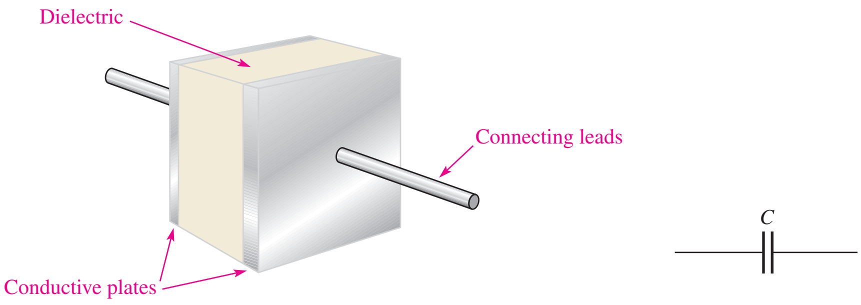
A **capacitor** is a passive electrical component that stores electrical charge and has the property of capacitance.

After completing this section, you should be able to

- **Describe the basic construction and characteristics of a capacitor**
    - Explain how a capacitor stores charge
    - Define *capacitance* and state its unit
    - Explain how a capacitor stores energy
    - Discuss voltage rating and temperature coefficient
    - Explain capacitor leakage
    - Specify how the physical characteristics affect the capacitance

Basic Construction

In its simplest form, a capacitor is an electrical device that stores electrical charge and is constructed of two parallel conductive plates separated by an insulating material called the **dielectric**. Connecting leads are attached to the parallel plates. A basic capacitor is shown in Figure 1(a), and a schematic symbol is shown in part (b).



(a) Construction (b) Symbol

FIGURE 1

The basic capacitor.

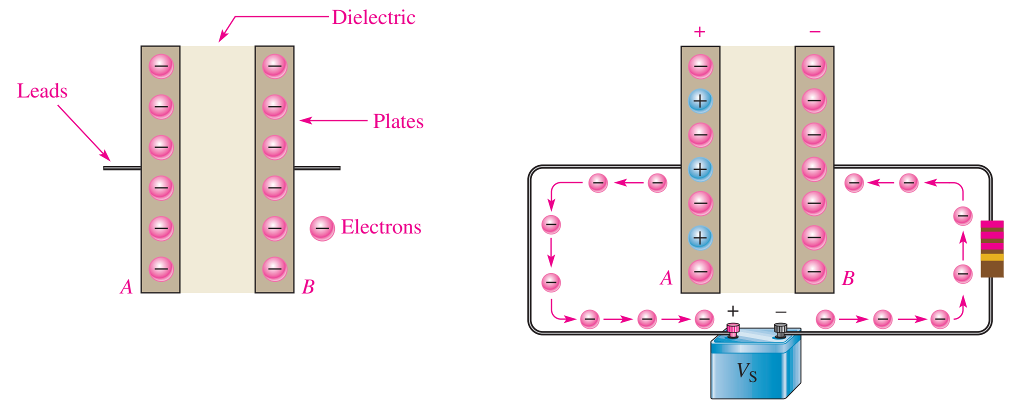
How a Capacitor Stores Charge

In the neutral state, both plates of a capacitor have an equal number of free electrons, as indicated in Figure 2(a). When the capacitor is connected to a voltage source through a resistor, as shown in part (b), electrons (negative charge) are removed from plate *A*, and an equal number are deposited on plate *B*. As plate *A* loses electrons and plate *B* gains electrons, plate *A* becomes positive with respect to plate *B*. During this charging process, electrons flow only through the connecting leads. No electrons flow through the dielectric of the capacitor because it is an insulator. The movement of electrons ceases when the voltage across the capacitor equals the source voltage, as indicated in Figure 2(c). If the capacitor is disconnected from the source, it retains the stored charge for a long period of time (the length of time depends on the type of capacitor) and still has the voltage across it, as shown in Figure 2(d). A charged capacitor can act as a temporary battery.

SAFETY NOTE

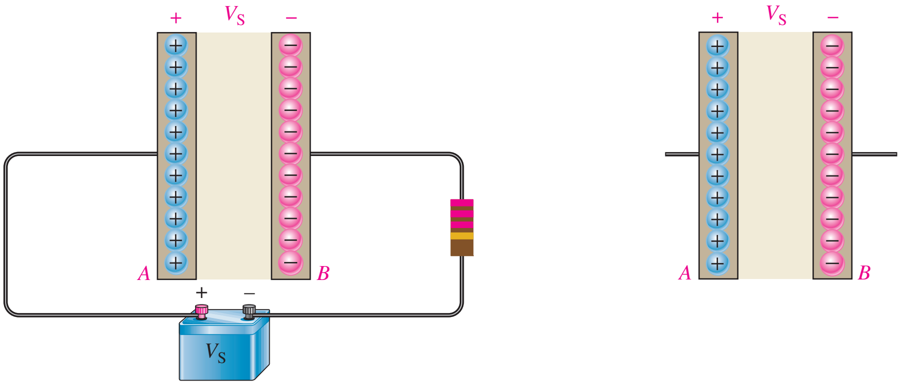
Capacitors are capable of storing electrical charge for a long time after power has been turned off in a circuit. Be careful when touching or handling capacitors in or out of a circuit. If you touch the leads, you may be in for a shock as the capacitor discharges through you! It is usually good practice to discharge a capacitor using a shorting tool with an insulated grip of some sort before handling the capacitor.

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.



(a) Neutral (uncharged) capacitor (same charge on both plates)

(b) Electrons flow from plate *A* to plate *B* as the capacitor charges when connected to a voltage source.



(c) After the capacitor charges to V_S, no electrons flow while connected to the voltage source.

(d) Ideally, the capacitor retains charge when disconnected from the voltage source.

▲ **FIGURE 2**

Illustration of a capacitor storing charge.

Capacitance

The amount of charge that a capacitor can store per unit of voltage across its plates is its capacitance, designated *C*. That is, **capacitance** is a measure of a capacitor's ability to store charge. The more charge per unit of voltage that a capacitor can store, the greater its capacitance, as expressed by the following formula:

Equation 1

$$C = \frac{Q}{V}$$

where C is capacitance, Q is charge, and V is voltage.

By rearranging the terms in Equation 1, you can obtain two other formulas.

Equation 2

$$Q = CV$$

Equation 3

$$V = \frac{Q}{C}$$

The Unit of Capacitance The farad (F) is the basic unit of capacitance. Recall that the coulomb (C) is the unit of electrical charge.

> **One farad is the amount of capacitance when one coulomb (C) of charge is stored with one volt across the plates.**

Most capacitors that are used in electronics work have capacitance values that are specified in microfarads (μF) and picofarads (pF). A microfarad is one-millionth of a farad

($1\ \mu F = 1 \times 10^{-6}$ F), and a picofarad is one-trillionth of a farad ($1\ pF = 1 \times 10^{-12}$ F). Conversions for farads, microfarads, and picofarads are given in Table 1.

TABLE 1

TO CONVERT FROM	TO	MOVE THE DECIMAL POINT
Farads	Microfarads	6 places to right ($\times 10^{6}$)
Farads	Picofarads	12 places to right ($\times 10^{12}$)
Microfarads	Farads	6 places to left ($\times 10^{-6}$)
Microfarads	Picofarads	6 places to right ($\times 10^{6}$)
Picofarads	Farads	12 places to left ($\times 10^{-12}$)
Picofarads	Microfarads	6 places to left ($\times 10^{-6}$)

EXAMPLE 1

(a) A certain capacitor stores 50 microcoulombs (50 μC) with 10 V across its plates. What is its capacitance in units of microfarads?

(b) A 2.2 μF capacitor has 100 V across its plates. How much charge does it store?

(c) Determine the voltage across a 1000 pF capacitor that is storing 20 microcoulombs (20 μC) of charge.

Solution

(a) $C = \frac{Q}{V} = \frac{50\ \mu C}{10\ V} = \mathbf{5\ \mu F}$

(b) $Q = CV = (2.2\ \mu F)(100\ V) = \mathbf{220\ \mu C}$

(c) $V = \frac{Q}{C} = \frac{20\ \mu C}{1000\ pF} = \mathbf{20\ kV}$

*Related Problem** Determine V if $C = 1000$ pF and $Q = 100\ \mu$C.

---

*Answers are at the end of the chapter.

EXAMPLE 2

Convert the following values to microfarads:

(a) 0.00001 F **(b)** 0.0047 F **(c)** 1000 pF **(d)** 220 pF

Solution

(a) $0.00001\ F \times 10^{6}\ \mu F/F = \mathbf{10\ \mu F}$ **(b)** $0.0047\ F \times 10^{6}\ \mu F/F = \mathbf{4700\ \mu F}$

(c) $1000\ pF \times 10^{-6}\ \mu F/pF = \mathbf{0.001\ \mu F}$ **(d)** $220\ pF \times 10^{-6}\ \mu F/pF = \mathbf{0.00022\ \mu F}$

Related Problem Convert 47,000 pF to microfarads.

EXAMPLE 3

Convert the following values to picofarads:

(a) 0.1×10^{-8} F **(b)** 0.000022 F **(c)** 0.01 μF **(d)** 0.0047 μF

Solution

(a) $0.1 \times 10^{-8}\ F \times 10^{12}\ pF/F = \mathbf{1000\ pF}$

(b) $0.000022\ F \times 10^{12}\ pF/F = \mathbf{22 \times 10^{6}\ pF}$

(c) $0.01\ \mu F \times 10^{6}\ pF/\mu F = \mathbf{10{,}000\ pF}$

(d) $0.0047\ \mu F \times 10^{6}\ pF/\mu F = \mathbf{4700\ pF}$

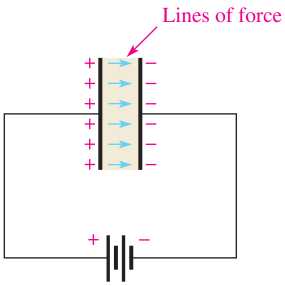
Related Problem Convert 100 μF to picofarads.

How a Capacitor Stores Energy

A capacitor stores energy in the form of an electric field that is established by the opposite charges stored on the two plates. The electric field is represented by lines of force between the positive and negative charges and is concentrated within the dielectric, as shown in Figure 3.

▶ FIGURE 3

The electric field stores energy in a capacitor.



The plates in Figure 3 have acquired a charge because they are connected to a battery. This creates an electric field between the plates, which stores energy. The energy stored in the electric field is directly related to the size of the capacitor and to the square of the voltage as given by the following equation for the energy stored:

Equation 4

$$W = \frac{1}{2}CV^2$$

When capacitance (C) is in farads and voltage (V) is in volts, the energy (W) is in joules.

Voltage Rating

Every capacitor has a limit on the amount of voltage that it can withstand across its plates. The voltage rating specifies the maximum dc voltage that can be applied without risk of damage to the device. If this maximum voltage, commonly called the *breakdown voltage* or *working voltage,* is exceeded, permanent damage to the capacitor can result.

You must consider both the capacitance and the voltage rating before you use a capacitor in a circuit application. The choice of capacitance value is based on particular circuit requirements. The voltage rating should always be above the maximum voltage expected in a particular application.

Dielectric Strength The breakdown voltage of a capacitor is determined by the **dielectric strength** of the dielectric material used. The dielectric strength is expressed in V/mil (1 mil $= 0.001$ in. $= 2.54 \times 10^{-5}$ m). Table 2 lists typical values for several materials. Exact values vary depending on the specific composition of the material.

▶ TABLE 2

Some common dielectric materials and their dielectric strengths.

MATERIAL	DIELECTRIC STRENGTH (V/MIL)
Air	80
Oil	375
Ceramic	1000
Paper (paraffined)	1200
Teflon®	1500
Mica	1500
Glass	2000

A capacitor's dielectric strength can best be explained by an example. Assume that a certain capacitor has a plate separation of 1 mil and that the dielectric material is ceramic. This particular capacitor can withstand a maximum voltage of 1000 V because its dielectric strength is 1000 V/mil. If the maximum voltage is exceeded, the dielectric may break down and conduct current, causing permanent damage to the capacitor. Similarly, if the ceramic capacitor has a plate separation of 2 mils, its breakdown voltage is 2000 V.

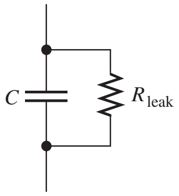
Temperature Coefficient

The **temperature coefficient** indicates the amount and direction of a change in capacitance value with temperature. A positive temperature coefficient means that the capacitance increases with an increase in temperature or decreases with a decrease in temperature. A negative coefficient means that the capacitance decreases with an increase in temperature or increases with a decrease in temperature.

Temperature coefficients are typically specified in parts per million per Celsius degree (ppm/°C). For example, a negative temperature coefficient of 150 ppm/°C for a 1 μF capacitor means that for every degree rise in temperature, the capacitance decreases by 150 pF (there are one million picofarads in one microfarad).

Leakage

No insulating material is perfect. The dielectric of any capacitor will conduct some very small amount of current. Thus, the charge on a capacitor will eventually leak off. Some types of capacitors, such as large electrolytic types, have higher leakages than others. An equivalent circuit for a nonideal capacitor is shown in Figure 4. The parallel resistor R_{leak} represents the extremely high resistance (several hundred kilohms or more) of the dielectric material through which there is leakage current.



▲ **FIGURE 4**

Equivalent circuit for a nonideal capacitor.

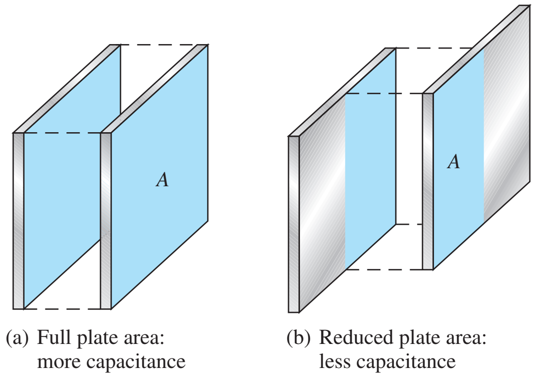
Physical Characteristics of a Capacitor

The following parameters are important in establishing the capacitance and the voltage rating of a capacitor: plate area, plate separation, and dielectric constant.

Plate Area *Capacitance is directly proportional to the physical size of the plates as determined by the plate area, A.* A larger plate area produces more capacitance, and a smaller plate area produces less capacitance. Figure 5(a) shows that the plate area of a parallel plate capacitor is the area of one of the plates. If the plates are moved in relation to each other, as shown in Figure 5(b), the overlapping area determines the effective plate area. This variation in effective plate area is the basis for a certain type of variable capacitor.

► **FIGURE 5**

Capacitance is directly proportional to plate area (A).



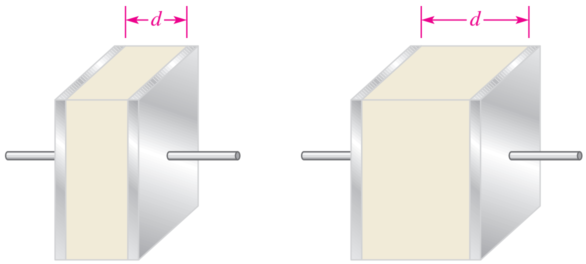
(a) Full plate area: more capacitance

(b) Reduced plate area: less capacitance

Plate Separation *Capacitance is inversely proportional to the distance between the plates.* The plate separation is designated d, as shown in Figure 6. A greater separation of the plates produces a smaller capacitance, as illustrated in the figure. As previously

▶ FIGURE 6

Capacitance is inversely proportional to the distance between the plates.



(a) Plates closer together: more capacitance

(b) Plates further apart: less capacitance

discussed, the breakdown voltage is directly proportional to the plate separation. The further the plates are separated, the greater the breakdown voltage.

Dielectric Constant As you know, the insulating material between the plates of a capacitor is called the *dielectric*. Dielectric materials tend to reduce the voltage between plates for a given charge and thus increase the capacitance. If the voltage is fixed, more charge can be stored due to the presence of a dielectric than can be stored without a dielectric. The measure of a material's ability to establish an electric field is called the **dielectric constant** or *relative permittivity,* symbolized by ε_r. (ε is the Greek letter epsilon.)

Capacitance is directly proportional to the dielectric constant. The dielectric constant of a vacuum is defined as 1 and that of air is very close to 1. These values are used as a reference, and all other materials have values of ε_r specified with respect to that of a vacuum or air. For example, a material with $\varepsilon_r = 8$ can result in a capacitance eight times greater than that of air with all other factors being equal.

Table 3 lists several common dielectric materials and a typical dielectric constant for each. The values can vary because it depends on the specific composition of the material.

▶ TABLE 3

Some common dielectric materials and their typical dielectric constants.

MATERIAL	TYPICAL ε_r VALUE
Air (vacuum)	1.0
Teflon®2.0	
Paper (paraffined)	2.5
Oil	4.0
Mica	5.0
Glass	7.5
Ceramic	1200

The dielectric constant (relative permittivity) is dimensionless because it is a relative measure. It is a ratio of the absolute permittivity of a material, ε, to the absolute permittivity of a vacuum, ε_0, as expressed by the following formula:

Equation 5

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

The value of ε_0 is 8.85×10^{-12} F/m (farads per meter).

Formula You have seen how capacitance is directly related to plate area, A, and the dielectric constant, ε_r, and inversely related to plate separation, d. An exact formula for calculating the capacitance in terms of these three quantities is

Equation 6

$$C = \frac{A\varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})}{d}$$

where A is in square meters (m^2), d is in meters (m), and C is in farads (F). Recall that the absolute permittivity of a vacuum, ε_0, is 8.85×10^{-12} F/m and that the absolute permittivity of a dielectric (ε), as derived from Equation (5), is

$$\varepsilon = \varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})$$

EXAMPLE 4

Determine the capacitance of a parallel plate capacitor having a plate area of 0.01 m^2 and a plate separation of 0.5 mil (1.27×10^{-5} m). The dielectric is mica, which has a dielectric constant of 5.0.

Solution Use Equation 6.

$$C = \frac{A\varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})}{d} = \frac{(0.01\ \text{m}^2)(5.0)(8.85 \times 10^{-12}\ \text{F/m})}{1.27 \times 10^{-5}\ \text{m}} = \mathbf{0.035\ \mu F}$$

Related Problem Determine C where $A = 3.6 \times 10^{-5}\text{m}^2$, $d = 1$ mil (2.54×10^{-5} m), and ceramic is the dielectric.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Define *capacitance*.
2. (a) How many microfarads are in one farad?
   (b) How many picofarads are in one farad?
   (c) How many picofarads are in one microfarad?
3. Convert 0.0015 μF to picofarads. To farads.
4. How much energy in joules is stored by a 0.01 μF capacitor with 15 V across its plates?
5. (a) When the plate area of a capacitor is increased, does the capacitance increase or decrease?
   (b) When the distance between the plates is increased, does the capacitance increase or decrease?
6. The plates of a ceramic capacitor are separated by 2 mils. What is the typical breakdown voltage?
7. A capacitor with a value of 2 μF at 25°C has a positive temperature coefficient of 50 ppm/°C. What is the capacitance value when the temperature increases to 125°C?

2 TYPES OF CAPACITORS

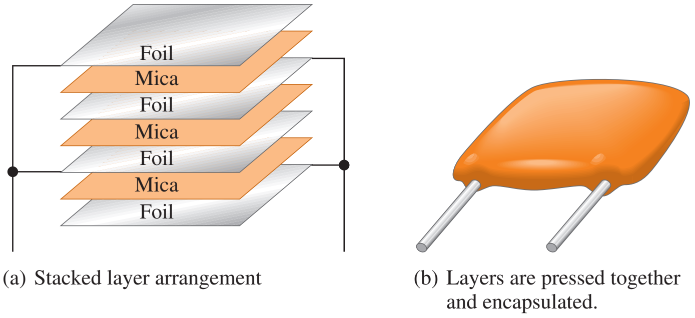
Capacitors normally are classified according to the type of dielectric material and whether they are polarized or nonpolarized. The most common types of dielectric materials are mica, ceramic, plastic-film, and electrolytic (aluminum oxide and tantalum oxide).

After completing this section, you should be able to

- **Discuss various types of capacitors**
  - Describe the characteristics of mica, ceramic, plastic-film, and electrolytic capacitors
  - Describe types of variable capacitors
  - Identify capacitor labeling
  - Discuss capacitance measurement

Fixed Capacitors

Mica Capacitors Two types of mica capacitors are stacked-foil and silver-mica. The basic construction of the stacked-foil type is shown in Figure 7. It consists of alternate layers of metal foil and thin sheets of mica. The metal foil forms the plate, with alternate foil sheets connected together to increase the plate area. More layers are used to increase the plate area, thus increasing the capacitance. The mica/foil stack is encapsulated in an insulating material such as Bakelite®, as shown in Figure 7(b). A silver-mica capacitor is formed in a similar way by stacking mica sheets with silver electrode material screened on them.


Foil
Mica
Foil
Mica
Foil
Mica
Foil

(a) Stacked layer arrangement

(b) Layers are pressed together and encapsulated.

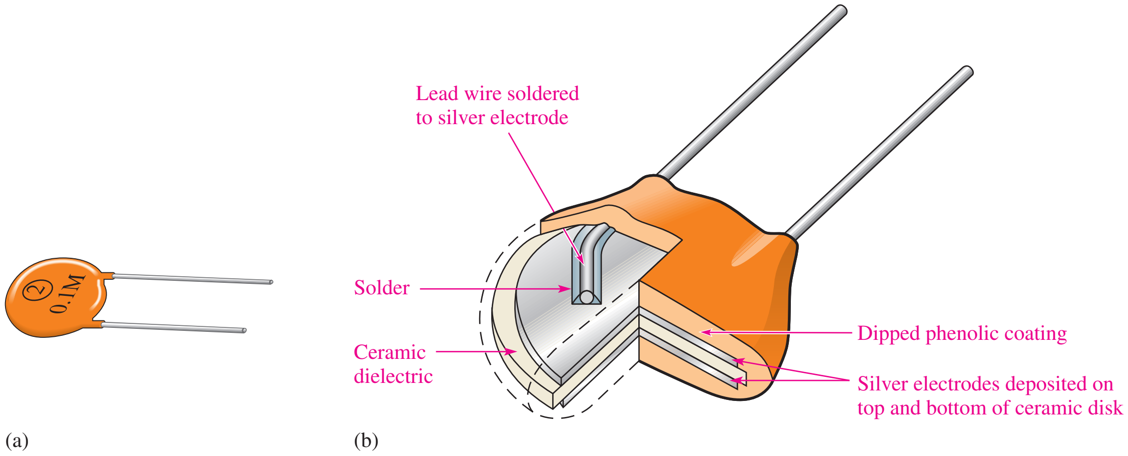
▲ FIGURE 7

Construction of a typical radial-lead mica capacitor.

Mica capacitors are available with capacitance values ranging from 1 pF to 0.1 μF and voltage ratings from 100 V dc to 2500 V dc. Mica has a typical dielectric constant of 5.

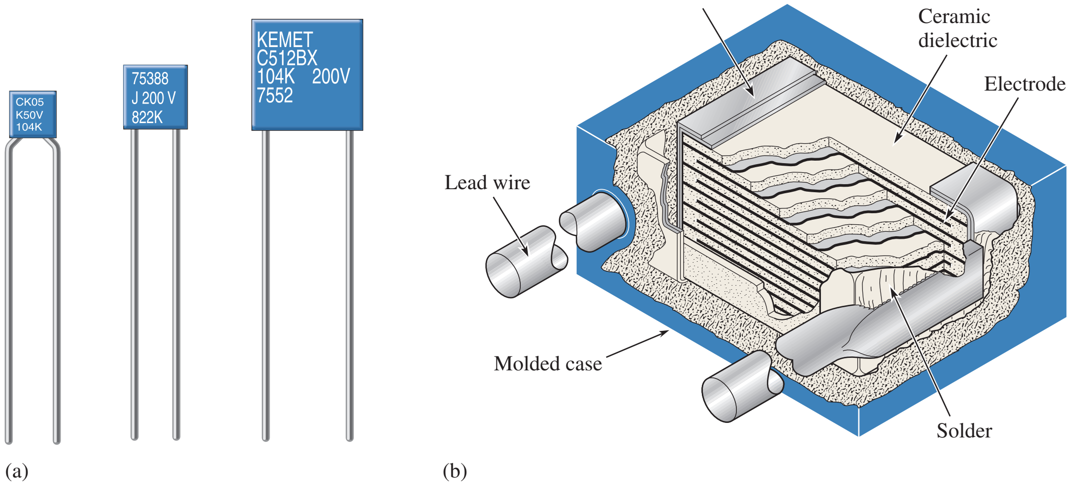
Ceramic Capacitors Ceramic dielectrics provide very high dielectric constants (1200 is typical). As a result, comparatively high capacitance values can be achieved in a small physical size. Ceramic capacitors are commonly available in a ceramic disk form, as shown in Figure 8, in a multilayer radial-lead configuration, as shown in Figure 9, or in a leadless ceramic chip, as shown in Figure 10, for surface mounting on printed circuit boards.

Ceramic capacitors typically are available in capacitance values ranging from 1 pF to 100 μF with voltage ratings up to 6 kV.


②
0.1M
Lead wire soldered to silver electrode
Solder
Ceramic dielectric
Dipped phenolic coating
Silver electrodes deposited on top and bottom of ceramic disk
(a)
(b)

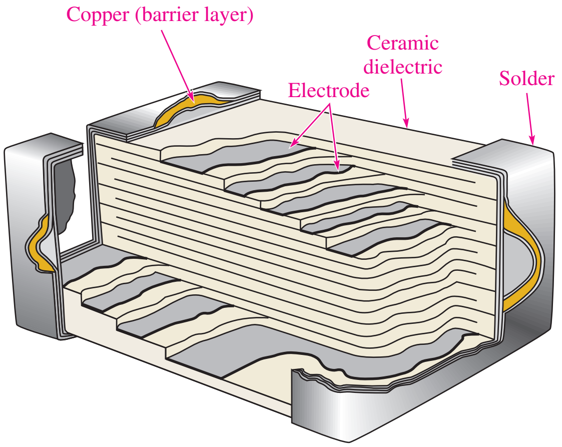
▲ FIGURE 8

A ceramic disk capacitor and its basic construction.


CK05
K50V
104K
75388
J 200 V
822K
KEMET
C512BX
104K 200V
7552
(a)
Solder
Ceramic dielectric
Electrode
Lead wire
Molded case
Solder
(b)

▲ **FIGURE 9**

(a) Typical ceramic capacitors. (b) Construction view.

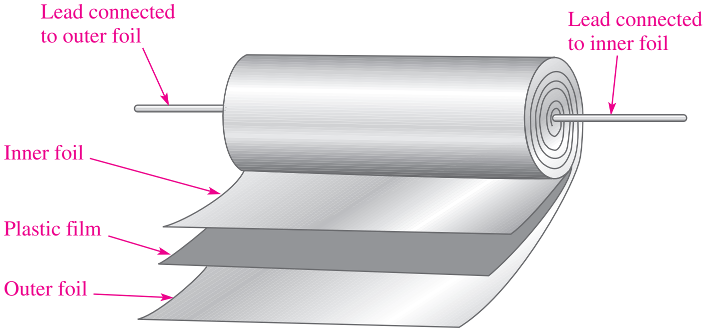

Copper (barrier layer)
Ceramic dielectric
Electrode
Solder

◀ **FIGURE 10**

Construction view of a typical ceramic chip capacitor used for surface mounting on printed circuit boards.

Plastic-Film Capacitors Common dielectric materials used in plastic-film capacitors include polycarbonate, propylene, polyester, polystyrene, polypropylene, and mylar. Some of these types have capacitance values up to 100 μF but most are less than 1 μF.

Figure 11 shows a common basic construction used in many plastic-film capacitors. A thin strip of plastic-film dielectric is sandwiched between two thin metal strips that act


Lead connected to outer foil
Lead connected to inner foil
Inner foil
Plastic film
Outer foil

▲ **FIGURE 11**

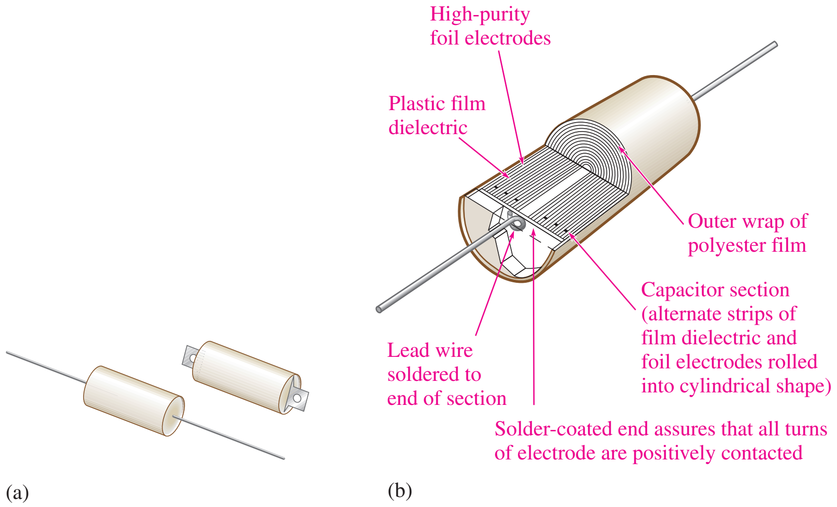
Basic construction of axial-lead tubular plastic-film dielectric capacitors.

TECH NOTE

Scientists are working with graphene, a carbon-based material that may be used for improved charge storage in rechargeable batteries and ultra-capacitors. The ability to store large amounts of charge is important in many applications from office copiers to improving the efficiency of electric and hybrid vehicles. The new technology may speed development of renewable energies such as wind and solar power, which need to be able to store large amounts of energy.

as plates. One lead is connected to the inner plate and one is connected to the outer plate as indicated. The strips are then rolled in a spiral configuration and encapsulated in a molded case. Thus, a large plate area can be packaged in a relatively small physical size, thereby achieving large capacitance values. Another method uses metal deposited directly on the film dielectric to form the plates.

Figure 12(a) shows typical plastic-film capacitors. Figure 12(b) shows a construction view for one type of plastic-film capacitor.



▲ **FIGURE 12**

(a) Typical capacitors. (b) Construction view of plastic-film capacitor.

SAFETY NOTE

Be extremely careful with electrolytic capacitors because it does make a difference which way an electrolytic capacitor is connected. Always observe the proper polarity. If a polarized capacitor is connected backwards, it may explode and cause injury.

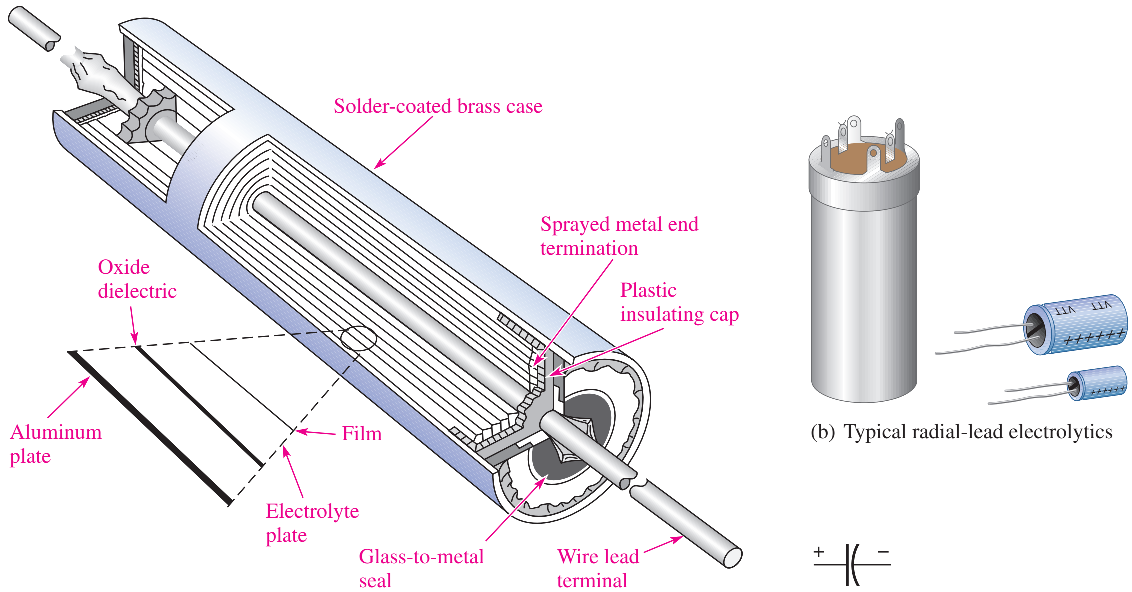
Electrolytic Capacitors Electrolytic capacitors are polarized so that one plate is positive and the other negative. These capacitors are used for capacitance values from 1 μF up to over 200,000 μF, but they have relatively low breakdown voltages (350 V is a typical maximum) and high amounts of leakage. In this text, capacitors with values of 1 μF or greater are considered to be polarized.

In recent years, manufacturers have developed new electrolytic capacitors with much larger capacitance values; however, these new capacitors have lower voltage ratings than smaller-value capacitors and tend to be expensive. Super capacitors with capacitances of hundreds of farads are available. These capacitors are useful for battery backup and for applications like small motor starters that require a very large capacitance.

Electrolytic capacitors offer much higher capacitance values than mica or ceramic capacitors, but their voltage ratings are typically lower. Aluminum electrolytics are probably the most commonly used type. While other capacitors use two similar plates, the electrolytic consists of one plate of aluminum foil and another plate made of a conducting electrolyte applied to a material such as plastic film. These two "plates" are separated by a layer of aluminum oxide that forms on the surface of the aluminum plate. Figure 13(a) illustrates the basic construction of a typical aluminum electrolytic capacitor with axial leads. Other electrolytics with radial leads are shown in Figure 13(b); the symbol for an electrolytic capacitor is shown in part (c).

Tantalum electrolytics can be in either a tubular configuration similar to Figure 13 or "tear drop" shape as shown in Figure 14. In the tear drop configuration, the positive plate is actually a pellet of tantalum powder rather than a sheet of foil. Tantalum pentoxide forms the dielectric, and manganese dioxide forms the negative plate.

Because of the process used for the insulating oxide dielectric, the metallic (aluminum or tantalum) plate must be connected so that it is always positive with respect to the electrolyte


Solder-coated brass case
Sprayed metal end termination
Plastic insulating cap
Oxide dielectric
Aluminum plate
Film
Electrolyte plate
Glass-to-metal seal
Wire lead terminal
+
−

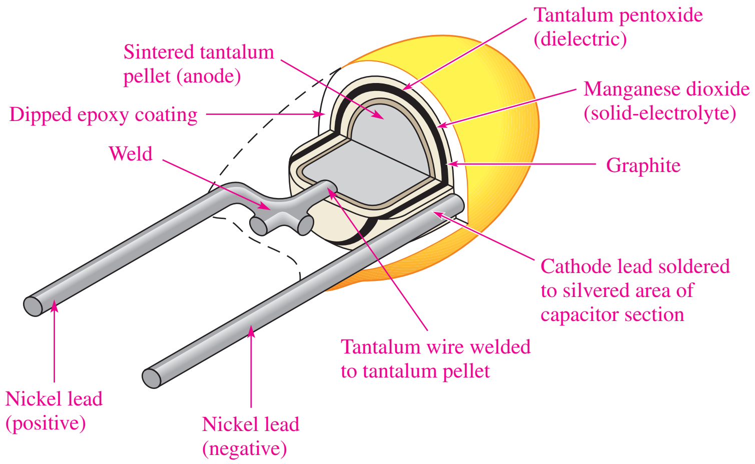
(a) Construction view of an axial-lead electrolytic capacitor

(b) Typical radial-lead electrolytics

(c) Symbol for an electrolytic capacitor. The straight plate is positive and the curved plate is negative, as indicated.

▲ **FIGURE 13**

Examples of electrolytic capacitors.


Tantalum pentoxide (dielectric)
Sintered tantalum pellet (anode)
Manganese dioxide (solid-electrolyte)
Dipped epoxy coating
Weld
Graphite
Cathode lead soldered to silvered area of capacitor section
Tantalum wire welded to tantalum pellet
Nickel lead (positive)
Nickel lead (negative)

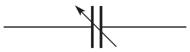
▲ **FIGURE 14**

Construction view of a typical “tear drop” shaped tantalum electrolytic capacitor.

plate, and, thus all electrolytic capacitors are polarized. The metal plate (positive lead) is usually indicated by a plus sign or some other obvious marking and must always be connected in a dc circuit where the voltage across the capacitor does not change polarity regardless of any ac present. Reversal of the polarity of the voltage will usually result in complete destruction of the capacitor.

The problem of dielectric absorption occurs mostly in electrolytic capacitors when they do not completely discharge during use and retain a residual charge. Approximately 25% of defective capacitors exhibit this condition.

Variable Capacitors



▲ **FIGURE 15**

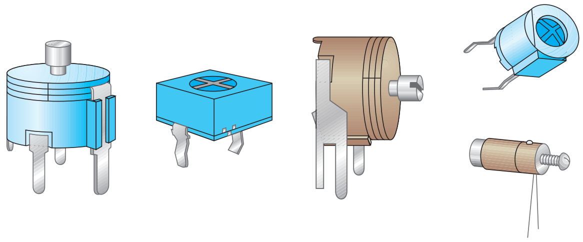
Schematic symbol for a variable capacitor.

Variable capacitors are used in a circuit when there is a need to adjust the capacitance value either manually or automatically. These capacitors are generally less than 300 pF but are available in larger values for specialized applications. The schematic symbol for a variable capacitor is shown in Figure 15.

Adjustable capacitors that normally have slotted screw-type adjustments and are used for very fine adjustments in a circuit are called **trimmers**. Ceramic or mica is a common dielectric in these types of capacitors, and the capacitance usually is changed by adjusting the plate separation. Generally, trimmer capacitors have values less than 100 pF. Figure 16 shows some typical devices.

▶ **FIGURE 16**

Examples of trimmer capacitors.



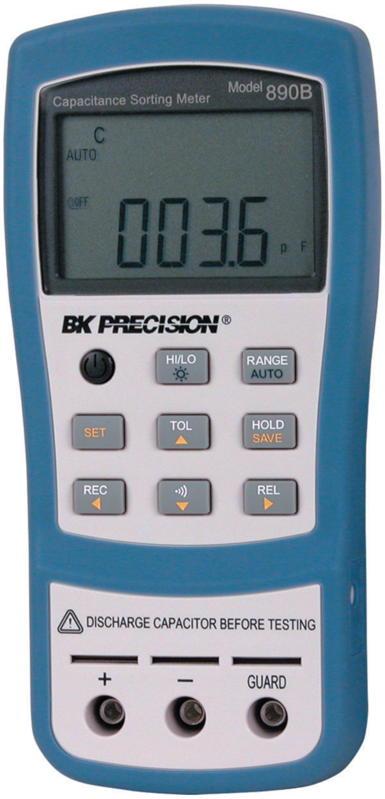
The **varactor** is a semiconductor device that exhibits a capacitance characteristic that is varied by changing the voltage across its terminals. This device usually is covered in detail in a course on electronic devices.

Capacitor Labeling

Capacitor values are indicated on the body of the capacitor either by typographical labels or by color codes. Typographical labels consist of letters and numbers that indicate various parameters such as capacitance, voltage rating, and tolerance.

Some capacitors carry no unit designation for capacitance. In these cases, the units are implied by the value indicated and are recognized by experience. For example, a ceramic capacitor marked .001 or .01 has units of microfarads because picofarad values that small are not available. As another example, a ceramic capacitor labeled 50 or 330 has units of picofarads because microfarad units that large normally are not available in this type. In some cases, a 3-digit designation is used. The first two digits are the first two digits of the capacitance value. The third digit is the number of zeros after the second digit. For example, 103 means 10,000 pF. In some instances, the units are labeled as pF or μF; sometimes the microfarad unit is labeled as MF or MFD.

A voltage rating appears on some types of capacitors with WV or WVDC and is omitted on others. When it is omitted, the voltage rating can be determined from information supplied by the manufacturer. The tolerance of the capacitor is usually labeled as a percentage, such as ±10%. The temperature coefficient is indicated by a *parts per million* marking. This type of label consists of a P or N followed by a number. For example, N750 means a negative temperature coefficient of 750 ppm/°C, and P30 means a positive temperature coefficient of 30 ppm/°C. An NP0 designation means that the positive and negative coefficients are zero; thus the capacitance does not change with temperature. Certain types of capacitors are color coded.



▲ **FIGURE 17**

A typical capacitance meter. (Courtesy of B+K Precision)

Capacitance Measurement

A capacitance meter such as the one shown in Figure 17 can be used to check the value of a capacitor. Also, many DMMs provide a capacitance measurement feature. Most capacitors change value over a period of time, some more than others. Ceramic capacitors, for example, often exhibit a 10% to 15% change in value during the first year. Electrolytic capacitors are particularly subject to value change due to drying of the electrolytic solution. In other cases, capacitors may be labeled incorrectly or the wrong value may have been installed in the circuit. Although a value change represents less than 25% of defective capacitors, a value check can quickly eliminate this as a source of trouble when troubleshooting a circuit.

Typically, values from 200 pF to 200,000 μF can be measured on a capacitance meter by simply connecting the capacitor, setting the switch, and reading the value on the display. Some capacitance meters can also be used to check for leakage current in capacitors. In order to check for leakage, a sufficient voltage must be applied across the capacitor to simulate operating conditions. This is automatically done by the test instrument. Over 40% of all defective capacitors have excessive leakage current and electrolytics are particularly susceptible to this problem.

SECTION 2 CHECKUP

1. Name one way capacitors can be classified.
2. What is the difference between a fixed and a variable capacitor?
3. What type of capacitor is polarized?
4. What precautions must be taken when installing a polarized capacitor in a circuit?

3 SERIES CAPACITORS

The total capacitance of a series connection of capacitors is less than the individual capacitance of any of the capacitors. Capacitors in series divide voltage across them in proportion to their capacitance.

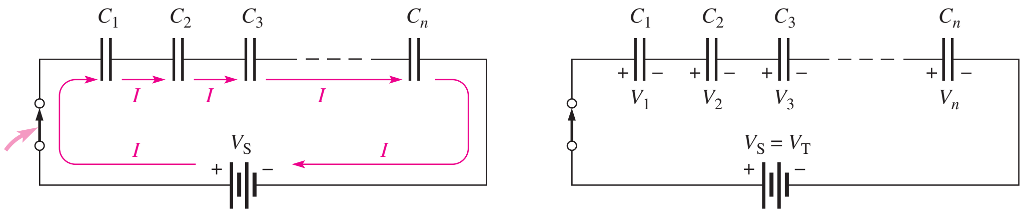
After completing this section, you should be able to

- **Analyze series capacitors**
  - Determine total capacitance
  - Determine capacitor voltages

Total Capacitance

When capacitors are connected in series, the total capacitance is less than the smallest capacitance value because the effective plate separation increases. The calculation of total series capacitance is analogous to the calculation of total resistance of parallel resistors.

Consider the generalized circuit in Figure 18(a), which has n capacitors in series with a voltage source and a switch. When the switch is closed, the capacitors charge as



(a) Charging current is same for each capacitor, $I = Q/t$.

(b) All capacitors store same amount of charge and $V = Q/C$.

▲ FIGURE 18

A series capacitive circuit.

current is established through the circuit. Since this is a series circuit, the current must be the same at all points, as illustrated. Since current is the rate of flow of charge, the amount of charge stored by each capacitor is equal to the total charge, expressed as

Equation 7

$$Q_T = Q_1 = Q_2 = Q_3 = \cdots = Q_n$$

Next, according to Kirchhoff's voltage law, the sum of the voltages across the charged capacitors must equal the total voltage, V_T, as shown in Figure 18(b). This is expressed in equation form as

$$V_T = V_1 + V_2 + V_3 + \cdots + V_n$$

From Equation 3, $V = Q/C$. When this relationship is substituted into each term of the voltage equation, the following result is obtained:

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \cdots + \frac{Q_n}{C_n}$$

Since the charges on all the capacitors are equal, the Q terms can be factored and canceled, resulting in

Equation 8

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

Taking the reciprocal of both sides of Equation 8 yields the following general formula for total series capacitance:

Equation 9

$$C_T = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots + \dfrac{1}{C_n}}$$

Remember,

The total series capacitance is always less than the smallest capacitance.

Two Capacitors in Series When only two capacitors are in series, a special form of Equation 8 can be used.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

Taking the reciprocal of the left and right terms gives the formula for total capacitance of two capacitors in series.

Equation 10

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors of Equal Value in Series This special case is another in which a formula can be developed from Equation 8. When all capacitor values are the same and equal to C, the formula is

$$\frac{1}{C_T} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \cdots + \frac{1}{C}$$

Adding all the terms on the right yields

$$\frac{1}{C_T} = \frac{n}{C}$$

where n is the number of equal-value capacitors. Taking the reciprocal of both sides yields

Equation 11

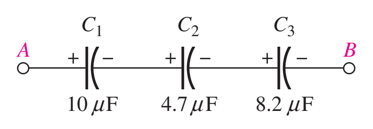
$$C_T = \frac{C}{n}$$

The capacitance value of the equal capacitors divided by the number of equal series capacitors gives the total capacitance.

EXAMPLE 5

Determine the total capacitance between points *A* and *B* in Figure 19.

► **FIGURE 19**



C_1
C_2
C_3
A
B
+
−
10 μF
4.7 μF
8.2 μF

Solution Use Equation 9.

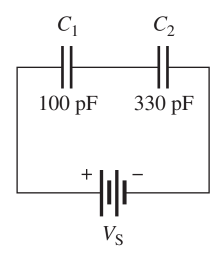
$$C_T = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}} = \frac{1}{\dfrac{1}{10\ \mu\text{F}} + \dfrac{1}{4.7\ \mu\text{F}} + \dfrac{1}{8.2\ \mu\text{F}}} = \mathbf{2.30\ \mu F}$$

Related Problem If a 4.7 μF capacitor is connected in series with the three existing capacitors in Figure 19, what is C_T?

EXAMPLE 6

Find the total capacitance, C_T, in Figure 20.

► **FIGURE 20**



C_1
C_2
100 pF
330 pF
+
−
V_S

Solution From Equation 10,

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{(100\ \text{pF})(330\ \text{pF})}{430\ \text{pF}} = \mathbf{76.7\ pF}$$

You can also use Equation 9.

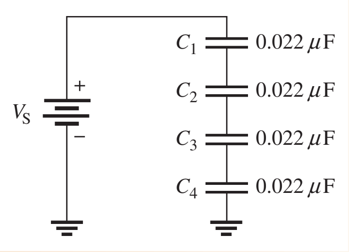
$$C_T = \frac{1}{\dfrac{1}{100\ \text{pF}} + \dfrac{1}{330\ \text{pF}}} = \mathbf{76.7\ pF}$$

Related Problem Determine C_T if $C_1 = 470$ pF and $C_2 = 680$ pF in Figure 20.

EXAMPLE 7

Determine C_T for the series capacitors in Figure 21.

► **FIGURE 21**



V_S
+
−
C_1
0.022 μF
C_2
0.022 μF
C_3
0.022 μF
C_4
0.022 μF

Solution Since $C_1 = C_2 = C_3 = C_4 = C$, use Equation 11,

$$C_T = \frac{C}{n} = \frac{0.022\ \mu F}{4} = \mathbf{0.0055\ \mu F}$$

Related Problem Determine C_T if the capacitor values in Figure 21 are doubled.

Capacitor Voltages

A series connection of charged capacitors acts as a voltage divider. The voltage across each capacitor in series is inversely proportional to its capacitance value, as shown by the formula $V = Q/C$. You can determine the voltage across any individual capacitor in series with the following formula:

Equation 12

$$V_x = \left(\frac{C_T}{C_x}\right)V_T$$

where C_x is any capacitor in series (such as C_1, C_2, or C_3), V_x is the voltage across C_x, and V_T is the total voltage across the capacitors. The derivation is as follows: Since the charge on any capacitor in series is the same as the total charge ($Q_x = Q_T$), and since $Q_x = V_xC_x$ and $Q_T = V_TC_T$, then

$$V_xC_x = V_TC_T$$

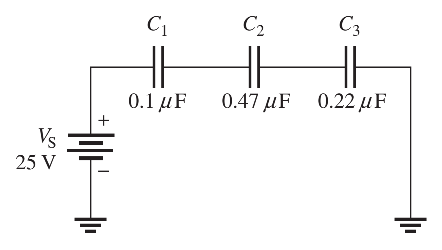
Solving for V_x yields

$$V_x = \frac{C_TV_T}{C_x}$$

The largest-value capacitor in a series connection will have the smallest voltage across it. The smallest-value capacitor will have the largest voltage across it.

EXAMPLE 8 Find the voltage across each capacitor in Figure 22.

▶ FIGURE 22



Solution Calculate the total capacitance.

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{0.1\ \mu F} + \frac{1}{0.47\ \mu F} + \frac{1}{0.22\ \mu F}} = 0.06\ \mu F$$

From Figure 22, $V_S = V_T = 25$ V. Therefore, use Equation 12 to calculate the voltage across each capacitor.

$$V_1 = \left(\frac{C_T}{C_1}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.1\ \mu\text{F}}\right)25\text{ V} = \mathbf{15.0\ V}$$

$$V_2 = \left(\frac{C_T}{C_2}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.47\ \mu\text{F}}\right)25\text{ V} = \mathbf{3.19\ V}$$

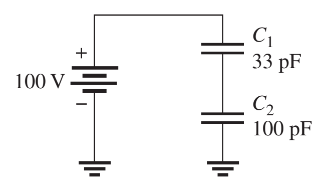
$$V_3 = \left(\frac{C_T}{C_3}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.22\ \mu\text{F}}\right)25\text{ V} = \mathbf{6.82\ V}$$

Related Problem Another 0.47 μF capacitor is connected in series with the existing capacitor in Figure 22. Determine the voltage across the new capacitor, assuming all the capacitors are initially uncharged.

SECTION 3 CHECKUP

1. Is the total capacitance of a series connection less than or greater than the value of the smallest capacitor?
2. The following capacitors are in series: 100 pF, 220 pF, and 560 pF. What is the total capacitance?
3. A 0.01 μF and a 0.015 μF capacitor are in series. Determine the total capacitance.
4. Five 100 pF capacitors are connected in series. What is C_T?
5. Determine the voltage across C_1 in Figure 23.

▶ FIGURE 23



4 PARALLEL CAPACITORS

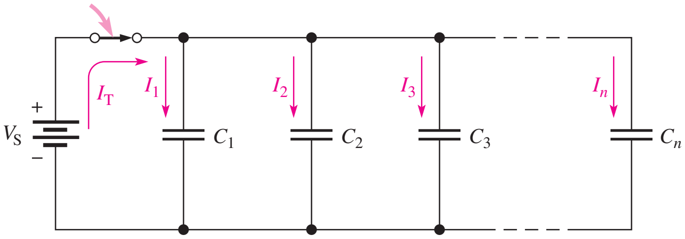
Capacitances add when capacitors are connected in parallel.

After completing this section, you should be able to

- **Analyze parallel capacitors**
  - Determine total capacitance

When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitances because the effective plate area increases. The calculation of total parallel capacitance is analogous to the calculation of total series resistance.

Consider what happens when the switch in Figure 24 is closed. The total charging current from the source divides at the junction of the parallel branches. There is a separate charging current through each branch so that a different charge can be stored by each capacitor. By Kirchhoff's current law, the sum of all of the charging currents is equal to the



▲ FIGURE 24

Capacitors in parallel.

total current. Therefore, the sum of the charges on the capacitors is equal to the total charge. Also, the voltages across all of the parallel branches are equal. These observations are used to develop a formula for total parallel capacitance as follows for the general case of n capacitors in parallel.

Equation 13

$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_n$$

From Equation 2, $Q = CV$. When this relationship is substituted into each term of Equation 13, the following result is obtained:

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 + \cdots + C_n V_n$$

Since $V_T = V_1 = V_2 = V_3 = \cdots = V_n$, the voltages can be factored and canceled, giving

Equation 14

$$C_T = C_1 + C_2 + C_3 + \cdots + C_n$$

Equation 14 is the general formula for total parallel capacitance where n is the number of capacitors. Remember,

The total parallel capacitance is the sum of all the capacitors in parallel.

For the special case when all of the capacitors have the same value, C, multiply the value by the number (n) of capacitors in parallel.

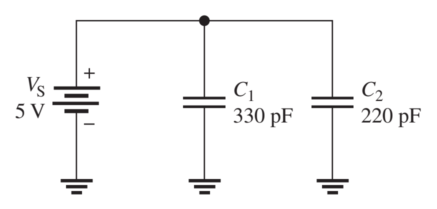
Equation 15

$$C_T = nC$$

EXAMPLE 9

What is the total capacitance in Figure 25? What is the voltage across each capacitor?

▶ FIGURE 25



Solution The total capacitance is

$$C_T = C_1 + C_2 = 330\text{ pF} + 220\text{ pF} = \mathbf{550\text{ pF}}$$

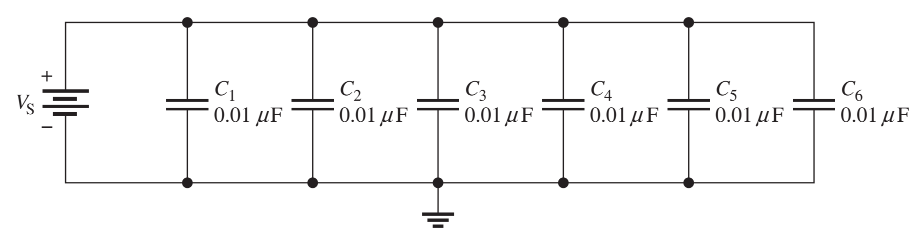
The voltage across each capacitor in parallel is equal to the source voltage.

$$V_S = V_1 = V_2 = \mathbf{5\text{ V}}$$

Related Problem What is C_T if a 100 pF capacitor is connected in parallel with C_2 in Figure 25?

EXAMPLE 10

Determine C_T in Figure 26.



▲ FIGURE 26

Solution There are six equal-value capacitors in parallel, so $n = 6$.

$$C_T = nC = (6)(0.01\ \mu\text{F}) = \mathbf{0.06\ \mu F}$$

Related Problem If three more 0.01 μF capacitors are connected in parallel in Figure 26, what is the total capacitance?

SECTION 4 CHECKUP

1. How is total parallel capacitance determined?
2. In a certain application, you need 0.05 μF. The only values available are 0.01 μF, which are available in large quantities. How can you get the total capacitance that you need?
3. The following capacitors are in parallel: 10 pF, 56 pF, 33 pF, and 68 pF. What is C_T?

5 CAPACITORS IN DC CIRCUITS

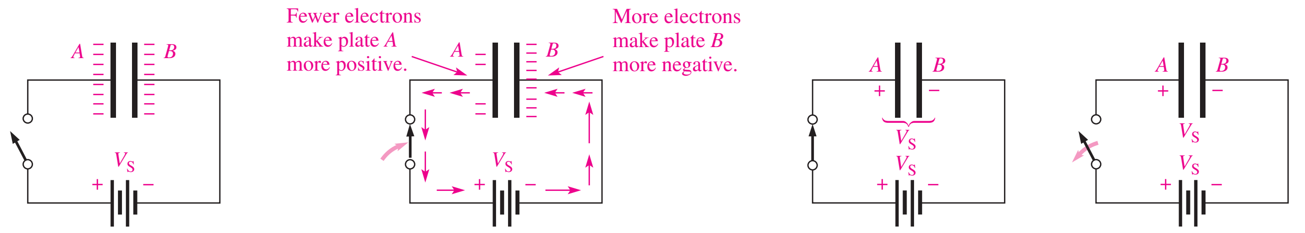
A capacitor will charge up when it is connected to a dc voltage source. The buildup of charge across the plates occurs in a predictable manner that is dependent on the capacitance and the resistance in a circuit.

After completing this section, you should be able to

- **Analyze capacitive dc switching circuits**
    - Describe the charging and discharging of a capacitor
    - Define *RC time constant*
    - Relate the time constant to charging and discharging of a capacitor
    - Write equations for the charging and discharging curves
    - Explain why a capacitor blocks dc

Charging a Capacitor

A capacitor will charge when it is connected to a dc voltage source, as shown in Figure 27. The capacitor in part (a) of the figure is uncharged; that is, plate *A* and plate *B* have equal numbers of free electrons. When the switch is closed, as shown in part (b), the source moves electrons away from plate *A* through the circuit to plate *B* as the arrows indicate. As



(a) Uncharged

(b) Charging (arrows indicate electron flow)

(c) Fully charged.
$I = 0$

(d) Retains charge

FIGURE 27

Charging a capacitor.

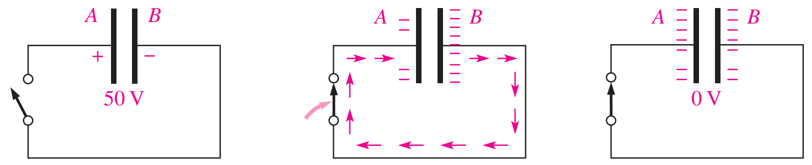
plate *A* loses electrons and plate *B* gains electrons, plate *A* becomes positive with respect to plate *B*. As this charging process continues, the voltage across the plates builds up rapidly until it is equal to the applied voltage, V_S, but opposite in polarity, as shown in part (c). When the capacitor is fully charged, there is no current.

A capacitor blocks constant dc.

When the charged capacitor is disconnected from the source, as shown in Figure 27(d), it remains charged for long periods of time, depending on its leakage resistance, and can cause severe electrical shock. The charge on an electrolytic capacitor generally leaks off more rapidly than in other types of capacitors.

Discharging a Capacitor

When a wire is connected across a charged capacitor, as shown in Figure 28, the capacitor will discharge. In this particular case, a very low resistance path (the wire) is connected across the capacitor with a switch. Before the switch is closed, the capacitor is charged to 50 V, as indicated in part (a). When the switch is closed, as shown in part (b), the excess electrons on plate *B* move through the circuit to plate *A* (indicated by the arrows); as a result of the electrons moving through the low resistance of the wire, the energy stored by the capacitor is dissipated in the wire. The charge is neutralized when the numbers of free electrons on both plates are again equal. At this time, the voltage across the capacitor is zero, and the capacitor is completely discharged, as shown in part (c).



(a) Retains charge

(b) Discharging (arrows indicate electron flow)

(c) Uncharged

FIGURE 28

Discharging a charged capacitor.

Current and Voltage During Charging and Discharging

Notice in Figures 27 and 28 that the direction of electron flow during discharge is opposite to that during charging. It is important to understand that ideally *there is no current through the dielectric of the capacitor during charging or discharging because the* *dielectric is an insulating material.* There is current from one plate to the other only through the external circuit.

Figure 29(a) shows a capacitor connected in series with a resistor and a switch to a dc voltage source. Initially, the switch is open and the capacitor is uncharged with zero volts across its plates. At the instant the switch is closed, the current jumps to its maximum value and the capacitor begins to charge. The current is maximum initially because the capacitor has zero volts across it and, therefore, effectively acts as a short; thus, the current is limited only by the resistance. As time passes and the capacitor charges, the current decreases and the voltage across the capacitor (V_C) increases. The resistor voltage is proportional to the current during this charging period.

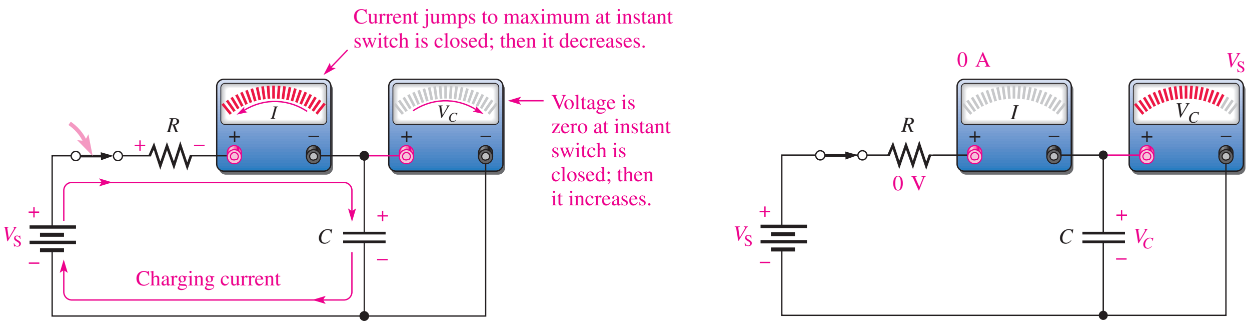
After a certain period of time, the capacitor reaches full charge. At this point, the current is zero and the capacitor voltage is equal to the dc source voltage, as shown in Figure 29(b). If the switch were opened now, the capacitor would retain its full charge (neglecting any leakage).

In Figure 29(c), the voltage source has been removed. When the switch is closed, the capacitor begins to discharge. Initially, the current jumps to a maximum but in a direction opposite to its direction during charging. As time passes, the current and capacitor voltage decrease. The resistor voltage is always proportional to the current. When the capacitor has fully discharged, the current and the capacitor voltage are zero.

Remember the following rules about capacitors in dc circuits:

1. A capacitor appears as an *open* to constant voltage.
2. A capacitor appears as a *short* to an instantaneous change in voltage.

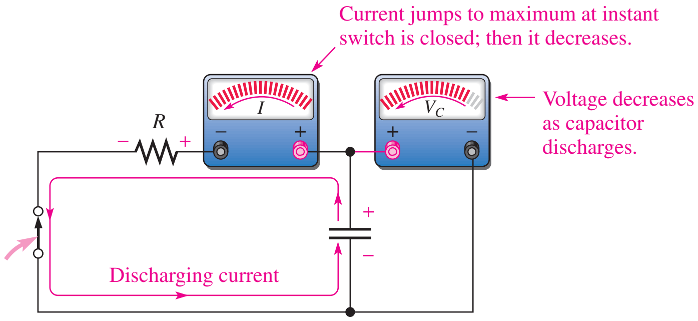
Now let's examine in more detail how the voltage and current change with time in a capacitive circuit.


Current jumps to maximum at instant switch is closed; then it decreases.

Voltage is zero at instant switch is closed; then it increases.

(a) Charging: Capacitor voltage increases as the current and resistor voltage decrease.

(b) Fully charged: Capacitor voltage equals source voltage. The current is zero.


Current jumps to maximum at instant switch is closed; then it decreases.

Voltage decreases as capacitor discharges.

(c) Discharging: Capacitor voltage, resistor voltage, and the current decrease from their initial maximum values. Note that the discharge current is opposite to the charge current.

▲ **FIGURE 29**

Current and voltage in a charging and discharging capacitor.

The *RC* Time Constant

In a practical situation, there cannot be capacitance without some resistance in a circuit. It may simply be the small resistance of a wire, a Thevenin source resistance, or it may be a physical resistor. Because of this, the charging and discharging characteristics of a capacitor must always be considered with the associated resistance. The resistance introduces the element of *time* in the charging and discharging of a capacitor.

When a capacitor charges or discharges through a resistance, a certain time is required for the capacitor to charge fully or discharge fully. The voltage across a capacitor cannot change instantaneously because a finite time is required to move charge from one point to another. The time constant of a series *RC* circuit determines the rate at which the capacitor charges or discharges.

The *RC* time constant is a fixed time interval that equals the product of the resistance and the capacitance in a series *RC* circuit.

The time constant is expressed in seconds when resistance is in ohms and capacitance is in farads. It is symbolized by τ (Greek letter tau), and the formula is

Equation 16

$$\tau = RC$$

Recall that $I = Q/t$. The current depends on the amount of charge moved in a given time. When the resistance is increased, the charging current is reduced, thus increasing the charging time of the capacitor. When the capacitance is increased, the amount of charge increases; thus, for the same current, more time is required to charge the capacitor.

EXAMPLE 11

A series *RC* circuit has a resistance of 1.0 MΩ and a capacitance of 4.7 μF. What is the time constant?

Solution

$$\tau = RC = (1.0 \times 10^{6}\ \Omega)(4.7 \times 10^{-6}\ \text{F}) = \mathbf{4.7\ s}$$

Related Problem A series *RC* circuit has a 270 kΩ resistor and a 3300 pF capacitor. What is the time constant?

When a capacitor is charging or discharging between two voltage levels, the charge on the capacitor changes by approximately 63% of the difference in the levels in one time constant. An uncharged capacitor charges to 63% of its fully charged voltage in one time constant. When a capacitor is discharging, its voltage drops to approximately $100\% - 63\% = 37\%$ of its initial value in one time constant, which is a 63% change.

The Charging and Discharging Curves

A capacitor charges and discharges following a nonlinear curve, as shown in Figure 30. In these graphs, the approximate percentage of full charge is shown at each time-constant interval. This type of curve follows a precise mathematical formula and is called an *exponential curve.* The charging curve is an increasing exponential, and the discharging curve is a decreasing exponential. It takes five time constants to change the voltage by 99% (considered 100%). This five time-constant interval is generally accepted as the time to fully charge or discharge a capacitor and is called the *transient time.*

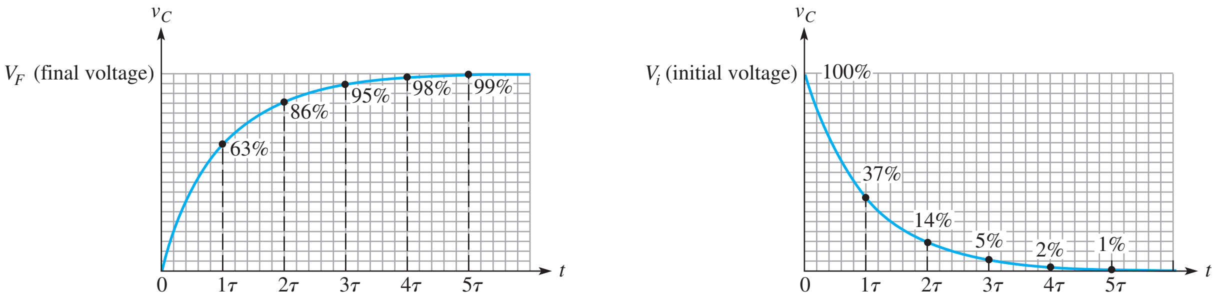
General Formula The general expressions for either increasing or decreasing exponential curves are given in the following equations for both instantaneous voltage and instantaneous current.

Equation 17

$$v = V_F + (V_i - V_F)e^{-t/\tau}$$

Equation 18

$$i = I_F + (I_i - I_F)e^{-t/\tau}$$



(a) Charging curve with percentages of the final voltage

(b) Discharging curve with percentages of the initial voltage

▲ FIGURE 30

Exponential voltage curves for the charging and discharging of an *RC* circuit.

where V_F and I_F are the final values of voltage and current, and V_i and I_i are the initial values of voltage and current. The lowercase italic letters v and i are the instantaneous values of the capacitor voltage and current at time t, and e is the base of natural logarithms. The e^x key on a calculator makes it easy to work with this exponential term.

Charging from Zero The formula for the special case in which an increasing exponential voltage curve begins at zero ($V_i = 0$), as shown in Figure 30(a), is given in Equation 19. It is developed as follows, starting with the general formula, Equation 17.

$$v = V_F + (V_i - V_F)e^{-t/\tau} = V_F + (0 - V_F)e^{-t/RC} = V_F - V_Fe^{-t/RC}$$

Factoring out V_F, you have

$$v = V_F(1 - e^{-t/RC})$$

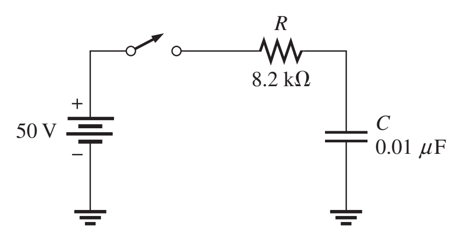
Equation 19

Using Equation 19, you can calculate the value of the charging voltage of a capacitor at any instant of time if it is initially uncharged. You can calculate an increasing current by substituting i for v and I_F for V_F in Equation 19.

EXAMPLE 12

In Figure 31, determine the capacitor voltage 50 μs after the switch is closed if the capacitor is initially uncharged. Draw the charging curve.

► FIGURE 31

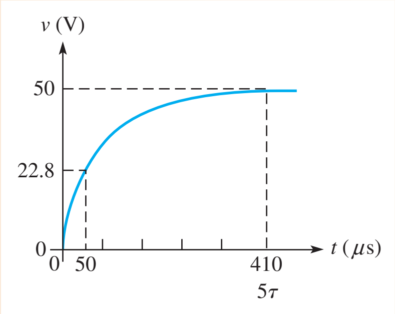


Solution The time constant is $RC = (8.2\text{ k}\Omega)(0.01\ \mu\text{F}) = 82\ \mu\text{s}$. The voltage to which the capacitor will fully charge is 50 V (this is V_F). The initial voltage is zero. Notice that 50 μs is less than one time constant; so the capacitor will charge less than 63% of the full voltage in that time.

$$v_C = V_F(1 - e^{-t/RC}) = (50\text{ V})(1 - e^{-50\mu\text{s}/82\mu\text{s}})$$
$$= (50\text{ V})(1 - e^{-0.61}) = (50\text{ V})(1 - 0.543) = \mathbf{22.8\ V}$$

The charging curve for the capacitor is shown in Figure 32.

► **FIGURE 32**



You can determine an exponential function on your calculator by using the e^x key and entering the value of the exponent of e.

Related Problem Determine the capacitor voltage 15 μs after switch closure in Figure 31.

Use Multisim file E12-12 to verify the calculated results in this example and to confirm your calculation for the related problem.

Discharging to Zero The formula for the special case in which a decreasing exponential voltage curve ends at zero ($V_F = 0$), as shown in Figure 30(b), is derived from the general formula as follows:

$$v = V_F + (V_i - V_F)e^{-t/\tau} = 0 + (V_i - 0)e^{-t/RC}$$

This reduces to

Equation 20

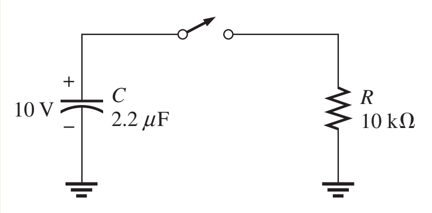
$$v = V_i e^{-t/RC}$$

where V_i is the voltage at the beginning of the discharge. You can use this formula to calculate the discharging voltage at any instant, as Example 13 illustrates.

EXAMPLE 13

Determine the capacitor voltage in Figure 33 at a point in time 6 ms after the switch is closed. Draw the discharging curve.

► **FIGURE 33**

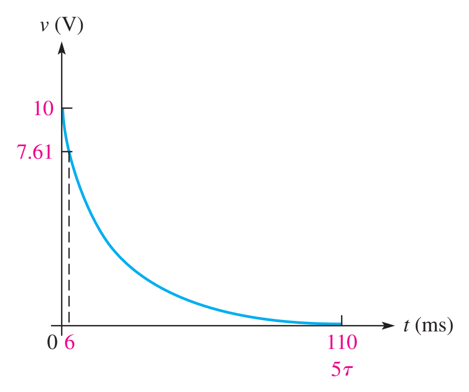


Solution The discharge time constant is $RC = (10\text{ k}\Omega)(2.2\text{ }\mu\text{F}) = 22\text{ ms}$. The initial capacitor voltage is 10 V. Notice that 6 ms is less than one time constant, so the capacitor will discharge less than 63%. Therefore, it will have a voltage greater than 37% of the initial voltage at 6 ms.

$$v_C = V_i e^{-t/RC} = (10\text{ V})e^{-6\text{ms}/22\text{ms}} = (10\text{ V})e^{-0.27} = (10\text{ V})(0.761) = \mathbf{7.61\text{ V}}$$

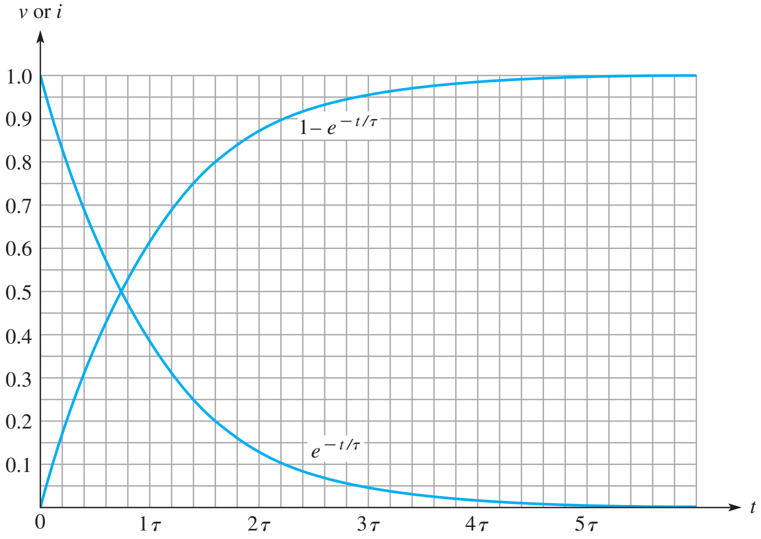
The discharging curve for the capacitor is shown in Figure 34.

▶ FIGURE 34



Related Problem In Figure 33, change R to 2.2 kΩ and determine the capacitor voltage 1 ms after the switch is closed.

Graphical Method Using Universal Exponential Curves The universal curves in Figure 35 provide a graphic solution of the charge and discharge of capacitors. Example 14 illustrates this graphical method.



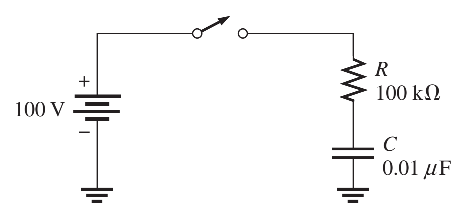
◀ FIGURE 35

Normalized universal exponential curves.

EXAMPLE 14

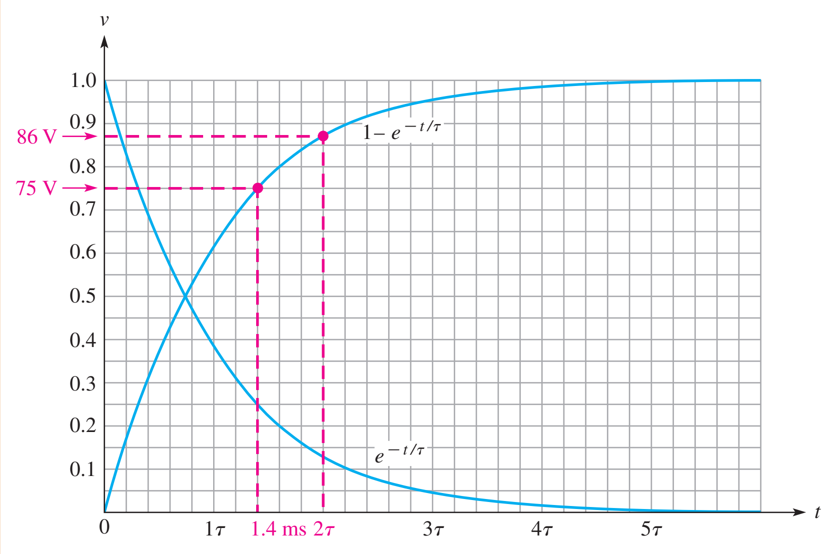
How long will it take the initially uncharged capacitor in Figure 36 to charge to 75 V? What is the capacitor voltage 2 ms after the switch is closed? Use the normalized universal exponential curves in Figure 35 to determine the answers.

▶ FIGURE 36



Solution The full charge voltage is 100 V, which is at the 100% level (1.0) on the normalized vertical scale of the graph. The value 75 V is 75% of maximum, or 0.75 on the graph. You can see that this value occurs at 1.4 time constants. In this circuit, one time constant is $RC = (100\ \text{k}\Omega)(0.01\ \mu\text{F}) = 1$ ms. Therefore, the capacitor voltage reaches 75 V at 1.4 ms after the switch is closed.

On the universal exponential curve, you see that the capacitor is at approximately 86 V (0.86 on the vertical axis) in 2 ms, which is 2 time constants. These graphic solutions are shown in Figure 37.



▲ FIGURE 37

Related Problem Using the normalized universal exponential curves, determine how long it will take the capacitor in Figure 36 to charge to 50 V? What is the capacitor voltage 3 ms after switch closure?

Use Multisim file E12-14 to verify the calculated results in this example and to confirm your calculation for the related problem. Use a square wave to replace the dc voltage source and the switch.

Time-Constant Percentage Tables The percentages of full charge or discharge at each time-constant interval can be calculated using the exponential formulas, or they can be extracted from the universal exponential curves. The results are summarized in Tables 4 and 5.

▶ TABLE 4

Percentage of final charge after each charging time-constant interval.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF FINAL CHARGE
1	63
2	86
3	95
4	98
5	99 (considered 100%)

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF INITIAL CHARGE
1	37
2	14
3	5
4	2
5	1 (considered 0)

TABLE 5

Percentage of initial charge after each discharging time-constant interval.

Solving for Time

Occasionally, it is necessary to determine how long it will take a capacitor to charge or discharge to a specified voltage. Equations 17 and 19 can be solved for t if v is specified. The natural logarithm (abbreviated ln) of $e^{-t/RC}$ is the exponent $-t/RC$. Therefore, taking the natural logarithm of both sides of the equation allows you to solve for time. This procedure is done as follows for the decreasing exponential formula when $V_F = 0$ (Equation 20).

$$v = V_i e^{-t/RC}$$

$$\frac{v}{V_i} = e^{-t/RC}$$

$$\ln\left(\frac{v}{V_i}\right) = \ln e^{-t/RC}$$

$$\ln\left(\frac{v}{V_i}\right) = \frac{-t}{RC}$$

$$t = -RC\ln\left(\frac{v}{V_i}\right)$$

Equation 21

The same procedure can be used for the increasing exponential formula in Equation 19 as follows:

$$v = V_F(1 - e^{-t/RC})$$

$$\frac{v}{V_F} = 1 - e^{-t/RC}$$

$$1 - \frac{v}{V_F} = e^{-t/RC}$$

$$\ln\left(1 - \frac{v}{V_F}\right) = \ln e^{-t/RC}$$

$$\ln\left(1 - \frac{v}{V_F}\right) = \frac{-t}{RC}$$

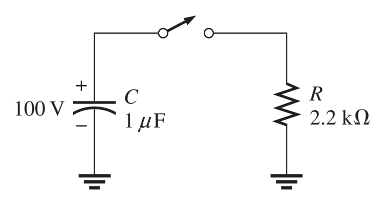
$$t = -RC\ln\left(1 - \frac{v}{V_F}\right)$$

Equation 22

EXAMPLE 15

In Figure 38, how long will it take the capacitor to discharge to 25 V when the switch is closed?

FIGURE 38



Solution Use Equation 21 to find the discharge time.

$$t = -RC\ln\left(\frac{v}{V_i}\right) = -(2.2\text{ k}\Omega)(1\ \mu\text{F})\ln\left(\frac{25\text{ V}}{100\text{ V}}\right)$$

$$= -(2.2\text{ ms})\ln(0.25) = -(2.2\text{ ms})(-1.39) = \mathbf{3.05\ ms}$$

You can determine ln(0.25) with your calculator by using the LN key.

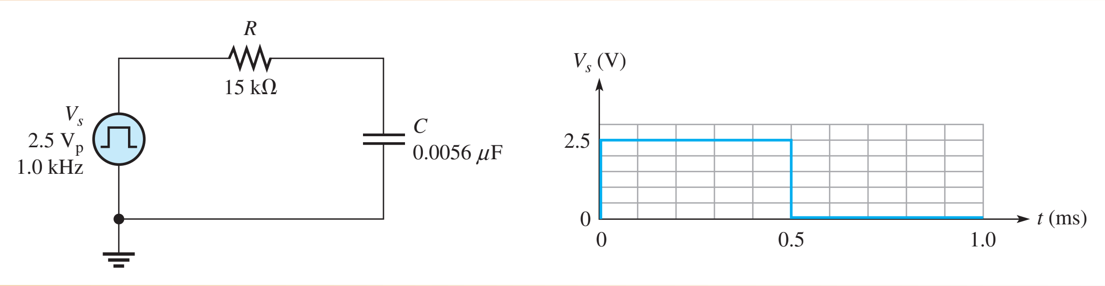
Related Problem How long will it take the capacitor in Figure 38 to discharge to 50 V?

Response to a Square Wave

A common case that illustrates the rising and falling exponential occurs when an *RC* circuit is driven with a square wave that has a long period compared to the time constant. The square wave provides on and off action but, unlike a single switch, it provides a discharge path back through the generator when the wave drops back to zero.

When the square wave rises, the voltage across the capacitor rises exponentially toward the maximum value of the square wave in a time that depends on the time constant. When the square wave returns to the zero level, the capacitor voltage decreases exponentially, again depending on the time constant. The Thevenin resistance of the generator is part of the *RC* time constant; however, it can be ignored if it is small compared to *R*. Example 16 shows the waveforms for the case where the period is long compared to the time constant.

EXAMPLE 16 In Figure 39, calculate the voltage across the capacitor every 0.1 ms for one complete period of the input. Then sketch the capacitor waveform. Assume the Thevenin resistance of the generator is negligible.



▲ FIGURE 39

Solution

$$\tau = RC = (15\text{ k}\Omega)(0.0056\ \mu\text{F}) = 0.084\text{ ms}$$

The period of the square wave is 1 ms, which is approximately 12τ. This means that 6τ will elapse after each change of the pulse, allowing the capacitor to fully charge and fully discharge.

For the increasing exponential,

$$v = V_F(1 - e^{-t/RC}) = V_F(1 - e^{-t/\tau})$$

At 0.1 ms: $v = 2.5\text{ V}(1 - e^{-0.1\text{ms}/0.084\text{ms}}) = 1.74\text{ V}$

At 0.2 ms: $v = 2.5\text{ V}(1 - e^{-0.2\text{ms}/0.084\text{ms}}) = 2.27\text{ V}$

At 0.3 ms: $v = 2.5\text{ V}(1 - e^{-0.3\text{ms}/0.084\text{ms}}) = 2.43\text{ V}$

At 0.4 ms: $v = 2.5\text{ V}(1 - e^{-0.4\text{ms}/0.084\text{ms}}) = 2.48\text{ V}$

At 0.5 ms: $v = 2.5\text{ V}(1 - e^{-0.5\text{ms}/0.084\text{ms}}) = 2.49\text{ V}$

For the decreasing exponential,

$$v = V_i(e^{-t/RC}) = V_i(e^{-t/\tau})$$

In the equation, time is shown from the point when the change occurs (subtracting 0.5 ms from the actual time). For example, at 0.6 ms, $t = 0.6\text{ ms} - 0.5\text{ ms} = 0.1\text{ ms}$.

At 0.6 ms: $v = 2.5\text{ V}(e^{-0.1\text{ms}/0.084\text{ms}}) = 0.76\text{ V}$

At 0.7 ms: $v = 2.5\text{ V}(e^{-0.2\text{ms}/0.084\text{ms}}) = 0.23\text{ V}$

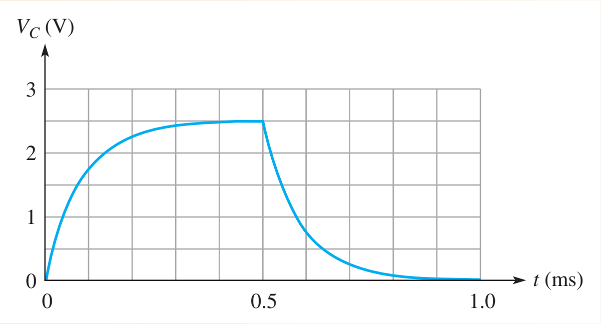
At 0.8 ms: $v = 2.5\text{ V}(e^{-0.3\text{ms}/0.084\text{ms}}) = 0.07\text{ V}$

At 0.9 ms: $v = 2.5\text{ V}(e^{-0.4\text{ms}/0.084\text{ms}}) = 0.02\text{ V}$

At 1.0 ms: $v = 2.5\text{ V}(e^{-0.5\text{ms}/0.084\text{ms}}) = 0.01\text{ V}$

Figure 40 is a plot of these results.

▶ **FIGURE 40**



Related Problem What is the capacitor voltage at 0.65 ms?

SECTION 5 CHECKUP

1. Determine the time constant when $R = 1.2\text{ k}\Omega$ and $C = 1000\text{ pF}$.
2. If the circuit mentioned in Question 1 is charged with a 5 V source, how long will it take the capacitor to reach full charge? At full charge, what is the capacitor voltage?
3. For a certain circuit, $\tau = 1\text{ ms}$. If it is charged with a 10 V battery, what will the capacitor voltage be at each of the following times: 2 ms, 3 ms, 4 ms, and 5 ms?
4. A capacitor is charged to 100 V. If it is discharged through a resistor, what is the capacitor voltage at one time constant?

6 CAPACITORS IN AC CIRCUITS

As you know, a capacitor blocks dc. A capacitor passes ac but with an amount of opposition, called capacitive reactance, that depends on the frequency of the ac.

After completing this section, you should be able to

- **Analyze capacitive ac circuits**
  - Explain why a capacitor causes a phase shift between voltage and current
  - Define *capacitive reactance*
  - Determine the value of capacitive reactance in a given circuit
  - Discuss instantaneous, true, and reactive power in a capacitor

To explain fully how capacitors work in ac circuits, the concept of the derivative must be introduced. *The derivative of a time-varying quantity is the instantaneous rate of change of that quantity.*

Recall that current is the rate of flow of charge (electrons). Therefore, instantaneous current, i, can be expressed as the instantaneous rate of change of charge, q, with respect to time, t.

Equation 23

$$i = \frac{dq}{dt}$$

The term dq/dt is the derivative of q with respect to time and represents the instantaneous rate of change of q. Also, in terms of instantaneous quantities, $q = Cv$. Therefore, from a basic rule of differential calculus, the derivative of q with respect to time is $dq/dt = C(dv/dt)$. Since $i = dq/dt$, we get the following relationship:

Equation 24

$$i = C\left(\frac{dv}{dt}\right)$$

This formula states

> **The instantaneous capacitor current is equal to the capacitance times the instantaneous rate of change of the voltage across the capacitor.**

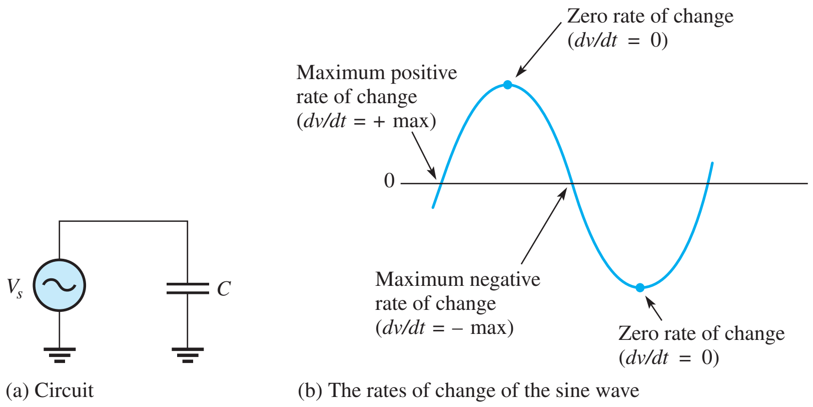
The faster the voltage across a capacitor changes, the greater the current.

Phase Relationship of Current and Voltage in a Capacitor

Consider what happens when a sinusoidal voltage is applied across a capacitor, as shown in Figure 41(a). The voltage waveform has a maximum rate of change ($dv/dt = \text{max}$) at the zero crossings and a zero rate of change ($dv/dt = 0$) at the peaks, as indicated in Figure 41(b).

The phase relationship between the current and the voltage for the capacitor can be established from Equation 24. When $dv/dt = 0$, i is also zero because $i = C(dv/dt) = C(0) = 0$. When dv/dt is a positive-going maximum, i is a positive maximum; when dv/dt is a negative-going maximum, i is a negative maximum.

A sinusoidal voltage always produces a sinusoidal current in a capacitive circuit. Therefore, you can plot the current with respect to the voltage if you know the points on the voltage curve at which the current is zero and those at which it is maximum. This relationship is shown in Figure 42(a). Notice that the current leads the voltage in phase by 90°. This is always true in a purely capacitive circuit. The relationship between the voltage and current phasors is shown in Figure 42(b).

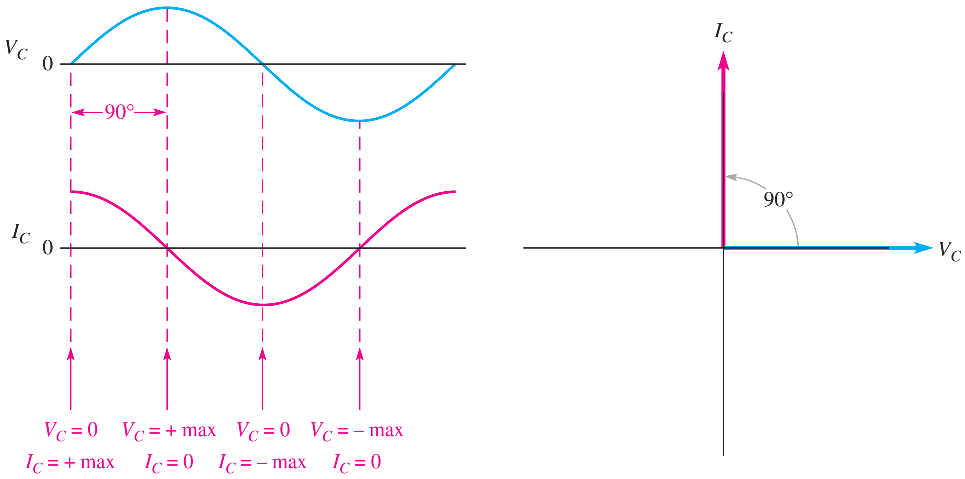


V_s
C
Zero rate of change
(dv/dt = 0)
Maximum positive
rate of change
(dv/dt = + max)
0
Maximum negative
rate of change
(dv/dt = – max)
Zero rate of change
(dv/dt = 0)

(a) Circuit (b) The rates of change of the sine wave

▲ **FIGURE 41**

A sine wave applied to a capacitor.



V_C
0
90°
I_C
0
$V_C = 0$
$I_C = +$ max
$V_C = +$ max
$I_C = 0$
$V_C = 0$
$I_C = -$ max
$V_C = -$ max
$I_C = 0$
I_C
90°
V_C

(a) Waveforms (b) Phasor diagram

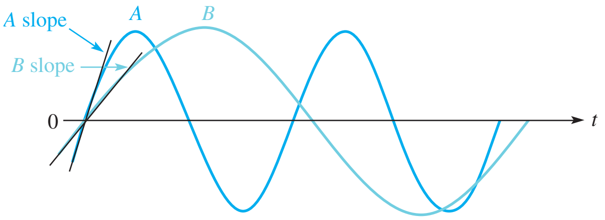
▲ **FIGURE 42**

Phase relation of V_C and I_C in a capacitor. Current always leads the capacitor voltage by 90°.

Capacitive Reactance, X_C

Capacitive reactance is the opposition to sinusoidal current, expressed in ohms. The symbol for capacitive reactance is X_C.

To develop a formula for X_C, we use the relationship $i = C(dv/dt)$ and the curves in Figure 43. The rate of change of voltage is directly related to frequency. The faster the voltage changes, the higher the frequency. For example, you can see that in Figure 43



A slope
B slope
A
B
0
t

▲ **FIGURE 43**

The higher frequency waveform (A) has a greater slope at its zero crossings, corresponding to a higher rate of change.

the slope of sine wave *A* at the zero crossings is steeper than that of sine wave *B*. The slope of a curve at a point indicates the rate of change at that point. Sine wave *A* has a higher frequency than sine wave *B*, as indicated by a greater maximum rate of change (dv/dt is greater at the zero crossings).

When frequency increases, dv/dt increases, and thus i increases. When frequency decreases, dv/dt decreases, and thus i decreases.

$$\overset{\uparrow}{i} = C(\overset{\uparrow}{dv/dt}) \quad \text{and} \quad \underset{\downarrow}{i} = C(\underset{\downarrow}{dv/dt})$$

An increase in i means that there is less opposition to current (X_C is less), and a decrease in i means a greater opposition to current (X_C is greater). Therefore, X_C is inversely proportional to i and thus inversely proportional to frequency.

X_C is inversely proportional to f, shown as $\frac{1}{f}$.

From the same relationship $i = C(dv/dt)$, you can see that if dv/dt is constant and C is varied, an increase in C produces an increase in i, and a decrease in C produces a decrease in i.

$$\overset{\uparrow}{i} = \overset{\uparrow}{C}(dv/dt) \quad \text{and} \quad \underset{\downarrow}{i} = \underset{\downarrow}{C}(dv/dt)$$

Again, an increase in i means less opposition (X_C is less), and a decrease in i means greater opposition (X_C is greater). Therefore, X_C is inversely proportional to i and thus inversely proportional to capacitance.

The capacitive reactance is inversely proportional to both f and C.

X_C is inversely proportional to fC, shown as $\frac{1}{fC}$.

Thus far, we have determined a proportional relationship between X_C and $1/fC$. Equation 25 is the complete formula for calculating X_C. The derivation is given in Appendix B.

Equation 25

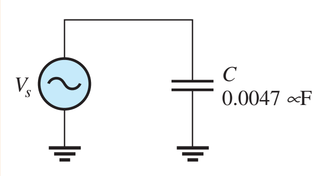
$$X_C = \frac{1}{2\pi fC}$$

Capacitive reactance, X_C, is in ohms when f is in hertz and C is in farads. Notice that 2π appears in the denominator as a constant of proportionality. This term is derived from the relationship of a sine wave to rotational motion.

EXAMPLE 17

A sinusoidal voltage is applied to a capacitor, as shown in Figure 44. The frequency of the sine wave is 1 kHz. Determine the capacitive reactance.

▶ FIGURE 44



Solution

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1 \times 10^3\,\text{Hz})(0.0047 \times 10^{-6}\,\text{F})} = \mathbf{33.9\,k\Omega}$$

Related Problem Determine the frequency required to make the capacitive reactance in Figure 44 equal to 10 kΩ.

Use Multisim file E12-17 to verify the calculated results in this example and to confirm your calculation for the related problem.

Reactance for Series Capacitors

When capacitors are in series in an ac circuit, the total capacitance is smaller than the smallest individual capacitance. Because the total capacitance is smaller, the total capacitive reactance (opposition to current) must be larger than any individual capacitive reactance. With series capacitors, the total capacitive reactance ($X_{C(tot)}$) is the sum of the individual reactances.

$$X_{C(tot)} = X_{C1} + X_{C2} + X_{C3} + \cdots + X_{Cn} \qquad \textbf{Equation 26}$$

Compare this formula with that for finding the total resistance of series resistors. In both cases, you simply add the individual oppositions.

Reactance for Parallel Capacitors

In ac circuits with parallel capacitors, the total capacitance is the sum of the capacitances. Recall that the capacitive reactance is inversely proportional to the capacitance. Because the total parallel capacitance is larger than any individual capacitances, the total capacitive reactance must be smaller than the reactance of any individual capacitor. With parallel capacitors, the total reactance is found by

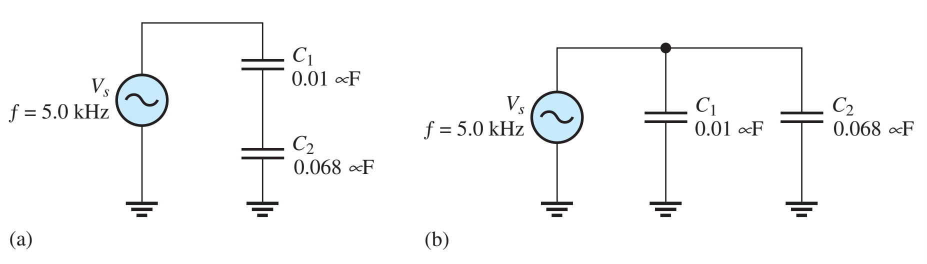
$$X_{C(tot)} = \frac{1}{\dfrac{1}{X_{C1}} + \dfrac{1}{X_{C2}} + \dfrac{1}{X_{C3}} + \cdots + \dfrac{1}{X_{Cn}}} \qquad \textbf{Equation 27}$$

Compare this formula with that for parallel resistors. As in the case of parallel resistors, the total opposition (resistance or reactance) is the reciprocal of the sum of the reciprocals of the individual oppositions.

For two capacitors in parallel, Equation 27 can be reduced to the product-over-sum form. This is useful because, for most practical circuits, more than two capacitors in parallel is not common.

$$X_{C(tot)} = \frac{X_{C1}X_{C2}}{X_{C1} + X_{C2}}$$

EXAMPLE 18 What is the total capacitive reactance of each circuit in Figure 45?



▲ **FIGURE 45**

Solution The reactances of the individual capacitors are the same in both circuits.

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(5.0\text{ kHz})(0.01\ \mu\text{F})} = 3.18\text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(5.0\text{ kHz})(0.068\ \mu\text{F})} = 468\ \Omega$$

The Series Circuit: For the capacitors in series in Figure 45 (a), the total reactance is the sum of X_{C1} and X_{C2}, as given in Equation 26.

$$X_{C(tot)} = X_{C1} + X_{C2} = 3.18\text{ k}\Omega + 468\ \Omega = \mathbf{3.65\text{ k}\Omega}$$

Alternatively, you can obtain the total series reactance by first finding the total capacitance using Equation 10. Then calculate the total reactance.

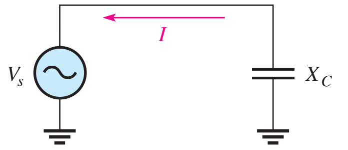
$$C_{tot} = \frac{C_1C_2}{C_1 + C_2} = \frac{(0.01\ \mu\text{F})(0.068\ \mu\text{F})}{0.01\ \mu\text{F} + 0.068\ \mu\text{F}} = 0.0087\ \mu\text{F}$$

$$X_{C(tot)} = \frac{1}{2\pi f C_{tot}} = \frac{1}{2\pi(5.0\text{ kHz})(0.0087\ \mu\text{F})} = \mathbf{3.65\text{ k}\Omega}$$

The Parallel Circuit: For the capacitors in parallel in Figure 45(b), determine the total reactance from the product-over-sum rule using X_{C1} and X_{C2}.

$$X_{C(tot)} = \frac{X_{C1}X_{C2}}{X_{C1} + X_{C2}} = \frac{(3.18\text{ k}\Omega)(468\ \Omega)}{3.18\text{ k}\Omega + 468\ \Omega} = \mathbf{408\ \Omega}$$

Related Problem Determine the total parallel capacitive reactance by first finding the total parallel capacitance.



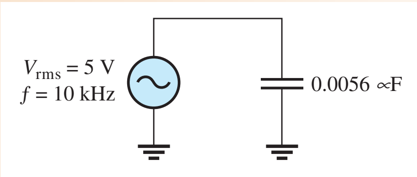
▲ FIGURE 46

Ohm's Law The reactance of a capacitor is analogous to the resistance of a resistor. In fact, both are expressed in ohms. Since both R and X_C are forms of opposition to current. Ohm's law applies to capacitive circuits as well as to resistive circuits; and it is stated as follows for Figure 46:

$$I = \frac{V_s}{X_C}$$

When applying Ohm's law in ac circuits, you must express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on.

EXAMPLE 19 Determine the rms current in Figure 47.



▲ FIGURE 47

Solution First, determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10 \times 10^3\,\text{Hz})(0.0056 \times 10^{-6}\,\text{F})} = 2.84\,\text{k}\Omega$$

Then apply Ohm's law.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{5\,\text{V}}{2.84\,\text{k}\Omega} = \mathbf{1.76\,mA}$$

Related Problem Change the frequency in Figure 47 to 25 kHz and determine the rms current.

Use Multisim file E12-19 to verify the calculated results in this example and to confirm your calculation for the related problem.

Capacitive Voltage Divider

In ac circuits, capacitors can be used in applications that require a voltage divider. (Some oscillator circuits use this method to tap off a fraction of the output.) The voltage across a series capacitor was given as Equation 12, which is repeated here ($V_{\text{T}} = V_s$).

$$V_x = \left(\frac{C_{\text{T}}}{C_x}\right)V_s$$

A resistive voltage divider is expressed in terms of a resistance ratio, which is a ratio of oppositions. You can think of the capacitive voltage divider by applying this idea from a resistive divider, but using reactance in place of resistance. The equation for the voltage across a capacitor in a capacitive voltage divider can be written as

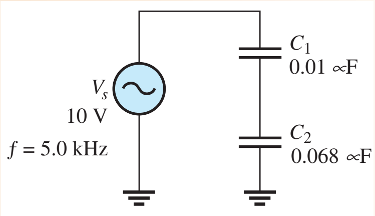
$$V_x = \left(\frac{X_{Cx}}{X_{C(tot)}}\right)V_s$$

Equation 28

where X_{Cx} is the reactance of capacitor C_x, $X_{C(tot)}$ is the total capacitive reactance, and V_x is the voltage across capacitor C_x. Either Equation 12 or Equation 28 can be used to find the voltage from a divider, as illustrated in Example 20.

EXAMPLE 20 What is the voltage across C_2 in the circuit of Figure 48?

FIGURE 48



Solution The reactance of the individual capacitors and the total reactance were determined in Example 18. Substituting into Equation 28.

$$V_2 = \left(\frac{X_{C2}}{X_{C(tot)}}\right)V_s = \left(\frac{468\,\Omega}{3.65\,\text{k}\Omega}\right)10\,\text{V} = \mathbf{1.28\,V}$$

Notice that the voltage across the larger capacitor is the smaller fraction of the total.

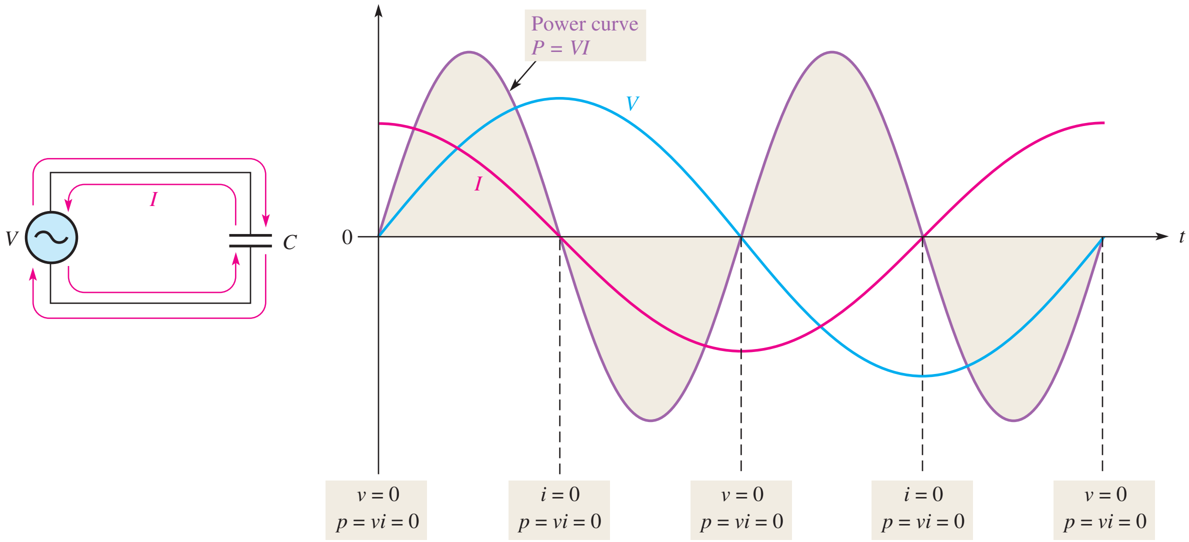
You can obtain the same result from Equation 12.

$$V_2 = \left(\frac{C_T}{C_2}\right)V_s = \left(\frac{0.0087\ \mu\text{F}}{0.068\ \mu\text{F}}\right)10\text{ V} = \mathbf{1.28\ V}$$

Related Problem Use Equation 28 to determine the voltage across C_1.

Power in a Capacitor

As discussed earlier in this chapter, a charged capacitor stores energy in the electric field within the dielectric. An ideal capacitor does not dissipate energy; it only stores it temporarily. When an ac voltage is applied to a capacitor, energy is stored by the capacitor during a portion of the voltage cycle; then the stored energy is returned to the source during another portion of the cycle. There is no net energy loss. Figure 49 shows the power curve that results from one cycle of capacitor voltage and current.



▲ **FIGURE 49**

Power curve.

Instantaneous Power (p) The product of v and i gives **instantaneous power**. At points where v or i is zero, p is also zero. When both v and i are positive, p is also positive. When either v or i is positive and the other is negative, p is negative. When both v and i are negative, p is positive. As you can see, the power follows a sinusoidal-shaped curve. Positive values of power indicate that energy is stored by the capacitor. Negative values of power indicate that energy is returned from the capacitor to the source. Note that the power fluctuates at a frequency twice that of the voltage or current as energy is alternately stored and returned to the source.

True Power (P_{true}) Ideally, all of the energy stored by a capacitor during the positive portion of the power cycle is returned to the source during the negative portion. No net energy is lost due to conversion to heat in the capacitor, so the **true power** is zero. Actually, because of leakage and foil resistance in a practical capacitor, a small percentage of the total power is dissipated in the form of true power.

Reactive Power (P_r) The rate at which a capacitor stores or returns energy is called its **reactive power**. The reactive power is a nonzero quantity, because at any instant in time, the capacitor is actually taking energy from the source or returning energy to it. Reactive power does not represent an energy loss. The following formulas apply:

$$P_r = V_{\text{rms}} I_{\text{rms}}$$

Equation 29

$$P_r = \frac{V_{\text{rms}}^2}{X_C}$$

Equation 30

$$P_r = I_{\text{rms}}^2 X_C$$

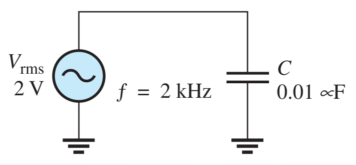
Equation 31

Notice that these equations are of the same form as those for power in a resistor. The voltage and current are expressed in rms. The unit of reactive power is **VAR (volt-ampere reactive)**.

EXAMPLE 21

Determine the true power and the reactive power in Figure 50.

► FIGURE 50



Solution The true power, P_{true}, is *always* **zero** for an ideal capacitor. The reactive power is determined by first finding the value for the capacitive reactance and then using Equation 30.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(2 \times 10^3\ \text{Hz})(0.01 \times 10^{-6}\ \text{F})} = 7.96\ \text{k}\Omega$$

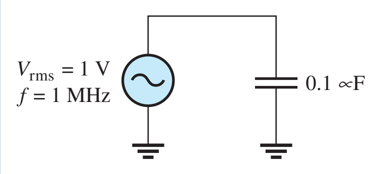
$$P_r = \frac{V_{\text{rms}}^2}{X_C} = \frac{(2\ \text{V})^2}{7.96\ \text{k}\Omega} = 503 \times 10^{-6}\ \text{VAR} = \mathbf{503\ \mu VAR}$$

Related Problem If the frequency is doubled in Figure 50, what are the true power and the reactive power?

SECTION 6 CHECKUP

1. State the phase relationship between current and voltage in a capacitor.
2. Calculate X_C for $f = 5$ kHz and $C = 50$ pF.
3. At what frequency is the reactance of a 0.1 μF capacitor equal to 2 kΩ ?
4. Calculate the rms current in Figure 51.
5. A 1 μF capacitor is connected to an ac voltage source of 12 V rms. What is the true power?
6. In Question 5, determine the reactive power at a frequency of 500 Hz.

► FIGURE 51



7 CAPACITOR APPLICATIONS

Capacitors are widely used in many electrical and electronic applications.

After completing this section, you should be able to

- **Discuss some capacitor applications**
    - Describe a power supply filter
    - Explain the purpose of coupling and bypass capacitors
    - Discuss the basics of capacitors applied to tuned circuits, timing circuits, and computer memories

If you pick up any circuit board, open any power supply, or look inside any piece of electronic equipment, chances are you will find capacitors of one type or another. These components are used for a variety of reasons in both dc and ac applications.

Electrical Storage

One of the most basic applications of a capacitor is as a backup voltage source for low-power circuits such as certain types of semiconductor memories in computers. This particular application requires a very high capacitance value and negligible leakage.

The storage capacitor is connected between the dc power supply input to the circuit and ground. When the circuit is operating from its normal power supply, the capacitor remains fully charged to the dc power supply voltage. If the normal power source is disrupted, effectively removing the power supply from the circuit, the storage capacitor temporarily becomes the power source for the circuit.

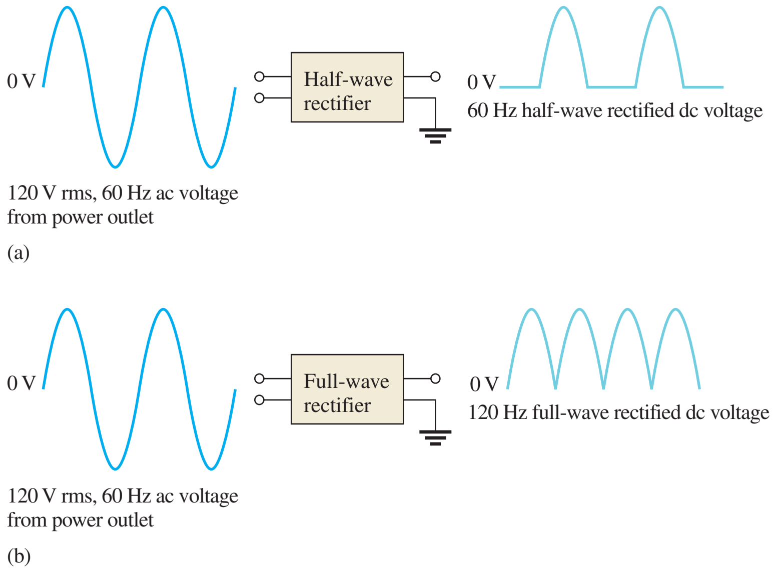
A capacitor provides voltage and current to a circuit as long as its charge remains sufficient. As current is drawn by the circuit, charge is removed from the capacitor and the voltage decreases. For this reason, the storage capacitor can only be used as a temporary power source. The length of time that a capacitor can provide sufficient power to the circuit depends on the capacitance and the amount of current drawn by the circuit. The smaller the current and the higher the capacitance, the longer the time a capacitor can provide power to a circuit.

Power Supply Filtering

A basic dc power supply consists of a circuit known as a **rectifier** followed by a **filter**. The rectifier converts the 120 V, 60 Hz sinusoidal voltage available at a standard outlet to a pulsating dc voltage that can be either a half-wave rectified voltage or a full-wave rectified voltage, depending on the type of rectifier circuit. As shown in Figure 52(a), a half-wave rectifier removes each negative half-cycle of the sinusoidal voltage. As shown in Figure 52(b), a full-wave rectifier actually reverses the polarity of the negative portion of each cycle. Both half-wave and full-wave rectified voltages are dc because, even though they are changing, they do not alternate polarity.

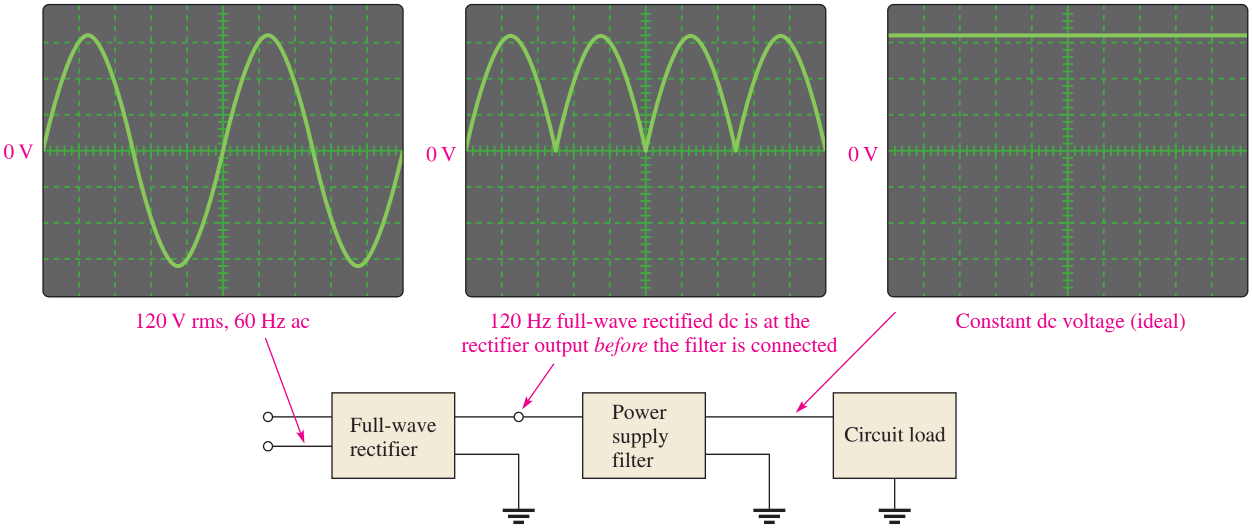
To be useful for powering electronic circuits, the rectified voltage must be changed to constant dc voltage because all circuits require constant power. The filter nearly eliminates the fluctuations in the rectified voltage and ideally provides a smooth constant-value dc voltage to the load that is the electronic circuit, as indicated in Figure 53.

The Capacitor as a Power Supply Filter Capacitors are used as filters in dc power supplies because of their ability to store electrical charge. Figure 54(a) shows a dc power supply with a full-wave rectifier and a capacitor filter. The operation can be described from a charging and discharging point of view as follows. Assume the capacitor is initially uncharged. When the power supply is first turned on and the first cycle of the rectified voltage


0 V
Half-wave rectifier
0 V
60 Hz half-wave rectified dc voltage
120 V rms, 60 Hz ac voltage from power outlet
(a)
0 V
Full-wave rectifier
0 V
120 Hz full-wave rectified dc voltage
120 V rms, 60 Hz ac voltage from power outlet
(b)

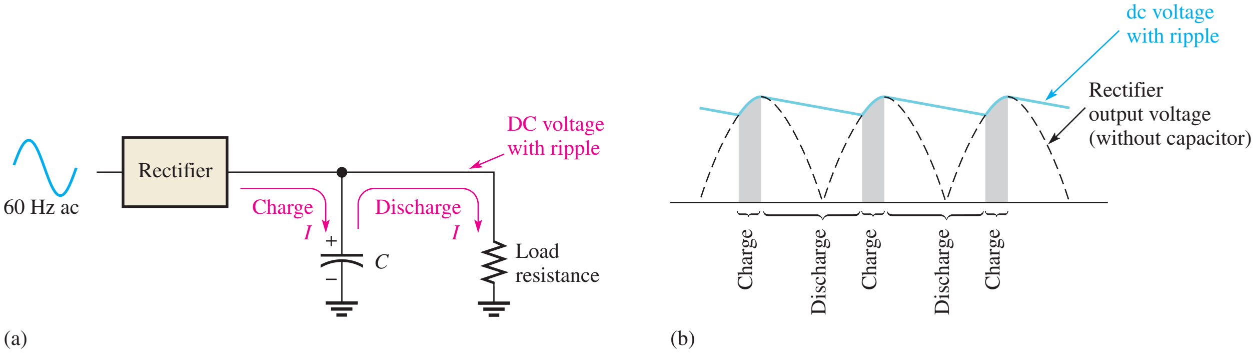
◀ FIGURE 52

Half-wave and full-wave rectifier operation.


0 V
0 V
0 V
120 V rms, 60 Hz ac
120 Hz full-wave rectified dc is at the rectifier output *before* the filter is connected
Constant dc voltage (ideal)
Full-wave rectifier
Power supply filter
Circuit load

▲ FIGURE 53

Basic waveforms showing the operation of a dc power supply.


60 Hz ac
Rectifier
DC voltage with ripple
Charge
I
+
−
C
Discharge
I
Load resistance
(a)
dc voltage with ripple
Rectifier output voltage (without capacitor)
Charge
Discharge
Charge
Discharge
Charge
(b)

▲ FIGURE 54

Basic operation of a power supply filter capacitor.

occurs, the capacitor will quickly charge through the low forward resistance of the rectifier. The capacitor voltage will follow the rectified voltage curve up to the peak of the rectified voltage. As the rectified voltage passes the peak and begins to decrease, the capacitor will begin to discharge very slowly through the high resistance of the load circuit, as indicated in Figure 54(b). The amount of discharge is typically very small and is exaggerated in the figure for purposes of illustration. The next cycle of the rectified voltage will recharge the capacitor back to the peak value by replenishing the small amount of charge lost since the previous peak. This pattern of a small amount of charging and discharging continues as long as the power is on.

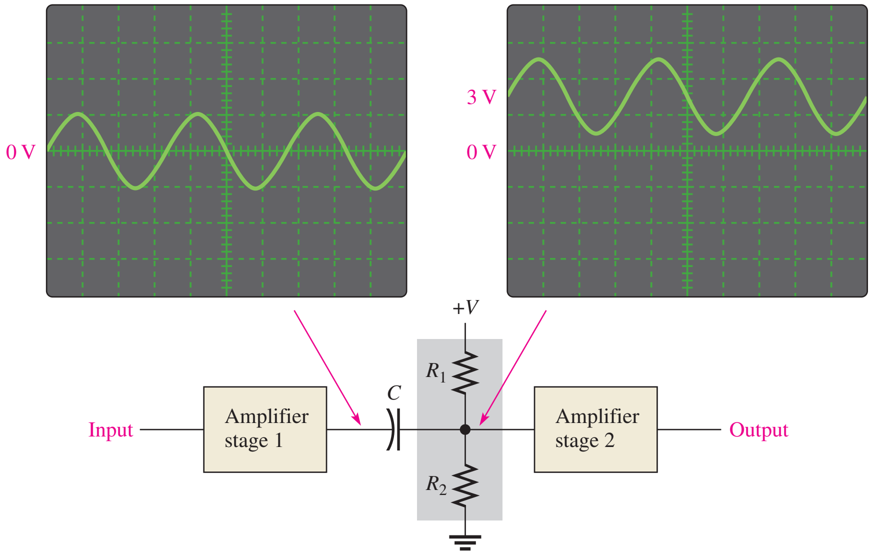
A rectifier is designed so that it allows current only in the direction to charge the capacitor. The capacitor will not discharge back through the rectifier but will only discharge a small amount through the relatively high resistance of the load. The small fluctuation in voltage due to the charging and discharging of the capacitor is called the **ripple voltage**. A good dc power supply has a very small amount of ripple on its dc output. The discharge time constant of a power supply filter capacitor depends on its capacitance and the resistance of the load; consequently, the higher the capacitance value, the longer the discharge time and therefore, the smaller the ripple voltage.

DC Blocking and AC Coupling

Capacitors are commonly used to block the constant dc voltage in one part of a circuit from getting to another part. As an example of this, a capacitor is connected between two stages of an amplifier to prevent the dc voltage at the output of stage 1 from affecting the dc voltage at the input of stage 2, as illustrated in Figure 55. Assume that, for proper operation, the output of stage 1 has a zero dc voltage and the input to stage 2 has a 3 V dc voltage. The capacitor prevents the 3 V dc at stage 2 from getting to the stage 1 output and affecting its zero value, and vice versa.

► FIGURE 55

An application of a capacitor to block dc and couple ac in an amplifier.



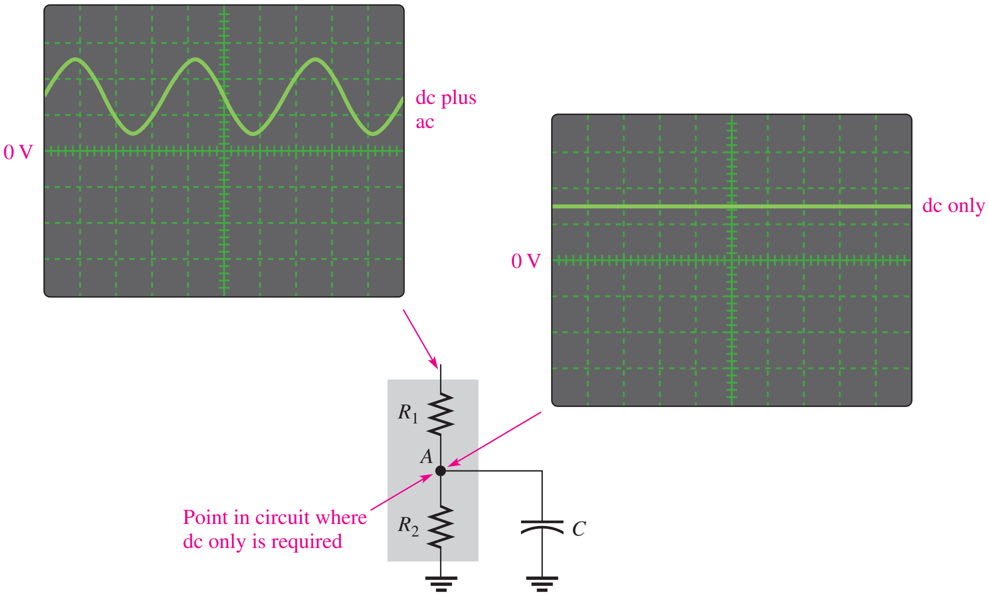
If a sinusoidal signal voltage is applied to the input to stage 1, the signal voltage is increased (amplified) and appears on the ouput of stage 1, as shown in Figure 55. The amplified signal voltage is then coupled through the capacitor to the input of stage 2 where it is superimposed on the 3 V dc level and then again amplified by stage 2. In order for the signal voltage to be passed through the capacitor without being reduced, the capacitor must be large enough so that its reactance at the frequency of the signal voltage is negligible. In this type of application, the capacitor is known as a *coupling capacitor,* which ideally appears as an open to dc and as a short to ac. As the signal frequency is reduced, the capacitive reactance increases and, at some point, the capacitive reactance becomes large enough to cause a significant reduction in ac voltage between stage 1 and stage 2.

Power Line Decoupling

Capacitors connected from the dc supply voltage line to ground are used on circuit boards to decouple unwanted voltage transients or spikes that occur on the dc supply voltage because of fast switching digital circuits. A voltage transient contains high frequencies that may affect the operation of the circuits. These transients are shorted to ground through the very low reactance of the decoupling capacitors. Several decoupling capacitors are often used at various points along the supply voltage line on a circuit board.

Bypassing

Another capacitor application is to bypass an ac voltage around a resistor in a circuit without affecting the dc voltage across the resistor. In amplifier circuits, for example, dc voltages called *bias voltages* are required at various points. For the amplifier to operate properly, certain bias voltages must remain constant and, therefore, any ac voltages must be removed. A sufficiently large capacitor connected from a bias point to ground provides a low reactance path to ground for ac voltages, leaving the constant dc bias voltage at the given point. At lower frequencies, the bypass capacitor becomes less effective because of its increased reactance. This bypass application is illustrated in Figure 56.



▲ **FIGURE 56**

Example of the operation of a bypass capacitor. Point *A* is at ac ground due to the low reactance path through the capacitor.

Signal Filters

Capacitors are essential to the operation of a class of circuits called *filters* that are used for selecting one ac signal with a certain specified frequency from a wide range of signals with many different frequencies or for selecting a certain band of frequencies and eliminating all others. A common example of this application is in radio and television receivers where it is necessary to select the signal transmitted from a given station and eliminate or filter out the signals transmitted from all the other stations in the area.

When you tune your radio or TV, you are actually changing the capacitance in the tuner circuit (which is a type of filter) so that only the signal from the station or channel you want passes through to the receiver circuitry. Capacitors are used in conjunction with resistors, inductors (covered in the next chapter), and other components in these types of filters.

The main characteristic of a filter is its frequency selectivity, which is based on the fact that the reactance of a capacitor depends on frequency ($X_C = 1/2\pi fC$).

Timing Circuits

Another important area in which capacitors are used is in timing circuits that generate specified time delays or produce waveforms with specific characteristics. Recall that the time constant of a circuit with resistance and capacitance can be controlled by selecting appropriate values for *R* and *C*. The charging time of a capacitor can be used as a basic time delay in various types of circuits. An example is the circuit that controls the turn indicators on your car where the light flashes on and off at regular intervals.

Computer Memories

Dynamic memories in computers use very tiny capacitors as the basic storage element for binary information, which consists of two binary digits, 1 and 0. A charged capacitor can represent a stored 1 and a discharged capacitor can represent a stored 0. Patterns of 1s and 0s that make up binary data are stored in a memory that consists of an array of capacitors with associated circuitry. You will study this topic in a computer or digital fundamentals course.

SECTION 7 CHECKUP

1. Explain how half-wave or full-wave rectified dc voltages are smoothed out by a filter capacitor.
2. Explain the purpose of a coupling capacitor.
3. How large must a coupling capacitor be?
4. Explain the purpose of a decoupling capacitor.
5. Discuss how the relationship of frequency and capacitive reactance is important in frequency-selective circuits such as signal filters.
6. What characteristic of a capacitor is most important in time-delay applications?

8 SWITCHED-CAPACITOR CIRCUITS

Another important application of capacitors is in programmable analog arrays, which are implemented in integrated circuit (IC) form. Switched-capacitors are used to implement various types of programmable analog circuits in which capacitors take the place of resistors. Capacitors can be implemented on an IC chip more easily than a resistor can, and they offer other advantages such as zero power dissipation. When a resistance is required in a circuit, the switched capacitor can be made to emulate a resistor. Using switched-capacitor emulation, resistor values can be readily changed by reprogramming and accurate and stable resistance values can be achieved.

After completing this section, you should be able to

- **Describe the basic operation of switched-capacitor circuits**
    - Explain how switched-capacitor circuits emulate resistors

Recall that current is defined in terms of charge Q and time t as

$$I = \frac{Q}{t}$$

This formula states that current is the rate at which charge flows through a circuit. Also recall that the basic definition of charge in terms of capacitance and voltage is

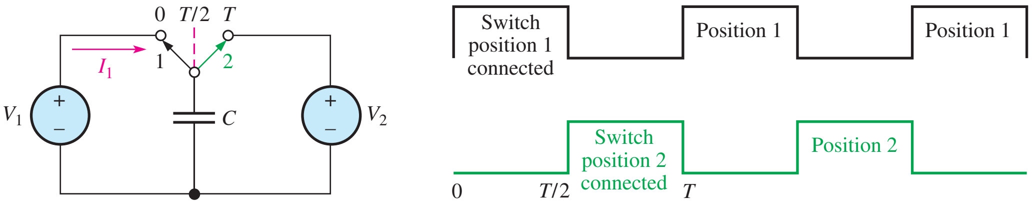
$$Q = CV$$

Substituting CV for Q, the current can be expressed as

$$I = \frac{CV}{t}$$

Basic Operation

A general model of a switched-capacitor circuit is shown in Figure 57. It consists of a capacitor, two arbitrary voltage sources (V_1 and V_2), and a two-pole switch. In actual circuits, the switch is implemented with transistors. Let's examine this circuit for a specified period of time, T, which is then repeated. Assume that V_1 and V_2 are constant during the time period T. Of particular interest is the average current I_1 from the source V_1 during the period of time, T.



(a) (b)

FIGURE 57

Basic operation of a switched-capacitor circuit. The voltage source symbol represents a time-varying symbol.

During the first half of the time period T, the switch is in position 1, as indicated in Figure 57. Therefore, there is a current I_1 due to V_1 that is charging the capacitor during the interval from $t = 0$ to $t = T/2$. During the second half of the time period, the switch is in position 2, as indicated, and there is no current from V_1; therefore, the average current from the source V_1 over the time period T is

$$I_{1(\text{avg})} = \frac{Q_{1(T/2)} - Q_{1(0)}}{T}$$

$Q_{1(0)}$ is the charge at $t = 0$ and $Q_{1(T/2)}$ is the charge at $t = T/2$. So, $Q_{1(T/2)} - Q_{1(0)}$ is the net charge transferred while the switch is in position 1.

The capacitor voltage at $T/2$ is equal to V_1, and the capacitor voltage at 0 or T is equal to V_2. Using the formula $Q = CV$ and substituting into the previous equation, you obtain

$$I_{1(\text{avg})} = \frac{CV_{1(T/2)} - CV_{2(0)}}{T} = \frac{C(V_{1(T/2)} - V_{2(0)})}{T}$$

Since V_1 and V_2 are assumed to be constant during T, the average current can be expressed as

$$I_{1(\text{avg})} = \frac{C(V_1 - V_2)}{T}$$ **Equation 32**

▶ FIGURE 58

Resistive circuit.

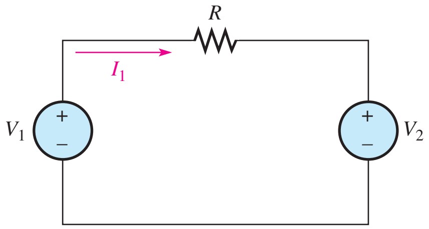


Figure 58 shows an equivalent circuit with a resistor instead of the capacitor and switches.

Applying Ohm's law to the resistive circuit, the current is

$$I_1 = \frac{V_1 - V_2}{R}$$

Setting $I_{1(avg)}$ in the switched-capacitor circuit equal to current in the resistive circuit, you have

$$\frac{C(V_1 - V_2)}{T} = \frac{V_1 - V_2}{R}$$

Canceling the $V_1 - V_2$ terms and solving for R gives the equivalent resistance.

Equation 33

$$R = \frac{T}{C}$$

This important result shows that a switched-capacitor circuit can emulate a resistor with a value determined by the time T and the capacitance C. Remember that the switch is in each position for one-half of the time period T and that T can be varied by varying the frequency at which the switches are operated. In a programmable analog device, the switching frequency is a programmable parameter for each emulated resistor and can be set to achieve a precise resistor value. Since $T = 1/f$, the resistance in terms of frequency is

Equation 34

$$R = \frac{1}{fC}$$

EXAMPLE 22

The input resistor, R, of an amplifier circuit is to be replaced with a switched-capacitor circuit. Assume that the switched capacitor value is 1000 pF. You want the switched-capacitor to emulate a 10 kΩ resistor. Determine the frequency at which the capacitor must be switched.

Solution Using the formula $R = T/C$,

$$T = RC = (10\,\text{k}\Omega)(1000\,\text{pF}) = 10\,\mu\text{s}$$

This means that the switch must be operated at a frequency of

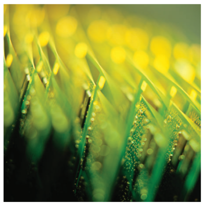
$$f = \frac{1}{T} = \frac{1}{10\,\mu\text{s}} = \mathbf{100\,kHz}$$

The duty cycle is 50% so that the switch is in each position half of the period.

Related Problem At what frequency must the 1000 pF capacitor be switched to emulate a 5.6 kΩ resistor?

SECTION 8 CHECKUP

1. How does a switched capacitor emulate a resistor?
2. What factors determine the resistance value that a given switched-capacitor circuit can emulate?
3. In a practical implementation, what devices are used for switches?



Application Activity

Capacitors are used in certain types of amplifiers to couple the ac signal while blocking the dc voltage. Capacitors are used in many other applications, but in this application, you will focus on the coupling capacitors in an amplifier circuit. This topic was introduced in Section 7. A knowledge of amplifier circuits is not necessary for this assignment.

All amplifier circuits contain transistors that require dc voltages to establish proper operating conditions for amplifying ac signals. These dc voltages are referred to as bias voltages. As indicated in Figure 59(a), a common type of dc bias circuit used in amplifiers is the voltage divider formed by R_1 and R_2, which sets up the proper dc voltage at the input to the amplifier.

When an ac signal voltage is applied to the amplifier, the input coupling capacitor, C_1, prevents the internal resistance of the ac source from changing the dc bias voltage. Without the capacitor, the internal source resistance would appear in parallel with R_2 and drastically change the value of the dc voltage.

The coupling capacitance is chosen so that its reactance (X_C) at the frequency of the ac signal is very small compared to the bias resistor values. The coupling capacitance therefore efficiently couples the ac signal from the source to the input of the amplifier. On the source side of the input coupling capacitor there is only ac but on the amplifier side there is ac plus dc (the signal voltage is riding on the dc bias voltage set by the voltage divider), as indicated in Figure 59(a). Capacitor C_2 is the output coupling capacitor, which couples the amplified ac signal to another amplifier stage that would be connected to the output.

You will check three amplifier boards like the one in Figure 59(b) for the proper input voltages using an oscilloscope. If the voltages are incorrect, you will determine the most likely fault. For all measurements, assume the amplifier has no dc loading effect on the voltage-divider bias circuit.

The Printed Circuit Board and the Schematic

1. Check the printed circuit board in Figure 59(b) to make sure it agrees with the amplifier schematic in part (a).

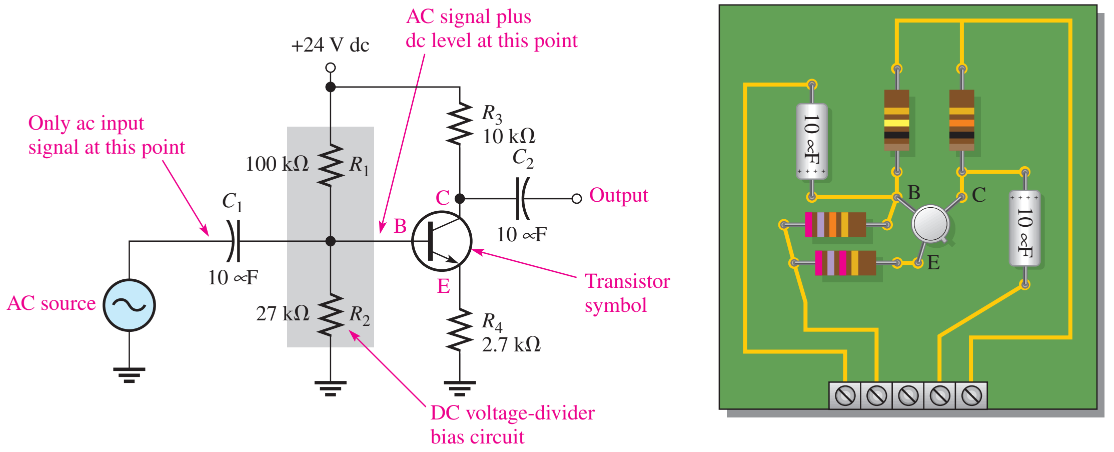
Testing Board 1

The oscilloscope probe is connected from channel 1 to the board as shown in Figure 60. The input signal from a sinusoidal voltage source is connected to the board and set to a frequency of 5 kHz with an amplitude of 1 V rms.

2. Determine if the voltage and frequency displayed on the scope are correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.

Testing Board 2

The oscilloscope probe is connected from channel 1 to board 2 the same as was shown in Figure 60 for board 1. The input signal from the sinusoidal voltage source is the same as it was for board 1.

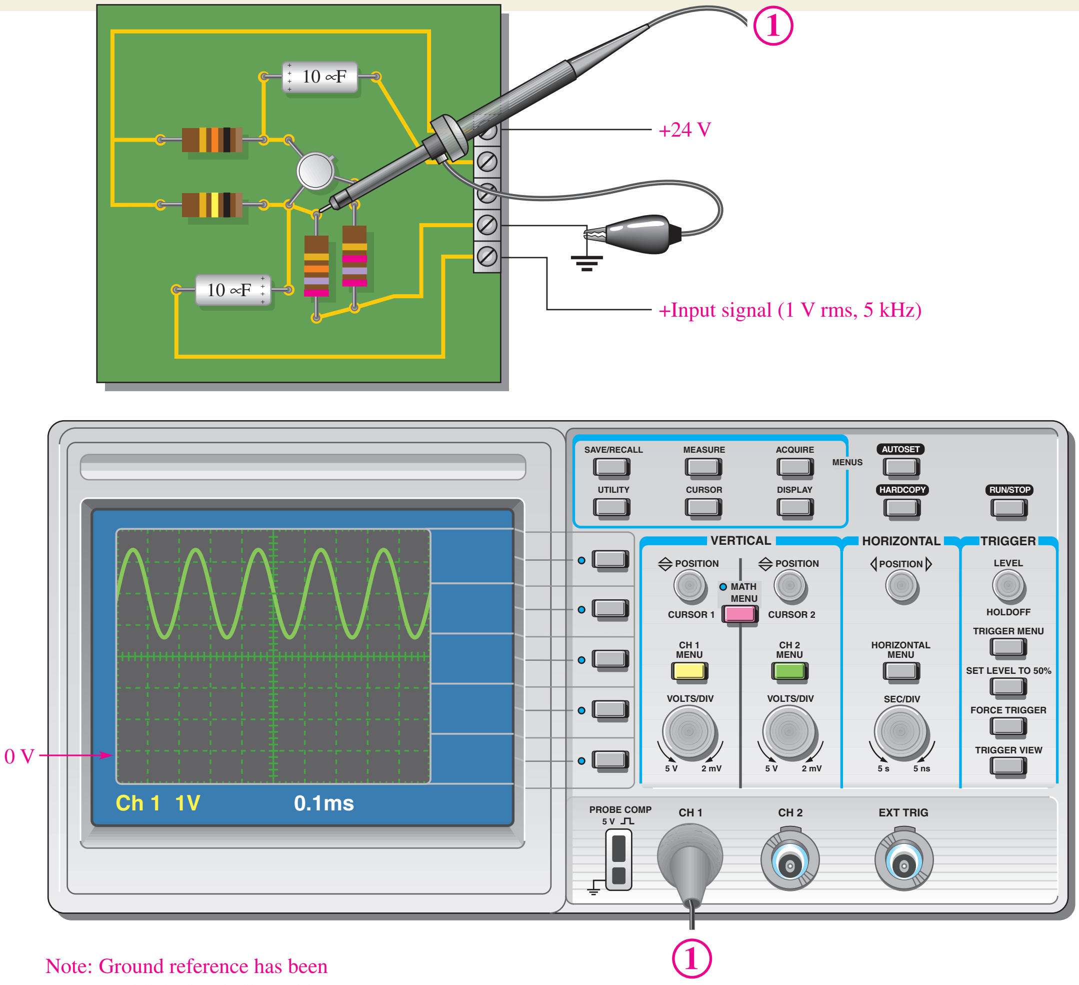


(a) Amplifier schematic

(b) Amplifier board

▲ **FIGURE 59**

A capacitively coupled amplifier.


①
10 μF
10 μF
+24 V
+Input signal (1 V rms, 5 kHz)
0 V
Ch 1 1V
0.1ms
SAVE/RECALL
MEASURE
ACQUIRE
MENUS
AUTOSET
UTILITY
CURSOR
DISPLAY
HARDCOPY
RUN/STOP
VERTICAL
HORIZONTAL
TRIGGER
POSITION
POSITION
POSITION
LEVEL
MATH MENU
CURSOR 1
CURSOR 2
HOLDOFF
TRIGGER MENU
CH 1 MENU
CH 2 MENU
HORIZONTAL MENU
SET LEVEL TO 50%
VOLTS/DIV
VOLTS/DIV
SEC/DIV
FORCE TRIGGER
5 V
2 mV
5 V
2 mV
5 s
5 ns
TRIGGER VIEW
PROBE COMP 5 V
CH 1
CH 2
EXT TRIG
①

Note: Ground reference has been established as indicated by 0 V.

▲ FIGURE 60

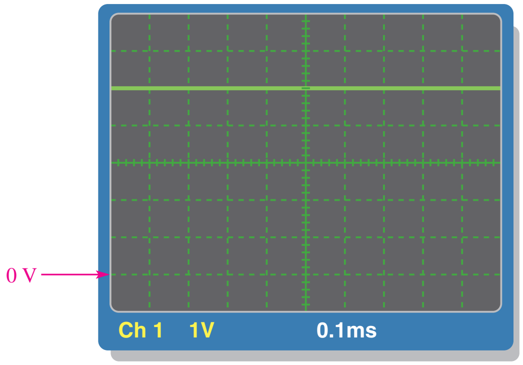
Testing board 1.

3. Determine if the scope display in Figure 61 is correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.

Testing Board 3

The oscilloscope probe is connected from channel 1 to board 3 the same as was shown in Figure 60 for board 1. The input signal from the sinusoidal voltage source is the same as before.

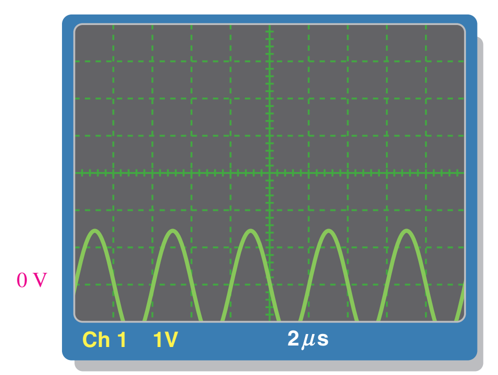
4. Determine if the scope display in Figure 62 is correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.


0 V
Ch 1 1V
0.1ms

Note: Ground reference has been established as indicated by 0 V.

▲ FIGURE 61

Testing board 2.



Note: Ground reference has been established as indicated by 0 V.

▲ FIGURE 62

Testing board 3.

Review

5. Explain why the input coupling capacitor is necessary when connecting an ac source to the amplifier.
6. Capacitor C_2 in Figure 59 is an output coupling capacitor. Generally, what would you expect to measure at the point in the circuit labelled C and at the output of the circuit when an ac input signal is applied to the amplifier?

SUMMARY

- A capacitor is composed of two parallel conducting plates separated by an insulating material called the *dielectric*.
- Energy is stored by a capacitor in the electric field between the plates.
- One farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates.
- Capacitance is directly proportional to the plate area and inversely proportional to the plate separation.
- The dielectric constant is an indication of the ability of a material to establish an electric field.
- The dielectric strength is one factor that determines the breakdown voltage of a capacitor.
- A capacitor blocks constant dc.
- The time constant for a series *RC* circuit is the resistance times the capacitance.
- In an *RC* circuit, the voltage and current in a charging or discharging capacitor make a 63% change during each time-constant interval.
- Five time constants are required for a capacitor to charge fully or to discharge fully. This is called the *transient time.*
- Charging and discharging follow exponential curves.
- Total series capacitance is less than that of the smallest capacitor in series.
- Capacitance adds in parallel.
- Current leads voltage by 90° in a capacitor.
- Capacitive reactance, X_C, is inversely proportional to frequency and capacitance.
- The total capacitive reactance of series capacitors is the sum of the individual reactances.
- The total capacitive reactance of parallel capacitors is the reciprocal of the sum of the reciprocals of the individual reactances.
- The true power in a capacitor is zero; that is, no energy is lost in an ideal capacitor due to conversion to heat.

KEY TERMS

Capacitive reactance The opposition of a capacitor to sinusoidal current. The unit is the ohm.

Capacitor An electrical device consisting of two conductive plates separated by an insulating material and possessing the property of capacitance.

Dielectric The insulating material between the plates of a capacitor.

Farad (F) The unit of capacitance.

Instantaneous power (p) The value of power in a circuit at any given instant of time.

***RC* time constant** A fixed time interval set by R and C values that determines the time response of a series RC circuit. It equals the product of the resistance and the capacitance.

Reactive power (P_r) The rate at which energy is alternately stored and returned to the source by a capacitor. The unit is the VAR.

Ripple voltage The small fluctuation in voltage due to the charging and discharging of a capacitor.

True power (P_{true}) The power that is dissipated in a circuit, usually in the form of heat.

VAR (volt-ampere reactive) The unit of reactive power.

FORMULAS

1	$C = \frac{Q}{V}$	Capacitance in terms of charge and voltage
2	$Q = CV$	Charge in terms of capacitance and voltage
3	$V = \frac{Q}{C}$	Voltage in terms of charge and capacitance
4	$W = \frac{1}{2}CV^2$	Energy stored by a capacitor
5	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	Dielectric constant (relative permittivity)
6	$C = \frac{A\varepsilon_r(8.85 \times 10^{-12}\text{ F/m})}{d}$	Capacitance in terms of physical parameters
7	$Q_T = Q_1 = Q_2 = Q_3 = \cdots = Q_n$	Total charge of series capacitors (general)
8	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$	Reciprocal of total series capacitance (general)
9	$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$	Total series capacitance (general)
10	$C_T = \frac{C_1C_2}{C_1 + C_2}$	Total capacitance of two capacitors in series
11	$C_T = \frac{C}{n}$	Total capacitance of equal-value capacitors in series
12	$V_x = \left(\frac{C_T}{C_x}\right)V_T$	Capacitor voltage for capacitors in series
13	$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_n$	Total charge of parallel capacitors (general)
14	$C_T = C_1 + C_2 + C_3 + \cdots + C_n$	Total parallel capacitance (general)
15	$C_T = nC$	Total capacitance of equal-value capacitors in parallel
16	$\tau = RC$	Time constant
17	$v = V_F + (V_i - V_F)e^{-t/\tau}$	Exponential voltage (general)
18	$i = I_F + (I_i - I_F)e^{-t/\tau}$	Exponential current (general)
19	$v = V_F(1 - e^{-t/RC})$	Increasing exponential voltage beginning at zero
20	$v = V_ie^{-t/RC}$	Decreasing exponential voltage ending at zero

21	$t = -RC\ln\left(\frac{v}{V_i}\right)$	Time on decreasing exponential ($V_F = 0$)
22	$t = -RC\ln\left(1 - \frac{v}{V_F}\right)$	Time on increasing exponential ($V_i = 0$)
23	$i = \frac{dq}{dt}$	Instantaneous current using charge derivative
24	$i = C\left(\frac{dv}{dt}\right)$	Instantaneous capacitor current using voltage derivative
25	$X_C = \frac{1}{2\pi fC}$	Capacitive reactance
26	$X_{C(tot)} = X_{C1} + X_{C2} + X_{C3} + \cdots + X_{Cn}$	Total series reactance
27	$X_{C(tot)} = \frac{1}{\frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}} + \cdots + \frac{1}{X_{Cn}}}$	Total parallel reactance
28	$V_x = \left(\frac{X_{Cx}}{X_{C(tot)}}\right)V_s$	Capacitive voltage divider
29	$P_r = V_{\mathrm{rms}}I_{\mathrm{rms}}$	Reactive power in a capacitor
30	$P_r = \frac{V_{\mathrm{rms}}^2}{X_C}$	Reactive power in a capacitor
31	$P_r = I_{\mathrm{rms}}^2X_C$	Reactive power in a capacitor
32	$I_{1(\mathrm{avg})} = \frac{C(V_1 - V_2)}{T}$	Switched-capacitor average current
33	$R = \frac{T}{C}$	Switched-capacitor equivalent resistance
34	$R = \frac{1}{fC}$	Switched-capacitor equivalent resistance

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The area of the plates of a capacitor is proportional to the capacitance.
2. A capacitance of 1200 pF is the same as 1.2 μF.
3. When two capacitors are in series with a voltage source, the smaller capacitor will have the larger voltage.
4. When two capacitors are in parallel with a voltage source, the smaller capacitor will have the larger voltage.
5. A capacitor appears as an open to a constant dc.
6. When a capacitor is charging or discharging between two levels, the charge on the capacitor changes by 63% of the difference in one time constant.
7. Capacitive reactance is proportional to the applied frequency.
8. The total reactance of series capacitors is the product-over-sum of the individual reactances.
9. Voltage leads current in a capacitor.
10. The unit of reactive power is the VAR.

SELF-TEST

Answers are at the end of the chapter.

1. Which of the following accurately describes a capacitor:?
   (a) The plates are conductive.
   (b) The dielectric is an insulator between the plates.

(c) There is constant direct current (dc) through a fully charged capacitor.

(d) A practical capacitor stores charge indefinitely when disconnected from the source.

(e) none of the above answers

(f) all of the above answers

(g) only answers (a) and (b)

2. Which one of the following statements is true?

(a) There is current through the dielectric of a charging capacitor.

(b) When a capacitor is connected to a dc voltage source, it will charge to the value of the source.

(c) An ideal capacitor can be discharged by disconnecting it from the voltage source.

3. A capacitance of 0.01 mF is larger than

(a) 0.00001 F **(b)** 100,000 pF **(c)** 1000 pF **(d)** all of these answers

4. A capacitance of 1000 pF is smaller than

(a) 0.01 μF **(b)** 0.001 μF **(c)** 0.00000001 F **(d)** both (a) and (c)

5. When the voltage across a capacitor is increased, the stored charge

(a) increases **(b)** decreases **(c)** remains constant **(d)** fluctuates

6. When the voltage across a capacitor is doubled, the stored charge

(a) stays the same **(b)** is halved **(c)** increases by four **(d)** doubles

7. The voltage rating of a capacitor is increased by

(a) increasing the plate separation **(b)** decreasing the plate separation

(c) increasing the plate area **(d)** answers (b) and (c)

8. The capacitance value is increased by

(a) decreasing the plate area **(b)** increasing the plate separation

(c) decreasing the plate separation **(d)** increasing the plate area

(e) answers (a) and (b) **(f)** answers (c) and (d)

9. A 1 μF, a 2.2 μF, and a 0.047 μF capacitor are connected in series. The total capacitance is less than

(a) 1 μF **(b)** 2.2 μF **(c)** 0.047 μF **(d)** 0.001 μF

10. Four 0.022 μF capacitors are in parallel. The total capacitance is

(a) 0.022 μF **(b)** 0.088 μF **(c)** 0.011 μF **(d)** 0.044 μF

11. An uncharged capacitor and a resistor are connected in series with a switch and a 12 V battery. At the instant the switch is closed, the voltage across the capacitor is

(a) 12 V **(b)** 6 V **(c)** 24 V **(d)** 0 V

12. In Question 11, the voltage across the capacitor when it is fully charged is

(a) 12 V **(b)** 6 V **(c)** 24 V **(d)** -6 V

13. In Question 11, the capacitor will reach full charge in a time equal to approximately

(a) RC **(b)** $5RC$ **(c)** $12RC$ **(d)** cannot be predicted

14. A sinusoidal voltage is applied across a capacitor. When the frequency of the voltage is increased, the current

(a) increases **(b)** decreases **(c)** remains constant **(d)** ceases

15. A capacitor and a resistor are connected in series to a sine wave generator. The frequency is set so that the capacitive reactance is equal to the resistance and, thus, an equal amount of voltage appears across each component. If the frequency is decreased,

(a) $V_R > V_C$ **(b)** $V_C > V_R$ **(c)** $V_R = V_C$

16. Two equal-value capacitors are connected in series. At any frequency, the total reactance is

(a) one-half the reactance of each capacitor

(b) equal to the reactance of each capacitor

(c) twice the reactance of each capacitor

17. Switched-capacitor circuits are used to

(a) increase capacitance **(b)** emulate inductance
(c) emulate resistance **(d)** generate sine wave voltages

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 70.

1. If the capacitors are initially uncharged and the switch is thrown to the closed position, the charge on C_1

(a) increases **(b)** decreases **(c)** stays the same

2. If C_4 is shorted with the switch closed, the charge on C_1

(a) increases **(b)** decreases **(c)** stays the same

3. If the switch is closed and C_2 fails to open, the charge on C_1

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 71.

4. Assume the switch is closed and C is allowed to fully charge. When the switch is opened, the voltage across C

(a) increases **(b)** decreases **(c)** stays the same

5. If C has failed open when the switch is closed, the voltage across C

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 74.

6. If the switch is closed allowing the capacitor to charge and then the switch is opened, the voltage across the capacitor

(a) increases **(b)** decreases **(c)** stays the same

7. If R_2 opens, the time it takes the capacitor to fully charge

(a) increases **(b)** decreases **(c)** stays the same

8. If R_4 opens, the maximum voltage to which the capacitor can charge

(a) increases **(b)** decreases **(c)** stays the same

9. If V_S is reduced, the time required for the capacitor to fully change

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 77(b).

10. If the frequency of the ac source is increased, the total current

(a) increases **(b)** decreases **(c)** stays the same

11. If C_1 opens, the current through C_2

(a) increases **(b)** decreases **(c)** stays the same

12. If the value of C_2 is changed to 1 μF, the current through it

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Basic Capacitor

1. **(a)** Find the capacitance when $Q = 50\ \mu\text{C}$ and $V = 10\ \text{V}$.
(b) Find the charge when $C = 0.001\ \mu\text{F}$ and $V = 1\ \text{kV}$.
(c) Find the voltage when $Q = 2\ \text{mC}$ and $C = 200\ \mu\text{F}$.

2. Convert the following values from microfarads to picofarads:
   (a) 0.1 μF **(b)** 0.0025 μF **(c)** 4.7 μF
3. Convert the following values from picofarads to microfarads:
   (a) 1000 pF **(b)** 3500 pF **(c)** 250 pF
4. Convert the following values from farads to microfarads:
   (a) 0.0000001 F **(b)** 0.0022 F **(c)** 0.0000000015 F
5. How much energy is stored in a 1000 μF capacitor that is charged to 500 V?
6. What size capacitor is capable of storing 10 mJ of energy with 100 V across its plates?
7. Calculate the absolute permittivity, ε, for each of the following materials. Refer to Table 3 for ε_r values.
   (a) air **(b)** oil **(c)** glass **(d)** Teflon®
8. A mica capacitor has square plates that are 3.8 cm on a side and separated by 2.5 mils. What is the capacitance?
9. An air capacitor has a total plate area of 0.05 m^2. The plates are separated by 4.5×10^{-4} m. Calculate the capacitance.

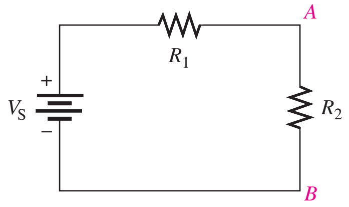
*10. A student wants to construct a 1 F capacitor out of two square plates for a science fair project. He plans to use a paper dielectric ($\varepsilon_r = 2.5$) that is 8×10^{-5} m thick. The science fair is to be held in the Astrodome. Will his capacitor fit in the Astrodome? What would be the size of the plates if it could be constructed?

11. A student decides to construct a capacitor using two conducting plates 30 cm on a side. He separates the plates with a paper dielectric ($\varepsilon_r = 2.5$) that is 8×10^{-5} m thick. What is the capacitance of his capacitor?
12. At ambient temperature (25°C), a certain capacitor is specified to be 1000 pF. It has a negative temperature coefficient of 200 ppm/°C. What is its capacitance at 75°C?
13. A 0.001 μF capacitor has a positive temperature coefficient of 500 ppm/°C. How much change in capacitance will a 25°C increase in temperature cause?

SECTION 2 Types of Capacitors

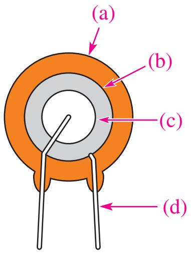
14. In the construction of a stacked-foil mica capacitor, how is the plate area increased?
15. Of mica or ceramic, which type of capacitor has the highest dielectric constant?
16. Show how to connect an electrolytic capacitor across R_2 between points A and B in Figure 63.

▶ FIGURE 63

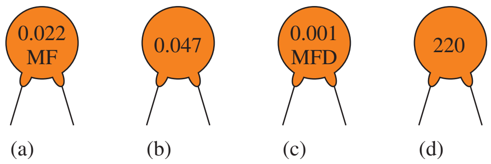


17. Name two types of electrolytic capacitors. How do electrolytics differ from other capacitors?
18. Identify the parts of the ceramic disk capacitor shown in the cutaway view of Figure 64.

▶ FIGURE 64



▶ FIGURE 65


0.022
MF
0.047
0.001
MFD
220
(a)
(b)
(c)
(d)

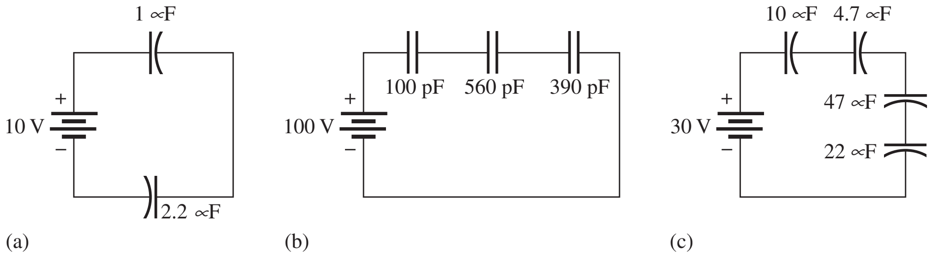
19. Determine the value of the ceramic disk capacitors in Figure 65.

SECTION 3 Series Capacitors

20. Five 1000 pF capacitors are in series. What is the total capacitance?

21. Find the total capacitance for each circuit in Figure 66.

22. For each circuit in Figure 66, determine the voltage across each capacitor.

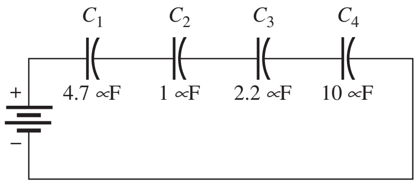

1 ∝F
+
10 V
−
2.2 ∝F
(a)
100 pF
560 pF
390 pF
+
100 V
−
(b)
10 ∝F
4.7 ∝F
+
30 V
−
47 ∝F
22 ∝F
(c)

▲ FIGURE 66

23. Two series capacitors (one 1 μF, the other of unknown value) are charged from a 12 V source. The 1 μF capacitor is charged to 8 V and the other to 4 V. What is the value of the unknown capacitor?

24. The total charge stored by the series capacitors in Figure 67 is 10 μC. Determine the voltage across each of the capacitors.

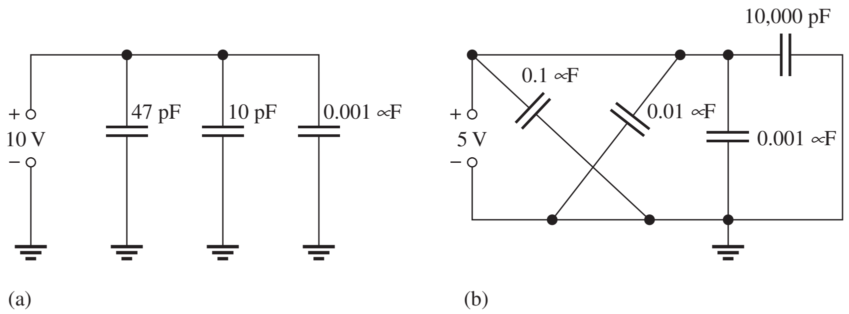
▶ FIGURE 67


C_1
C_2
C_3
C_4
+
−
4.7 ∝F
1 ∝F
2.2 ∝F
10 ∝F

SECTION 4 Parallel Capacitors

25. Determine C_T for each circuit in Figure 68.

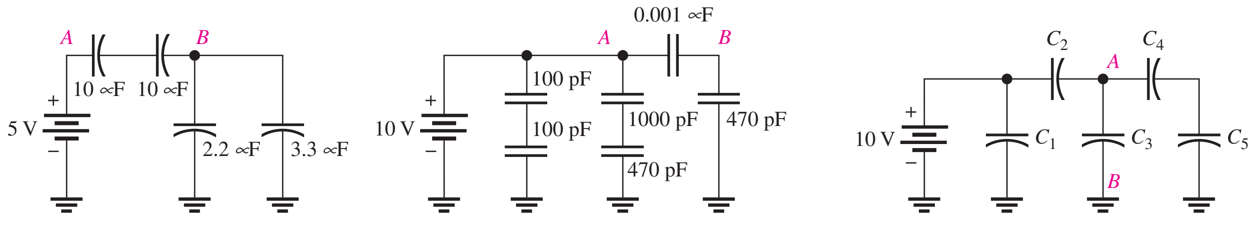
26. What is the charge on each capacitor in Figure 68?


+
10 V
−
47 pF
10 pF
0.001 ∝F
(a)
10,000 pF
0.1 ∝F
0.01 ∝F
+
5 V
−
0.001 ∝F
(b)

▲ FIGURE 68

27. Determine C_T for each circuit in Figure 69.

28. What is the voltage between nodes A and B in each circuit in Figure 69?


A
B
10 ∝F
10 ∝F
+
5 V
–
2.2 ∝F
3.3 ∝F
0.001 ∝F
A
B
+
10 V
–
100 pF
100 pF
1000 pF
470 pF
470 pF
C_2
C_4
A
+
10 V
–
C_1
C_3
B
C_5

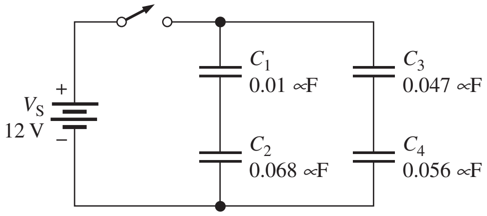
(a) (b) (c) C = 1 ∝F for each capacitor

▲ **FIGURE 69**

****29.** Initially, the capacitors in the circuit in Figure 70 are uncharged.

- **(a)** After the switch is closed, what total charge is supplied by the source?
- **(b)** What is the voltage across each capacitor?

▶ **FIGURE 70**


+
V_S
12 V
–
C_1
0.01 ∝F
C_2
0.068 ∝F
C_3
0.047 ∝F
C_4
0.056 ∝F

SECTION 5 Capacitors in DC Circuits

30. Determine the time constant for each of the following series RC combinations:

- **(a)** $R = 100\ \Omega, C = 1\ \mu F$ **(b)** $R = 10\ M\Omega, C = 47\ pF$
- **(c)** $R = 4.7\ k\Omega, C = 0.0047\ \mu F$ **(d)** $R = 1.5\ M\Omega, C = 0.01\ \mu F$

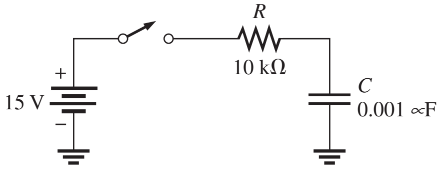
31. Determine how long it takes the capacitor to reach full charge for each of the following combinations:

- **(a)** $R = 56\ \Omega, C = 47\ \mu F$ **(b)** $R = 3300\ \Omega, C = 0.015\ \mu F$
- **(c)** $R = 22\ k\Omega, C = 100\ pF$ **(d)** $R = 5.6\ M\Omega, C = 10\ pF$

32. In the circuit of Figure 71, the capacitor is initially uncharged. Determine the capacitor voltage at the following times after the switch is closed:

(a) 10 μs **(b)** 20 μs **(c)** 30 μs **(d)** 40 μs **(e)** 50 μs

▶ **FIGURE 71**


+
15 V
–
R
10 kΩ
C
0.001 ∝F

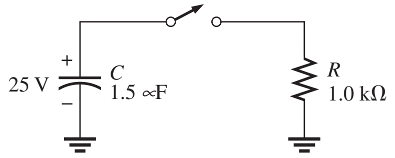
33. In Figure 72, the capacitor is charged to 25 V. When the switch is closed, what is the capacitor voltage after the following times?

(a) 1.5 ms **(b)** 4.5 ms **(c)** 6 ms **(d)** 7.5 ms

34. Repeat Problem 32 for the following time intervals:

(a) 2 μs **(b)** 5 μs **(c)** 15 μs

▶ FIGURE 72

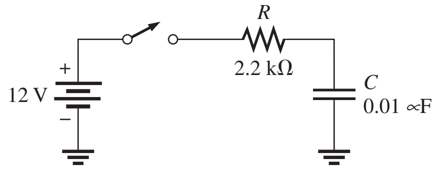


35. Repeat Problem 33 for the following times:

(a) 0.5 ms **(b)** 1 ms **(c)** 2 ms

*__36.__ Derive the formula for finding the time at any point on an increasing exponential voltage curve. Use this formula to find the time at which the voltage in Figure 73 reaches 6 V after switch closure.

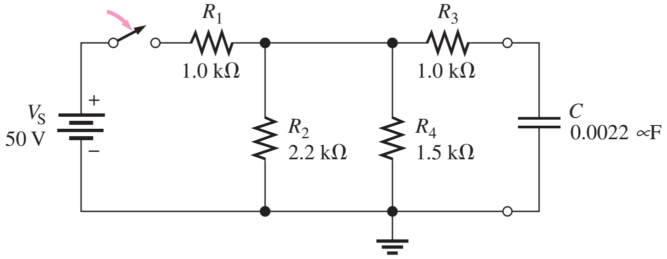
▶ FIGURE 73



37. How long does it take C to charge to 8 V in Figure 71?

38. How long does it take C to discharge to 3 V in Figure 72?

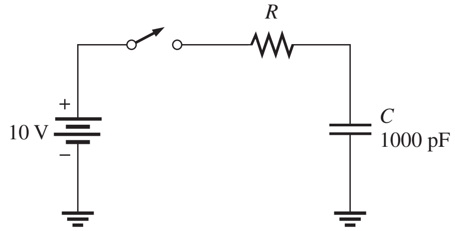
39. Determine the time constant for the circuit in Figure 74.



▲ FIGURE 74

*__40.__ In Figure 75, the capacitor is initially uncharged. At $t = 10\ \mu s$ after the switch is closed, the instantaneous capacitor voltage is 7.2 V. Determine the value of R.

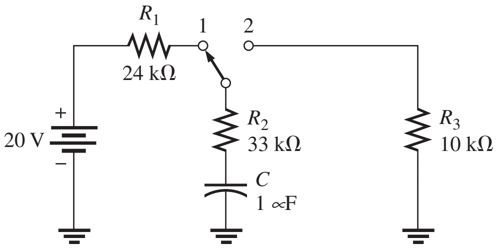
▶ FIGURE 75



*__41.__ **(a)** The capacitor in Figure 76 is uncharged when the switch is thrown into position 1. The switch remains in position 1 for 10 ms and is then thrown into position 2, where it remains indefinitely. Draw the complete waveform for the capacitor voltage.

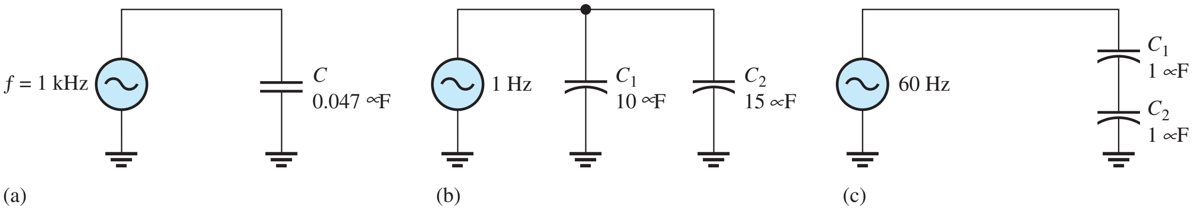
(b) If the switch is thrown back to position 1 after 5 ms in position 2, and then left in position 1, how would the waveform appear?

▶ FIGURE 76


R_1
24 kΩ
1
2
+
20 V
−
R_2
33 kΩ
C
1 µF
R_3
10 kΩ

SECTION 6 Capacitors in AC Circuits

42. What is the value of the total capacitive reactance in each circuit in Figure 77?


f = 1 kHz
C
0.047 µF
(a)
1 Hz
C_1
10 µF
C_2
15 µF
(b)
60 Hz
C_1
1 µF
C_2
1 µF
(c)

▲ FIGURE 77

43. In Figure 69, each dc voltage source is replaced by a 10 V rms, 2 kHz ac source. Determine the total reactance in each case.

44. In each circuit of Figure 77, what frequency is required to produce an X_C of 100 Ω? An X_C of 1 kΩ?

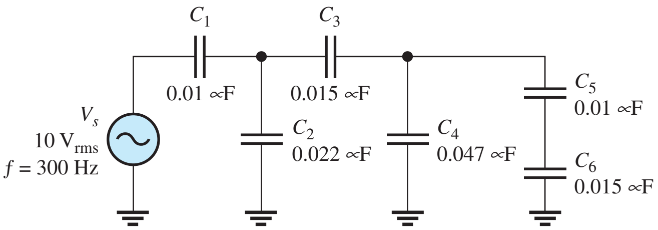
45. A sinusoidal voltage of 20 V rms produces an rms current of 100 mA when connected to a certain capacitor. What is the reactance?

46. A 10 kHz voltage is applied to a 0.0047 µF capacitor, and 1 mA of rms current is measured. What is the value of the voltage?

47. Determine the true power and the reactive power in Problem 46.

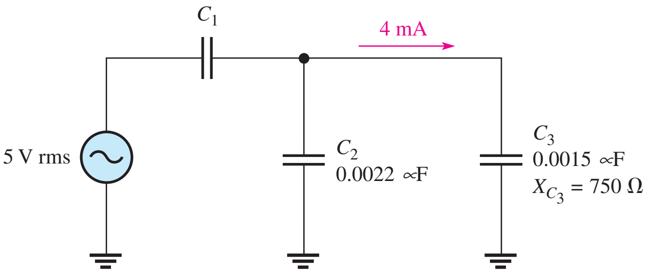
***48.** Determine the ac voltage across each capacitor and the current in each branch of the circuit in Figure 78.

▶ FIGURE 78


C_1
C_3
0.01 µF
0.015 µF
V_s
10 V_{rms}
f = 300 Hz
C_2
0.022 µF
C_4
0.047 µF
C_5
0.01 µF
C_6
0.015 µF

49. Find the value of C_1 in Figure 79.

▶ FIGURE 79


C_1
4 mA
5 V rms
C_2
0.0022 µF
C_3
0.0015 µF
X_{C_3} = 750 Ω

*50. If C_4 in Figure 78 opened, determine the voltages that would be measured across the other capacitors.

SECTION 7 Capacitor Applications

51. If another capacitor is connected in parallel with the existing capacitor in the power supply filter of Figure 54, how is the ripple voltage affected?

52. Ideally, what should the reactance of a bypass capacitor be in order to eliminate a 10 kHz ac voltage at a given point in an amplifier circuit?

SECTION 8 Switched-Capacitor Circuits

53. The capacitor in a switched-capacitor circuit has a value of 2200 pF and is switched with a waveform having a period of 10 μs. Determine the value of the resistor that it emulates.

54. In a switched-capacitor circuit, the 100 pF capacitor is switched at a frequency of 8 kHz. What resistor value is emulated?

Multisim Troubleshooting and Analysis

These problems require Multisim.

55. Open file P12-55 and measure the voltage across each capacitor.

56. Open file P12-56 and measure the voltage across each capacitor.

57. Open file P12-57 and measure the current. Decrease the frequency by one-half and measure the current again. Double the original frequency and measure the current again. Explain your observations.

58. Open file P12-58 and find the open capacitor if there is one.

59. Open file P12-59 and find the shorted capacitor if there is one.

ANSWERS

SECTION CHECKUPS

SECTION 1 The Basic Capacitor

1. Capacitance is the ability (capacity) to store electrical charge.
2. **(a)** 1,000,000 μF in 1 F **(b)** 1×10^{12} pF in 1 F **(c)** 1,000,000 pF in 1 μF
3. $0.0015\ \mu\text{F} = 1500\ \text{pF}; 0.0015\ \mu\text{F} = 0.0000000015\ \text{F}$
4. $W = \frac{1}{2}CV^2 = 1.125\ \mu\text{J}$
5. **(a)** C increases. **(b)** C decreases.
6. (1000 V/mil) (2 mil) = 2 kV
7. $C = 2.01\ \mu\text{F}$

SECTION 2 Types of Capacitors

1. Capacitors can be classified by the dielectric material.
2. The capacitance value of a fixed capacitor cannot be changed; the capacitance value of a variable capacitor can be changed.
3. Electrolytic capacitors are polarized.
4. When connecting a polarized capacitor, make sure the voltage rating is sufficient. Connect the positive end to the positive side of the circuit.

SECTION 3 Series Capacitors

1. Series C_T is less than smallest C.
2. $C_T = 61.2$ pF
3. $C_T = 0.006\ \mu$F

4. $C_T = 20\text{ pF}$
5. $V_{C1} = 75.2\text{ V}$

SECTION 4 Parallel Capacitors

1. The values of the individual capacitors are added in parallel.
2. Achieve C_T by using five 0.01 μF capacitors in parallel.
3. $C_T = 167\text{ pF}$

SECTION 5 Capacitors in DC Circuits

1. $\tau = RC = 1.2\ \mu\text{s}$
2. $5\tau = 6\ \mu\text{s}$; $V_C = 4.97\text{ V}$
3. $v_{2\text{ms}} = 8.65\text{ V}$; $v_{3\text{ms}} = 9.50\text{ V}$; $v_{4\text{ms}} = 9.82\text{ V}$; $v_{5\text{ms}} = 9.93\text{ V}$
4. $v_C = 36.8\text{ V}$

SECTION 6 Capacitors in AC Circuits

1. Current leads voltage by 90° in a capacitor.
2. $X_C = 1/2\pi fC = 637\text{ k}\Omega$
3. $f = 1/2\pi X_C C = 796\text{ Hz}$
4. $I_{\text{rms}} = 629\text{ mA}$
5. $P_{\text{true}} = 0\text{ W}$
6. $P_r = 0.453\text{ VAR}$

SECTION 7 Capacitor Applications

1. Once the capacitor charges to the peak voltage, it discharges very little before the next peak, thus smoothing the rectified voltage.
2. A coupling capacitor allows ac to pass from one point to another, but blocks constant dc.
3. A coupling capacitor must be large enough to have a negligible reactance at the frequency that is to be passed without opposition.
4. A decoupling capacitor shorts power line voltage transients to ground.
5. X_C is inversely proportional to frequency and so is the filter's ability to pass ac signals.
6. The charging time

SECTION 8 Switched-Capacitor Circuits

1. By moving the same amount of charge corresponding to the current in the equivalent resistance
2. Switching frequency and capacitance value
3. Transistors

RELATED PROBLEMS FOR EXAMPLES

1 100 kV
2 0.047 μF
3 100×10^6 pF
4 62.7 pF
5 1.54 μF
6 278 pF
7 0.011 μF
8 2.83 V
9 650 pF
10 0.09 μF
11 891 μs

12 8.36 V

13 8.13 V

14 ≈0.74 ms; 95 V

15 1.52 ms

16 0.42 V

17 3.39 kHz

18 **(a)** 1.83 kΩ **(b)** 204 Ω

19 4.40∠90° mA

20 8.72 V

21 0 W; 1.01 mVAR

22 178.6 kHz

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** F **5.** T

6. T **7.** F **8.** F **9.** F **10.** T

SELF-TEST

1. (g) **2.** (b) **3.** (c) **4.** (d) **5.** (a) **6.** (d) **7.** (a)

8. (f) **9.** (c) **10.** (b) **11.** (d) **12.** (a) **13.** (b) **14.** (a)

15. (b) **16.** (c) **17.** (c)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (b)

7. (a) **8.** (a) **9.** (c) **10.** (a) **11.** (c) **12.** (b)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. **(a)** 5 μF **(b)** 1 μC **(c)** 10 V

3. **(a)** 0.001 μF **(b)** 0.0035 μF **(c)** 0.00025 μF

5. 125 J

7. **(a)** 8.85×10^{-12} F/m **(b)** 35.4×10^{-12} F/m
(c) 66.4×10^{-12} F/m **(d)** 17.7×10^{-12} F/m

9. 983 pF

11. 0.0249 μF

13. 12.5 pF increase

15. Ceramic

17. Aluminum, tantalum; they are polarized.

19. **(a)** 0.022 μF **(b)** 0.047 μF
(c) 0.001 μF **(d)** 220 pF

21. **(a)** 0.688 μF **(b)** 69.7 pF **(c)** 2.64 μF

23. 2 μF

25. **(a)** 1057 pF **(b)** 0.121 μF

27. **(a)** 2.62 μF **(b)** 689 pF **(c)** 1.6 μF

29. **(a)** 0.411 μC
(b) $V_1 = 10.47$ V
$V_2 = 1.54$ V
$V_3 = 6.52$ V
$V_4 = 5.48$ V

31. **(a)** 13.2 ms **(b)** 247.5 μs **(c)** 11 μs **(d)** 280 μs

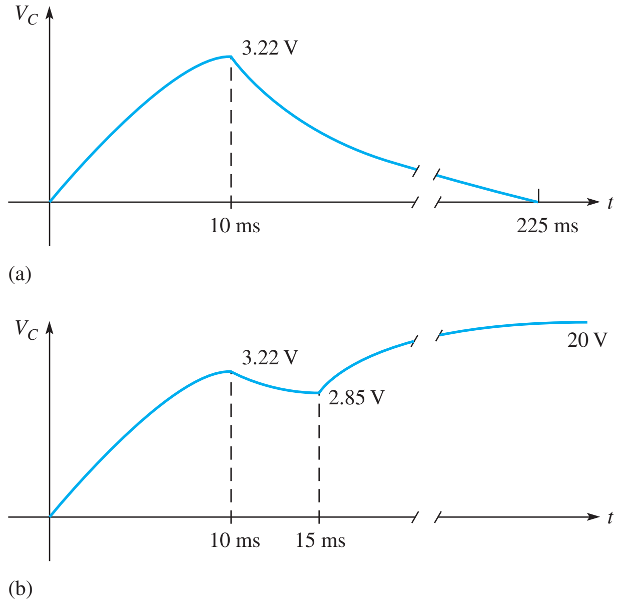
33. **(a)** 9.20 V **(b)** 1.24 V **(c)** 0.458 V **(d)** 0.168 V

35. **(a)** 17.9 V **(b)** 12.8 V **(c)** 6.59 V

37. 7.62 μs

39. 3.00 μs

41. See Figure 80.



▲ **FIGURE 80**

43. **(a)** 30.4 Ω **(b)** 116 kΩ **(c)** 49.7 Ω

45. 200 Ω

47. 0 W, 3.39 mVAR

49. 0.00541 μF

51. The ripple is reduced.

53. 4.55 kΩ

55. $V_1 = 3.103$ V; $V_2 = 6.828$ V; $V_3 = 2.069$ V

57. I_C @ 1 kHz = 1.383 mA; I_C @ 500 Hz = 0.691 mA
I_C @ 2 kHz = 2.768 mA

59. C_4 shorted

INDUCTORS

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INDUCTORS

CHAPTER OUTLINE

1 The Basic Inductor
2 Types of Inductors
3 Series and Parallel Inductors
4 Inductors in DC Circuits
5 Inductors in AC Circuits
6 Inductor Applications
Application Activity

CHAPTER OBJECTIVES

- Describe the basic construction and characteristics of an inductor
- Discuss various types of inductors
- Analyze series and parallel inductors
- Analyze inductive dc switching circuits
- Analyze inductive ac circuits
- Discuss some inductor applications

KEY TERMS

- Inductor
- Winding
- Induced voltage
- Inductance
- Henry (H)
- *RL* time constant
- Inductive reactance
- Quality factor (**Q**)

APPLICATION ACTIVITY PREVIEW

In this application activity, the inductance of coils is determined by measuring the time constant of a test circuit using oscilloscope waveforms.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, you will learn about a type of basic passive component, the *inductor,* and study its characteristics.

The basic construction and electrical properties of inductors are discussed, and the effects of connecting them in series and in parallel are analyzed. How an inductor works in both dc and ac circuits is an important part of this coverage and forms the basis for the study of reactive circuits in terms of both frequency response and time response. You will also learn how to check for a faulty inductor.

The inductor, which is basically a coil of wire, is based on the principle of electromagnetic induction. Inductance is the property of a coil of wire that opposes a change in current. The basis for inductance is the electromagnetic field that surrounds any conductor when there is current through it. The electrical component designed to have the property of inductance is called an *inductor, coil,* or in certain applications a *choke.* All of these terms refer to the same type of device.

1 THE BASIC INDUCTOR

An **inductor** is a passive electrical component formed by a coil of wire and which exhibits the property of inductance.

After completing this section, you should be able to

- **Describe the basic construction and characteristics of an inductor**
    - Define *inductance* and state its unit
    - Discuss induced voltage
    - Explain how an inductor stores energy
    - Specify how the physical characteristics affect inductance
    - Discuss winding resistance and winding capacitance
    - State Faraday's law
    - State Lenz's law

When a length of wire is formed into a coil, as shown in Figure 1, it becomes an inductor. The terms *coil* and *inductor* are used interchangeably. Current through the coil produces an electromagnetic field, as illustrated. The magnetic lines of force around each loop (turn) in the **winding** of the coil effectively add to the lines of force around the adjoining loops, forming a strong electromagnetic field within and around the coil. The net direction of the total electromagnetic field creates a north and a south pole.

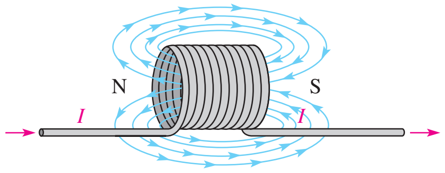
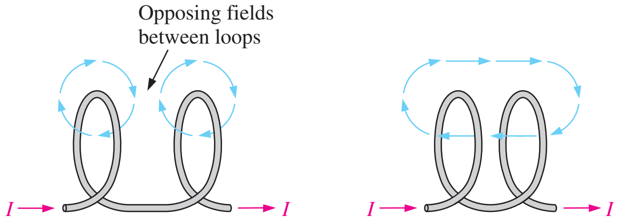


FIGURE 1

A coil of wire forms an inductor. When there is current through it, a three-dimensional electromagnetic field is created, surrounding the coil in all directions.

To understand the formation of the total electromagnetic field in a coil, consider the interaction of the electromagnetic fields around two adjacent loops. The magnetic lines of force around adjacent loops are each deflected into a single outer path when the loops are brought close together. This effect occurs because the magnetic lines of force are in opposing directions between adjacent loops and therefore cancel out when the loops are close together, as illustrated in Figure 2. The total electromagnetic field for the two loops is



(a) Separated

(b) Closely adjacent loops; opposing fields between loops cancel.

FIGURE 2

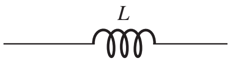
Interaction of magnetic lines of force in two adjacent loops of a coil.

HISTORY NOTE

Joseph Henry 1797–1878

Henry began his career as a professor at a small school in Albany, NY, and later became the first director of the Smithsonian Institution. He was the first American since Franklin to undertake original scientific experiments. He was the first to superimpose coils of wire wrapped on an iron core and first observed the effects of electromagnetic induction in 1830, a year before Faraday, but he did not publish his findings. Henry did obtain credit for the discovery of self-induction, however. The unit of inductance is named in his honor. (Photo credit: Courtesy of the Smithsonian Institution. Photo number 59,054.)

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.



▲ FIGURE 3

Symbol for inductor.

depicted in part (b). This effect is additive for many closely adjacent loops in a coil; that is, each additional loop adds to the strength of the electromagnetic field. For simplicity, only single lines of force are shown, although there are many. Figure 3 shows a schematic symbol for an inductor.

Inductance

When there is current through an inductor, an electromagnetic field is established. When the current changes, the electromagnetic field also changes. An increase in current expands the electromagnetic field, and a decrease in current reduces it. Therefore, a changing current produces a changing electromagnetic field around the inductor. In turn, the changing electromagnetic field causes an **induced voltage** across the coil in a direction to oppose the change in current. This property is called *self-inductance* but is usually referred to as simply *inductance,* symbolized by L.

> **Inductance is a measure of a coil's ability to establish an induced voltage as a result of a change in its current, and that induced voltage is in a direction to oppose the change in current.**

The inductance (L) of a coil and the time rate of change of the current (di/dt) determine the induced voltage (v_{ind}). A change in current causes a change in the electromagnetic field, which, in turn, induces a voltage across the coil, as you know. The induced voltage is directly proportional to L and di/dt, as stated by the following formula:

Equation 1

$$v_{\text{ind}} = L\left(\frac{di}{dt}\right)$$

This formula indicates that the greater the inductance, the greater the induced voltage. Also, it shows that the faster the coil current changes (greater di/dt), the greater the induced voltage. Notice the similarity of Equation 1 to $i = C(dv/dt)$.

The Unit of Inductance The **henry (H)** is the basic unit of inductance. By definition, the inductance of a coil is one henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The henry is a large unit, so in practical applications, millihenries (mH) and microhenries (μH) are the more common units.

EXAMPLE 1

Determine the induced voltage across a 1 henry (1 H) inductor when the current is changing at a rate of 2 A/s.

Solution

$$v_{\text{ind}} = L\left(\frac{di}{dt}\right) = (1\text{ H})(2\text{ A/s}) = \mathbf{2\text{ V}}$$

*Related Problem**

Determine the inductance when a current changing at a rate of 10 A/s causes 50 V to be induced.

---

*Answers are at the end of the chapter.

Energy Storage An inductor stores energy in the electromagnetic field created by the current. The energy stored is expressed as follows:

Equation 2

$$W = \frac{1}{2}LI^2$$

As you can see, the energy stored is proportional to the inductance and the square of the current. When current (I) is in amperes and inductance (L) is in henries, energy (W) is in joules.

Physical Characteristics of an Inductor

The following parameters are important in establishing the inductance of a coil: permeability of the core material, number of turns of wire, core length, and cross-sectional area of the core.

Core Material As discussed earlier, an inductor is basically a coil of wire that surrounds a magnetic or nonmagnetic material called the **core**. Examples of magnetic materials are iron, nickel, steel, cobalt, or alloys. These materials have permeabilities that are hundreds or thousands of times greater than that of a vacuum and are classified as *ferromagnetic*. A ferromagnetic core provides a better path for the magnetic lines of force and thus permits a stronger magnetic field. Examples of nonmagnetic materials are air, copper, plastic, and glass. The permeabilities of these materials are the same as for a vacuum.

The permeability (μ) of the core material determines how easily a magnetic field can be established, and is measured in Wb/At · m, which is the same as H/m. The inductance is directly proportional to the permeability of the core material.

Physical Parameters As indicated in Figure 4, the number of turns of wire, the length, and the cross-sectional area of the core are factors in setting the value of inductance. The inductance is inversely proportional to the length of the core and directly proportional to the cross-sectional area. Also, the inductance is directly related to the number of turns squared. This relationship is as follows:

$$L = \frac{N^2 \mu A}{l}$$

Equation 3

where L is the inductance in henries (H), N is the number of turns of wire, μ is the permeability in henries per meter (H/m), A is the cross-sectional area in meters squared, and l is the core length in meters (m).

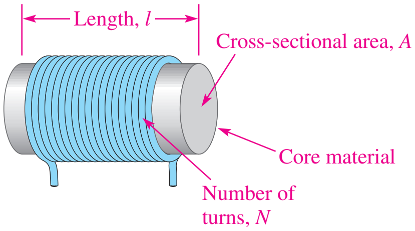


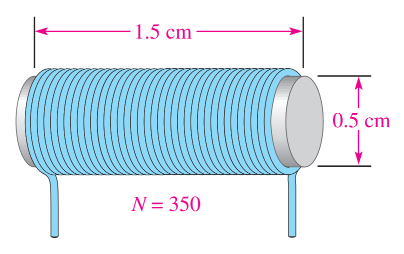
FIGURE 4

Physical parameters of an inductor.

EXAMPLE 2

Determine the inductance of the coil in Figure 5. The permeability of the core is 0.25×10^{-3} H/m.

FIGURE 5



Solution First determine the length and area in meters.

$$l = 1.5\text{ cm} = 0.015\text{ m}$$
$$A = \pi r^2 = \pi(0.25 \times 10^{-2}\text{ m})^2 = 1.96 \times 10^{-5}\text{ m}^2$$

The inductance of the coil is

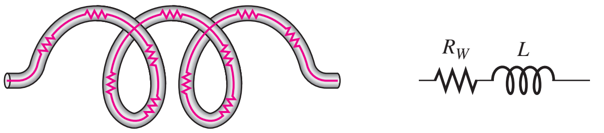
$$L = \frac{N^2 \mu A}{l} = \frac{(350)^2(0.25 \times 10^{-3}\ \text{H/m})(1.96 \times 10^{-5}\ \text{m}^2)}{0.015\ \text{m}} = \mathbf{40\ mH}$$

Related Problem Determine the inductance of a coil with 90 turns around a core that is 1.0 cm long and has a diameter of 0.8 cm. The permeability is 0.25×10^{-3} H/m.

Winding Resistance

When a coil is made of a certain material, for example, insulated copper wire, that wire has a certain resistance per unit of length. When many turns of wire are used to construct a coil, the total resistance may be significant. This inherent resistance is called the *dc resistance* or the *winding resistance* (R_W).

Although this resistance is distributed along the length of the wire, it effectively appears in series with the inductance of the coil, as shown in Figure 6. In many applications, the winding resistance may be small enough to be ignored and the coil can be considered an ideal inductor. In other cases, the resistance must be considered.



(a) The wire has resistance distributed along its length.

(b) Equivalent circuit

▲ **FIGURE 6**

Winding resistance of a coil.

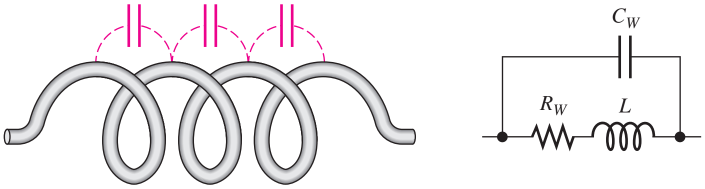
SAFETY NOTE

Be careful when working with inductors because high induced voltages can be developed due to a rapidly changing magnetic field. This occurs when the current is interrupted or its value abruptly changed.

Winding Capacitance

When two conductors are placed side by side, there is always some capacitance between them. Thus, when many turns of wire are placed close together in a coil, a certain amount of stray capacitance, called *winding capacitance* (C_W), is a natural side effect. In many applications, this winding capacitance is very small and has no significant effect. In other cases, particularly at high frequencies, it may become quite important.

The equivalent circuit for an inductor with both its winding resistance (R_W) and its winding capacitance (C_W) is shown in Figure 7. The capacitance effectively acts in parallel. The total of the stray capacitances between each loop of the wind ing is indicated in a schematic as a capacitance appearing in parallel with the coil and its winding resistance, as shown in Figure 7(b).



(a) Stray capacitance between each loop appears as a total parallel capacitance (C_W).

(b) Equivalent circuit

▲ **FIGURE 7**

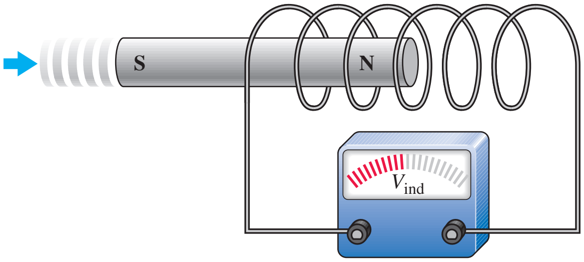
Winding capacitance of a coil.

Review of Faraday's Law

Faraday's law is reviewed here because of its importance in the study of inductors. Michael Faraday found that by moving a magnet through a coil of wire, a voltage was induced across the coil and that when a complete path was provided, the induced voltage caused an induced current. Faraday observed that

> **The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil.**

This principle is illustrated in Figure 8, where a bar magnet is moved through a coil of wire. An induced voltage is indicated by the voltmeter connected across the coil. The faster the magnet is moved, the greater the induced voltage.



◀ FIGURE 8

Induced voltage created by a changing magnetic field.

When a wire is formed into a certain number of loops or turns and is exposed to a changing magnetic field, a voltage is induced across the coil. The induced voltage is proportional to the number of turns of wire in the coil, N, and to the rate at which the magnetic field changes. The rate of change of the magnetic field is designated $d\phi/dt$, where ϕ is the magnetic flux. The ratio $d\phi/dt$ is expressed in webers/second (Wb/s). Faraday's law states that the induced voltage across a coil is equal to the number of turns (loops) times the rate of flux change. It is stated here.

$$v_{ind} = N\left(\frac{d\phi}{dt}\right)$$

EXAMPLE 3

Apply Faraday's law to find the induced voltage across a coil with 500 turns located in a magnetic field that is changing at a rate of 5 Wb/s.

Solution

$$v_{ind} = N\left(\frac{d\phi}{dt}\right) = (500\text{ t})(5\text{ Wb/s}) = \mathbf{2.5\text{ kV}}$$

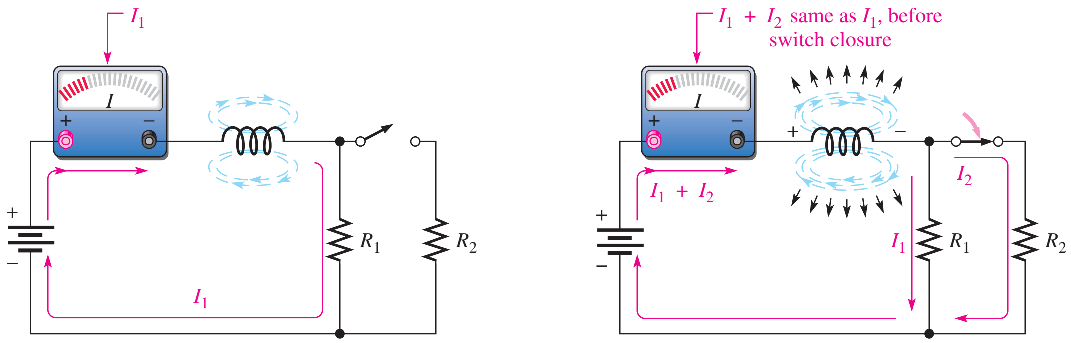
Related Problem A 1000 turn coil has an induced voltage of 500 V across it. What is the rate of change of the magnetic field?

Lenz's Law

Lenz's law is stated here.

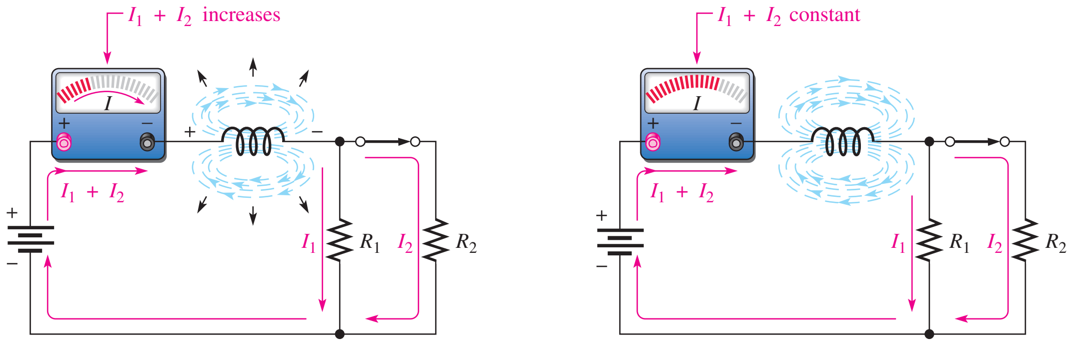
> **When the current through a coil changes, an induced voltage is created as a result of the changing electromagnetic field and the polarity of the induced voltage is such that it always opposes the change in current.**

Figure 9 illustrates Lenz's law. In part (a), the current is constant and is limited by R_1. There is no induced voltage because the electromagnetic field is unchanging. In part (b), the


I_1
I
+
−
R_1
R_2
I_1
$I_1 + I_2$ same as I_1, before switch closure
$I_1 + I_2$
I_2
I_1

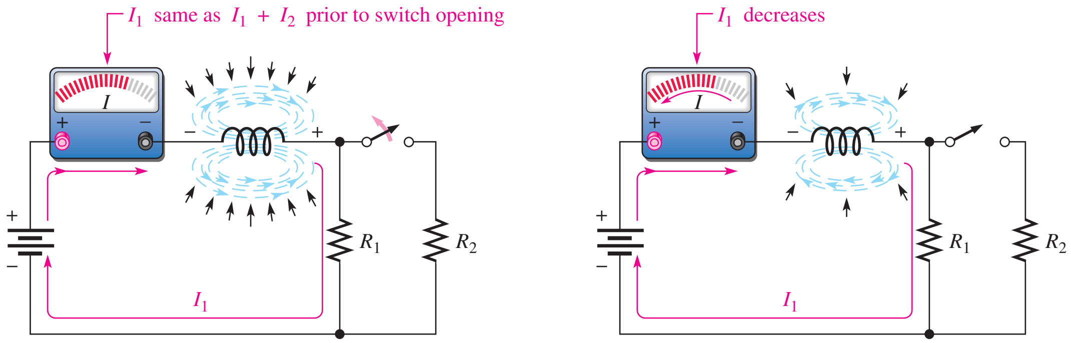
(a) Switch open: Constant current and constant magnetic field; no induced voltage.

(b) At instant of switch closure: Expanding magnetic field induces voltage, which opposes an increase in total current. The total current remains the same at this instant.


$I_1 + I_2$ increases
I
+
−
$I_1 + I_2$
I_1
R_1
I_2
R_2
$I_1 + I_2$ constant
$I_1 + I_2$
I_1
I_2

(c) Right after switch closure: The rate of expansion of the magnetic field decreases, allowing the current to increase exponentially as induced voltage decreases.

(d) Switch remains closed: Current and magnetic field reach constant value.


I_1 same as $I_1 + I_2$ prior to switch opening
I
+
−
R_1
R_2
I_1
I_1 decreases

(e) At instant of switch opening: Magnetic field begins to collapse, creating an induced voltage, which opposes a decrease in current.

(f) After switch opening: Rate of collapse of magnetic field decreases, allowing current to decrease exponentially back to original value.

FIGURE 9

Demonstration of Lenz's law in an inductive circuit: When the current tries to change suddenly, the electromagnetic field changes and induces a voltage in a direction that opposes that change in current.

switch suddenly is closed, placing R_2 in parallel with R_1 and thus reducing the resistance. Naturally, the current tries to increase and the electromagnetic field begins to expand, but the induced voltage opposes this attempted increase in current for an instant.

In Figure 9(c), the induced voltage gradually decreases, allowing the current to increase. In part (d), the current has reached a constant value as determined by the parallel resistors, and the induced voltage is zero. In part (e), the switch has been suddenly opened, and, for an instant, the induced voltage prevents any decrease in current, and arcing between the switch contacts results. In part (f), the induced voltage gradually decreases, allowing the current to decrease back to a value determined by R_1. Notice that the induced voltage has a polarity that opposes any current change. The polarity of the induced voltage is opposite that of the battery voltage for an increase in current and aids the battery voltage for a decrease in current.

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. List the parameters that contribute to the inductance of a coil.
2. The current through a 15 mH inductor is changing at the rate of 500 mA/s. What is the induced voltage?
3. Describe what happens to L when
   (a) N is increased
   (b) The core length is increased
   (c) The cross-sectional area of the core is decreased
   (d) A ferromagnetic core is replaced by an air core
4. Explain why inductors have some winding resistance.
5. Explain why inductors have some winding capacitance.

2 TYPES OF INDUCTORS

Inductors normally are classified according to the type of core material.

After completing this section, you should be able to

- **Discuss various types of inductors**
  - Describe the basic types of fixed inductors
  - Distinguish between fixed and variable inductors

Inductors are made in a variety of shapes and sizes. Basically, they fall into two general categories: fixed and variable. The standard schematic symbols are shown in Figure 10.

Both fixed and variable inductors can be classified according to the type of core material. Three common types are the air core, the iron core, and the ferrite core. Each has a unique symbol, as shown in Figure 11.

Adjustable (variable) inductors usually have a screw-type adjustment that moves a sliding core in and out, thus changing the inductance. A wide variety of inductors exist, and some are shown in Figure 12. Small fixed inductors are frequently encapsulated in an insulating material that protects the fine wire in the coil. Encapsulated inductors have an appearance similar to a resistor.

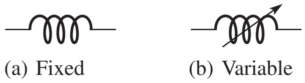

(a) Fixed (b) Variable

FIGURE 10

Symbols for fixed and variable inductors.

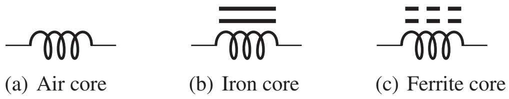
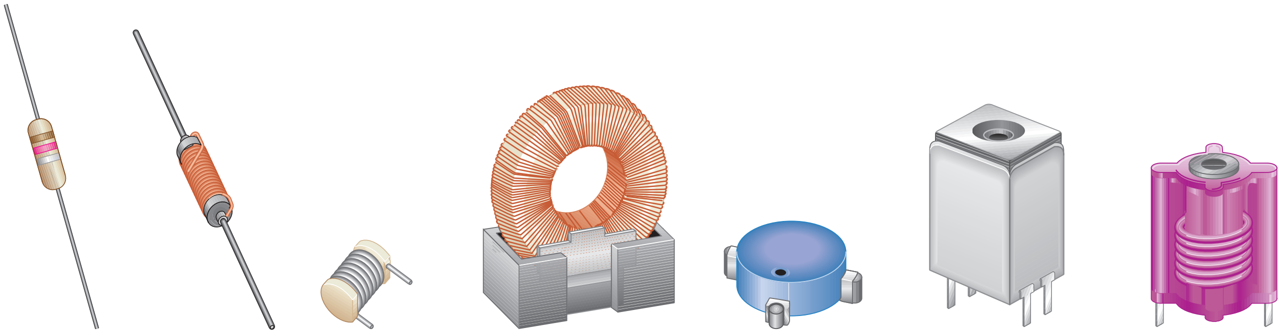

(a) Air core (b) Iron core (c) Ferrite core

FIGURE 11

Inductor symbols.



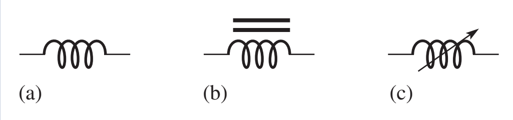
▲ FIGURE 12

Typical inductors.

SECTION 2 CHECKUP

1. Name two general categories of inductors.
2. Identify the inductor symbols in Figure 13.

▶ FIGURE 13



3 SERIES AND PARALLEL INDUCTORS

When inductors are connected in series, the total inductance increases. When inductors are connected in parallel, the total inductance decreases.

After completing this section, you should be able to

- **Analyze series and parallel inductors**
  - Determine total series inductance
  - Determine total parallel inductance

Total Series Inductance

When inductors are connected in series, as in Figure 14, the total inductance, L_T, is the sum of the individual inductances. The formula for L_T is expressed in the following equation for the general case of n inductors in series:

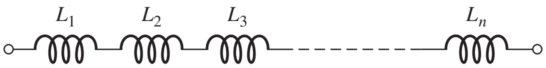
Equation 4

$$L_T = L_1 + L_2 + L_3 + \cdots + L_n$$

Notice that the calculation of total inductance in series is analogous to the calculations of total resistance in series and total capacitance in parallel.

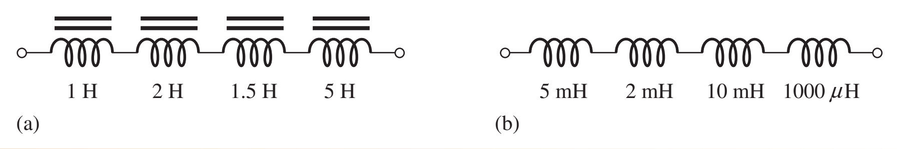
▶ FIGURE 14

Inductors in series.



EXAMPLE 4

Determine the total inductance for each of the series connections in Figure 15.



▲ FIGURE 15

Solution In Figure 15(a),

$$L_T = 1\text{ H} + 2\text{ H} + 1.5\text{ H} + 5\text{ H} = \mathbf{9.5\text{ H}}$$

In Figure 15(b),

$$L_T = 5\text{ mH} + 2\text{ mH} + 10\text{ mH} + 1\text{ mH} = \mathbf{18\text{ mH}}$$

Note: $1000\ \mu\text{H} = 1\text{ mH}$

Related Problem What is the total inductance of three 50 μH inductors in series?

Total Parallel Inductance

When inductors are connected in parallel, as in Figure 16, the total inductance is less than the smallest inductance. The general formula states that the reciprocal of the total inductance is equal to the sum of the reciprocals of the individual inductances.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

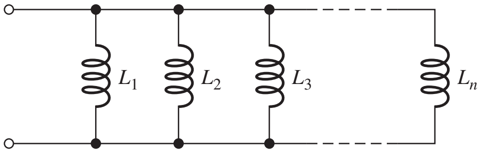
Equation 5

You can calculate total inductance, L_T, by taking the reciprocal of both sides of Equation 5.

$$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right) + \cdots + \left(\frac{1}{L_n}\right)}$$

Equation 6

The calculation for total inductance in parallel is analogous to the calculations for total parallel resistance and total series capacitance. For series-parallel combinations of inductors, determine the total inductance in the same way as total resistance in series-parallel resistive circuits.

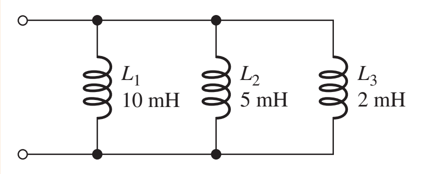


◀ FIGURE 16

Inductors in parallel.

EXAMPLE 5 Determine L_T in Figure 17.

▶ FIGURE 17



Solution Use Equation 6 to determine the total inductance.

$$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right)} = \frac{1}{\frac{1}{10\text{ mH}} + \frac{1}{5\text{ mH}} + \frac{1}{2\text{ mH}}} = \mathbf{1.25\text{ mH}}$$

Related Problem Determine L_T for a parallel connection of 50 μH, 80 μH, 100 μH, and 150 μH.

SECTION 3 CHECKUP

1. State the rule for combining inductors in series.
2. What is L_T for a series connection of 100 μH, 500 μH, and 2 mH?
3. Five 100 mH coils are connected in series. What is the total inductance?
4. Compare the total inductance in parallel with the smallest-value individual inductor.
5. The calculation of total parallel inductance is analogous to that for total parallel resistance. (T or F)
6. Determine L_T for each parallel combination:
   (a) 40 μH and 60 μH
   (b) 100 mH, 50 mH, and 10 mH

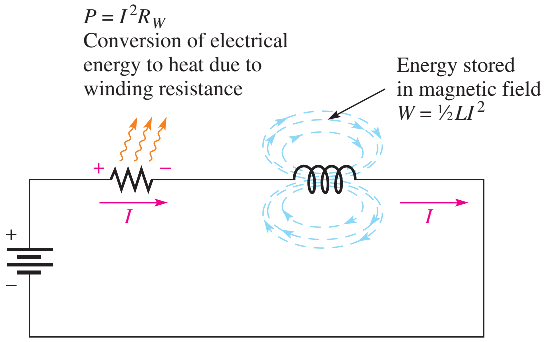
4 INDUCTORS IN DC CIRCUITS

Energy is stored in the electromagnetic field of an inductor when it is connected to a dc voltage source. The buildup of current through the inductor occurs in a predictable manner, which is dependent on the time constant of the circuit. The time constant is determined by the inductance and the resistance in a circuit.

After completing this section, you should be able to

- **Analyze inductive dc switching circuits**
  - Describe the increase and decrease of current an inductor
  - Define *RL time constant*
  - Describe induced voltage
  - Write the exponential equations for current in an inductor

When there is constant direct current in an inductor, there is no induced voltage. There is, however, a voltage drop due to the winding resistance of the coil. The inductance itself appears as a short to dc. Energy is stored in the electromagnetic field according to the formula previously stated in Equation 2, $W = \frac{1}{2}LI^2$. The only energy conversion to heat occurs in the winding resistance ($P = I^2R_W$). This condition is illustrated in Figure 18.


$P = I^2R_W$
Conversion of electrical energy to heat due to winding resistance
Energy stored in magnetic field $W = \frac{1}{2}LI^2$
+
−
I
I
+
−

◀ FIGURE 18

Energy storage and conversion to heat in an inductor in a dc circuit.

The *RL* Time Constant

Because the inductor's basic action is to develop a voltage that opposes a change in its current, it follows that current cannot change instantaneously in an inductor. A certain time is required for the current to make a change from one value to another. The rate at which the current changes is determined by the *RL* time constant.

The *RL* time constant is a fixed time interval that equals the ratio of the inductance to the resistance.

The formula is

$$\tau = \frac{L}{R}$$

Equation 7

where τ is in seconds when inductance (L) is in henries and resistance (R) is in ohms.

EXAMPLE 6

A series *RL* circuit has a resistance of 1.0 kΩ and an inductance of 1 mH. What is the time constant?

Solution

$$\tau = \frac{L}{R} = \frac{1\text{ mH}}{1.0\text{ k}\Omega} = \frac{1 \times 10^{-3}\text{ H}}{1 \times 10^{3}\ \Omega} = 1 \times 10^{-6}\text{ s} = \mathbf{1\ \mu s}$$

Related Problem Find the time constant for $R = 2.2\text{ k}\Omega$ and $L = 500\ \mu\text{H}$.

Current in an Inductor

Increasing Current In a series *RL* circuit, the current will increase to approximately 63% of its full value in one time-constant interval after voltage is applied. This buildup of current is analogous to the buildup of capacitor voltage during the charging in an *RC* circuit; they both follow an exponential curve and reach the approximate percentages of the final current as indicated in Table 1 and as illustrated in Figure 19.

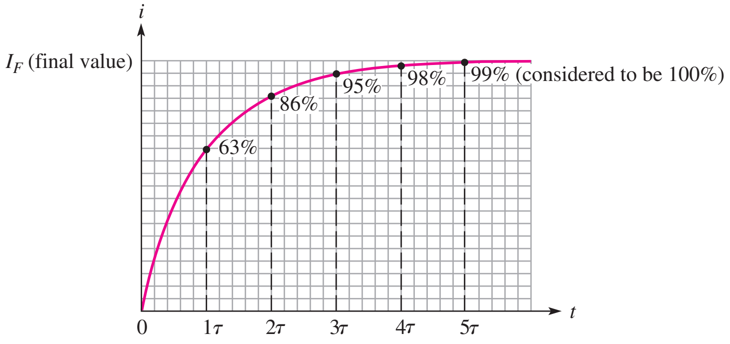
The change in current over five time-constant intervals is illustrated in Figure 20. When the current reaches its final value at 5τ, it ceases to change. At this time, the inductor acts as a short (except for winding resistance) to the constant current. The final value of the current is

$$I_F = \frac{V_S}{R} = \frac{10\text{ V}}{1.0\text{ k}\Omega} = 10\text{ mA}$$

▶ TABLE 1

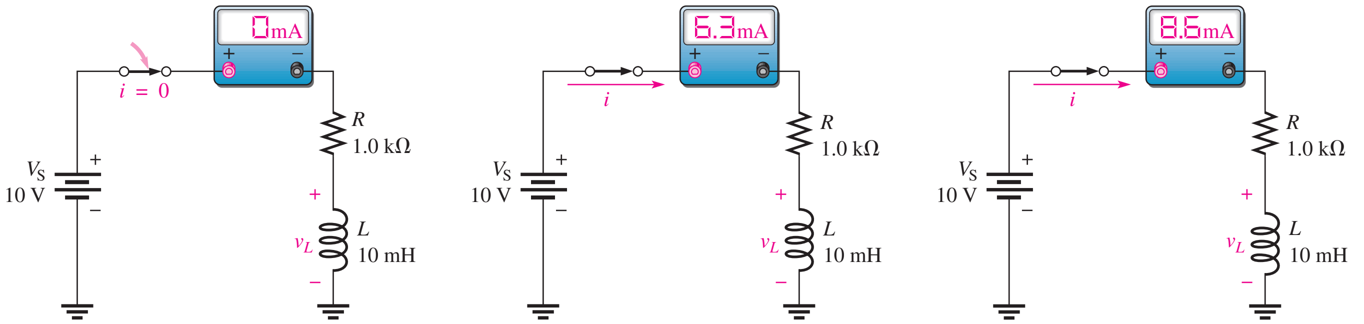
Percentage of the final current after each time-constant interval during current buildup.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF FINAL CURRENT
1	63
2	86
3	95
4	98
5	99 (considered 100%)



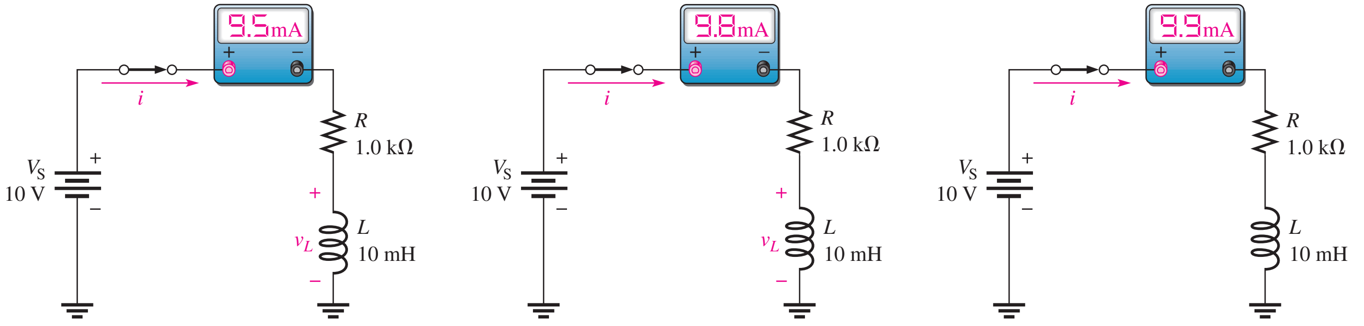
▲ FIGURE 19

Increasing current in an inductor.



0mA
6.3mA
8.6mA
i = 0
i
V_S
10 V
R
1.0 kΩ
L
10 mH
v_L
+
−

(a) Initially ($t = 0$) (b) At $t = 1\tau$ (c) At $t = 2\tau$



9.5mA
9.8mA
9.9mA
i
V_S
10 V
R
1.0 kΩ
L
10 mH
v_L
+
−

(d) At $t = 3\tau$ (e) At $t = 4\tau$ (f) At $t = 5\tau$

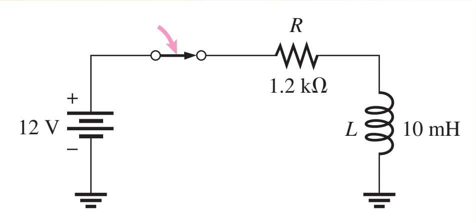
▲ FIGURE 20

Illustration of the exponential buildup of current in an inductor. The current increases approximately 63% during each time-constant interval after the switch is closed. A voltage (v_L) is induced in the coil that tends to oppose the increase in current.

EXAMPLE 7

Calculate the *RL* time constant for Figure 21. Then determine the current and the time at each time-constant interval, measured from the instant the switch is closed.

▶ **FIGURE 21**



Solution The *RL* time constant is

$$\tau = \frac{L}{R} = \frac{10\text{ mH}}{1.2\text{ k}\Omega} = \mathbf{8.33\ \mu s}$$

The current at each time-constant interval is a certain percentage of the final current. The final current is

$$I_F = \frac{V_S}{R} = \frac{12\text{ V}}{1.2\text{ k}\Omega} = 10\text{ mA}$$

Using the time-constant percentage values from Figure 1,

$$\text{At } 1\tau\text{: } i = 0.63(10\text{ mA}) = \mathbf{6.3\ mA};\ t = \mathbf{8.33\ \mu s}$$

$$\text{At } 2\tau\text{: } i = 0.86(10\text{ mA}) = \mathbf{8.6\ mA};\ t = \mathbf{16.7\ \mu s}$$

$$\text{At } 3\tau\text{: } i = 0.95(10\text{ mA}) = \mathbf{9.5\ mA};\ t = \mathbf{25.0\ \mu s}$$

$$\text{At } 4\tau\text{: } i = 0.98(10\text{ mA}) = \mathbf{9.8\ mA};\ t = \mathbf{33.3\ \mu s}$$

$$\text{At } 5\tau\text{: } i = 0.99(10\text{ mA}) = 9.9\text{ mA} \cong \mathbf{10\ mA};\ t = \mathbf{41.7\ \mu s}$$

Related Problem Repeat the calculations if R is 680 Ω and L is 100 μH.

Use Multisim file E13-07 to verify the calculated results in this example and to confirm your calculation for the related problem. Use a square wave to replace the dc voltage source and the switch.

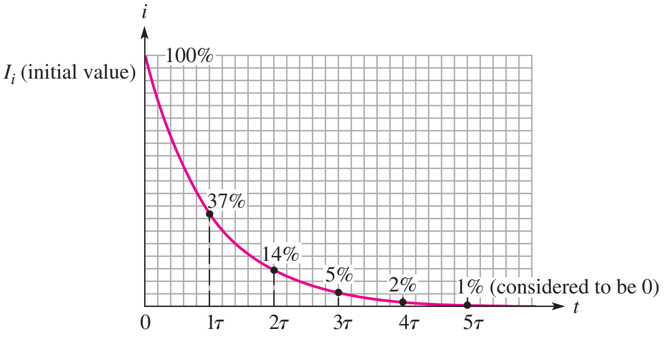
Decreasing Current Current in an inductor decreases exponentially according to the approximate percentage values in Table 2 and in Figure 22.

The change in current over five time-constant intervals is illustrated in Figure 23. When the current reaches its final value of approximately 0 A, it ceases to change. Before

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF INITIAL CURRENT
1	37
2	14
3	5
4	2
5	1 (considered 0)

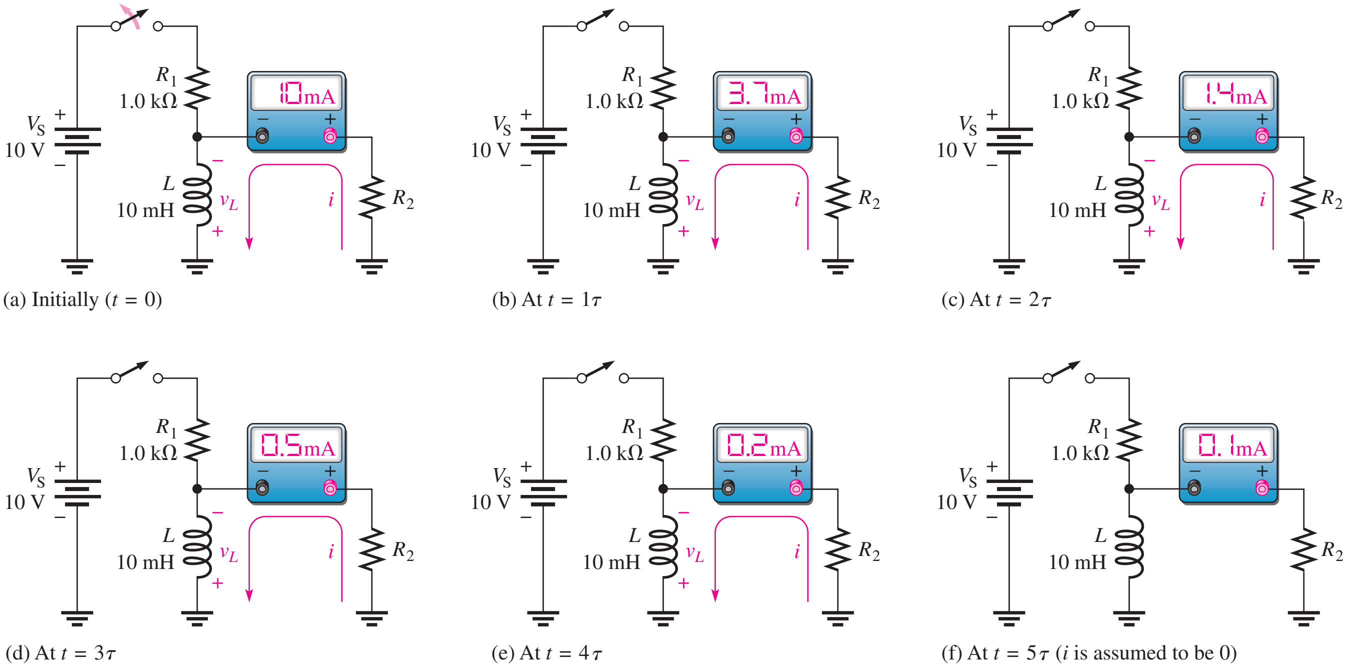
◀ **TABLE 2**

Percentage of initial current after each time-constant interval while current is decreasing.



▲ **FIGURE 22**

Decreasing current in an inductor.



(a) Initially ($t = 0$)

(b) At $t = 1\tau$

(c) At $t = 2\tau$

(d) At $t = 3\tau$

(e) At $t = 4\tau$

(f) At $t = 5\tau$ (i is assumed to be 0)

▲ **FIGURE 23**

Illustration of the exponential decrease of current in an inductor. The current decreases approximately 63% during each time-constant interval after the switch is opened. A voltage (v_L) is induced in the coil that tends to oppose the decrease in current.

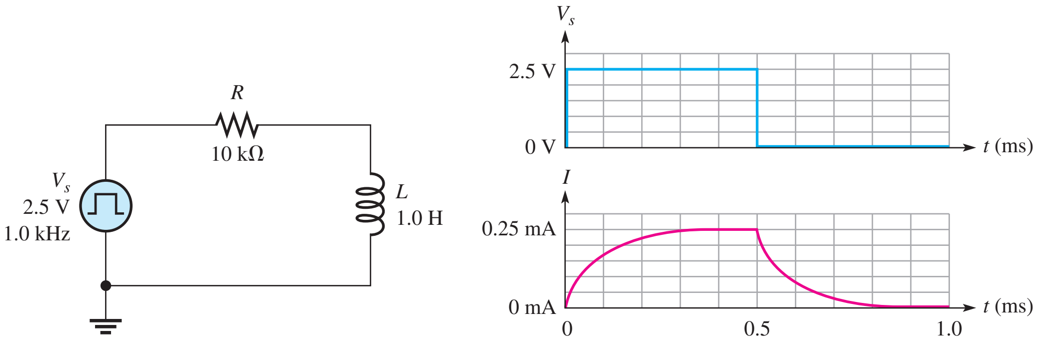
the switch is opened, the current through L is at a constant value of 10 mA, which is determined by R_1 because L acts ideally as a short. When the switch is opened, the induced inductor voltage initially provides 10 mA through R_2. The current then decreases by 63% during each time constant interval.

Response to a Square Wave

A good way to demonstrate both increasing and decreasing current in an *RL* circuit is to use a square wave voltage as the input. The square wave is a useful signal for observing the dc response of a circuit because it automatically provides on and off action similar to a switch. When the square wave goes from its low level to its high level, the current in the circuit responds by exponentially rising to its final value. When the square wave returns to the zero level, the current in the circuit responds by exponentially decreasing to its zero value. Figure 24 shows input voltage and current waveforms.

TECH NOTE

To measure the current waveform in a series *RL* circuit, you can measure the voltage across the resistor and apply Ohm's law. If the resistor is ungrounded, as in Figure 24, you can use a difference measurement to view the resistor voltage by connecting one oscilloscope probe across each end of the resistor and selecting ADD and Invert on the oscilloscope. Both channels on the oscilloscope should be set to the same VOLTS/DIV setting. Alternatively, you can reverse the components (see the Multisim problem in Example 8).



▲ **FIGURE 24**

EXAMPLE 8 For the circuit in Figure 24, what is the current at 0.1 ms and 0.6 ms?

Solution The *RL* time constant for the circuit is

$$\tau = \frac{L}{R} = \frac{1.0\text{ H}}{10\text{ k}\Omega} = 0.1\text{ ms}$$

If the square-wave generator period is long enough for the current to reach its maximum value in 5τ, the current will increase exponentially and during each time constant interval will have a value equal to the percentage of the final current given in Table 1. The final current is

$$I_F = \frac{V_S}{R} = \frac{2.5\text{ V}}{10\text{ k}\Omega} = 0.25\text{ mA}$$

The current at 0.1 ms is

$$i = 0.63(0.25\text{ mA}) = \mathbf{0.158\text{ mA}}$$

At 0.6 ms, the square-wave input has been at the 0 V level for 0.1 ms, or 1τ; and the current decreases from the maximum value toward its final value of 0 mA by 63%. Therefore,

$$i = 0.25\text{ mA} - 0.63(0.25\text{ mA}) = \mathbf{0.092\text{ mA}}$$

Related Problem What is the current at 0.2 ms and 0.8 ms?

Open the Multisim file E13-08. Notice that the inductor and resistor are reversed in order to ground one side of the resistor and make the measurement of the resistor voltage simpler. The shape of the current in the circuit is the same as the shape of the resistor voltage. By applying Ohm's law to the resistor voltage, you can find the current in the circuit at any instant in time. Confirm that the current at a time 0.1 ms is close to the calculated value.

Voltages in a Series *RL* Circuit

As you know, when current changes in an inductor, a voltage is induced. Let's examine what happens to the induced voltage across the inductor in the series circuit in Figure 25 during one complete cycle of a square wave input. Keep in mind that the generator produces a level that is like switching a dc source on and then puts an "automatic" low resistance (ideally zero) path across the source when it returns to its zero level.

An ammeter placed in the circuit shows the current in the circuit at any instant in time. The V_L waveform is the voltage across the inductor. In Figure 25(a), the square wave

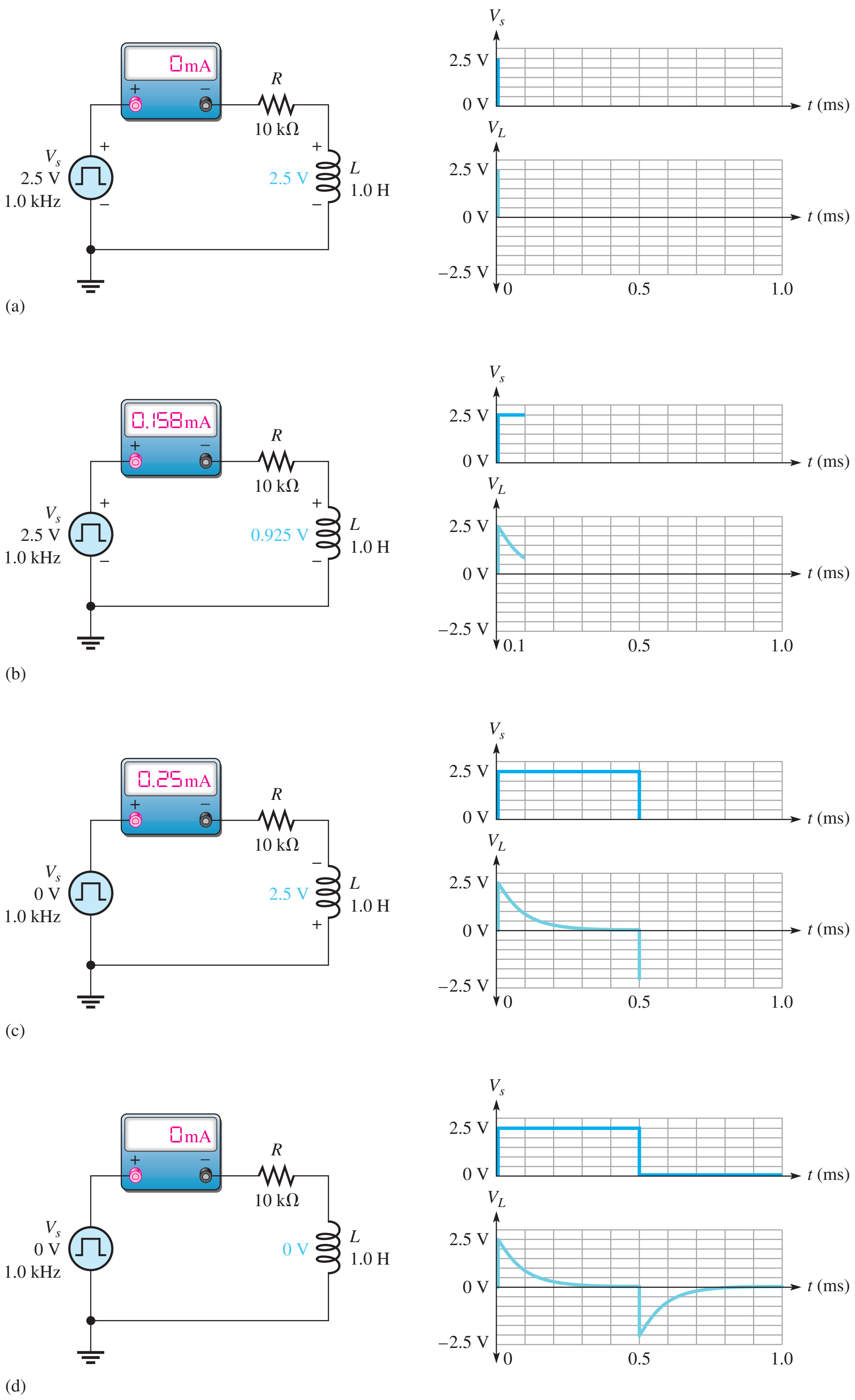

0mA
R
10 kΩ
V_s
2.5 V
1.0 kHz
2.5 V
L
1.0 H
(a)
0.158mA
0.925 V
(b)
0.25mA
0 V
(c)
0mA
0 V
(d)
V_s
V_L
2.5 V
0 V
−2.5 V
t (ms)
0
0.1
0.5
1.0

FIGURE 25

has just transitioned from zero to its maximum value of 2.5 V. In accordance with Lenz's law, a voltage is induced across the inductor that opposes this *change* as the magnetic field surrounding the inductor builds up. There is no current in the circuit due to the equal but opposing voltages.

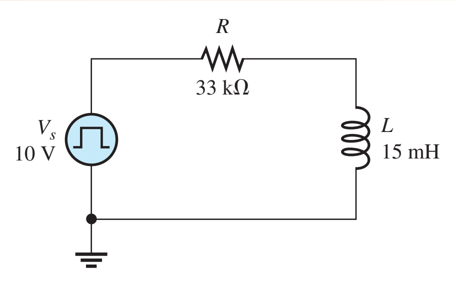
As the magnetic field builds up, the induced voltage across the inductor decreases, and current is in the circuit. After 1τ, the induced voltage across the inductor has decreased by 63%, which causes the current to increase by 63% to 0.158 mA. This is shown in Figure 25(b) at the end of one time constant (0.1 ms).

The voltage on the inductor continues to exponentially decrease to zero, at which point the current is limited only by the circuit resistance. Then the square wave goes back to zero (at $t = 0.5$ ms) as shown in Figure 25(c). Again a voltage is induced across the inductor opposing this change. This time, the polarity of the inductor voltage is reversed due to the collapsing magnetic field. Although the source voltage is 0, the collapsing magnetic field maintains current in the same direction until the current decreases to zero, as shown in Figure 25(d).

EXAMPLE 9

(a) The circuit in Figure 26 has a square wave input. What is the highest frequency that can be used and still observe the complete waveform across the inductor?

(b) Assume the generator is set to the frequency determined in (a). Describe the voltage waveform across the resistor?



▲ FIGURE 26

Solution

(a) $$\tau = \frac{L}{R} = \frac{15\text{ mH}}{33\text{ k}\Omega} = 0.454\ \mu\text{s}$$

The period needs to be 10 times longer than τ to observe the entire wave.

$$T = 10\tau = 4.54\ \mu\text{s}$$

$$f = \frac{1}{T} = \frac{1}{4.54\ \mu\text{s}} = \mathbf{220\ kHz}$$

(b) The voltage across the resistor has the same shape as the current waveform. The general shape was shown in Figure 24 and has a maximum value of 10 V (the same V_s assuming no winding resistance).

Related Problem What is the maximum voltage across the resistor for $f = 220$ kHz?

Use Multisim file E13-09 to verify the calculated results in this example and to confirm your calculation for the related problem.

The Exponential Formulas

The formulas for the exponential current and voltage in an *RL* circuit are similar to those for the *RC* circuit, and universal exponential curves apply to inductors as well as capacitors. The general formulas for *RL* circuits are stated as follows:

Equation 8

$$v = V_F + (V_i - V_F)e^{-Rt/L}$$

Equation 9

$$i = I_F + (I_i - I_F)e^{-Rt/L}$$

where V_F and I_F are the final values of voltage and current, V_i and I_i are the initial values of voltage and current. The lowercase italic letters v and i are the instantaneous values of the inductor voltage and current at time t.

Increasing Current The formula for the special case in which an increasing exponential current curve begins at zero is derived by setting $I_i = 0$ in Equation 9.

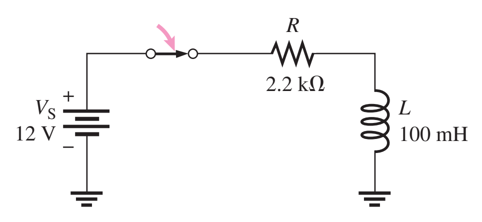
Equation 10

$$i = I_F(1 - e^{-Rt/L})$$

Using Equation 10, you can calculate the value of the increasing inductor current at any instant of time. You can calculate voltage by substituting v for i and V_F for I_F in Equation 10. Notice that the exponent Rt/L can also be written as $t/(L/R) = t/\tau$.

EXAMPLE 10

In Figure 27, determine the inductor current 30 μs after the switch is closed.



▲ FIGURE 27

Solution The *RL* time constant is

$$\tau = \frac{L}{R} = \frac{100\text{ mH}}{2.2\text{ k}\Omega} = 45.5\ \mu\text{s}$$

The final current is

$$I_F = \frac{V_S}{R} = \frac{12\text{ V}}{2.2\text{ k}\Omega} = 5.45\text{ mA}$$

The initial current is zero. Notice that 30 μs is less than one time constant, so the current will reach less than 63% of its final value in that time.

$$i_L = I_F(1 - e^{-Rt/L}) = 5.45\text{ mA}(1 - e^{-0.66}) = 5.45\text{ mA}(1 - 0.517) = \mathbf{2.63\text{ mA}}$$

Related Problem In Figure 27, determine the inductor current 55 μs after the switch is closed.

Decreasing Current The formula for the special case in which a decreasing exponential current has a final value of zero is derived by setting $I_F = 0$ in Equation 9.

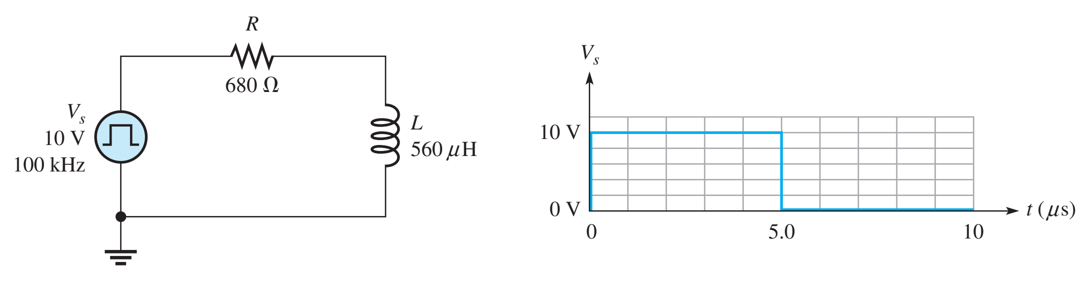
$$i = I_i e^{-Rt/L}$$

Equation 11

This formula can be used to calculate the decreasing inductor current at any instant, as the next example shows.

EXAMPLE 11

In Figure 28, what is the current at each microsecond interval for one complete cycle of the input square wave, V_s After calculating the current at each time, sketch the current waveform.



▲ FIGURE 28

Solution

$$\tau = \frac{L}{R} = \frac{560\ \mu\text{H}}{680\ \Omega} = 0.824\ \mu\text{s}$$

When the pulse goes from 0 V to 10 V at $t = 0$, the final current is

$$I_F = \frac{V_s}{R} = \frac{10\ \text{V}}{680\ \Omega} = 14.7\ \text{mA}$$

For the increasing current,

$$i = I_F(1 - e^{-Rt/L}) = I_F(1 - e^{-t/\tau})$$

At 1 μs: $i = 14.7\ \text{mA}(1 - e^{-1\mu\text{s}/0.824\mu\text{s}}) = \mathbf{10.3\ mA}$

At 2 μs: $i = 14.7\ \text{mA}(1 - e^{-2\mu\text{s}/0.824\mu\text{s}}) = \mathbf{13.4\ mA}$

At 3 μs: $i = 14.7\ \text{mA}(1 - e^{-3\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.3\ mA}$

At 4 μs: $i = 14.7\ \text{mA}(1 - e^{-4\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.6\ mA}$

At 5 μs: $i = 14.7\ \text{mA}(1 - e^{-5\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.7\ mA}$

When the pulse goes from 10 V to 0 V at $t = 5$ μs, the current decreases exponentially. For the decreasing current,

$$i = I_i(e^{-Rt/L}) = I_i(e^{-t/\tau})$$

The initial current is the value at 5 μs, which is 14.7 mA.

At 6 μs: $i = 14.7\ \text{mA}(e^{-1\mu\text{s}/0.824\mu\text{s}}) = \mathbf{4.37\ mA}$

At 7 μs: $i = 14.7\ \text{mA}(e^{-2\mu\text{s}/0.824\mu\text{s}}) = \mathbf{1.30\ mA}$

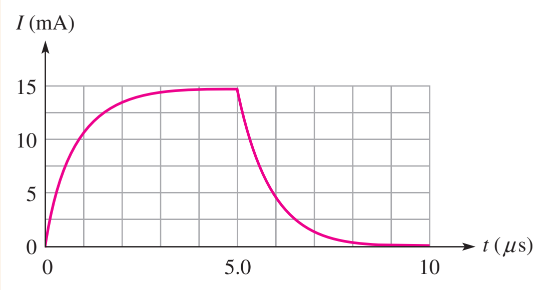
At 8 μs: $i = 14.7\ \text{mA}(e^{-3\mu\text{s}/0.824\mu\text{s}}) = \mathbf{0.38\ mA}$

At 9 μs: $i = 14.7\ \text{mA}(e^{-4\mu\text{s}/0.824\mu\text{s}}) = \mathbf{0.11\ mA}$

At 10 μs: $i = 14.7\ \text{mA}(e^{-5\ \mu\text{s}/0.824\ \mu\text{s}}) = \mathbf{0.03\ mA}$

Figure 29 is a graph of these results.

▶ **FIGURE 29**



Related Problem What is the current at 0.5 μs?

Use Multisim file E13-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 4 CHECKUP

1. A 15 mH inductor with a winding resistance of 10 Ω has a constant direct current of 10 mA through it. What is the voltage drop across the inductor?
2. A 20 V dc source is connected to a series *RL* circuit with a switch. At the instant of switch closure, what are the values of *i* and v_L?
3. In the same circuit as in Question 2, after a time interval equal to 5τ from switch closure, what is v_L?
4. In a series *RL* circuit where $R = 1.0\ \text{k}\Omega$ and $L = 500\ \mu\text{H}$, what is the time constant? Determine the current 0.25 μs after a switch connects 10 V across the circuit.

5 INDUCTORS IN AC CIRCUITS

An inductor passes ac with an amount of opposition called inductive reactance that depends on the frequency of the ac. The concept of the derivative should already be familiar to you; the expression for induced voltage in an inductor was stated earlier in Equation 1. These are used again in this section.

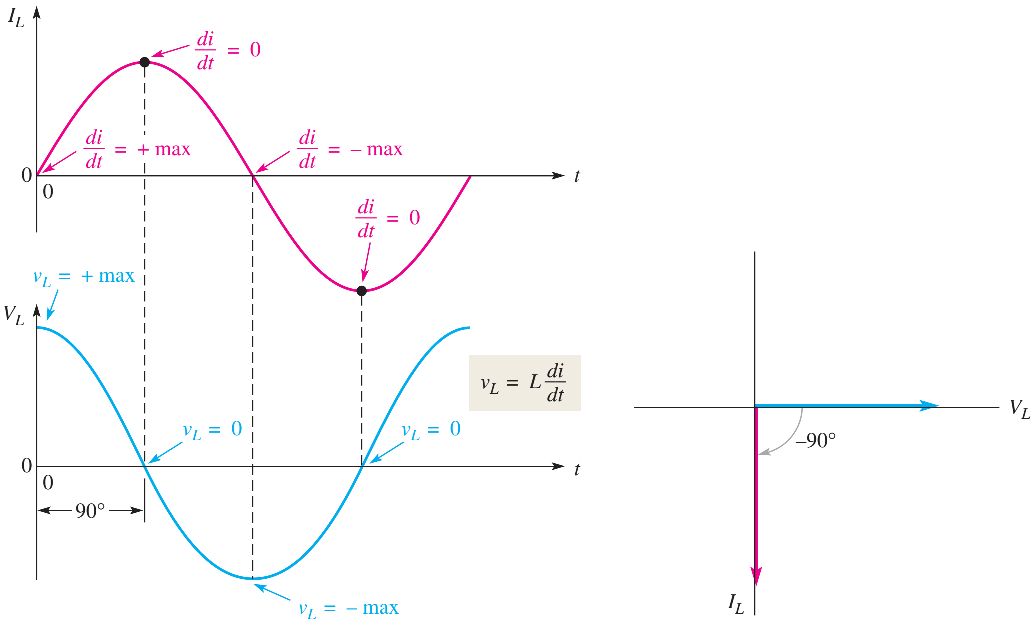
After completing this section, you should be able to

- **Analyze inductive ac circuits**
  - Explain why an inductor causes a phase shift between voltage and current
  - Define *inductive reactance*
  - Determine the value of inductive reactance in a given circuit
  - Discuss instantaneous, true, and reactive power in an inductor

Phase Relationship of Current and Voltage in an Inductor

From Equation 1, the formula for induced voltage, you can see that the faster the current through an inductor changes, the greater the induced voltage will be. For example, if the rate of change of current is zero, the voltage is zero [$v_{\text{ind}} = L(di/dt) = L(0) = 0\text{ V}$]. When di/dt is a positive-going maximum, v_{ind} is a positive maximum; when di/dt is a negative-going maximum, v_{ind} is a negative maximum.

A sinusoidal current always induces a sinusoidal voltage in inductive circuits. Therefore, you can plot the voltage with respect to the current if you know the points on the current curve at which the voltage is zero and those at which it is maximum. This phase relationship is shown in Figure 30(a). Notice that the voltage leads the current by 90°. This is always true in a purely inductive circuit. The current and voltage of this relationship is shown by the phasors in Figure 30(b).



(a)

(b)

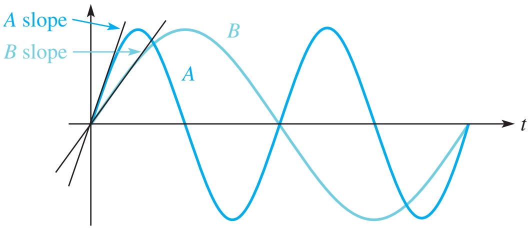
▲ FIGURE 30

Phase relation of V_L and I_L in an inductor. Current always lags the inductor voltage by 90°.

Inductive Reactance, X_L

Inductive reactance is the opposition to sinusoidal current, expressed in ohms. The symbol for inductive reactance is X_L.

To develop a formula for X_L, we use the relationship $v_{\text{ind}} = L(di/dt)$ and the curves in Figure 31. The rate of change of current is directly related to frequency. The faster the current changes, the higher the frequency. For example, you can see that in Figure 31, the slope of sine wave A at the zero crossings is steeper than that of sine wave B. Recall that the slope of a curve at a point indicates the rate of change at that point. Sine wave A has a higher frequency than sine wave B, as indicated by a greater maximum rate of change (di/dt is greater at the zero crossings).



◀ FIGURE 31

Slope indicates rate of change. Sine wave A has a greater rate of change at the zero crossing than B, and thus A has a higher frequency.

When frequency increases, *di*/*dt* increases, and thus v_{ind} increases. When frequency decreases, *di*/*dt* decreases, and thus v_{ind} decreases. The induced voltage is directly dependent on frequency.

$$\overset{\uparrow}{v_{\text{ind}}} = L(\overset{\uparrow}{di/dt}) \qquad \text{and} \qquad \underset{\downarrow}{v_{\text{ind}}} = L(\underset{\downarrow}{di/dt})$$

An increase in induced voltage means more opposition (X_L is greater). Therefore, X_L is directly proportional to induced voltage and thus directly proportional to frequency.

X_L is proportional to f.

Now, if *di*/*dt* is constant and the inductance is varied, an increase in L produces an increase in v_{ind}, and a decrease in L produces a decrease in v_{ind}.

$$\overset{\uparrow}{v_{\text{ind}}} = \overset{\uparrow}{L}(di/dt) \qquad \text{and} \qquad \underset{\downarrow}{v_{\text{ind}}} = \underset{\downarrow}{L}(di/dt)$$

Again, an increase in v_{ind} means more opposition (greater X_L). Therefore, X_L is directly proportional to induced voltage and thus directly proportional to inductance. The inductive reactance is directly proportional to both f and L.

X_L is proportional to fL.

The formula for inductive reactance, X_L, is

Equation 12

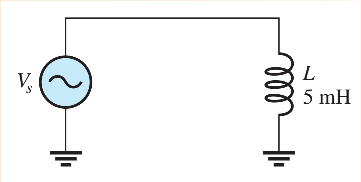
$$X_L = 2\pi fL$$

Inductive reactance, X_L, is in ohms when f is in hertz and L is in henries. As with capacitive reactance, the 2π term is a constant factor in the equation, which comes from the relationship of a sine wave to rotational motion.

EXAMPLE 12

A sinusoidal voltage is applied to the circuit in Figure 32. The frequency is 10 kHz. Determine the inductive reactance.

▶ **FIGURE 32**



Solution Convert 10 kHz to 10×10^3 Hz and 5 mH to 5×10^{-3} H. Therefore, the inductive reactance is

$$X_L = 2\pi fL = 2\pi(10 \times 10^3\ \text{Hz})(5 \times 10^{-3}\ \text{H}) = \mathbf{314\ \Omega}$$

Related Problem What is X_L in Figure 32 if the frequency is increased to 35 kHz?

Reactance for Series Inductors

As given in Equation 4, the total inductance of series inductors is the sum of the individual inductances. Because reactance is directly proportional to the inductance, the total reactance of series inductors is the sum of the individual reactances.

Equation 13

$$X_{L(tot)} = X_{L1} + X_{L2} + X_{L3} + \cdots + X_{Ln}$$

Notice that Equation 13 has the same form as Equation 4. It also has the same form as the formulas for finding the total opposition to current such as the total resistance of series resistors or the total reactance of series capacitors. When combining the resistance or reactance of the same type of component in series (resistors, inductors, or capacitors), you simply add the individual oppositions to obtain the total.

Reactance for Parallel Inductors

In an ac circuit with parallel inductors, Equation 6 was given to find the total inductance. It stated that the total inductance is the reciprocal of the sum of the reciprocals of the inductors. Likewise, the total inductive reactance is the reciprocal of the sum of the reciprocals of the individual reactances.

Equation 14

$$X_{L(tot)} = \frac{1}{\dfrac{1}{X_{L1}} + \dfrac{1}{X_{L2}} + \dfrac{1}{X_{L3}} + \cdots + \dfrac{1}{X_{Ln}}}$$

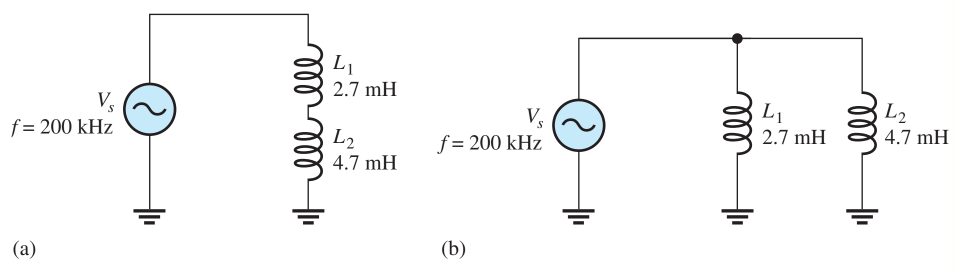
Notice that Equation 14 has the same form as Equation 6. It also has the same form as the formulas for finding the total resistance of parallel resistors or the total reactance of parallel capacitors. When combining the resistance or reactance of the same type of component in parallel (resistors, inductors, or capacitors), you take the reciprocal of the sum of the reciprocals to obtain the total opposition.

For two inductors in parallel, Equation 14 can be reduced to the product-over-sum form. Thus,

$$X_{L(tot)} = \frac{X_{L1}X_{L2}}{X_{L1} + X_{L2}}$$

EXAMPLE 13

What is the total inductive reactance of each circuit of Figure 33?



▲ FIGURE 33

Solution The reactances of the individual inductors are the same in both circuits. From Equation 12,

$$X_{L1} = 2\pi fL_1 = 2\pi(200\text{ kHz})(2.7\text{ mH}) = 3.39\text{ k}\Omega$$
$$X_{L2} = 2\pi fL_2 = 2\pi(200\text{ kHz})(4.7\text{ mH}) = 5.91\text{ k}\Omega$$

For the series inductors in Figure 33(a), the total reactance is the sum of X_{L1} and X_{L2}, as given in Equation 13.

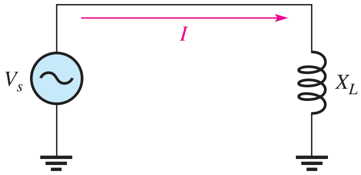
$$X_{L(tot)} = X_{L1} + X_{L2} = 3.39\text{ k}\Omega + 5.91\text{ k}\Omega = \mathbf{9.30\text{ k}\Omega}$$

For the inductors in parallel in Figure 33(b), determine the total reactance by Equation 14 or from the product-over-sum rule using X_{L1} and X_{L2}.

$$X_{L(tot)} = \frac{X_{L1}X_{L2}}{X_{L1} + X_{L2}} = \frac{(3.39\text{ k}\Omega)(5.91\text{ k}\Omega)}{3.39\text{ k}\Omega + 5.91\text{ k}\Omega} = \mathbf{2.15\text{ k}\Omega}$$

You can also obtain the total reactance for either series or parallel inductors by first finding the total inductance and then substituting in Equation 12 to find the total reactance.

Related Problem What is the total inductive reactance for each circuit in Figure 33 if $L_1 = 1$ mH and L_2 is unchanged?



▲ FIGURE 34

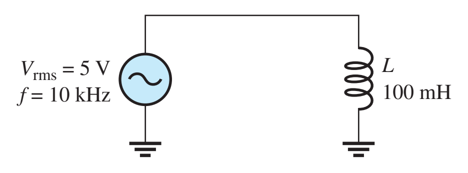
Ohm's Law The reactance of an inductor is analogous to the resistance of a resistor. In fact, X_L, just like X_C and R, is expressed in ohms. Since inductive reactance is a form of opposition to current, Ohm's law applies to inductive circuits as well as to resistive circuits and capacitive circuits; and it is stated as follows for Figure 34.

$$I = \frac{V_s}{X_L}$$

When applying Ohm's law in ac circuits, you must express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on.

EXAMPLE 14 Determine the rms current in Figure 35.

▶ FIGURE 35



Solution Convert 10 kHz to 10×10^3 Hz and 100 mH to 100×10^{-3} H. Then calculate X_L.

$$X_L = 2\pi fL = 2\pi(10 \times 10^3\text{ Hz})(100 \times 10^{-3}\text{ H}) = 6283\ \Omega$$

Apply Ohm's law to determine the rms current.

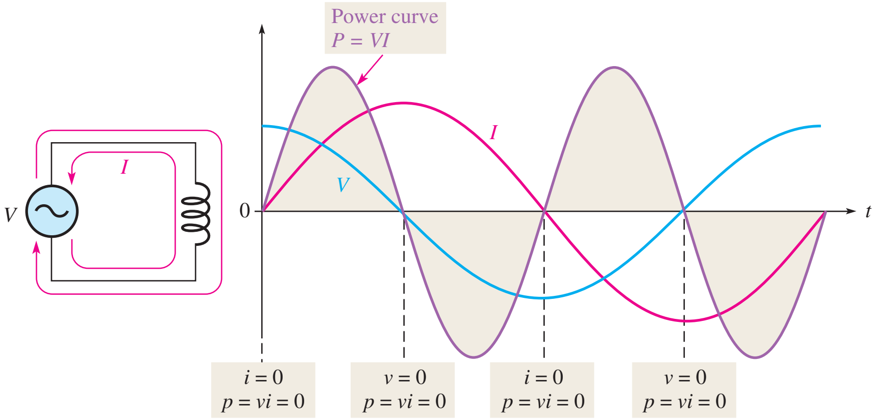
$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{5\text{ V}}{6283\ \Omega} = \mathbf{796\ \mu A}$$

Related Problem Determine the rms current in Figure 35 for the following values: $V_{rms} = 12$ V, $f =$ 4.9 kHz, and $L = 680$ mH.

Use Multisim file E13-14 to verify the calculated results in this example and to confirm your calculation for the related problem.

Power in an Inductor

As discussed earlier, an inductor stores energy in its magnetic field when there is current through it. An ideal inductor (assuming no winding resistance) does not dissipate energy; it only stores it. When an ac voltage is applied to an ideal inductor, energy is stored by the inductor during a portion of the cycle; then the stored energy is returned to the source during another portion of the cycle. No net energy is lost in an ideal inductor due to conversion to heat. Figure 36 shows the power curve that results from one cycle of inductor current and voltage.



▲ FIGURE 36

Power curve.

Instantaneous Power (p) The product of v and i gives instantaneous power. At points where v or i is zero, p is also zero. When both v and i are positive, p is also positive. When either v or i is positive and the other negative, p is negative. When both v and i are negative, p is positive. As you can see in Figure 36, the power follows a sinusoidal-shaped curve. Positive values of power indicate that energy is stored by the inductor. Negative values of power indicate that energy is returned from the inductor to the source. Note that the power fluctuates at a frequency twice that of the voltage or current as energy is alternately stored and returned to the source.

True Power (P_{true}) Ideally, all of the energy stored by an inductor during the positive portion of the power cycle is returned to the source during the negative portion. No net energy is lost due to conversion to heat in the inductor, so the true power is zero. Actually, because of winding resistance in a practical inductor, some power is always dissipated; and there is a very small amount of true power, which can normally be neglected.

$$P_{\text{true}} = (I_{\text{rms}})^2 R_W$$ Equation 15

Reactive Power (P_r) The rate at which an inductor stores or returns energy is called its **reactive power**, with the unit of VAR (volt-ampere reactive). The reactive power is a nonzero quantity because at any instant in time the inductor is actually taking energy from the source or returning energy to it. Reactive power does not represent an energy loss due to conversion to heat. The following formulas apply:

$$P_r = V_{\text{rms}} I_{\text{rms}}$$ Equation 16

$$P_r = \frac{V_{\text{rms}}^2}{X_L}$$ Equation 17

$$P_r = I_{\text{rms}}^2 X_L$$ Equation 18

EXAMPLE 15

A 10 V rms signal with a frequency of 1 kHz is applied to a 10 mH coil with a negligible winding resistance. Determine the reactive power (P_r).

Solution First, calculate the inductive reactance and current values.

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(10\text{ mH}) = 62.8\ \Omega$$

$$I = \frac{V_s}{X_L} = \frac{10\text{ V}}{62.8\ \Omega} = 159\text{ mA}$$

Then, use Equation 18.

$$P_r = I^2X_L = (159\text{ mA})^2(62.8\ \Omega) = \mathbf{1.59\ VAR}$$

Related Problem What happens to the reactive power if the frequency increases?

The Quality Factor (Q) of a Coil

The **quality factor (Q)** is the ratio of the reactive power in an inductor to the true power in the winding resistance of the coil or the resistance in series with the coil. It is a ratio of the power in L to the power in R_W. The quality factor is important in resonant circuits. A formula for Q is developed as follows:

$$Q = \frac{\text{reactive power}}{\text{true power}} = \frac{I^2X_L}{I^2R_W}$$

The current is the same in L and R_W; thus, the I^2 terms cancel, leaving

Equation 19

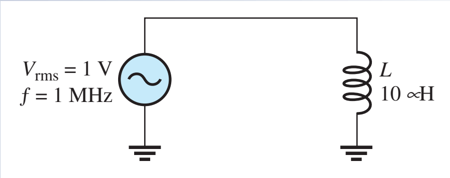
$$Q = \frac{X_L}{R_W}$$

When the resistance is just the winding resistance of the coil, the circuit Q and the coil Q are the same. Note that Q is a ratio of like units and, therefore, has no unit itself. The quality factor is also known as unloaded Q because it is defined with no load across the coil.

SECTION 5 CHECKUP

1. State the phase relationship between current and voltage in an inductor.
2. Calculate X_L for $f = 5$ kHz and $L = 100$ mH.
3. At what frequency is the reactance of a 50 μH inductor equal to 800 Ω?
4. Calculate the rms current in Figure 37.
5. An ideal 50 mH inductor is connected to a 12 V rms source. What is the true power? What is the reactive power at a frequency of 1 kHz?

▶ FIGURE 37



6 INDUCTOR APPLICATIONS

Inductors are not as versatile as capacitors and tend to be more limited in their application due, in part, to size, cost factors, and nonideal behavior (internal resistance, etc.). One of the most common applications for inductors is noise reduction applications.

After completing this section, you should be able to

- **Discuss some inductor applications**
  - Discuss two ways in which noise enters a circuit
  - Describe the suppression of electromagnetic interference (EMI)
  - Explain how a ferrite bead is used
  - Discuss the basics of tuned circuits

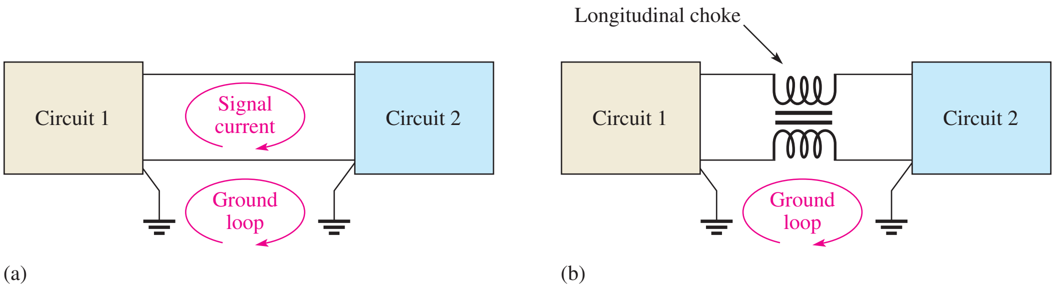
Noise Suppression

One of the most important applications of inductors has to do with suppressing unwanted electrical noise. The inductors used in these applications are generally wound on a closed core to avoid having the inductor become a source of radiated noise itself. Two types of noise are conductive noise and radiated noise.

Conductive Noise Many systems have common conductive paths connecting different parts of the system, which can conduct high frequency noise from one part of the system to another. Consider the case of two circuits connected with common lines as shown in Figure 38(a). A path for high frequency noise exists though the common grounds, creating a condition known as a *ground loop*. Ground loops are particularly a problem in instrumentation systems, where a transducer may be located a distance from the recording system and noise current in the ground can affect the signal.

If the signal of interest changes slowly, a special inductor, called a *longitudinal choke,* can be installed in the signal line as shown in Figure 38(b). The longitudinal choke is a form of transformer that acts as inductors in each signal line. The ground loop sees a high impedance path, thus reducing the noise, while the low-frequency signal is coupled through the low impedance of the choke.

Switching circuits also tend to generate high-frequency noise (above 10 MHz) by virtue of the high-frequency components present. (Recall that a pulse waveform



▲ FIGURE 38

contains many high-frequency harmonics). Certain types of power supplies use high-speed switching circuits that are a source of conductive and radiated noise.

Because an inductor's impedance increases with frequency, inductors are good for blocking electrical noise from these supplies, which should carry only dc. Inductors are frequently installed in the power supply lines to suppress this conductive noise, so that one circuit does not adversely affect another circuit. One or more capacitors may also be used in conjunction with the inductor to improve filtering action.

Radiated Noise Noise can also enter a circuit by way of an electromagnetic field. The noise source can be an adjacent circuit or a nearby power supply. There are several approaches to reducing the effects of radiated noise. Usually, the first step is to determine the cause of the noise and isolate it using shielding or filtering.

Inductors are widely employed in filters that are used to suppress radio-frequency noise. The inductor used for noise suppression must be carefully selected so as not to become a source of radiated noise itself. For high frequencies($>$20 MHz), inductors wound on highly permeable toroidal cores are widely used, as they tend to keep the magnetic flux restricted to the core.

RF Chokes

Inductors used for the purpose of blocking very high frequencies are called *radio frequency (RF) chokes*. RF chokes are used for conductive or radiated noise. They are special inductors designed to block high frequencies from getting into or leaving parts of a system by providing a high impedance path for high frequencies. In general, the choke is placed in series with the line for which RF suppression is required. Depending on the frequency of the interference, different types of chokes are required. A common type of electromagnetic interference (EMI) filter wraps the signal line on a toroidal core several times. The toroidal configuration is desired because it contains the magnetic field so that the choke does not become a source of noise itself.

Another common type of RF choke is a ferrite bead. All wires have inductance, and the ferrite bead is a small ferromagnetic material that is strung onto the wire to increase its inductance. The impedance presented by the bead is a function of both the material and the frequency, as well as the size of the bead. It is an effective and inexpensive "choke" for high frequencies. Ferrite beads are common in highfrequency communication systems. Sometimes several are strung together in series to increase the effective inductance.

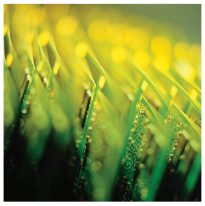
Tuned Circuits

Inductors are used in conjunction with capacitors to provide frequency selection in communications systems. These tuned circuits allow a narrow band of frequencies to be selected while all other frequencies are rejected. The tuners in your TV and radio receivers are based on this principle and permit you to select one channel or station out of the many that are available.

Frequency selectivity is based on the fact that the reactances of both capacitors and inductors depend on the frequency and on the interaction of these two components when connected in series or parallel. Since the capacitor and the inductor produce opposite phase shifts, their combined opposition to current can be used to obtain a desired response at a selected frequency.

SECTION 6
CHECKUP

1. Name two types of unwanted noise.
2. What do the letters *EMI* stand for?
3. How is a ferrite bead used?



Application Activity

In this application, you will see how you can test coils for their unknown inductance values using a test setup consisting of a square wave generator and an oscilloscope. Two coils for which the inductance values are not known are tested using simple laboratory instruments to determine the inductance values. The method is to place the coil in series with a resistor with a known value and measure the time constant. Knowing the time constant and the resistance value, the value of *L* can be calculated.

The method of determining the time constant is to apply a square wave to the circuit and measure the resulting voltage across the resistor. Each time the square wave input voltage goes high, the inductor is energized and each time the square wave goes back to zero, the inductor is deenergized. The time it takes for the exponential resistor voltage to increase to approximately its final value equals five time constants. This operation is illustrated in Figure 39. To make sure that the winding resistance of the coil can be neglected, it must be measured and the value of the resistor used in the circuit must be selected to be considerably larger than the winding and source resistances.

The Winding Resistance

Assume that the winding resistance of the coil in Figure 40 has been measured with an ohmmeter and found to be $85\ \Omega$. To make the winding and source resistances negligible for time constant measurement, a $10\ \text{k}\Omega$ series resistor is used in the circuit.

1. If 10 V dc is connected with the clip leads as shown, how much current is in the circuit after $t = 5\tau$?

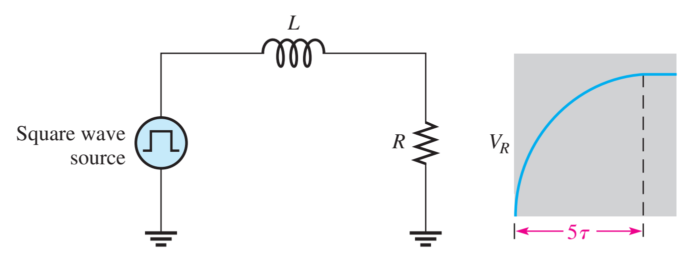
Inductance of Coil 1

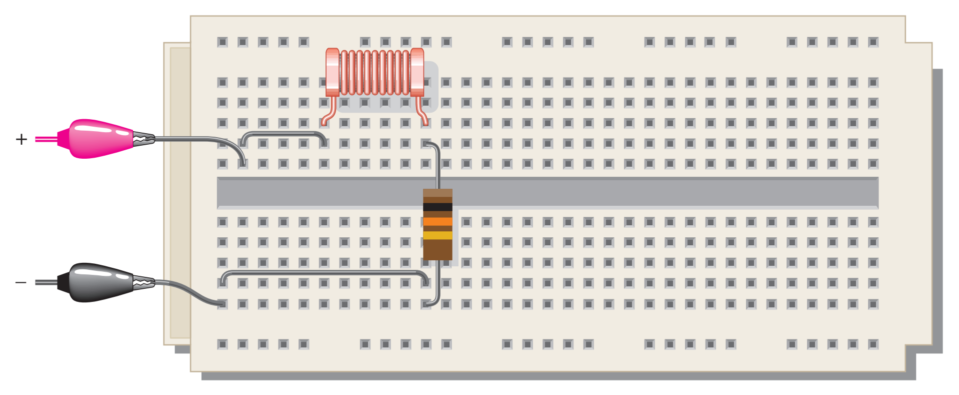
Refer to Figure 41. To measure the inductance of coil 1, a square wave voltage is applied to the circuit. The amplitude of the square wave is adjusted to 10 V. The frequency is adjusted so that the inductor has time to fully energize during each square wave pulse; the scope is set to view a complete energizing curve as shown.

2. Determine the approximate circuit time constant.
3. Calculate the inductance of coil 1.

▶ FIGURE 39

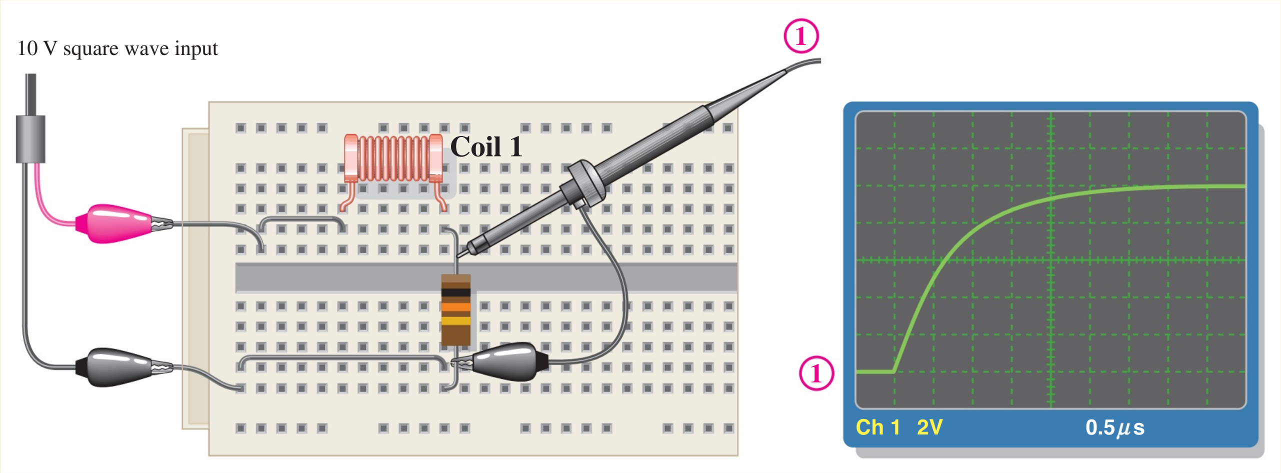
Circuit for time-constant measurement.





▲ FIGURE 40

Breadboard setup for measuring the time constant.



▲ FIGURE 41

Testing coil 1.

The Inductance of Coil 2

Refer to Figure 42 in which coil 2 replaces coil 1. To determine the inductance, a 10 V square wave is applied to the breadboarded circuit. The frequency of the square wave is adjusted so that the inductor has time to fully energize during each square wave pulse; the scope is set to view a complete energizing curve as shown.

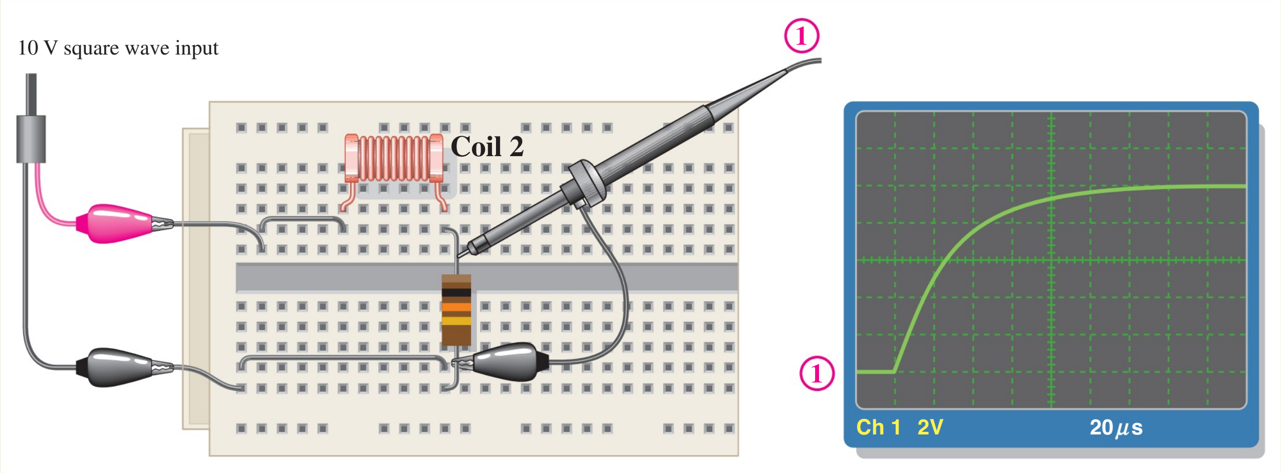
4. Determine the approximate circuit time constant.
5. Calculate the inductance of coil 2.
6. Discuss any difficulty in using this method.
7. Specify how you can use a sinusoidal input voltage instead of a square wave to determine inductance.

Review

8. What is the maximum square wave frequency that can be used in Figure 41?
9. What is the maximum square wave frequency that can be used in Figure 42?
10. What happens if the frequency exceeds the maximum you determined in Questions 8 and 9? Explain how your measurements would be affected.

Multisim Analysis

Open your Multisim software. Connect the *RL* circuit using the value of the resistance shown and the value of the inductance determined in activity 3. Verify the time constant by measurement. Repeat for the inductance determined in activity 5.



▲ FIGURE 42

Testing coil 2.

SUMMARY

- Inductance is a measure of a coil's ability to establish an induced voltage as a result of a change in its current.
- An inductor opposes a change in its own current.
- Faraday's law states that relative motion between a magnetic field and a coil induces a voltage across the coil.
- The amount of induced voltage is directly proportional to the inductance and to the rate of change in current.
- Lenz's law states that the polarity of induced voltage is such that the resulting induced current is in a direction that opposes the change in the magnetic field that produced it.
- Energy is stored by an inductor in its magnetic field.
- One henry is the amount of inductance when current, changing at the rate of one ampere per second, induces one volt across the inductor.
- Inductance is directly proportional to the square of the number of turns, the permeability, and the cross-sectional area of the core. It is inversely proportional to the length of the core.
- The permeability of a core material is an indication of the ability of the material to establish a magnetic field.
- The time constant for a series *RL* circuit is the inductance divided by the resistance.
- In an *RL* circuit, the increasing or decreasing voltage and current in an inductor make a 63% change during each time-constant interval.
- Increasing and decreasing voltages and currents follow exponential curves.
- Inductors add in series.
- Total parallel inductance is less than that of the smallest inductor in parallel.
- Voltage leads current by 90° in an inductor.
- Inductive reactance, X_L, is directly proportional to frequency and inductance.
- The true power in an inductor is zero; that is, no energy is lost in an ideal inductor due to conversion to heat, only in its winding resistance.

KEY TERMS

Henry (H) The unit of inductance.

Induced voltage Voltage produced as a result of a changing magnetic field.

Inductance The property of an inductor whereby a change in current causes the inductor to produce a voltage that opposes the change in current.

Inductive reactance The opposition of an inductor to sinusoidal current. The unit is the ohm.

Inductor An electrical device formed by a coil of wire having the property of inductance; also known as *coil.*

Quality factor (*Q*) The ratio of reactive power to true power in an inductor.

***RL* time constant** A fixed time interval, set by the *L* and *R* values, that determines the time response of a circuit and is equal to *L*/*R*.

Winding The loops or turns of wire in an inductor.

FORMULAS

1	$v_{ind} = L\left(\frac{di}{dt}\right)$	Induced voltage
2	$W = \frac{1}{2}LI^2$	Energy stored by an inductor
3	$L = \frac{N^2\mu A}{l}$	Inductance in terms of physical parameters

4	$L_T = L_1 + L_2 + L_3 + \cdots + L_n$	Series inductance
5	$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$	Reciprocal of total parallel inductance
6	$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right) + \cdots + \left(\frac{1}{L_n}\right)}$	Total parallel inductance
7	$\tau = \frac{L}{R}$	Time constant
8	$v = V_F + (V_i - V_F)e^{-Rt/L}$	Exponential voltage (general)
9	$i = I_F + (I_i - I_F)e^{-Rt/L}$	Exponential current (general)
10	$i = I_F(1 - e^{-Rt/L})$	Increasing exponential current beginning at zero
11	$i = I_i e^{-Rt/L}$	Decreasing exponential current ending at zero
12	$X_L = 2\pi fL$	Inductive reactance
13	$X_{L(tot)} = X_{L1} + X_{L2} + X_{L3} + \cdots + X_{Ln}$	Series inductive reactance
14	$X_{L(tot)} = \frac{1}{\frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} + \cdots + \frac{1}{X_{Ln}}}$	Parallel inductive reactance
15	$P_{true} = (I_{rms})^2 R_W$	True power
16	$P_r = V_{rms} I_{rms}$	Reactive power
17	$P_r = \frac{V_{rms}^2}{X_L}$	Reactive power
18	$P_r = I_{rms}^2 X_L$	Reactive power
19	$Q = \frac{X_L}{R_W}$	Quality factor

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. An inductor opposes any change in its current.
2. An inductance of 10 mH is equivalent to 1000 μH.
3. Inductance is proportional to the square of the number of windings.
4. Inductors have no resistance.
5. Ideally, an inductor appears as a short to dc.
6. The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil.
7. Inductive reactance is inversely proportional to frequency.
8. The total reactance of series inductors is the sum of the individual reactances.
9. Voltage lags current in an inductor.
10. The unit of inductive reactance is the ohm.

SELF-TEST

Answers are at the end of the chapter.

1. An inductance of 0.05 μH is larger than

   (a) 0.0000005 H **(b)** 0.000005 H **(c)** 0.000000008 H **(d)** 0.00005 mH

2. An inductance of 0.33 mH is smaller than

   (a) 33 μH **(b)** 330 μH **(c)** 0.05 mH **(d)** 0.0005 H

3. When the current through an inductor increases, the amount of energy stored in the electromagnetic field

   (a) decreases **(b)** remains constant **(c)** increases **(d)** doubles

4. When the current through an inductor doubles, the stored energy

(a) doubles **(b)** quadruples **(c)** is halved **(d)** does not change

5. The winding resistance of a coil can be decreased by

(a) reducing the number of turns **(b)** using a larger wire

(c) changing the core material **(d)** either answer (a) or (b)

6. The inductance of an iron-core coil increases if

(a) the number of turns is increased **(b)** the iron core is removed

(c) the length of the core is increased **(d)** larger wire is used

7. Four 10 mH inductors are in series. The total inductance is

(a) 40 mH **(b)** 2.5 mH **(c)** 40,000 μH **(d)** answers (a) and (c)

8. A 1 mH, a 3.3 mH, and a 0.1 mH inductor are connected in parallel. The total inductance is

(a) 4.4 mH **(b)** greater than 3.3 mH **(c)** less than 0.1 mH **(d)** answers (a) and (b)

9. An inductor, a resistor, and a switch are connected in series to a 12 V battery. At the instant the switch is closed, the inductor voltage is

(a) 0 V **(b)** 12 V **(c)** 6 V **(d)** 4 V

10. A sinusoidal voltage is applied across an inductor. When the frequency of the voltage is increased, the current

(a) decreases **(b)** increases **(c)** does not change **(d)** momentarily goes to zero

11. An inductor and a resistor are in series with a sinusoidal voltage source. The frequency is set so that the inductive reactance is equal to the resistance. If the frequency is increased, then

(a) $V_R > V_L$ **(b)** $V_L < V_R$ **(c)** $V_L = V_R$ **(d)** $V_L > V_R$

12. An ohmmeter is connected across an inductor and the pointer indicates an infinite value. The inductor is

(a) good **(b)** open **(c)** shorted **(d)** resistive

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 45.

1. The switch is in position 1. When it is thrown into position 2, the inductance between *A* and *B*

(a) increases **(b)** decreases **(c)** stays the same

2. If the switch is moved from position 3 to position 4, the inductance between *A* and *B*

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 48.

3. If *R* were 10 kΩ instead of 1.0 kΩ and the switch is closed, the time it takes for the current to reach its maximum value

(a) increases **(b)** decreases **(c)** stays the same

4. If *L* is decreased from 10 mH to 1 mH and the swtich is closed, the time constant

(a) increases **(b)** decreases **(c)** stays the same

5. If the source voltage drops from +15 V to +10 V, the time constant

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 51.

6. If the frequency of the source voltage is increased, the total current

(a) increases **(b)** decreases **(c)** stays the same

7. If L_2 opens, the current through L_1

(a) increases **(b)** decreases **(c)** stays the same

8. If the frequency of the source voltage is decreased, the ratio of the values of the currents through L_2 and L_3

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 52.

9. If the frequency of the source voltage increases, the voltage across L_1

(a) increases **(b)** decreases **(c)** stays the same

10. If L_3 opens, the voltage across L_2

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Basic Inductor

1. Convert the following to millihenries:

(a) 1 H **(b)** 250 μH **(c)** 10 μH **(d)** 0.0005 H

2. Convert the following to microhenries:

(a) 300 mH **(b)** 0.08 H **(c)** 5 mH **(d)** 0.00045 mH

3. What is the voltage across a coil when $di/dt = 10$ mA/μs and $L = 5\ \mu$H?

4. Fifty volts are induced across a 25 mH coil. At what rate is the current changing?

5. The current through a 100 mH coil is changing at a rate of 200 mA/s. How much voltage is induced across the coil?

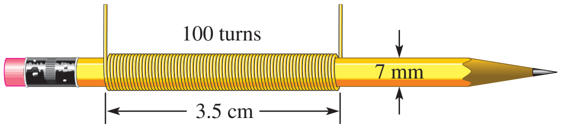
6. How many turns are required to produce 30 mH with a coil wound on a cylindrical core having a cross-sectional area of 10×10^{-5} m^2 and a length of 0.05 m? The core has a permeability of 1.2×10^{-6} H/m.

7. What amount of energy is stored in a 4.7 mH inductor when the current is 20 mA?

8. Compare the inductance of two inductors that are identical except that inductor 2 has twice the number of turns as inductor 1.

9. Compare the inductance of two inductors that are identical except that inductor 2 is wound on an iron coil (relative permeability = 150) and inductor 1 is wound on a low carbon steel core (relative permeability = 200).

10. A student wraps 100 turns of wire on a pencil that is 7 mm in diameter as shown in Figure 43. The pencil is a nonmagnetic core so has the same permeability as a vacuum ($4\pi \times 10^{-6}$ H/m). Determine the inductance of the coil that is formed.



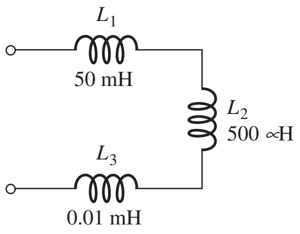
▲ FIGURE 43

SECTION 3 Series and Parallel Inductors

11. Five inductors are connected in series. The lowest value is 5 μH. If the value of each inductor is twice that of the preceding one, and if the inductors are connected in order of ascending values, what is the total inductance?

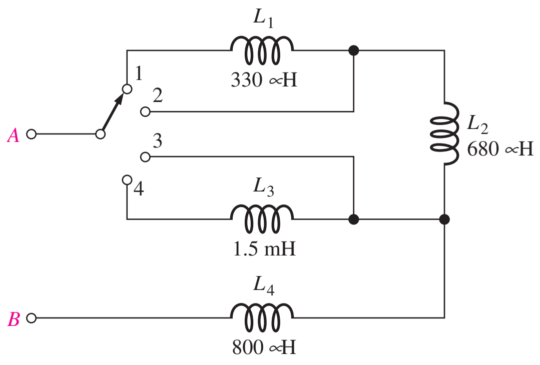
12. Suppose that you require a total inductance of 50 mH. You have available a 10 mH coil and a 22 mH coil. How much additional inductance do you need?

13. Determine the total inductance in Figure 44.



L_1
50 mH
L_2
500 ∝H
L_3
0.01 mH

▲ FIGURE 44



L_1
330 ∝H
1
2
3
4
A
L_2
680 ∝H
L_3
1.5 mH
L_4
800 ∝H
B

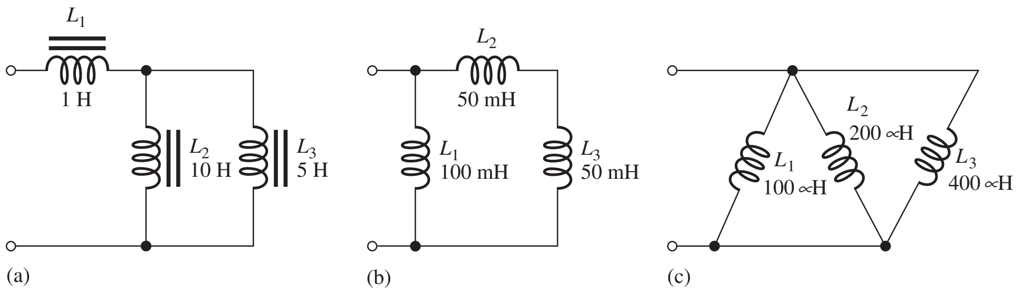
▲ FIGURE 45

14. What is the total inductance between points *A* and *B* for each switch position in Figure 45?

15. Determine the total parallel inductance for the following coils in parallel: 75 μH, 50 μH, 25 μH, and 15 μH.

16. You have a 12 mH inductor, and it is your smallest value. You need an inductance of 8 mH. What value can you use in parallel with the 12 mH to obtain 8 mH?

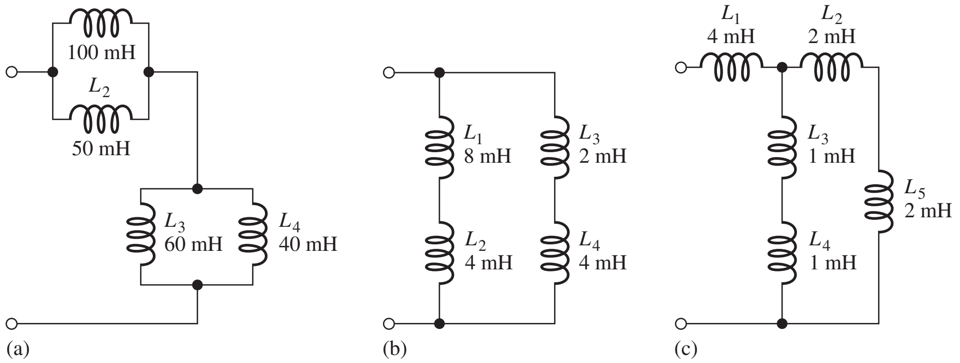
17. Determine the total inductance of each circuit in Figure 46.



L_1
1 H
L_2
10 H
L_3
5 H
(a)
L_2
50 mH
L_1
100 mH
L_3
50 mH
(b)
L_2
200 ∝H
L_1
100 ∝H
L_3
400 ∝H
(c)

▲ FIGURE 46

18. Determine the total inductance of each circuit in Figure 47.



L_1
100 mH
L_2
50 mH
L_3
60 mH
L_4
40 mH
(a)
L_1
8 mH
L_3
2 mH
L_2
4 mH
L_4
4 mH
(b)
L_1
4 mH
L_2
2 mH
L_3
1 mH
L_5
2 mH
L_4
1 mH
(c)

▲ FIGURE 47

SECTION 4 Inductors in DC Circuits

19. Determine the time constant for each of the following series *RL* combinations:

(a) $R = 100\ \Omega, L = 100\ \mu\text{H}$ **(b)** $R = 4.7\ \text{k}\Omega, L = 10\ \text{mH}$

(c) $R = 1.5\ \text{M}\Omega, L = 3\ \text{H}$

20. In a series *RL* circuit, determine how long it takes the current to build up to its full value for each of the following:

(a) $R = 56\ \Omega, L = 50\ \mu\text{H}$ **(b)** $R = 3300\ \Omega, L = 15\ \text{mH}$

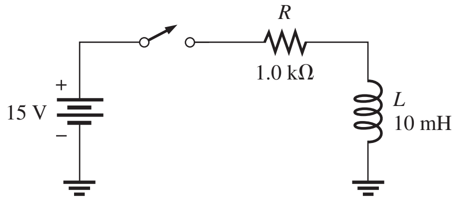
(c) $R = 22\ \text{k}\Omega, L = 100\ \text{mH}$

21. In the circuit of Figure 48, there is initially no current. Determine the inductor voltage at the following times after the switch is closed:

(a) 10 μs **(b)** 20 μs **(c)** 30 μs **(d)** 40 μs **(e)** 50 μs

22. Determine the inductor current in Figure 48 at each of the times specified in Problem 21.

▶ FIGURE 48



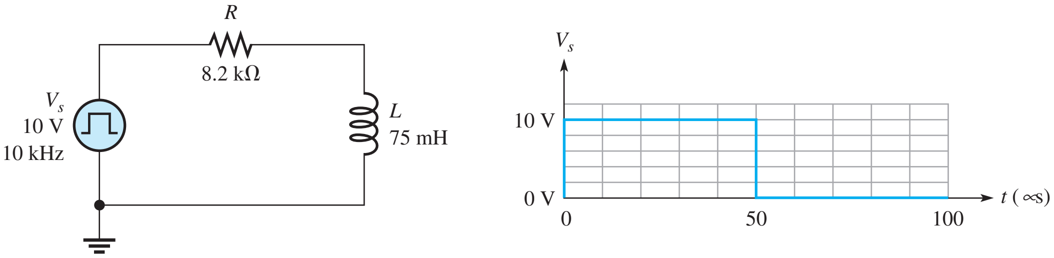
23. Determine the time constant for the circuit in Figure 49.

***24.** For the ideal inductor in Figure 49, calculate the current at each of the following times:

(a) 10 μs **(b)** 20 μs **(c)** 30 μs

25. Repeat Problem 21 for the following times:

(a) 2 μs **(b)** 5 μs **(c)** 15 μs



▲ FIGURE 49

***26.** Repeat Problem 24 for the following times:

(a) 65 μs **(b)** 75 μs **(c)** 85 μs

27. In Figure 48, at what time after switch closure does the inductor voltage reach 5 V?

28. **(a)** What is the polarity of the induced voltage across the inductor in Figure 49 when the square wave is rising?

(b) What is the current just before the square wave drops to zero?

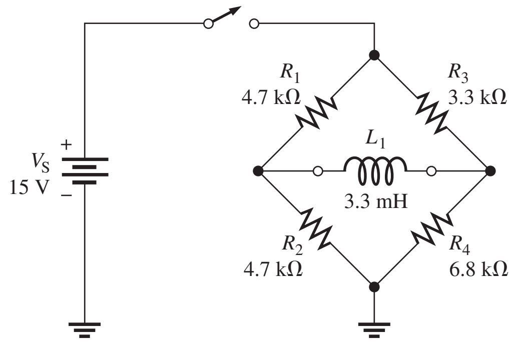
29. Determine the time constant for the circuit in Figure 50.

***30.** **(a)** What is the current in the inductor 1.0 μs after the switch closes in Figure 50?

(b) What is the current after 5τ have elapsed?

***31.** For the circuit in Figure 50, assume the switch has been closed for more than 5τ and is opened. What is the current in the inductor 1.0 μs after the switch is opened?

▶ FIGURE 50



SECTION 5 Inductors in AC Circuits

32. Find the total reactance for each circuit in Figure 46 when a voltage with a frequency of 5 kHz is applied across the terminals.

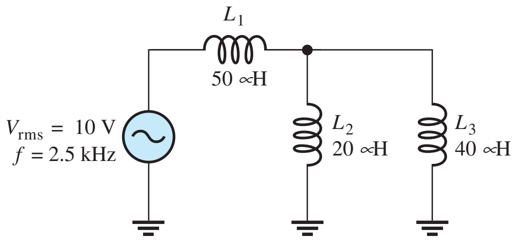
33. Find the total reactance for each circuit in Figure 47 when a 400 Hz voltage is applied.

34. Determine the total rms current in Figure 51. What are the currents through L_2 and L_3?

35. What frequency will produce 500 mA total rms current in each circuit of Figure 47 with an rms input voltage of 10 V?

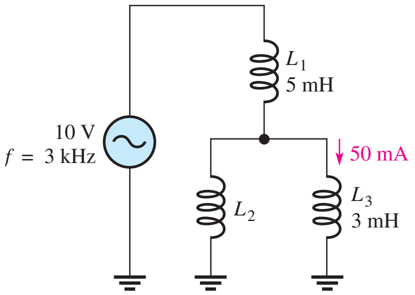
36. Determine the reactive power in Figure 51.

▶ FIGURE 51



37. Determine I_{L2} in Figure 52.

▶ FIGURE 52



Multisim Troubleshooting and Analysis

These problems require Multisim.

38. Open file P13-38 and measure the voltage across each inductor.

39. Open file P13-39 and measure the voltage across each inductor.

40. Open file P13-40 and measure the current. Double the frequency and measure the current again. Reduce the original frequency by one-half and measure the current. Explain your observations.

41. Open file P13-41 and determine the fault if there is one.

42. Open file P13-42 and find the fault if there is one.

ANSWERS

SECTION CHECKUPS

SECTION 1 The Basic Inductor

1. Inductance depends on number of turns of wire, permeability, cross-sectional area, and core length.
2. $v_{\text{ind}} = 7.5\text{ mV}$
3. **(a)** L increases when N increases.
   (b) L decreases when core length increases.
   (c) L decreases when core cross-sectional area decreases.
   (d) L decreases when ferromagnetic core is replaced by air core.
4. All wire has some resistance, and because inductors are made from turns of wire, there is always resistance.
5. Adjacent turns in a coil act as plates of a capacitor.

SECTION 2 Types of Inductors

1. Two categories of inductors are fixed and variable.
2. **(a)** air core **(b)** iron core **(c)** variable

SECTION 3 Series and Parallel Inductors

1. Inductances are added in series.
2. $L_T = 2.60\text{ mH}$
3. $L_T = 5(100\text{ mH}) = 500\text{ mH}$
4. The total parallel inductance is smaller than that of the smallest-value inductor in parallel.
5. True, calculation of parallel inductance is similar to parallel resistance.
6. **(a)** $L_T = 24\ \mu\text{H}$ **(b)** $L_T = 7.69\text{ mH}$

SECTION 4 Inductors in DC Circuits

1. $V_L = IR_W = 100\text{ mV}$
2. $i = 0\text{ V}, v_L = 20\text{ V}$
3. $v_L = 0\text{ V}$
4. $\tau = 500\text{ ns}, i_L = 3.93\text{ mA}$

SECTION 5 Inductors in AC Circuits

1. Voltage leads current by 90 degrees in an inductor.
2. $X_L = 2\pi fL = 3.14\text{ k}\Omega$
3. $f = X_L/2\pi L = 2.55\text{ MHz}$
4. $I_{\text{rms}} = 15.9\text{ mA}$
5. $P_{\text{true}} = 0\text{ W}; P_r = 458\text{ mVAR}$

SECTION 6 Inductor Applications

1. Conductive and radiated
2. Electromagnetic interference
3. A ferrite bead is placed on a wire to increase its inductance, creating an RF choke.

RELATED PROBLEMS FOR EXAMPLES

1 5 H
2 10.1 mH
3 0.5 Wb/s

4 150 μH

5 20.3 μH

6 227 ns

7 $I_F = 17.6$ mA, $\tau = 147$ ns
at 1τ: $i = 11.1$ mA; $t = 147$ ns
at 2τ: $i = 15.1$ mA; $t = 294$ ns
at 3τ: $i = 16.7$ mA; $t = 441$ ns
at 4τ: $i = 17.2$ mA; $t = 588$ ns
at 5τ: $i = 17.4$ mA; $t = 735$ ns

8 at 0.2 ms, $i = 0.215$ mA
at 0.8 ms, $i = 0.035$ mA

9 10 V

10 3.83 mA

11 6.7 mA

12 1.1 kΩ

13 **(a)** 7.17 kΩ; **(b)** 1.04 kΩ

14 573 mA

15 P_r decreases.

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** F **5.** T
6. T **7.** F **8.** T **9.** F **10.** T

SELF-TEST

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (d) **6.** (a)
7. (d) **8.** (c) **9.** (b) **10.** (a) **11.** (d) **12.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (b) **4.** (b) **5.** (b)
6. (b) **7.** (b) **8.** (c) **9.** (c) **10.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. **(a)** 1000 mH **(b)** 0.25 mH **(c)** 0.01 mH **(d)** 0.5 mH

3. 50 mV

5. 20 mV

7. 0.94 μJ

9. Inductor 2 has three-fourths the inductance of inductor 1.

11. 155 μH

13. 50.5 mH

15. 7.14 μH

17. **(a)** 4.33 H **(b)** 50 mH **(c)** 57.1 μH

19. **(a)** 1 μs **(b)** 2.13 μs **(c)** 2 μs

21. **(a)** 5.52 V **(b)** 2.03 V **(c)** 747 mV
(d) 275 mV **(e)** 101 mV

23. 9.15 μs

25. **(a)** 12.3 V **(b)** 9.10 V **(c)** 3.35 V

27. 11.0 μs

29. 0.722 μs

31. 136 μA

33. **(a)** 144 Ω **(b)** 10.1 Ω **(c)** 13.4 Ω

35. **(a)** 55.5 Hz **(b)** 796 Hz **(c)** 597 Hz

37. 26.1 mA

39. $V_1 = 12.953$ V; $V_2 = 11.047$ V; $V_3 = 5.948$ V;
$V_4 = 5.099$ V; $V_5 = 5.099$ V

41. L_3 open

RC CIRCUITS

CHAPTER OUTLINE

1 The Complex Number System

SERIES CIRCUITS

2 Sinusoidal Response of Series *RC* Circuits
3 Impedance of Series *RC* Circuits
4 Analysis of Series *RC* Circuits

PARALLEL CIRCUITS

5 Impedance and Admittance of Parallel *RC* Circuits
6 Analysis of Parallel *RC* Circuits

SERIES-PARALLEL CIRCUITS

7 Analysis of Series-Parallel *RC* Circuits

SPECIAL TOPICS

8 Power in *RC* Circuits
9 Basic Applications
10 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Use complex numbers to express phasor quantities

SERIES CIRCUITS

- Describe the relationship between current and voltage in a series *RC* circuit
- Determine the impedance of a series *RC* circuit
- Analyze a series *RC* circuit

PARALLEL CIRCUITS

- Determine impedance and admittance in a parallel *RC* circuit
- Analyze a parallel *RC* circuit

SERIES-PARALLEL CIRCUITS

- Analyze series-parallel *RC* circuits

SPECIAL TOPICS

- Determine power in *RC* circuits
- Discuss some basic *RC* applications
- Troubleshoot *RC* circuits

KEY TERMS

- Complex plane
- Real number
- Imaginary number
- Rectangular form
- Polar form
- Apparent power (P_a)
- Power factor
- Filter
- Impedance
- *RC* lag circuit
- *RC* lead circuit
- Capacitive susceptance (B_C)
- Admittance (Y)
- Frequency response
- Cutoff frequency
- Bandwidth

APPLICATION ACTIVITY PREVIEW

The frequency response of the *RC* input circuit in an amplifier is the subject of this chapter's application activity.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

An *RC* circuit contains both resistance and capacitance. In this chapter, basic series and parallel *RC* circuits and their responses to sinusoidal ac voltages are presented. Series-parallel combinations are also analyzed. True, reactive, and apparent power in *RC* circuits are discussed and some basic *RC* circuit applications are introduced. Applications of *RC* circuits include filters, amplifier coupling, oscillators, and wave-shaping circuits. Troubleshooting is also covered in this chapter.

Section 1 provides an introduction to complex numbers, an important tool for the analysis of ac circuits. The complex number system is a way to mathematically express a phasor quantity and allows phasor quantities to be added, subtracted, multiplied, and divided.

From Chapter 15 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 THE COMPLEX NUMBER SYSTEM

Complex numbers allow mathematical operations with phasor quantities and are useful in the analysis of ac circuits. With the complex number system, you can add, subtract, multiply, and divide quantities that have both magnitude and angle, such as sine waves and other ac circuit quantities. Most scientific calculators can perform operations with complex numbers. Consult your user's manual for the exact procedure.

After completing this section, you should be able to

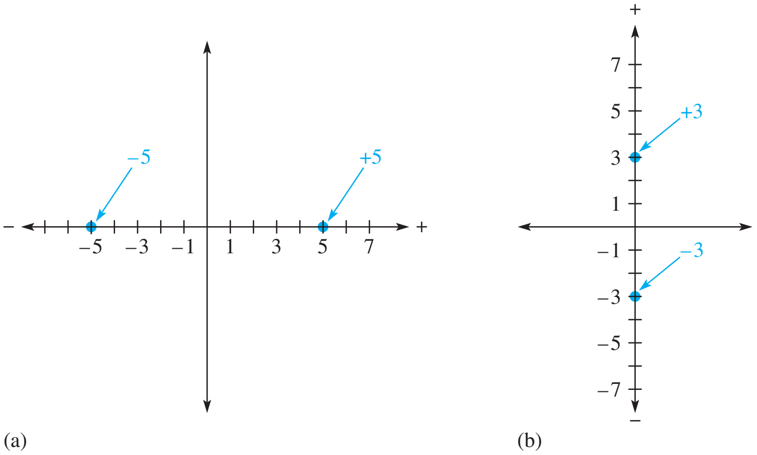
- **Use complex numbers to express phasor quantities**
  - Describe the complex plane
  - Represent a point on the complex plane
  - Discuss real and imaginary numbers
  - Express phasor quantities in both rectangular and polar forms
  - Convert between rectangular and polar forms
  - Do arithmetic operations with complex numbers

Positive and Negative Numbers

Positive numbers are represented by points to the right of the origin on the horizontal axis of a graph, and negative numbers are represented by points to the left of the origin, as illustrated in Figure 1(a). Also, positive numbers are represented by points on the vertical axis above the origin, and negative numbers are represented by points below the origin, as shown in Figure 1(b).

FIGURE 1

Graphic representation of positive and negative numbers.



(a) (b)

The Complex Plane

To distinguish between values on the horizontal axis and values on the vertical axis, a **complex plane** is used. In the complex plane, the horizontal axis is called the *real axis,* and the vertical axis is called the *imaginary axis,* as shown in Figure 2. In electrical circuit work, a $\pm j$ prefix is used to designate numbers that lie on the imaginary axis in order to distinguish them from numbers lying on the real axis. This prefix is known as the j

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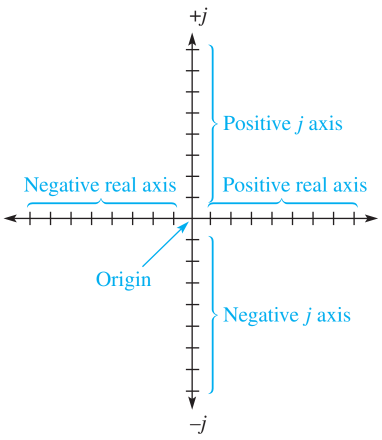
This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

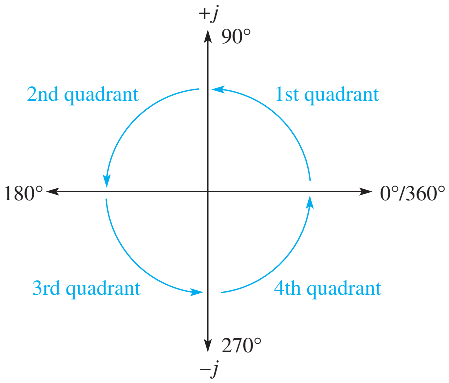

+j
Positive j axis
Negative real axis
Positive real axis
Origin
Negative j axis
–j

◀ **FIGURE 2**

The complex plane.

operator. In mathematics, an *i* is used instead of a *j*, but in electric circuits, the *i* can be confused with instantaneous current, so *j* is used.

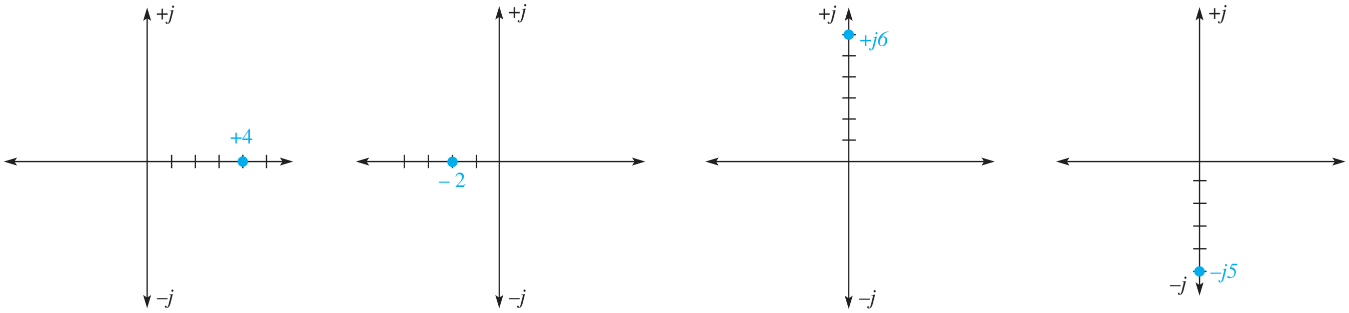
Angular Position on the Complex Plane Angular positions are represented on the complex plane, as shown in Figure 3. The positive real axis represents zero degrees. Proceeding counterclockwise, the $+j$ axis represents 90°, the negative real axis represents 180°, the $-j$ axis is the 270° point, and, after a full rotation of 360°, you are back to the positive real axis. Notice that the plane is divided into four quadrants.


+j
90°
2nd quadrant
1st quadrant
180°
0°/360°
3rd quadrant
4th quadrant
270°
–j

◀ **FIGURE 3**

Angles on the complex plane.

Representing a Point on the Complex Plane A point located on the complex plane is classified as real, imaginary ($\pm j$), or a combination of the two. For example, a point located 4 units from the origin on the positive real axis is the positive **real number**, +4, as shown in Figure 4(a). A point 2 units from the origin on the negative real



(a) Real number, +4
(b) Real number, – 2
(c) Imaginary number, +j6
(d) Imaginary number, –j5

▲ **FIGURE 4**

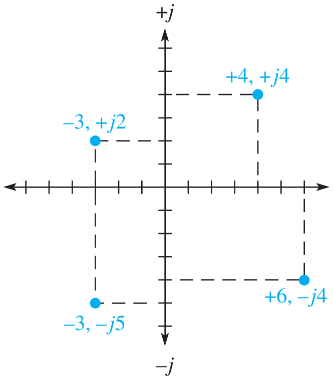
Real and imaginary (j) numbers on the complex plane.

axis is the negative real number, −2, as shown in part (b). A point on the $+j$ axis 6 units from the origin, as shown in part (c), is the positive **imaginary number**, $+j6$. Finally, a point 5 units along the $-j$ axis is the negative imaginary number, $-j5$, as shown in part (d).

When a point lies not on any axis but somewhere in one of the four quadrants, it is a complex number and is defined by its coordinates. For example, in Figure 5, the point located in the first quadrant has a real value of +4 and a j value of $+j4$ and is expressed as $+4, +j4$. The point located in the second quadrant has coordinates −3 and $+j2$. The point located in the third quadrant has coordinates −3 and $-j5$. The point located in the fourth quadrant has coordinates of +6 and $-j4$.

▶ **FIGURE 5**

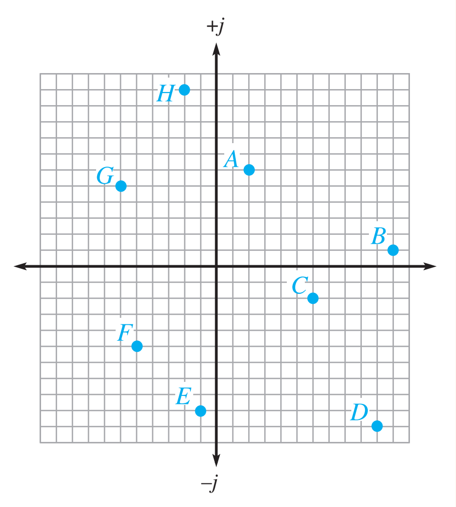
Coordinate points on the complex plane.



EXAMPLE 1

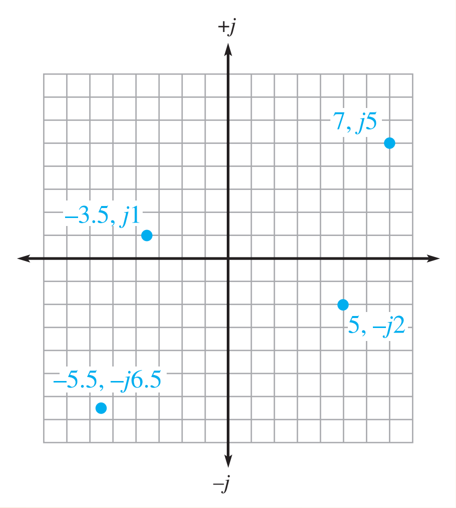
(a) Locate the following points on the complex plane: 7, j 5; 5, $-j2$; −3.5, j 1; and −5.5, $-j6.5$.

(b) Determine the coordinates for each point in Figure 6.



▲ **FIGURE 6**

Solution **(a)** See Figure 7.



▲ FIGURE 7

(b) A: $\mathbf{2, j6}$ B: $\mathbf{11, j1}$ C: $\mathbf{6, -j2}$ D: $\mathbf{10, -j10}$

E: $\mathbf{-1, -j9}$ F: $\mathbf{-5, -j5}$ G: $\mathbf{-6, j5}$ H: $\mathbf{-2, j11}$

*Related Problem** In what quadrant is each of the following points located?

(a) $+2.5, +j1$ **(b)** $7, -j5$ **(c)** $-10, -j5$ **(d)** $-11, +j6.8$

---

*Answers are at the end of the chapter.

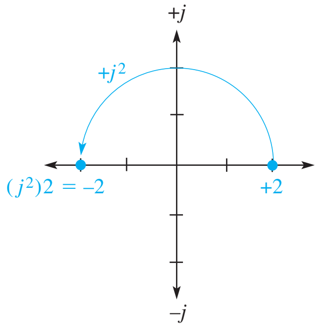
Value of *j*

If you multiply the positive real value of $+2$ by j, the result is $+j2$. This multiplication has effectively moved the $+2$ through a 90° angle to the $+j$ axis. Similarly, multiplying $+2$ by $-j$ rotates it $-90°$ to the $-j$ axis. Thus, j is considered a rotational operator.

Mathematically, the j operator has a value of $\sqrt{-1}$. If $+j2$ is multiplied by j, you get

$$j^2 2 = (\sqrt{-1})(\sqrt{-1})(2) = (-1)(2) = -2$$

This calculation effectively places the value on the negative real axis. Therefore, multiplying a positive real number by j^2 converts it to a negative real number, which, in effect, is a rotation of 180° on the complex plane. This operation is illustrated in Figure 8.



◀ FIGURE 8

Effect of the *j* operator on location of a number on the complex plane.

Rectangular and Polar Forms

Rectangular and polar are two forms of complex numbers that are used to represent phasor quantities. Each has certain advantages when used in circuit analysis, depending on the particular application. A phasor quantity contains both *magnitude* and angular position or *phase.* In this text, italic letters such as V and I are used to represent magnitude only, and boldfaced nonitalic letters such as **V** and **I** are used to represent complete phasor quantities.

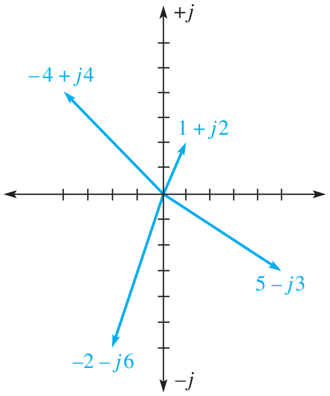
Rectangular Form A phasor quantity is represented in **rectangular form** by the algebraic sum of the real value (A) of the coordinate and the j value (B) of the coordinate, expressed in the following general form:

$$A + jB$$

Examples of phasor quantities are $1 + j2$, $5 - j3$, $-4 + j4$, and $-2 - j6$, which are shown on the complex plane in Figure 9. As you can see, the rectangular coordinates describe the phasor in terms of its values projected onto the real axis and the j axis. An "arrow" drawn from the origin to the coordinate point in the complex plane represents graphically the phasor quantity.

▶ FIGURE 9

Examples of phasors specified by rectangular coordinates.



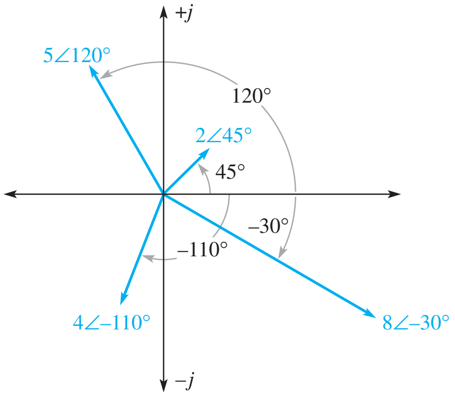
Polar Form Phasor quantities can also be expressed in **polar form**, which consists of the phasor magnitude (C) and the angular position relative to the positive real axis (θ), expressed in the following general form:

$$C\angle\pm\theta$$

Examples are $2\angle 45°$, $5\angle 120°$, $4\angle -110°$, and $8\angle -30°$. The first number is the magnitude, and the symbol $\angle$ precedes the value of the angle. Figure 10 shows these phasors on the complex plane. The length of the phasor, of course, represents the magnitude of the

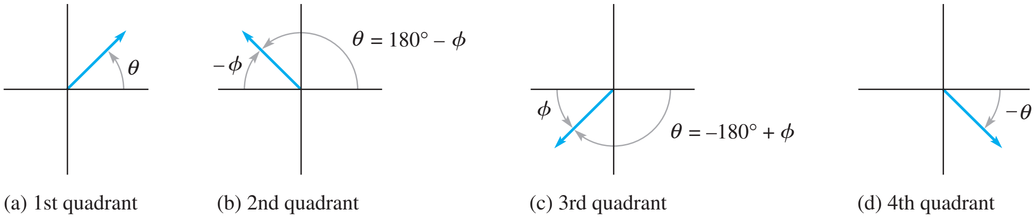
▶ FIGURE 10

Examples of phasors specified by polar values.



quantity. Keep in mind that for every phasor expressed in polar form, there is also an equivalent expression in rectangular form.

Conversion from Rectangular to Polar Form A phasor can exist in any of the four quadrants of the complex plane, as indicated in Figure 11. The phase angle θ in each case is measured relative to the positive real axis (0°), and ϕ (phi) is the angle in the 2nd and 3rd quadrants relative to the negative real axis, as shown.



(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant

▲ FIGURE 11

All possible phasor quadrant locations.

The first step to convert from rectangular form to polar form is to determine the magnitude of the phasor. A phasor can be visualized as forming a right triangle in the complex plane, as indicated in Figure 12, for each quadrant location. The horizontal side of the triangle is the real value, A, and the vertical side is the j value, B. The hypotenuse of the triangle is the length of the phasor, C, representing the magnitude, and can be expressed, using the Pythagorean theorem, as

$$C = \sqrt{A^2 + B^2} \qquad \text{Equation 1}$$

Next, the angle θ indicated in parts (a) and (d) of Figure 12 is expressed as an inverse tangent function.

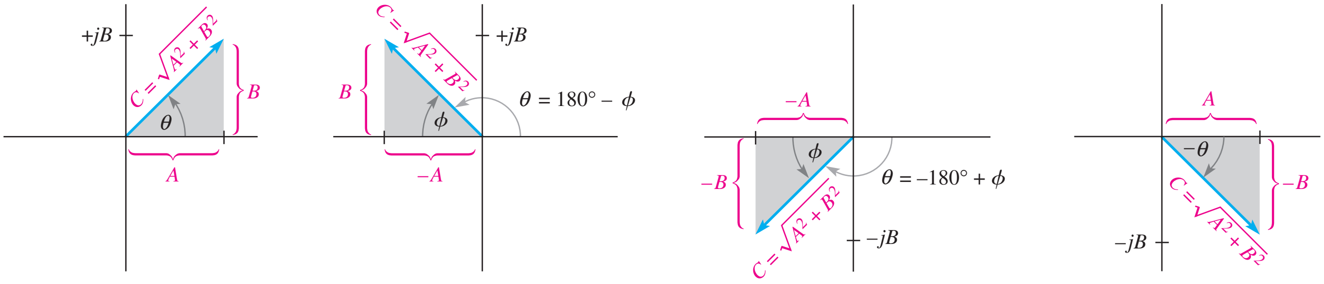
$$\theta = \tan^{-1}\left(\frac{\pm B}{A}\right) \qquad \text{Equation 2}$$

The angle θ indicated in parts (b) and (c) of Figure 12 is

$$\theta = \pm 180^\circ \mp \phi$$

which includes both conditions as indicated by the dual signs.

$$\theta = \pm 180^\circ \mp \tan^{-1}\left(\frac{B}{A}\right)$$



(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant

▲ FIGURE 12

Right angle relationships in the complex plane.

In each case the appropriate signs must be used in the calculation.

The general formula for converting from rectangular to polar is

Equation 3

$$\pm A \pm jB = C\angle \pm\theta$$

Example 2 illustrates the conversion procedure.

EXAMPLE 2

Convert the following complex numbers from rectangular form to polar form by determining the magnitude and angle:

(a) $8 + j6$ **(b)** $10 - j5$

Solution

(a) The magnitude of the phasor represented by $8 + j6$ is

$$C = \sqrt{A^2 + B^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Since the phasor is in the first quadrant, use Equation 2. The angle is

$$\theta = \tan^{-1}\left(\frac{\pm B}{A}\right) = \tan^{-1}\left(\frac{6}{8}\right) = 36.9^\circ$$

θ is the angle relative to the positive real axis. The polar form of $8 + j6$ is

$$C\angle\theta = \mathbf{10\angle 36.9^\circ}$$

(b) The magnitude of the phasor represented by $10 - j5$ is

$$C = \sqrt{10^2 + (-5)^2} = \sqrt{125} = 11.2$$

Since the phasor is in the fourth quadrant, use Equation 2. The angle is

$$\theta = \tan^{-1}\left(\frac{-5}{10}\right) = -26.6^\circ$$

θ is the angle relative to the positive real axis. The polar form of $10 - j5$ is

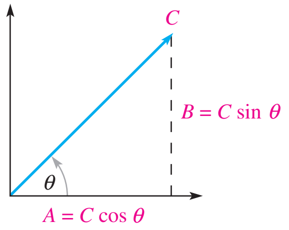
$$C\angle\theta = \mathbf{11.2\angle -26.6^\circ}$$

Related Problem Convert $18 + j23$ to polar form.

Conversion from Polar to Rectangular Form The polar form gives the magnitude and angle of a phasor quantity, as indicated in Figure 13.

FIGURE 13

Polar components of a phasor.



To get the rectangular form, you must find sides A and B of the triangle, using the rules from trigonometry stated below:

Equation 4

$$A = C\cos\theta$$

Equation 5

$$B = C\sin\theta$$

The polar-to-rectangular conversion formula is

Equation 6

$$C\angle\theta = A + jB$$

The following example demonstrates this conversion.

EXAMPLE 3 Convert the following polar quantities to rectangular form:

(a) $10\angle 30^\circ$ **(b)** $200\angle -45^\circ$

Solution **(a)** The real part of the phasor represented by $10\angle 30^\circ$ is

$$A = C\cos\theta = 10\cos 30^\circ = 10(0.866) = 8.66$$

The j part of this phasor is

$$jB = jC\sin\theta = j10\sin 30^\circ = j10(0.5) = j5$$

The rectangular form of $10\angle 30^\circ$ is

$$A + jB = \mathbf{8.66 + \mathit{j}5}$$

(b) The real part of the phasor represented by $200\angle -45^\circ$ is

$$A = 200\cos(-45^\circ) = 200(0.707) = 141$$

The j part is

$$jB = j200\sin(-45^\circ) = j200(-0.707) = -j141$$

The rectangular form of $200\angle -45^\circ$ is

$$A + jB = \mathbf{141 - \mathit{j}141}$$

Related Problem Convert $78\angle -26^\circ$ to rectangular form.

Mathematical Operations

Addition Complex numbers must be in rectangular form in order to add them. The rule is

Add the real parts of each complex number to get the real part of the sum. Then add the *j* parts of each complex number to get the *j* part of the sum.

EXAMPLE 4 Add the following sets of complex numbers:

(a) $8 + j5$ and $2 + j1$ **(b)** $20 - j10$ and $12 + j6$

Solution **(a)** $(8 + j5) + (2 + j1) = (8 + 2) + j(5 + 1) = \mathbf{10 + \mathit{j}6}$

(b) $(20 - j10) + (12 + j6) = (20 + 12) + j(-10 + 6) = 32 + j(-4) = \mathbf{32 - \mathit{j}4}$

Related Problem Add $5 - j11$ and $-6 + j3$.

Subtraction As in addition, the numbers must be in rectangular form to be subtracted. The rule is

Subtract the real parts of the numbers to get the real part of the difference. Then subtract the *j* parts of the numbers to get the *j* part of the difference.

EXAMPLE 5 Perform the following subtractions:

(a) Subtract $1 + j2$ from $3 + j4$.

(b) Subtract $10 - j8$ from $15 + j15$.

Solution **(a)** $(3 + j4) - (1 + j2) = (3 - 1) + j(4 - 2) = \mathbf{2 + j2}$

(b) $(15 + j15) - (10 - j8) = (15 - 10) + j[15 - (-8)] = \mathbf{5 + j23}$

Related Problem Subtract $3.5 - j4.5$ from $-10 - j9$.

Multiplication Multiplication of two complex numbers in rectangular form is accomplished by multiplying, in turn, each term in one number by both terms in the other number and then combining the resulting real terms and the resulting j terms (recall that $j \times j = -1$). As an example,

$$(5 + j3)(2 - j4) = 10 - j20 + j6 + 12 = 22 - j14$$

Multiplication of two complex numbers is easier when both numbers are in polar form, so it is best to convert to polar form before multiplying. The rule is

Multiply the magnitudes, and add the angles algebraically.

EXAMPLE 6 Perform the following multiplications:

(a) $10\angle 45^\circ$ times $5\angle 20^\circ$ **(b)** $2\angle 60^\circ$ times $4\angle -30^\circ$

Solution **(a)** $(10\angle 45^\circ)(5\angle 20^\circ) = (10)(5)\angle(45^\circ + 20^\circ) = \mathbf{50\angle 65^\circ}$

(b) $(2\angle 60^\circ)(4\angle -30^\circ) = (2)(4)\angle[60^\circ + (-30^\circ)] = \mathbf{8\angle 30^\circ}$

Related Problem Multiply $50\angle 10^\circ$ times $30\angle -60^\circ$.

Division Division of two complex numbers in rectangular form is accomplished by multiplying both the numerator and the denominator by the **complex conjugate** of the denominator and then combining terms and simplifying. The complex conjugate of a number is found by changing the sign of the j term. As an example,

$$\frac{10 + j5}{2 + j4} = \frac{(10 + j5)(2 - j4)}{(2 + j4)(2 - j4)} = \frac{20 - j30 + 20}{4 + 16} = \frac{40 - j30}{20} = 2 - j1.5$$

Like multiplication, division is easier when the numbers are in polar form, so it is best to convert to polar form before dividing. The rule is

Divide the magnitude of the numerator by the magnitude of the denominator to get the magnitude of the quotient. Then subtract the denominator angle from the numerator angle to get the angle of the quotient.

EXAMPLE 7 Perform the following divisions:

(a) Divide $100\angle 50^\circ$ by $25\angle 20^\circ$.

(b) Divide $15\angle 10^\circ$ by $3\angle -30^\circ$.

Solution **(a)** $\dfrac{100\angle 50^\circ}{25\angle 20^\circ} = \left(\dfrac{100}{25}\right)\angle(50^\circ - 20^\circ) = \mathbf{4\angle 30^\circ}$

(b) $\dfrac{15\angle 10^\circ}{3\angle -30^\circ} = \left(\dfrac{15}{3}\right)\angle[10^\circ - (-30^\circ)] = \mathbf{5\angle 40^\circ}$

Related Problem Divide $24\angle -30^\circ$ by $6\angle 12^\circ$.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Convert $2 + j2$ to polar form. In which quadrant does this phasor lie?
2. Convert $5\angle -45^\circ$ to rectangular form. In which quadrant does this phasor lie?
3. Add $1 + j2$ and $3 - j1$.
4. Subtract $12 + j18$ from $15 + j25$.
5. Multiply $8\angle 45^\circ$ times $2\angle 65^\circ$.
6. Divide $30\angle 75^\circ$ by $6\angle 60^\circ$.

SERIES CIRCUITS

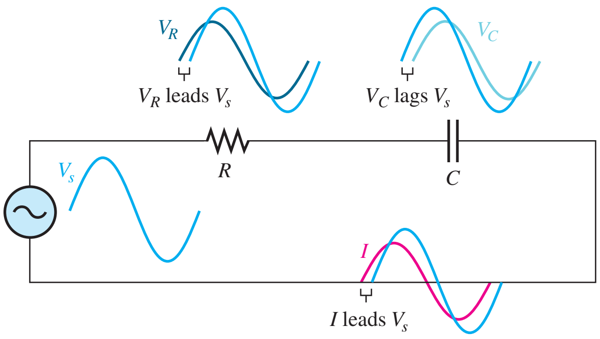
2 SINUSOIDAL RESPONSE OF SERIES *RC* CIRCUITS

When a sinusoidal voltage is applied to a series *RC* circuit, each resulting voltage drop and the current in the circuit are also sinusoidal and have the same frequency as the applied voltage. The capacitance causes a phase shift between the voltage and current that depends on the relative values of the resistance and the capacitive reactance.

After completing this section, you should be able to

- **Describe the relationship between current and voltage in a series *RC* circuit**
    - Discuss voltage and current waveforms
    - Discuss phase shift

As shown in Figure 14, the resistor voltage (V_R), the capacitor voltage (V_C), and the current (I) are all sine waves with the frequency of the source. Phase shifts are introduced because of the capacitance. The resistor voltage and current *lead* the source voltage, and the capacitor voltage *lags* the source voltage. The phase angle between the current and the capacitor voltage is always 90°. These generalized phase relationships are indicated in Figure 14.



▶ FIGURE 14

Illustration of sinusoidal response with general phase relationships of V_R, V_C, and I relative to the source voltage. V_R and I are in phase while V_R and V_C are 90° out of phase.

The amplitudes and the phase relationships of the voltages and current depend on the values of the resistance and the **capacitive reactance**. When a circuit is purely resistive, the phase angle between the applied (source) voltage and the total current is zero. When a circuit is purely capacitive, the phase angle between the applied voltage and the total current is 90°, with the current leading the voltage. When there is a combination of both resistance and capacitive reactance in a circuit, the phase angle between the applied voltage and the total current is somewhere between 0° and 90°, depending on the relative values of the resistance and the capacitive reactance.

SECTION 2 CHECKUP

1. A 60 Hz sinusoidal voltage is applied to an *RC* circuit. What is the frequency of the capacitor voltage? What is the frequency of the current?
2. What causes the phase shift between V_s and I in a series *RC* circuit?
3. When the resistance in a series *RC* circuit is greater than the capacitive reactance, is the phase angle between the applied voltage and the total current closer to 0° or to 90°?

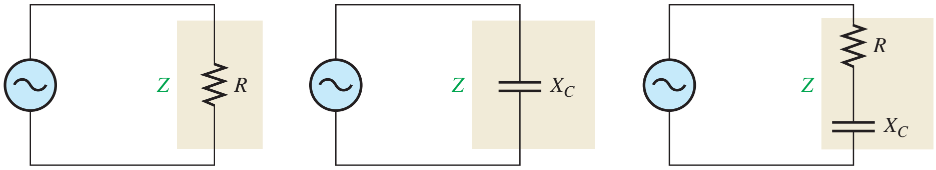
3 IMPEDANCE OF SERIES *RC* CIRCUITS

Impedance is the total opposition to sinusoidal current. Its unit is the ohm. The impedance of a series *RC* circuit consists of resistance and capacitive reactance. It causes a phase difference between the total current and the source voltage. Therefore, the impedance consists of a magnitude component and a phase angle component.

After completing this section, you should be able to

- **Determine the impedance of a series *RC* circuit**
  - Define *impedance*
  - Express capacitive reactance in complex form
  - Express total impedance in complex form
  - Draw an impedance triangle
  - Calculate impedance magnitude and the phase angle

In a purely resistive circuit, the impedance is simply equal to the total resistance. In a purely capacitive circuit, the impedance is equal to the total capacitive reactance. The impedance of a series *RC* circuit is determined by both the resistance and the capacitive reactance. These cases are illustrated in Figure 15. The magnitude of the impedance is symbolized by Z.



(a) $Z = R$ (b) $Z = X_C$ (c) Z includes both R and X_C

▲ FIGURE 15

Three cases of impedance.

Capacitive reactance is a phasor quantity and is expressed as a complex number in rectangular form as

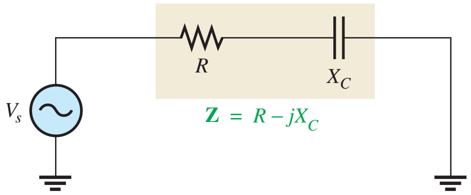
$$\mathbf{X}_C = -jX_C$$

where boldface $\mathbf{X}_C$ designates a phasor quantity (representing both magnitude and angle) and X_C is just the magnitude.

In the series *RC* circuit of Figure 16, the total impedance is the phasor sum of *R* and $-jX_C$ and is expressed as

Equation 7

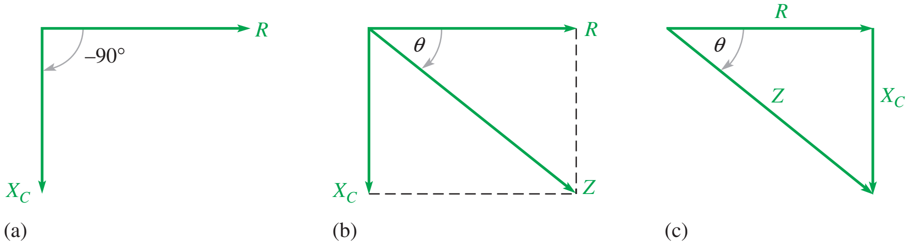
$$\mathbf{Z} = R - jX_C$$



▲ **FIGURE 16**

Impedance in a series *RC* circuit.

In ac analysis, both *R* and X_C are as shown in the phasor diagram of Figure 17(a), with X_C appearing at a $-90°$ angle with respect to *R*. This relationship comes from the fact that the capacitor voltage in a series *RC* circuit lags the current, and thus the resistor voltage, by 90°. Since $\mathbf{Z}$ is the phasor sum of *R* and $-jX_C$, its phasor representation is as shown in Figure 17(b). A repositioning of the phasors, as shown in part (c), forms a right triangle called the *impedance triangle*. The length of each phasor represents the magnitude in ohms, and the angle θ is the phase angle of the *RC* circuit and represents the phase difference between the applied voltage and the current.



▲ **FIGURE 17**

Development of the impedance triangle for a series *RC* circuit.

From right-angle trigonometry (Pythagorean theorem), the magnitude (length) of the impedance can be expressed in terms of the resistance and reactance as

$$Z = \sqrt{R^2 + X_C^2}$$

The italic letter *Z* represents the magnitude of the phasor quantity $\mathbf{Z}$ and is expressed in ohms.

The phase angle, θ, is expressed as

$$\theta = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

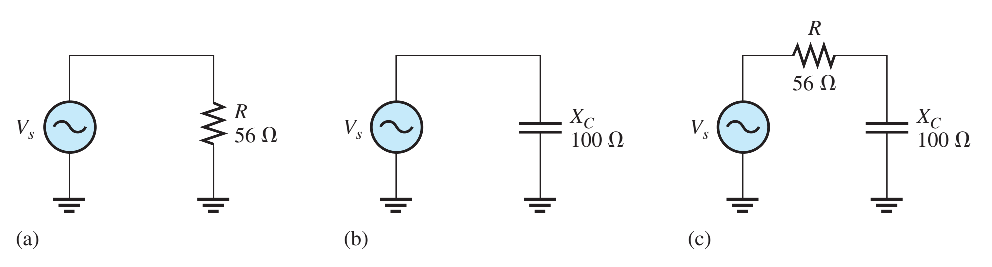
The symbol $\tan^{-1}$ stands for inverse tangent. You can find the $\tan^{-1}$ value on your calculator. Combining the magnitude and angle, the phasor expression for impedance in polar form is

Equation 8

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

EXAMPLE 8

For each circuit in Figure 18, write the phasor expression for the impedance in both rectangular form and polar form.



▲ **FIGURE 18**

Solution For the circuit in Figure 18(a), the impedance is

$$\mathbf{Z} = R - j0 = R = \mathbf{56\ \Omega} \qquad \text{in rectangular form } (X_C = 0)$$

$$\mathbf{Z} = R\angle 0^\circ = \mathbf{56\angle 0^\circ\ \Omega} \qquad \text{in polar form}$$

The impedance is simply the resistance, and the phase angle is zero because pure resistance does not cause a phase shift between the voltage and current.

For the circuit in Figure 18(b), the impedance is

$$\mathbf{Z} = 0 - jX_C = \mathbf{-j100\ \Omega} \qquad \text{in rectangular form } (R = 0)$$

$$\mathbf{Z} = X_C\angle -90^\circ = \mathbf{100\angle -90^\circ\ \Omega} \qquad \text{in polar form}$$

The impedance is simply the capacitive reactance, and the phase angle is -90° because the capacitance causes the current to lead the voltage by 90°.

For the circuit in Figure 18(c), the impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = \mathbf{56\ \Omega - j100\ \Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(56\ \Omega)^2 + (100\ \Omega)^2}\angle -\tan^{-1}\left(\frac{100\ \Omega}{56\ \Omega}\right) = \mathbf{115\angle -60.8^\circ\ \Omega}$$

In this case, the impedance is the phasor sum of the resistance and the capacitive reactance. The phase angle is fixed by the relative values of X_C and R. Rectangular to polar conversion can be done on a calculator (refer to your user's manual).

Related Problem Use your calculator to convert the impedance in Figure 18(c) from rectangular to polar form. Draw the impedance phasor diagram.

SECTION 3 CHECKUP

1. The impedance of a certain *RC* circuit is $150\ \Omega - j220\ \Omega$. What is the value of the resistance? The capacitive reactance?
2. A series *RC* circuit has a total resistance of 33 kΩ and a capacitive reactance of 50 kΩ. Write the phasor expression for the impedance in rectangular form.
3. For the circuit in Question 2, what is the magnitude of the impedance? What is the phase angle?

4 ANALYSIS OF SERIES *RC* CIRCUITS

Ohm's law and Kirchhoff's voltage law are used in the analysis of series *RC* circuits to determine voltage, currents and impedance. Also, in this section *RC* lead and lag circuits are examined.

After completing this section, you should be able to

- **Analyze a series *RC* circuit**
  - Apply Ohm's law and Kirchhoff's voltage law to series *RC* circuits
  - Express the voltages and current as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Analyze the *RC* lag circuit
  - Analyze the *RC* lead circuit

Ohm's Law

The application of Ohm's law to series *RC* circuits involves the use of the phasor quantities of **Z**, **V**, and **I**. Keep in mind that the use of boldface nonitalic letters indicates phasor quantities where both magnitude and angle are included. The three equivalent forms of Ohm's law are as follows:

Equation 9

$$\mathbf{V} = \mathbf{IZ}$$

Equation 10

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$

Equation 11

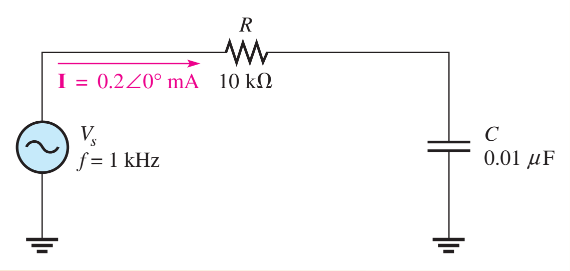
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Recall that multiplication and division are most easily accomplished with the polar forms. Since Ohm's law calculations involve multiplications and divisions, you should express the voltage, current, and impedance in polar form. The following two examples show the relationship between the source voltage and source current. In Example 9, the current is the reference and in Example 10, the voltage is the reference. Notice that the reference is drawn along the x-axis in both cases.

EXAMPLE 9

The current in Figure 19 is expressed in polar form as $\mathbf{I} = 0.2\angle 0^\circ$ mA. Determine the source voltage expressed in polar form, and draw a phasor diagram showing the relation between source voltage and current.

▶ FIGURE 19



Solution The magnitude of the capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1000\text{ Hz})(0.01\ \mu\text{F})} = 15.9\text{ k}\Omega$$

The total impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = 10\text{ k}\Omega - j15.9\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

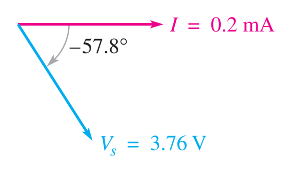
$$= \sqrt{(10\text{ k}\Omega)^2 + (15.9\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{15.9\text{ k}\Omega}{10\text{ k}\Omega}\right) = 18.8\angle -57.8^\circ\text{ k}\Omega$$

Use Ohm's law to determine the source voltage.

$$\mathbf{V}_s = \mathbf{IZ} = (0.2\angle 0^\circ\text{ mA})(18.8\angle -57.8^\circ\text{ k}\Omega) = \mathbf{3.76\angle -57.8^\circ\ V}$$

The magnitude of the source voltage is 3.76 V at an angle of −57.8° with respect to the current; that is, the voltage lags the current by 57.8°, as shown in the phasor diagram of Figure 20.

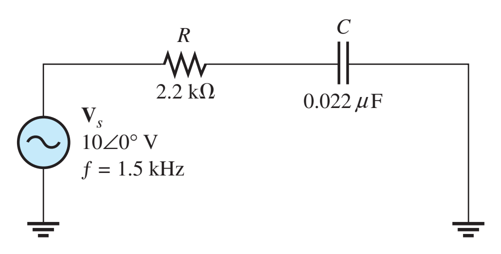
▶ **FIGURE 20**



Related Problem Determine $\mathbf{V}_s$ in Figure 19 if $f = 2$ kHz and $\mathbf{I} = 0.2\angle 0^\circ$ A.

EXAMPLE 10 Determine the current in the circuit of Figure 21, and draw a phasor diagram showing the relation between source voltage and current.

▶ **FIGURE 21**



Solution The magnitude of the capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.5\text{ kHz})(0.022\ \mu\text{F})} = 4.82\text{ k}\Omega$$

The total impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = 2.2\text{ k}\Omega - j4.82\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

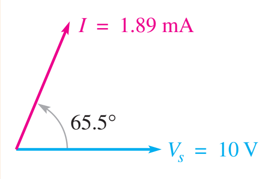
$$= \sqrt{(2.2\text{ k}\Omega)^2 + (4.82\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{4.82\text{ k}\Omega}{2.2\text{ k}\Omega}\right) = 5.30\angle -65.5^\circ\text{ k}\Omega$$

Use Ohm's law to determine the current.

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\angle 0^\circ\text{ V}}{5.30\angle -65.5^\circ\text{ k}\Omega} = \mathbf{1.89\angle 65.5^\circ\text{ mA}}$$

The magnitude of the current is 1.89 mA. The positive phase angle of 65.5° indicates that the current leads the voltage by that amount, as shown in the phasor diagram of Figure 22.

▶ **FIGURE 22**



Related Problem Determine **I** in Figure 21 if the frequency is increased to 5 kHz.

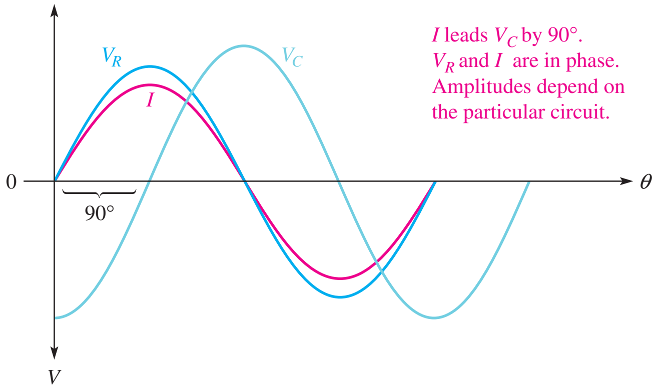
Use Multisim file E15-10 to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Current and Voltages

In a series *RC* circuit, the current is the same through both the resistor and the capacitor. Thus, the resistor voltage is in phase with the current, and the capacitor voltage lags the current by 90°. Therefore, there is a phase difference of 90° between the resistor voltage, V_R, and the capacitor voltage, V_C, as shown in the waveform diagram of Figure 23.

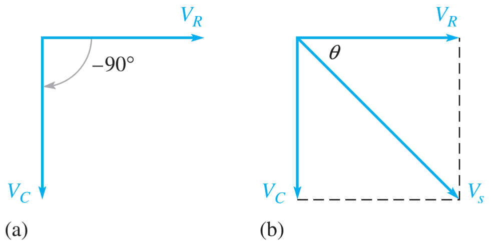
▶ **FIGURE 23**

Phase relation of voltages and current in a series *RC* circuit.



I leads V_C by 90°.
V_R and I are in phase.
Amplitudes depend on the particular circuit.

From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage. However, since V_R and V_C are not in phase with each other, they must be added as phasor quantities, with V_C lagging V_R by 90°, as shown in Figure 24(a). As shown in



▲ **FIGURE 24**

Voltage phasor diagram for a series *RC* circuit.

Figure 24(b), $\mathbf{V}_s$ is the phasor sum of V_R and V_C, as expressed in rectangular form in the following equation:

Equation 12

$$\mathbf{V}_s = V_R - jV_C$$

This equation can be expressed in polar form as

Equation 13

$$\mathbf{V}_s = \sqrt{V_R^2 + V_C^2}\angle -\tan^{-1}\left(\frac{V_C}{V_R}\right)$$

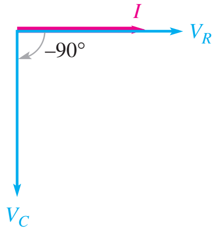
where the magnitude of the source voltage is

$$V_s = \sqrt{V_R^2 + V_C^2}$$

and the phase angle between the resistor voltage and the source voltage is

$$\theta = -\tan^{-1}\left(\frac{V_C}{V_R}\right)$$

Since the resistor voltage and the current are in phase, θ also represents the phase angle between the source voltage and the current. Figure 25 shows a complete voltage and current phasor diagram that represents the waveform diagram of Figure 23.



▲ **FIGURE 25**

Voltage and current phasor diagram for the waveforms in Figure 23.

Variation of Impedance and Phase Angle with Frequency

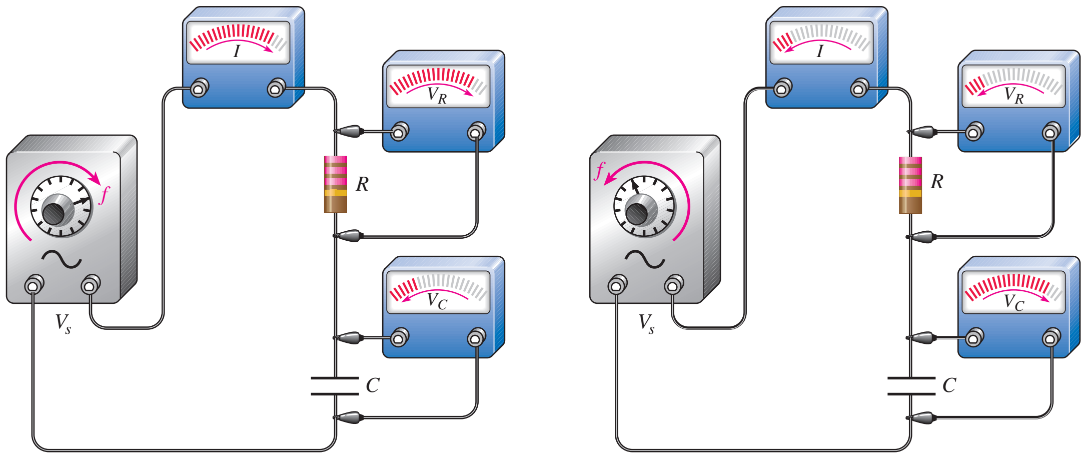
As you know, capacitive reactance varies inversely with frequency. Since $Z = \sqrt{R^2 + X_C^2}$, you can see that when X_C increases, the entire term under the square root sign increases and thus the magnitude of the total impedance also increases; and when X_C decreases, the magnitude of the total impedance also decreases. Therefore, *in a series RC circuit, Z is inversely dependent on frequency.*

Figure 26 illustrates how the voltages and current in a series *RC* circuit vary as the frequency increases or decreases, with the source voltage held at a constant value. In part (a), as the frequency is increased, X_C decreases; so less voltage is dropped across the capacitor. Also, Z decreases as X_C decreases, causing the current to increase. An increase in the current causes more voltage across R.

In Figure 26(b), as the frequency is decreased, X_C increases; so more voltage is dropped across the capacitor. Also, Z increases as X_C increases, causing the current to decrease. A decrease in the current causes less voltage across R.

Changes in Z and X_C can be observed as shown in Figure 27. As the frequency increases, the voltage across Z remains constant because V_s is constant. Also, the voltage across C decreases. The increasing current indicates that Z is decreasing. It does so because of the inverse relationship stated in Ohm's law ($Z = V_Z/I$). The increasing current also indicates that X_C is decreasing ($X_C = V_C/I$). The decrease in V_C corresponds to the decrease in X_C.

Since X_C is the factor that introduces the phase angle in a series *RC* circuit, a change in X_C produces a change in the phase angle. As the frequency is increased, X_C becomes

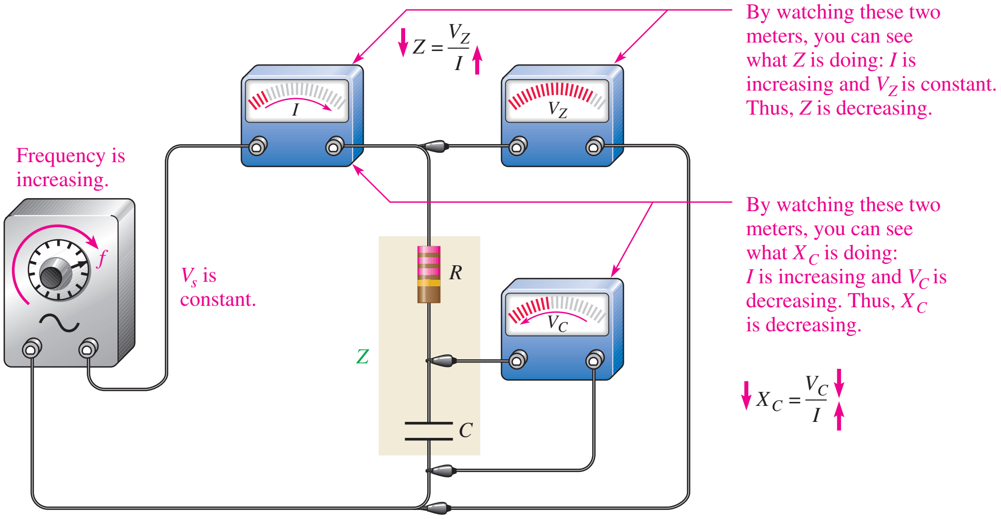


(a) As frequency is increased, Z decreases as X_C decreases, causing I and V_R to increase and V_C to decrease.

(b) As frequency is decreased, Z increases as X_C increases, causing I and V_R to decrease and V_C to increase.

▲ **FIGURE 26**

An illustration of how the variation of impedance affects the voltages and current as the source frequency is varied. The source voltage is held at a constant amplitude.


$\downarrow Z = \frac{V_Z}{I}\uparrow$
I
V_Z
R
Z
V_C
C
f
$\downarrow X_C = \frac{V_C \downarrow}{I \uparrow}$

Frequency is increasing.

V_s is constant.

By watching these two meters, you can see what Z is doing: I is increasing and V_Z is constant. Thus, Z is decreasing.

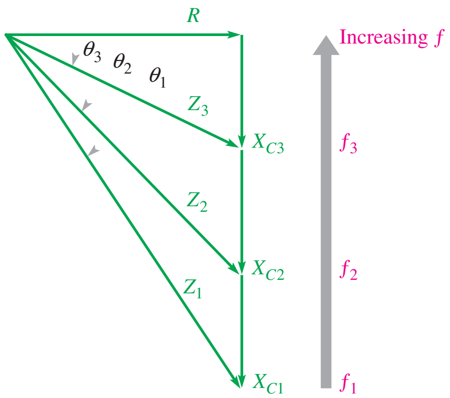
By watching these two meters, you can see what X_C is doing: I is increasing and V_C is decreasing. Thus, X_C is decreasing.

▲ **FIGURE 27**

An illustration of how Z and X_C change with frequency.

smaller, and thus the phase angle decreases. As the frequency is decreased, X_C becomes larger, and thus the phase angle increases. The angle between V_s and V_R is the phase angle of the circuit because I is in phase with V_R. By measuring the phase of V_R, you are effectively measuring the phase of I. An oscilloscope is normally used to observe the phase angle by measuring the phase angle between V_s and one of the component voltages.

Figure 28 uses the impedance triangle to illustrate the variations in X_C, Z, and θ as the frequency changes. Of course, R remains constant. The main point is that because X_C varies inversely with the frequency, so also do the magnitude of the total impedance and the phase angle. Example 11 illustrates this.



▲ FIGURE 28

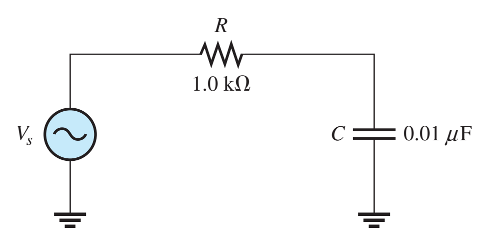
As the frequency increases, X_C decreases, Z decreases, and θ decreases. Each value of frequency can be visualized as forming a different impedance triangle.

EXAMPLE 11

For the series *RC* circuit in Figure 29, determine the magnitude of the total impedance and the phase angle for each of the following values of input frequency:

(a) 10 kHz **(b)** 20 kHz **(c)** 30 kHz

► FIGURE 29



Solution **(a)** For $f = 10\text{ kHz}$,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.59\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{1.59\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.88\angle -57.8^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{1.88\text{ k}\Omega}$ and $\theta = \mathbf{-57.8^\circ}$.

(b) For $f = 20\text{ kHz}$,

$$X_C = \frac{1}{2\pi(20\text{ kHz})(0.01\ \mu\text{F})} = 796\ \Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (796\ \Omega)^2}\angle -\tan^{-1}\left(\frac{796\ \Omega}{1.0\text{ k}\Omega}\right) = 1.28\angle -38.5^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{1.28\text{ k}\Omega}$ and $\theta = \mathbf{-38.5^\circ}$.

(c) For $f = 30\text{ kHz}$,

$$X_C = \frac{1}{2\pi(30\text{ kHz})(0.01\ \mu\text{F})} = 531\ \Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (531\ \Omega)^2}\angle -\tan^{-1}\left(\frac{531\ \Omega}{1.0\text{ k}\Omega}\right) = 1.13\angle -28.0°\text{ k}\Omega$$

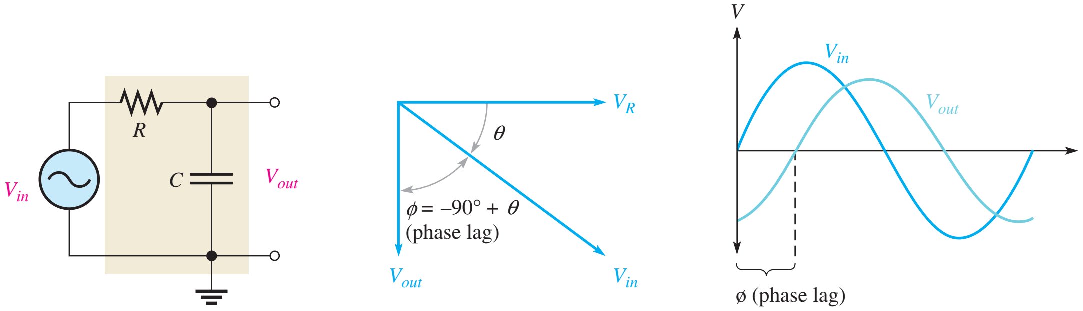
Thus, $Z = \mathbf{1.13\ k\Omega}$ and $\theta = \mathbf{-28.0°}$.

Notice that as the frequency increases, X_C, Z, and θ decrease.

Related Problem Find the magnitude of the total impedance and the phase angle in Figure 29 for $f = 1\text{ kHz}$.

The *RC* Lag Circuit

An ***RC* lag circuit** is a phase shift circuit in which the output voltage lags the input voltage by a specified amount. Figure 30(a) shows a series *RC* circuit with the output voltage taken across the capacitor. The source voltage is the input, V_{in}. As you know, θ, the phase angle between the current and the input voltage, is also the phase angle between the resistor voltage and the input voltage because V_R and I are in phase with each other.



(a) A basic *RC* lag circuit

(b) Phasor voltage diagram showing the phase lag between V_{in} and V_{out}

(c) Input and output voltage waveforms

▲ **FIGURE 30**

RC lag circuit ($V_{out} = V_C$).

Since V_C lags V_R by 90°, the phase angle between the capacitor voltage and the input voltage is the difference between $-90°$ and θ, as shown in Figure 30(b). The capacitor voltage is the output, and it lags the input, thus creating a basic lag circuit.

The input and output voltage waveforms of the lag circuit are shown in Figure 30(c). The amount of phase difference, designated ϕ, between the input and the output is dependent on the relative sizes of the capacitive reactance and the resistance, as is the magnitude of the output voltage.

Phase Difference Between Input and Output As already established, θ is the phase angle between I and V_{in}. The angle between V_{out} and V_{in} is designated ϕ (phi) and is developed as follows.

The polar expressions for the input voltage and the current are $V_{in}\angle 0°$ and $I\angle\theta$, respectively. The output voltage in polar form is

$$\mathbf{V}_{out} = (I\angle\theta)(X_C\angle -90°) = IX_C\angle(-90° + \theta)$$

The preceding equation states that the output voltage is at an angle of $-90° + \theta$ with respect to the input voltage. Since $\theta = -\tan^{-1}(X_C/R)$, the angle ϕ between the input and output is

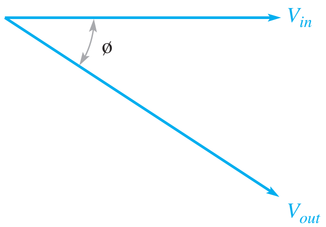
$$\phi = -90° + \tan^{-1}\left(\frac{X_C}{R}\right)$$

Equivalently, this angle can be expressed as

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

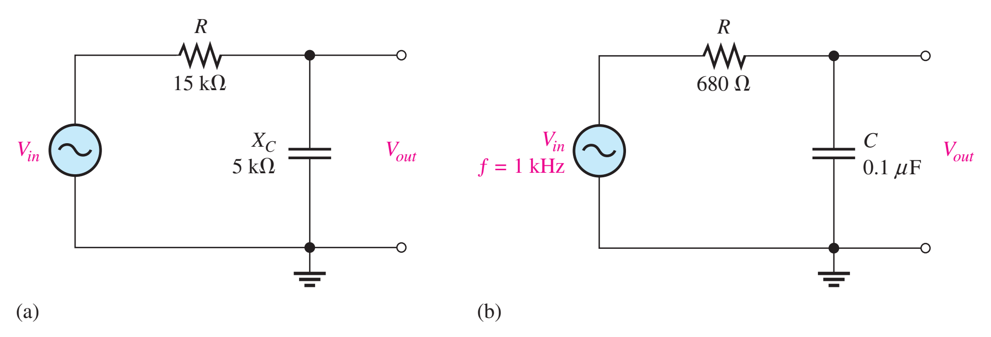
Equation 14

This angle is always negative, indicating that the output voltage lags the input voltage, as shown in Figure 31.



◀ FIGURE 31

EXAMPLE 12 Determine the amount of phase lag from input to output in each lag circuit in Figure 32.



▲ FIGURE 32

Solution For the lag circuit in Figure 32(a),

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{15\text{ k}\Omega}{5\text{ k}\Omega}\right) = \mathbf{-71.6°}$$

The output lags the input by 71.6°.

For the lag circuit in Figure 32(b), first determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.1\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{680\ \Omega}{1.59\text{ k}\Omega}\right) = \mathbf{-23.2°}$$

The output lags the input by 23.2°.

Related Problem In a lag circuit, what happens to the phase lag if the frequency increases?

Use Multisim files E15-12A, E15-12B, and E15-12C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage To evaluate the output voltage in terms of its magnitude, visualize the *RC* lag circuit as a voltage divider. A portion of the total input voltage is dropped across the resistor and a portion across the capacitor. Because the output voltage is the voltage across the capacitor, it can be calculated using either Ohm's law ($V_{out} = IX_C$) or the voltage divider formula.

Equation 15

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

The phasor expression for the output voltage of an *RC* lag circuit is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 13

For the lag circuit in Figure 32(b) (Example 12), determine the output voltage in phasor form when the input voltage has an rms value of 10 V. Draw the input and output voltage waveforms showing the proper phase relationship. The capacitive reactance X_C (1.59 kΩ) and ϕ (−23.2°) were found in Example 12.

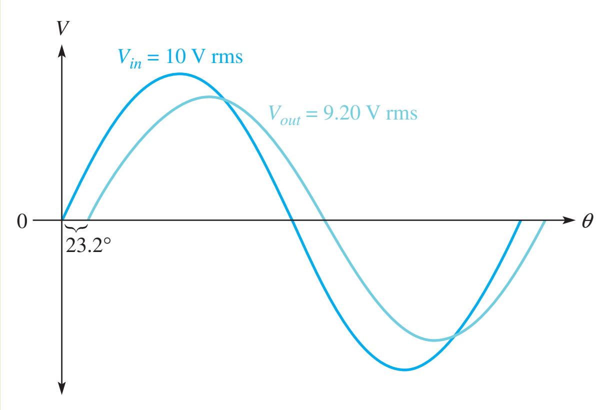
Solution The output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.59\text{ k}\Omega}{\sqrt{(680\ \Omega)^2 + (1.59\text{ k}\Omega)^2}}\right)10\angle-23.2^\circ\text{ V} = \mathbf{9.20\angle-23.2^\circ\text{ V rms}}$$

The waveforms are shown in Figure 33. Notice that the output voltage lags the input voltage by 23.2°.

▶ FIGURE 33

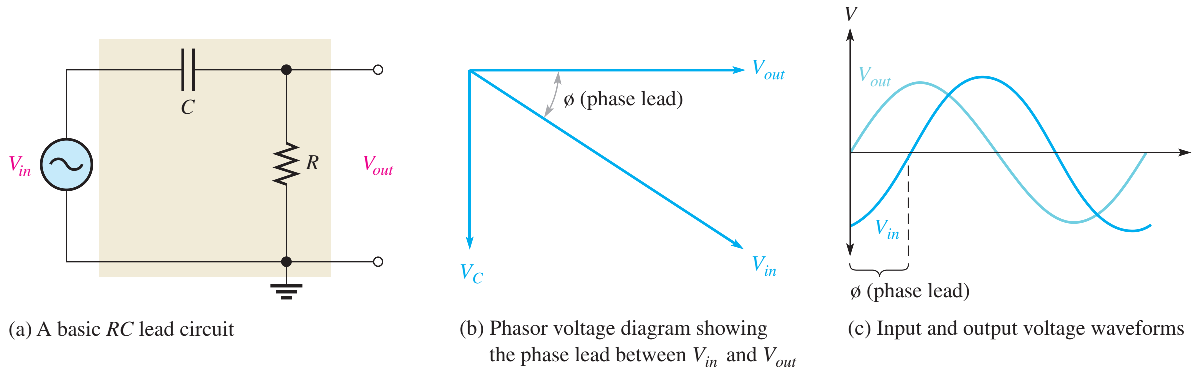


Related Problem In a lag circuit, what happens to the output voltage if the frequency increases?

Use Multisim files E15-13A and E15-13B to verify the calculated results in this example and to confirm your calculation for the related problem.

The *RC* Lead Circuit

An ***RC* lead circuit** is a phase shift circuit in which the output voltage leads the input voltage by a specified amount. When the output of a series *RC* circuit is taken across the resistor rather than across the capacitor, as shown in Figure 34(a), it becomes a lead circuit.



(a) A basic *RC* lead circuit

(b) Phasor voltage diagram showing the phase lead between V_{in} and V_{out}

(c) Input and output voltage waveforms

▲ **FIGURE 34**

RC lead circuit ($V_{out} = V_R$).

Phase Difference Between Input and Output In a series *RC* circuit, the current leads the input voltage. Also, as you know, the resistor voltage is in phase with the current. Since the output voltage is taken across the resistor, the output leads the input, as indicated by the phasor diagram in Figure 34(b). The waveforms are shown in Figure 34(c).

As in the lag circuit, the amount of phase difference between the input and output and the magnitude of the output voltage in the lead circuit are dependent on the relative values of the resistance and the capacitive reactance. When the input voltage is assigned a reference angle of 0°, the angle of the output voltage is the same as θ (the angle between total current and applied voltage) because the resistor voltage (output) and the current are in phase with each other. Therefore, since $\phi = \theta$ in this case, the expression is

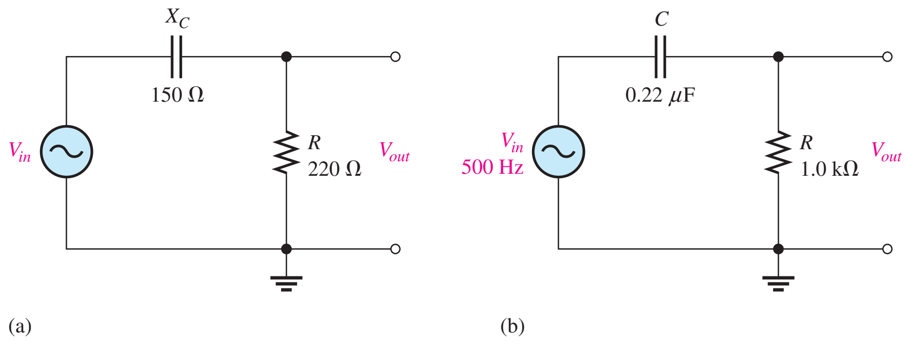
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Equation 16

This angle is positive because the output leads the input.

EXAMPLE 14

Calculate the output phase angle for each circuit in Figure 35.



▲ **FIGURE 35**

Solution For the lead circuit in Figure 35(a),

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{150\ \Omega}{220\ \Omega}\right) = \mathbf{34.3^\circ}$$

The output leads the input by 34.3°.

For the lead circuit in Figure 35(b), first determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(500\text{ Hz})(0.22\ \mu\text{F})} = 1.45\text{ k}\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1.45\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = \mathbf{55.4^\circ}$$

The output leads the input by 55.4°.

Related Problem In a lead circuit, what happens to the phase lead if the frequency increases?

Use Multisim files E15-14A, E15-14B, and E15-14C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage Since the output voltage of an *RC* lead circuit is taken across the resistor, the magnitude can be calculated using either Ohm's law ($V_{out} = IR$) or the voltage-divider formula.

Equation 17

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

The expression for the output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 15 The input voltage in Figure 35(b) (Example 14) has an rms value of 10 V. Determine the phasor expression for the output voltage. Draw the waveform relationships for the input and output voltages showing peak values. The phase angle (55.4°) and X_C (1.45 kΩ) were found in Example 14.

Solution The phasor expression for the output voltage is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.0\text{ k}\Omega}{1.76\text{ k}\Omega}\right)10\angle 55.4^\circ\text{ V} = \mathbf{5.68\angle 55.4^\circ\ V\ rms}$$

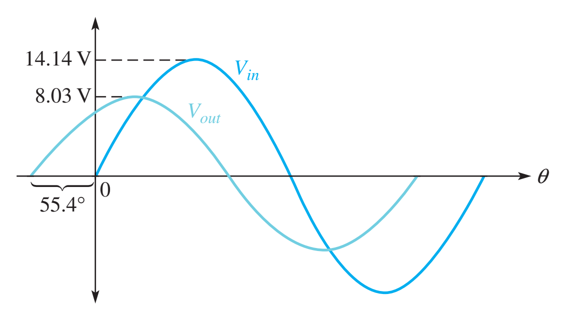
The peak value of the input voltage is

$$V_{in(p)} = 1.414V_{in(rms)} = 1.414(10\text{ V}) = 14.14\text{ V}$$

The peak value of the output voltage is

$$V_{out(p)} = 1.414V_{out(rms)} = 1.414(5.68\text{ V}) = 8.03\text{ V}$$

The waveforms are shown in Figure 36.



▲ **FIGURE 36**

Related Problem In a lead circuit, what happens to the output voltage if the frequency is reduced?

Use Multisim files E15-15A and E15-15B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 4 CHECKUP

1. In a certain series *RC* circuit, $V_R = 4$ V, and $V_C = 6$ V. What is the magnitude of the source voltage?
2. In Question 1, what is the phase angle between the source voltage and the current?
3. What is the phase difference between the capacitor voltage and the resistor voltage in a series *RC* circuit?
4. When the frequency of the applied voltage in a series *RC* circuit is increased, what happens to the capacitive reactance? What happens to the magnitude of the total impedance? What happens to the phase angle?
5. A certain *RC* lag circuit consists of a 4.7 kΩ resistor and a 0.022 μF capacitor. Determine the phase shift between input and output at a frequency of 3 kHz.
6. An *RC* lead circuit has the same component values as the lag circuit in Question 5. What is the magnitude of the output voltage at 3 kHz when the input is 10 V rms?

PARALLEL CIRCUITS

5 IMPEDANCE AND ADMITTANCE OF PARALLEL *RC* CIRCUITS

In this section, you will learn how to determine the impedance and phase angle of a parallel *RC* circuit. The impedance consists of a magnitude component and a phase angle component. Also, capacitive susceptance and admittance of a parallel *RC* circuit are introduced.

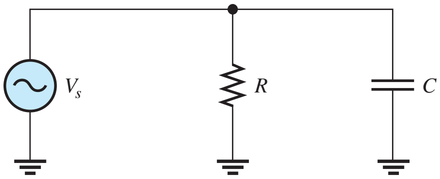
After completing this section, you should be able to

- **Determine impedance and admittance in a parallel *RC* circuit**
    - Express total impedance in complex form
    - Define and calculate *conductance, capacitive susceptance,* and *admittance*

Figure 37 shows a basic parallel *RC* circuit connected to an ac voltage source.

▶ FIGURE 37

Basic parallel *RC* circuit.



The expression for the total impedance is developed as follows, using complex numbers. Since there are only two circuit components, R and C, the total impedance can be found from the product-over-sum rule.

$$\mathbf{Z} = \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C}$$

By multiplying the magnitudes, adding the angles in the numerator, and converting the denominator to polar form, you get

$$\mathbf{Z} = \frac{RX_C\angle(0^\circ - 90^\circ)}{\sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\dfrac{X_C}{R}\right)}$$

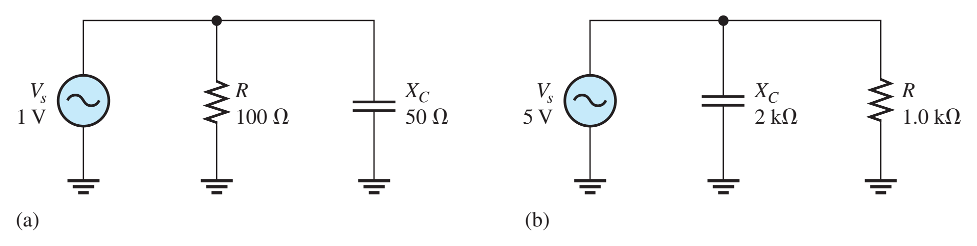
Now, by dividing the magnitude expression in the numerator by that in the denominator, and by subtracting the angle in the denominator from that in the numerator, you get

$$\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle\left(-90^\circ + \tan^{-1}\left(\frac{X_C}{R}\right)\right)$$

Equivalently, this expression can be written as

$$\mathbf{Z} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$ **Equation 18**

EXAMPLE 16 For each circuit in Figure 38, determine the magnitude and phase angle of the total impedance.



▲ FIGURE 38

Solution For the circuit in Figure 38(a), the total impedance is

$$\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$

$$= \left(\frac{(100\ \Omega)(50\ \Omega)}{\sqrt{(100\ \Omega)^2 + (50\ \Omega)^2}}\right)\angle -\tan^{-1}\left(\frac{100\ \Omega}{50\ \Omega}\right) = \mathbf{44.7\angle -63.4^\circ\ \Omega}$$

Thus, $Z = 44.7\ \Omega$ and $\theta = -63.4^\circ$.

For the circuit in Figure 38(b), the total impedance is

$$\mathbf{Z} = \left(\frac{(1.0\ \text{k}\Omega)(2\ \text{k}\Omega)}{\sqrt{(1.0\ \text{k}\Omega)^2 + (2\ \text{k}\Omega)^2}}\right)\angle -\tan^{-1}\left(\frac{1.0\ \text{k}\Omega}{2\ \text{k}\Omega}\right) = \mathbf{894\angle -26.6^\circ\ \Omega}$$

Thus, $Z = 894\ \Omega$ and $\theta = -26.6^\circ$.

Related Problem Determine **Z** in Figure 38(a) if the frequency is doubled.

Conductance, Susceptance, and Admittance

Recall that **conductance, *G***, is the reciprocal of resistance. The phasor expression for conductance is expressed as

$$\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

Two new terms are now introduced for use in parallel *RC* circuits. **Capacitive susceptance (B_C)** is the reciprocal of capacitive reactance. The phasor expression for capacitive susceptance is

$$\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = +jB_C$$

Admittance (Y) is the reciprocal of impedance. The phasor expression for admittance is

$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta$$

The unit of each of these terms is the siemens (S), which is the reciprocal of the ohm.

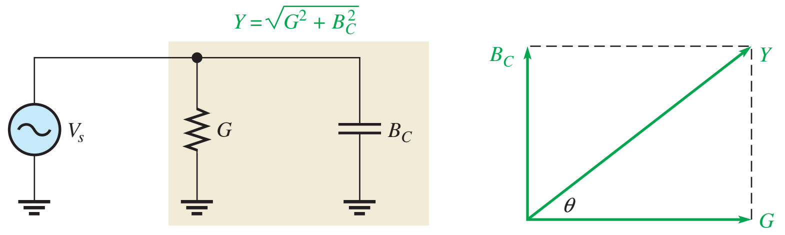
In working with parallel circuits, it is often easier to use conductance (G), capacitive susceptance (B_C), and admittance (Y) rather than resistance (R), capacitive reactance (X_C), and impedance (Z). In a parallel RC circuit, as shown in Figure 39, the total admittance is simply the phasor sum of the conductance and the capacitive susceptance.

Equation 19

$$\mathbf{Y} = G + jB_C$$

▶ FIGURE 39

Admittance in a parallel RC circuit.

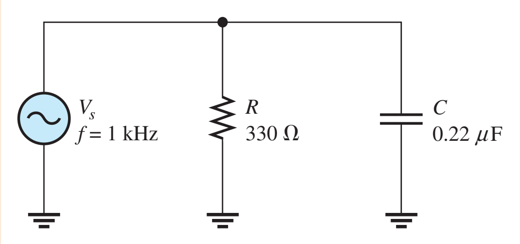


(a) (b)

EXAMPLE 17

Determine the total admittance ($\mathbf{Y}$) and then convert it to total impedance ($\mathbf{Z}$) in Figure 40. Draw the admittance phasor diagram.

▶ FIGURE 40



Solution From Figure 40, $R = 330\ \Omega$; thus $G = 1/R = 1/330\ \Omega = 3.03$ mS. The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1000\text{ Hz})(0.22\ \mu\text{F})} = 723\ \Omega$$

The capacitive susceptance magnitude is

$$B_C = \frac{1}{X_C} = \frac{1}{723\ \Omega} = 1.38\text{ mS}$$

The total admittance is

$$\mathbf{Y}_{tot} = G + jB_C = \mathbf{3.03\ mS + \mathit{j}1.38\ mS}$$

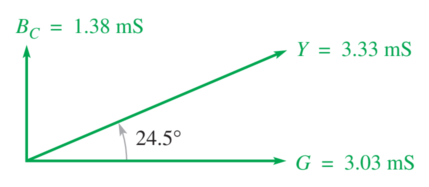
which can be expressed in polar form as

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(3.03\text{ mS})^2 + (1.38\text{ mS})^2}\angle\tan^{-1}\left(\frac{1.38\text{ mS}}{3.03\text{ mS}}\right) = \mathbf{3.33\angle 24.5^\circ\ mS}$$

The admittance phasor diagram is shown in Figure 41.

▶ **FIGURE 41**



Convert total admittance to total impedance as follows:

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}_{tot}} = \frac{1}{(3.33\angle 24.5^\circ \text{ mS})} = \mathbf{300\angle -24.5\ \Omega}$$

Related Problem Calculate the total admittance in Figure 40 if f is increased to 2.5 kHz.

SECTION 5 CHECKUP

1. Define *conductance, capacitive susceptance,* and *admittance.*
2. If $Z = 100\ \Omega$, what is the value of Y?
3. In a certain parallel RC circuit, $R = 47\ \text{k}\Omega$ and $X_C = 75\ \text{k}\Omega$. Determine $\mathbf{Y}$.
4. In Question 3, what is $\mathbf{Z}$?

6 ANALYSIS OF PARALLEL *RC* CIRCUITS

Ohm's law and Kirchhoff's current law are used in the analysis of *RC* circuits. Current and voltage relationships in a parallel *RC* circuit are examined.

After completing this section, you should be able to

- **Analyze a parallel *RC* circuit**
  - Apply Ohm's law and Kirchhoff's current law to parallel *RC* circuits
  - Express the voltages and currents as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Convert from a parallel circuit to an equivalent series circuit

For convenience in the analysis of parallel circuits, the Ohm's law formulas using impedance, previously stated, can be rewritten for admittance using the relation $Y = 1/Z$. Remember, the use of boldface nonitalic letters indicates phasor quantities.

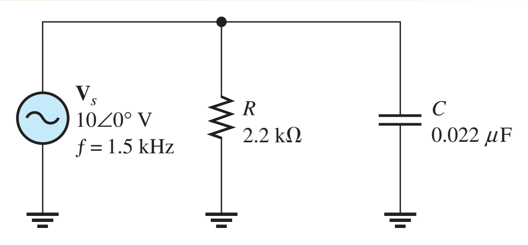
$$\mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}}$$ **Equation 20**

$$\mathbf{I} = \mathbf{VY}$$ **Equation 21**

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$$ **Equation 22**

EXAMPLE 18

Determine the total current and phase angle in Figure 42. Draw a phasor diagram showing the relationship of $\mathbf{V}_s$ and $\mathbf{I}_{tot}$.



▲ FIGURE 42

Solution The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.5\text{ kHz})(0.022\ \mu\text{F})} = 4.82\text{ k}\Omega$$

The capacitive susceptance magnitude is

$$B_C = \frac{1}{X_C} = \frac{1}{4.82\text{ k}\Omega} = 207\ \mu\text{S}$$

The conductance magnitude is

$$G = \frac{1}{R} = \frac{1}{2.2\text{ k}\Omega} = 455\ \mu\text{S}$$

The total admittance is

$$\mathbf{Y}_{tot} = G + jB_C = 455\ \mu\text{S} + j207\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(455\ \mu\text{S})^2 + (207\ \mu\text{S})^2}\angle\tan^{-1}\left(\frac{207\ \mu\text{S}}{455\ \mu\text{S}}\right) = 500\angle 24.5°\ \mu\text{S}$$

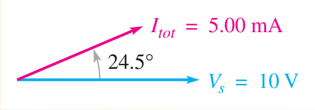
The phase angle is 24.5°.

Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \mathbf{V}_s\mathbf{Y}_{tot} = (10\angle 0°\text{ V})(500\angle 24.5°\ \mu\text{S}) = \mathbf{5.00\angle 24.5°\ mA}$$

The magnitude of the total current is 5.00 mA, and it leads the applied voltage by **24.5°**, as the phasor diagram in Figure 43 indicates.

▶ FIGURE 43

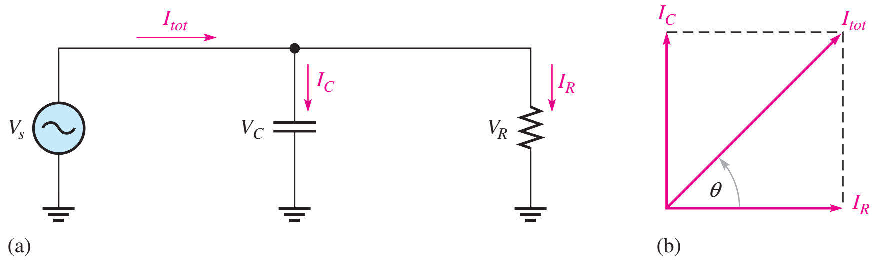


Related Problem What is the total current (in polar form) if *f* is doubled?

Use Multisim files E15-18A and E15-18B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Currents and Voltages

Figure 44(a) shows all the currents in a basic parallel *RC* circuit. The total current, I_{tot}, divides at the junction into the two branch currents, I_R and I_C. The applied voltage, V_s, appears across both the resistive and the capacitive branches, so V_s, V_R, and V_C are all in phase and of the same magnitude.



▲ **FIGURE 44**

Currents in a parallel *RC* circuit. The current directions shown in (a) are instantaneous and, of course, reverse when the source voltage reverses.

The current through the resistor is in phase with the voltage. The current through the capacitor leads the voltage, and thus the resistive current, by 90°. By Kirchhoff's current law, the total current is the phasor sum of the two branch currents, as shown by the phasor diagram in Figure 44(b). The total current is expressed as

$$\mathbf{I}_{tot} = I_R + jI_C$$

Equation 23

This equation can be expressed in polar form as

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

Equation 24

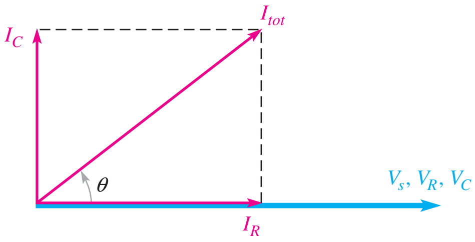
where the magnitude of the total current is

$$I_{tot} = \sqrt{I_R^2 + I_C^2}$$

and the phase angle between the resistor current and the total current is

$$\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

Since the resistor current and the applied voltage are in phase, θ also represents the phase angle between the total current and the applied voltage. Figure 45 shows a complete current and voltage phasor diagram.

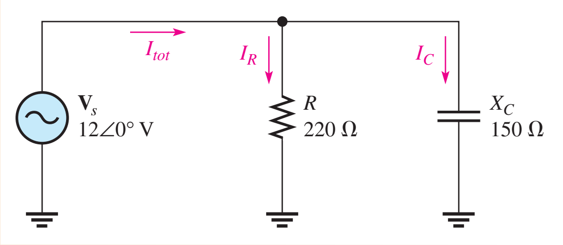


▲ **FIGURE 45**

Current and voltage phasor diagram for a parallel *RC* circuit (amplitudes depend on the particular circuit).

EXAMPLE 19 Determine the value of each current in Figure 46, and describe the phase relationship of each with the applied voltage. Draw the current phasor diagram.

▶ FIGURE 46



Solution The resistor current, the capacitor current, and the total current are expressed as follows:

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{12\angle 0°\text{ V}}{220\angle 0°\ \Omega} = \mathbf{54.5\angle 0°\ mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}_s}{\mathbf{X}_C} = \frac{12\angle 0°\text{ V}}{150\angle -90°\ \Omega} = \mathbf{80\angle 90°\ mA}$$

$$\mathbf{I}_{tot} = I_R + jI_C = 54.5\text{ mA} + j80\text{ mA}$$

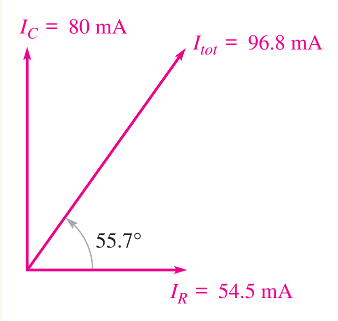
Converting $\mathbf{I}_{tot}$ to polar form yields

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$= \sqrt{(54.5\text{ mA})^2 + (80\text{ mA})^2}\angle\tan^{-1}\left(\frac{80\text{ mA}}{54.5\text{ mA}}\right) = \mathbf{96.8\angle 55.7°\ mA}$$

As the results show, the resistor current is 54.5 mA and is in phase with the voltage. The capacitor current is 80 mA and leads the voltage by 90°. The total current is 96.8 mA and leads the voltage by 55.7°. The phasor diagram in Figure 47 illustrates these relationships.

▶ FIGURE 47



Related Problem In a parallel circuit, $\mathbf{I}_R = 100\angle 0°$ mA and $\mathbf{I}_C = 60\angle 90°$ mA. Determine the total current.

Conversion from Parallel to Series Form

For every parallel *RC* circuit, there is an equivalent series *RC* circuit for a given frequency. Two circuits are considered equivalent when they both present an equal impedance at their terminals; that is, the magnitude of impedance and the phase angle are identical.

To obtain the equivalent series circuit for a given parallel *RC* circuit, first find the impedance and phase angle of the parallel circuit. Then use the values of *Z* and θ to construct an impedance triangle, shown in Figure 48. The vertical and horizontal sides of the triangle represent the equivalent series resistance and capacitive reactance as indicated. These values can be found using the following trigonometric relationships:

$$R_{eq} = Z\cos\theta \qquad \text{Equation 25}$$

$$X_{C(eq)} = Z\sin\theta \qquad \text{Equation 26}$$

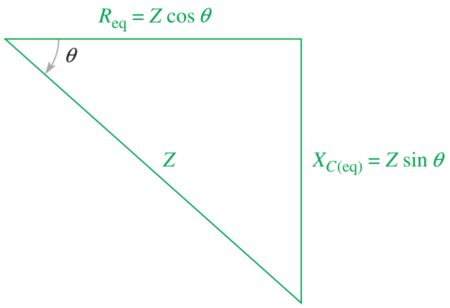
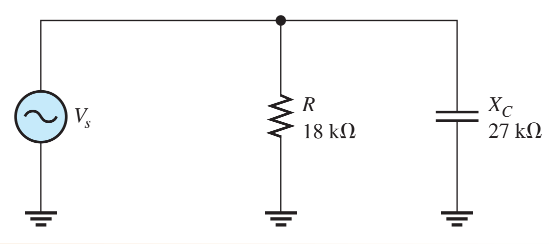


FIGURE 48

Impedance triangle for the series equivalent of a parallel *RC* circuit. *Z* and θ are the known values for the parallel circuit. R_{eq} and $X_{C(eq)}$ are the series equivalent values.

EXAMPLE 20 Convert the parallel circuit in Figure 49 to a series form.

FIGURE 49



Solution First, find the admittance of the parallel circuit as follows:

$$G = \frac{1}{R} = \frac{1}{18\text{ k}\Omega} = 55.6\ \mu\text{S}$$

$$B_C = \frac{1}{X_C} = \frac{1}{27\text{ k}\Omega} = 37.0\ \mu\text{S}$$

$$\mathbf{Y} = G + jB_C = 55.6\ \mu\text{S} + j37.0\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(55.6\ \mu\text{S})^2 + (37.0\ \mu\text{S})^2}\angle\tan^{-1}\left(\frac{37.0\ \mu\text{S}}{55.6\ \mu\text{S}}\right) = 66.8\angle 33.6^\circ\ \mu\text{S}$$

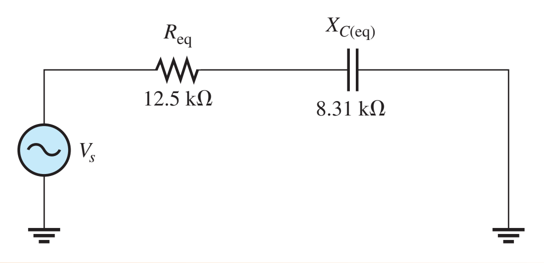
Then, the total impedance is

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}} = \frac{1}{66.8\angle 33.6^\circ\ \mu\text{S}} = 15.0\angle -33.6^\circ\text{ k}\Omega$$

Converting to rectangular form yields

$$\begin{aligned}\mathbf{Z}_{tot} &= Z\cos\theta - jZ\sin\theta = R_{\text{eq}} - jX_{C(\text{eq})}\\ &= 15.0\text{ k}\Omega\cos(-33.6°) - j15.0\text{ k}\Omega\sin(-33.6°) = 12.5\text{ k}\Omega - j8.31\text{ k}\Omega\end{aligned}$$

The equivalent series *RC* circuit is a 12.5 kΩ resistor in series with a capacitive reactance of 8.31 kΩ. This is shown in Figure 50.



▲ FIGURE 50

Related Problem The impedance of a parallel *RC* circuit is $\mathbf{Z} = 10\angle -26°$ kΩ. Convert to an equivalent series circuit.

SECTION 6 CHECKUP

1. The admittance of a parallel *RC* circuit is 3.50 mS, and the applied voltage is 6 V. What is the total current?
2. In a certain parallel *RC* circuit, the resistor current is 10 mA, and the capacitor current is 15 mA. Determine the magnitude and phase angle of the total current. This phase angle is measured with respect to what?
3. What is the phase angle between the capacitor current and the applied voltage in a parallel *RC* circuit?

SERIES-PARALLEL CIRCUITS

7 ANALYSIS OF SERIES-PARALLEL *RC* CIRCUITS

The concepts studied with respect to series and parallel circuits are used to analyze circuits with combinations of both series and parallel *R* and *C* components.

After completing this section, you should be able to

- **Analyze series-parallel *RC* circuits**
  - Determine total impedance
  - Calculate currents and voltages
  - Measure impedance and phase angle

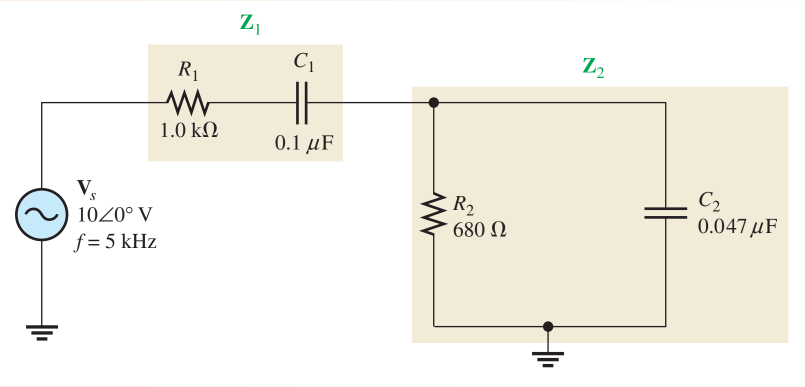
The impedance of series components is most easily expressed in rectangular form, and the impedance of parallel components is best found by using polar form. The steps for analyzing a circuit with a series and a parallel component are illustrated in Example 21. First express the impedance of the series part of the circuit in rectangular form and the impedance of the parallel part in polar form. Next, convert the impedance of the parallel part to rectangular form and add it to the impedance of the series part. Once you determine the rectangular form of the total impedance, you can convert it to polar form in order to see the magnitude and phase angle and to calculate the current.

EXAMPLE 21

In the series-parallel *RC* circuit of Figure 51, determine the following:

(a) total impedance **(b)** total current **(c)** phase angle by which I_{tot} leads V_s

FIGURE 51



Solution **(a)** First, calculate the magnitudes of capacitive reactance.

$$X_{C1} = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\text{ kHz})(0.1\ \mu\text{F})} = 318\ \Omega$$

$$X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\text{ kHz})(0.047\ \mu\text{F})} = 677\ \Omega$$

One approach is to find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance. The impedance of the series combination of R_1 and C_1 is

$$\mathbf{Z}_1 = R_1 - jX_{C1} = 1.0\text{ k}\Omega - j318\ \Omega$$

To determine the impedance of the parallel portion, first determine the admittance of the parallel combination of R_2 and C_2.

$$G_2 = \frac{1}{R_2} = \frac{1}{680\ \Omega} = 1.47\text{ mS}$$

$$B_{C2} = \frac{1}{X_{C2}} = \frac{1}{677\ \Omega} = 1.48\text{ mS}$$

$$\mathbf{Y}_2 = G_2 + jB_{C2} = 1.47\text{ mS} + j1.48\text{ mS}$$

Converting to polar form yields

$$\mathbf{Y}_2 = \sqrt{G_2^2 + B_{C2}^2}\angle\tan^{-1}\left(\frac{B_{C2}}{G_2}\right)$$

$$= \sqrt{(1.47\text{ mS})^2 + (1.48\text{ mS})^2}\angle\tan^{-1}\left(\frac{1.48\text{ mS}}{1.47\text{ mS}}\right) = 2.09\angle 45.2^\circ\text{ mS}$$

Then, the impedance of the parallel portion is

$$\mathbf{Z}_2 = \frac{1}{\mathbf{Y}_2} = \frac{1}{2.09\angle 45.2^\circ\text{ mS}} = 478\angle -45.2^\circ\ \Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta - jZ_2\sin\theta$$

$$= (478\ \Omega)\cos(-45.2^\circ) - j(478\ \Omega)\sin(-45.2^\circ) = 337\ \Omega - j339\ \Omega$$

The series portion and the parallel portion are in series with each other. Combine $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get the total impedance.

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$= (1.0\text{ k}\Omega - j318\ \Omega) + (337\ \Omega - j339\ \Omega) = 1337\ \Omega - j657\ \Omega$$

Expressing $\mathbf{Z}_{tot}$ in polar form yields

$$\mathbf{Z}_{tot} = \sqrt{Z_1^2 + Z_2^2}\angle -\tan^{-1}\left(\frac{Z_2}{Z_1}\right)$$

$$= \sqrt{(1338\ \Omega)^2 + (657\ \Omega)^2}\angle -\tan^{-1}\left(\frac{657\ \Omega}{1337\ \Omega}\right) = \mathbf{1.49\angle -26.2^\circ\ k\Omega}$$

(b) Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{10\angle 0^\circ\text{ V}}{1.49\angle -26.2^\circ\text{ k}\Omega} = \mathbf{6.71\angle 26.2^\circ\ mA}$$

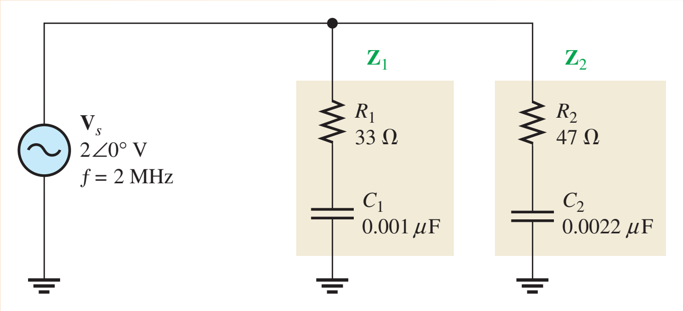
(c) The total current leads the applied voltage by **26.2°.**

Related Problem Determine the voltages across $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure 51 and express in polar form.

Use Multisim files E15-21A, E15-21B, and E15-21C to verify the calculated results in the (b) part of this example and to confirm your calculation for the related problem.

Example 22 shows two sets of series components in parallel. The approach is to first express each branch impedance in rectangular form and then convert each of these impedances to polar form. Next, calculate each branch current using polar notation. Once you know the branch currents, you can find the total current by adding the two branch currents in rectangular form. In this particular case, the total impedance is not required.

EXAMPLE 22 Determine all currents in Figure 52. Draw a current phasor diagram.



▲ FIGURE 52

Solution First, calculate X_{C1} and X_{C2}.

$$X_{C1} = \frac{1}{2\pi fC} = \frac{1}{2\pi(2\text{ MHz})(0.001\ \mu\text{F})} = 79.6\ \Omega$$

$$X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi(2\text{ MHz})(0.0022\ \mu\text{F})} = 36.2\ \Omega$$

Next, determine the impedance of each of the two parallel branches.

$$\mathbf{Z}_1 = R_1 - jX_{C1} = 33\ \Omega - j79.6\ \Omega$$
$$\mathbf{Z}_2 = R_2 - jX_{C2} = 47\ \Omega - j36.2\ \Omega$$

Convert these impedances to polar form.

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_{C1}^2}\angle -\tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$
$$= \sqrt{(33\ \Omega)^2 + (79.6\ \Omega)^2}\angle -\tan^{-1}\left(\frac{79.6\ \Omega}{33\ \Omega}\right) = 86.2\angle -67.5°\ \Omega$$
$$\mathbf{Z}_2 = \sqrt{R_2^2 + X_{C2}^2}\angle -\tan^{-1}\left(\frac{X_{C2}}{R_2}\right)$$
$$= \sqrt{(47\ \Omega)^2 + (36.2\ \Omega)^2}\angle -\tan^{-1}\left(\frac{36.2\ \Omega}{47\ \Omega}\right) = 59.3\angle -37.6°\ \Omega$$

Calculate each branch current.

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{Z}_1} = \frac{2\angle 0^\circ\ \text{V}}{86.2\angle -67.5^\circ\ \Omega} = \mathbf{23.2\angle 67.5^\circ\ mA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_s}{\mathbf{Z}_2} = \frac{2\angle 0^\circ\ \text{V}}{59.3\angle -37.6^\circ\ \Omega} = \mathbf{33.7\angle 37.6^\circ\ mA}$$

To get the total current, express each branch current in rectangular form so that they can be added.

$$\mathbf{I}_1 = 8.89\ \text{mA} + j21.4\ \text{mA}$$
$$\mathbf{I}_2 = 26.7\ \text{mA} + j20.6\ \text{mA}$$

The total current is

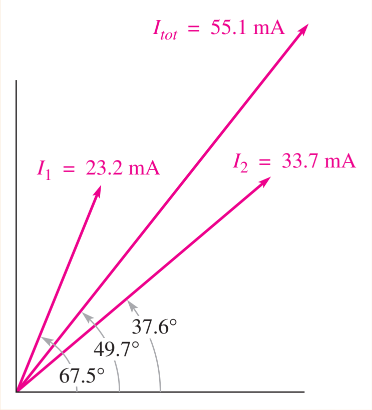
$$\begin{aligned}\mathbf{I}_{tot} &= \mathbf{I}_1 + \mathbf{I}_2\\ &= (8.89\ \text{mA} + j21.4\ \text{mA}) + (26.7\ \text{mA} + j20.6\ \text{mA}) = 35.6\ \text{mA} + j42.0\ \text{mA}\end{aligned}$$

Converting $\mathbf{I}_{tot}$ to polar form yields

$$\mathbf{I}_{tot} = \sqrt{(35.6\ \text{mA})^2 + (42.0\ \text{mA})^2}\angle\tan^{-1}\left(\frac{42.0\ \Omega}{35.6\ \Omega}\right) = \mathbf{55.1\angle 49.7^\circ\ mA}$$

The current phasor diagram is shown in Figure 53.

FIGURE 53



Related Problem Determine the voltages across each component in Figure 52 and draw a voltage phasor diagram.

Use Multisim files E15-22A through E15-22G to verify the calculated results in this example and to confirm your calculations for the related problem.

Measurement of Z_{tot}

Now, let's see how the value of Z_{tot} for the circuit in Example 21 can be determined by measurement. First, the total impedance is measured as outlined in the following steps and as illustrated in Figure 54 (other ways are also possible):

Step 1: Using a sine wave generator, set the source voltage to a known value (10 V) and the frequency to 5 kHz. If your generator is not accurate, then it is advisable to check the voltage with an ac voltmeter and the frequency with a frequency counter rather than relying on the marked values on the generator controls.

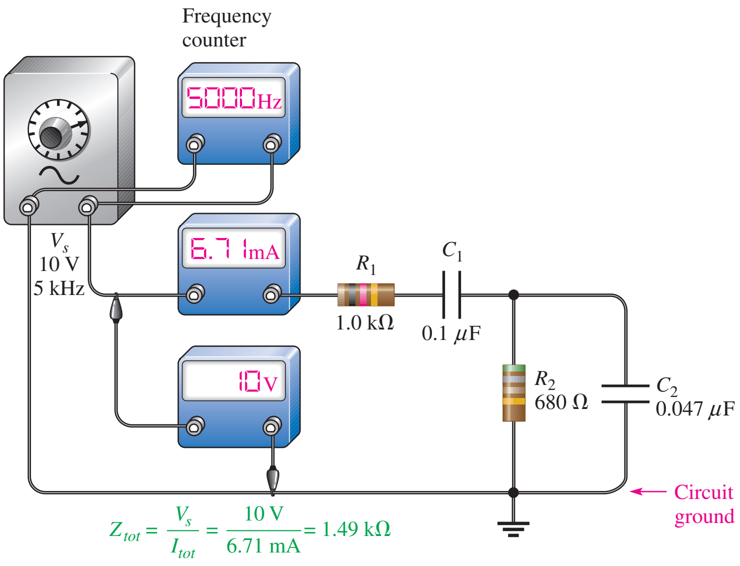


FIGURE 54

Determining Z_{tot} by measurement of V_s and I_{tot}.

Step 2: Connect an ac ammeter as shown in Figure 54, and measure the total current. Alternatively, you can measure the voltage across R_1 with a voltmeter and calculate the current.

Step 3: Calculate the total impedance by using Ohm's law.

Measurement of Phase Angle, θ

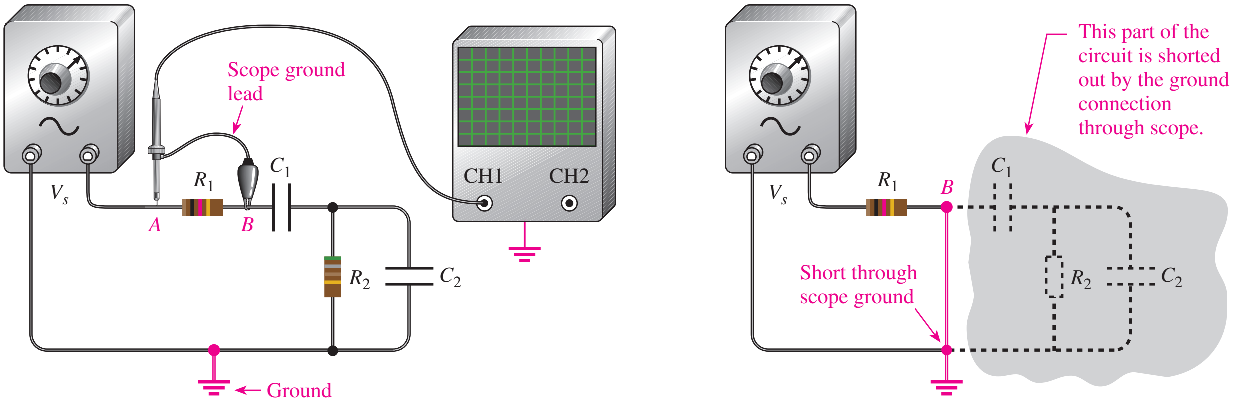
To measure the phase angle, the source voltage and the total current must be displayed on an oscilloscope screen in the proper time relationship. Two basic types of scope probes are available to measure the quantities with an oscilloscope: the voltage probe and the current probe. The current probe is a convenient device, but it is often not as readily available as a voltage probe. We will confine our phase measurement technique to the use of voltage probes in conjunction with the oscilloscope. Although there are special isolation methods, a typical oscilloscope voltage probe has two points that are connected to the circuit: the probe tip and the ground lead. Thus, all voltage measurements must be referenced to ground.

Since only voltage probes are to be used, the total current cannot be measured directly. However, for phase measurement, the voltage across R_1 is in phase with the total current and can be used to establish the phase angle of the current.

Before proceeding with the actual phase measurement, there is a problem with displaying V_{R1}. If the scope probe is connected across the resistor, as indicated in Figure 55(a), the ground lead of the scope will short point B to ground, thus bypassing the rest of the components and effectively removing them from the circuit electrically, as illustrated in Figure 55(b) (assuming that the scope is not isolated from power line ground).

To avoid this problem, you can switch the generator output terminals so that one end of R_1 is connected to the ground terminal, as shown in Figure 56(a). Now the scope can be connected across it to display V_{R1}, as indicated in Figure 56(b). The other probe is connected across the voltage source to display V_s as indicated. Now channel 1 of the scope has V_{R1} as an input, and channel 2 has V_s. The scope should be triggered from the source voltage (channel 2 in this case).

Before connecting the probes to the circuit, you should align the two horizontal lines (traces) so that they appear as a single line across the center of the scope screen. To do so, ground the probe tips and adjust the vertical position knobs to move the traces toward the center line of the screen until they are superimposed. This procedure ensures that both waveforms have the same zero crossing so that an accurate phase measurement can be made.



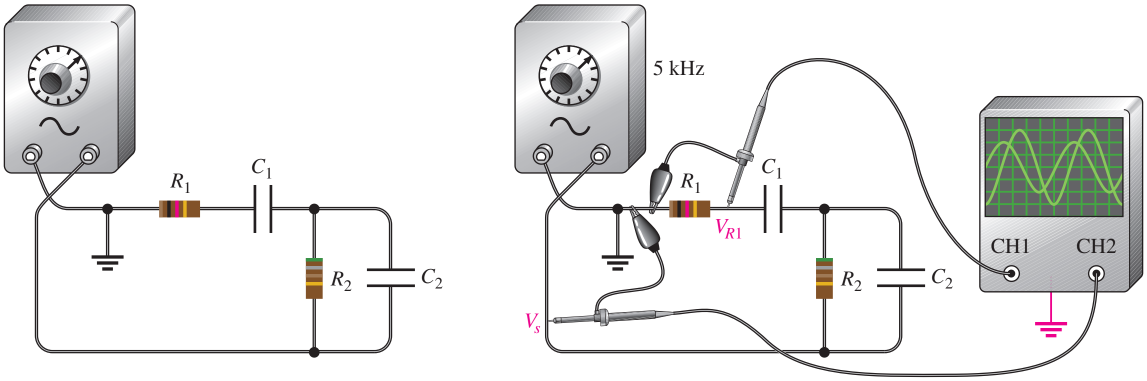
Scope ground lead
V_s
R_1
C_1
A
B
CH1
CH2
R_2
C_2
Ground
This part of the circuit is shorted out by the ground connection through scope.
V_s
R_1
B
C_1
Short through scope ground
R_2
C_2

(a) Ground lead on scope probe grounds point *B*.

(b) The effect of grounding point *B* is to short out the rest of the circuit.

▲ **FIGURE 55**

Effects of measuring directly across a component when the instrument and the circuit are grounded.



R_1
C_1
R_2
C_2
5 kHz
R_1
C_1
V_{R1}
V_s
CH1
CH2
R_2
C_2

(a) Ground repositioned so that one end of R_1 is grounded.

(b) The scope displays V_{R1} and V_s. V_{R1} represents the phase of the total current.

▲ **FIGURE 56**

Repositioning ground so that a direct voltage measurement can be made with respect to ground without shorting out part of the circuit.

Once you have stabilized the waveforms on the scope screen, you can measure the period of the source voltage. Next, use the Volts/Div controls to adjust the amplitudes of the waveforms until they both appear to have the same amplitude. Now, spread the waveforms horizontally by using the Sec/Div control to expand the distance between them. This horizontal distance represents the time between the two waveforms. The number of divisions between the waveforms along any horizontal lines times the Sec/Div setting is equal to the time between them, Δt. Also, you can use the cursors to determine Δt if your oscilloscope has this feature.

Once you have determined the period, T, and the time between the waveforms, Δt, you can calculate the phase angle with the following equation:

Equation 27

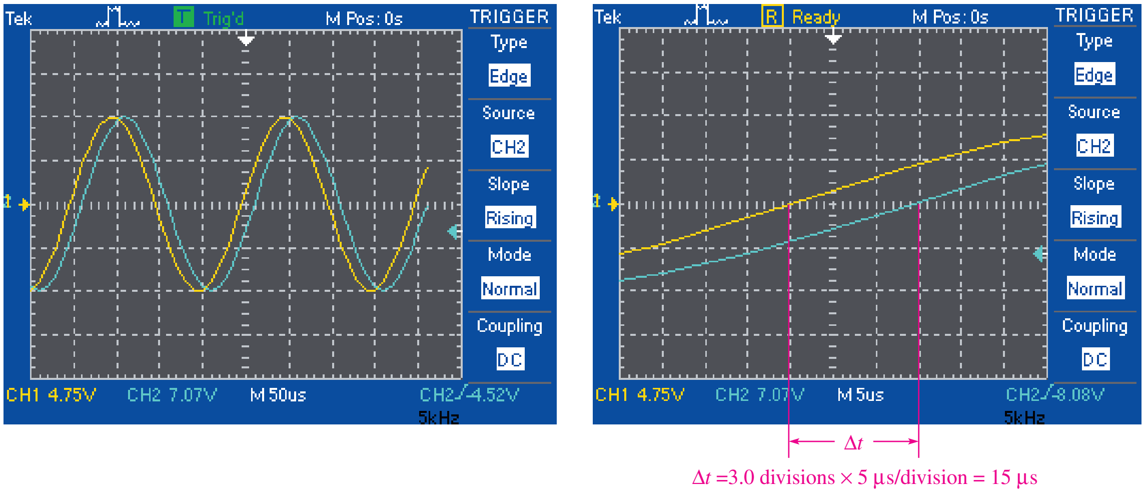
$$\theta = \left(\frac{\Delta t}{T}\right)360^\circ$$

Figure 57 shows simulated screen displays for an oscilloscope in Multisim. In Figure 57(a), the waveforms are aligned and set to the same apparent amplitude by adjusting the fine Volts/Div control. The period of these waveforms is 200 μs. The Sec/Div control is adjusted to spread the waveforms out to read Δt more accurately. As shown in part (b), there are 3.0 divisions between the crossings on the center line. The Sec/Div control is set to 5.0 μs and there are 3.0 divisions between the waveforms.

$$\Delta t = 3.0\,\text{divisions} \times 5.0\,\mu\text{s/division} = 15\,\mu\text{s}$$

The phase angle is

$$\theta = \left(\frac{\Delta t}{T}\right)360^\circ = \left(\frac{15\,\mu\text{s}}{200\,\mu\text{s}}\right)360^\circ = 27^\circ$$


Tek
Trig'd
M Pos: 0s
TRIGGER
Type
Edge
Source
CH2
Slope
Rising
Mode
Normal
Coupling
DC
CH1 4.75V
CH2 7.07V
M 50us
CH2 ╱ 4.52V
5kHz
Tek
Ready
M Pos: 0s
TRIGGER
Type
Edge
Source
CH2
Slope
Rising
Mode
Normal
Coupling
DC
CH1 4.75V
CH2 7.07V
M 5us
CH2 ╱ 8.08V
5kHz
Δt
Δt =3.0 divisions × 5 µs/division = 15 µs

(a) (b)

▲ **FIGURE 57**

Determining the phase angle on the oscilloscope.

SECTION 7 CHECKUP

1. What is the equivalent series *RC* circuit for the series-parallel circuit in Figure 51?
2. What is the total impedance in polar form of the circuit in Figure 52?

SPECIAL TOPICS

8 POWER IN *RC* CIRCUITS

In a purely resistive ac circuit, all of the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely capacitive ac circuit, all of the energy delivered by the source is stored by the capacitor during a portion of the voltage cycle and then returned to the source during another portion of the cycle so that there is no net energy conversion to heat. When there is both resistance and capacitance, some of the energy is alternately stored and returned by the capacitance and some is dissipated by the resistance. The amount of energy converted to heat is determined by the relative values of the resistance and the capacitive reactance.

After completing this section, you should be able to

- **Determine power in *RC* circuits**
  - Explain true and reactive power
  - Draw the power triangle
  - Define *power factor*
  - Explain apparent power
  - Calculate power in an *RC* circuit

When the resistance in a series *RC* circuit is greater than the capacitive reactance, more of the total energy delivered by the source is converted to heat by the resistance than is stored by the capacitor. Likewise, when the reactance is greater than the resistance, more of the total energy is stored and returned than is converted to heat.

The formulas for power in a resistor, sometimes called *true power* (P_{true}), and the power in a capacitor, called *reactive power* (P_r), are restated here. The unit of true power is the watt, and the unit of reactive power is the VAR (volt-ampere reactive).

Equation 28

$$P_{\text{true}} = I^2R$$

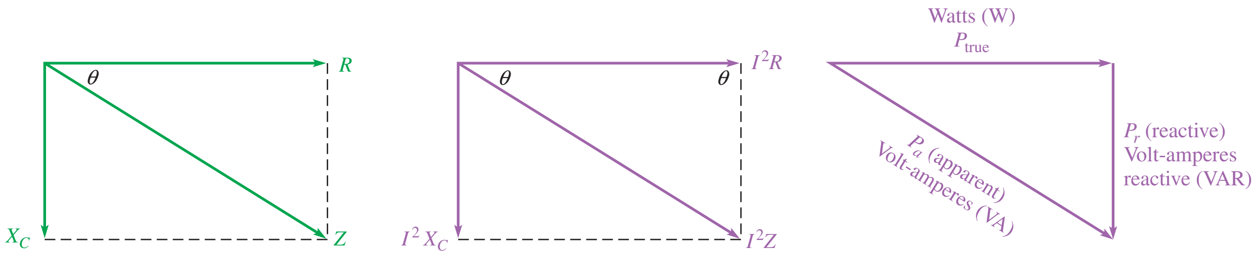
Equation 29

$$P_r = I^2X_C$$

Power Triangle for *RC* Circuits

The generalized impedance phasor diagram for a series *RC* circuit is shown in Figure 58(a). A phasor relationship for the powers can also be represented by a similar diagram because the respective magnitudes of the powers, P_{true} and P_r, differ from R and X_C by a factor of I^2. This is shown in Figure 58(b).

The resultant power phasor, I^2Z, represents the **apparent power P_a**. At any instant in time P_a is the total power that appears to be transferred between the source and the *RC*



(a) Impedance phasors (b) Power phasors (c) Power triangle

▲ **FIGURE 58**

Development of the power triangle for a series *RC* circuit.

circuit. The unit of apparent power is the volt-ampere, VA. The expression for apparent power is

$$P_a = I^2Z$$

Equation 30

The power phasor diagram in Figure 58(b) can be rearranged in the form of a right triangle, as shown in Figure 58(c). This is called the *power triangle.* Using the rules of trigonometry, P_{true} can be expressed as

$$P_{true} = P_a \cos\theta$$

Since P_a equals I^2Z or VI, the equation for the true power dissipation in an *RC* circuit can be written as

$$P_{true} = VI\cos\theta$$

Equation 31

where V is the applied voltage and I is the total current.

For the case of a purely resistive current, $\theta = 0°$ and $\cos 0° = 1$, so P_{true} equals VI. For the case of a purely capacitive circuit, $\theta = 90°$ and $\cos 90° = 0$, so P_{true} is zero. As you already know, there is no power dissipation in an ideal capacitor.

Power Factor

The term $\cos\theta$ is called the **power factor** and is stated as

$$PF = \cos\theta$$

Equation 32

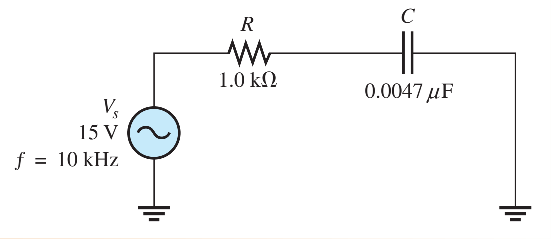
As the phase angle between applied voltage and total current increases, the power factor decreases, indicating an increasingly reactive circuit. The smaller the power factor, the smaller the power dissipation.

The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit. In an *RC* circuit, the power factor is referred to as a leading power factor because the current leads the voltage.

EXAMPLE 23

Determine the power factor and the true power in the circuit of Figure 59.

▶ **FIGURE 59**



Solution The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.0047\ \mu\text{F})} = 3.39\text{ k}\Omega$$

The total impedance of the circuit in rectangular form is

$$\mathbf{Z} = R - jX_C = 1.0\text{ k}\Omega - j3.39\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (3.39\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{3.39\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 3.53\angle -73.6^\circ\text{ k}\Omega$$

The angle associated with the impedance is θ, the angle between the applied voltage and the total current; therefore, the power factor is

$$PF = \cos\theta = \cos(-73.6^\circ) = \mathbf{0.282}$$

The current magnitude is

$$I = \frac{V_s}{Z} = \frac{15\text{ V}}{3.53\text{ k}\Omega} = 4.25\text{ mA}$$

The true power is

$$P_{\text{true}} = V_sI\cos\theta = (15\text{ V})(4.25\text{ mA})(0.282) = \mathbf{18.0\ mW}$$

Related Problem What is the power factor if f is reduced by half in Figure 59?

Significance of Apparent Power

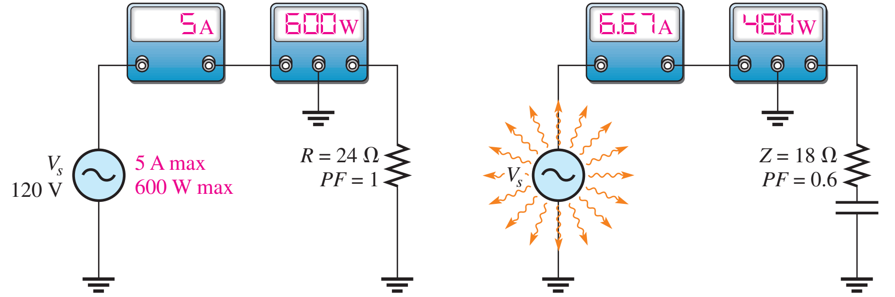
As mentioned, apparent power is the power that appears to be transferred between the source and the load, and it consists of two components—a true power component and a reactive power component.

In all electrical and electronic systems, it is the true power that does the work. The reactive power is simply shuttled back and forth between the source and load. Ideally, in terms of performing useful work, all of the power transferred to the load should be true power and none of it reactive power. However, in most practical situations the load has some reactance associated with it, and therefore you must deal with both power components.

For any reactive load, there are two components of the total current: the resistive component and the reactive component. If you consider only the true power (watts) in a load, you are dealing with only a portion of the total current that the load demands from a source. In order to have a realistic picture of the actual current that a load will draw, you must consider apparent power (VA).

A source such as an ac generator can provide current to a load up to some maximum value. If the load draws more than this maximum value, the source can be damaged. Figure 60(a) shows a 120 V generator that can deliver a maximum current of 5 A to a load. Assume that the generator is rated at 600 W and is connected to a purely resistive load of 24 Ω (power factor of 1). The ammeter shows that the current is 5 A, and the wattmeter indicates that the power is 600 W. The generator has no problem under these conditions, although it is operating at maximum current and power.

Now, consider what happens if the load is changed to a reactive one with an impedance of 18 Ω and a power factor of 0.6, as indicated in Figure 60(b). The current is



Ammeter indicates that current is excessive.

Wattmeter indicates that power is below rated value.

(a) Generator operating at its limits with a resistive load.

(b) Generator is in danger of internal damage due to excess current, even though the wattmeter indicates that the power is below the maximum wattage rating.

▲ **FIGURE 60**

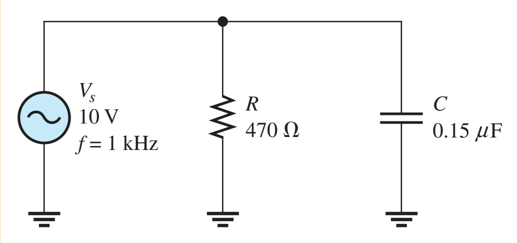
Wattage rating of a source is inappropriate when the load is reactive. The rating should be in VA rather than in watts.

120 V/18 Ω = 6.67 A, which *exceeds* the maximum. Even though the wattmeter reads 480 W, which is less than the power rating of the generator, the excessive current probably will cause damage. This illustration shows that a true power rating can be deceiving and is inappropriate for ac sources. The ac generator should be rated at 600 VA, a rating that manufacturers generally use, rather than 600 W.

EXAMPLE 24

For the circuit in Figure 61, find the true power, the reactive power, and the apparent power.

► **FIGURE 61**



Solution The capacitive reactance and currents through R and C are

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1000\text{ Hz})(0.15\ \mu\text{F})} = 1061\ \Omega$$

$$I_R = \frac{V_s}{R} = \frac{10\text{ V}}{470\ \Omega} = 21.3\text{ mA}$$

$$I_C = \frac{V_s}{X_C} = \frac{10\text{ V}}{1061\ \Omega} = 9.43\text{ mA}$$

The true power is

$$P_{\text{true}} = I_R^2R = (21.3\text{ mA})^2(470\ \Omega) = \mathbf{213\ mW}$$

The reactive power is

$$P_r = I_C^2 X_C = (9.43\text{ mA})^2(1061\ \Omega) = \mathbf{94.3\ mVAR}$$

The apparent power is

$$P_a = \sqrt{P_{\text{true}}^2 + P_r^2} = \sqrt{(213\text{ mW})^2 + (94.3\text{ mVAR})^2} = \mathbf{233\ mVA}$$

Related Problem What is the true power in Figure 61 if the frequency is changed to 2 kHz?

SECTION 8 CHECKUP

1. To which component in an *RC* circuit is the power dissipation due?
2. The phase angle, θ, is 45°. What is the power factor?
3. A certain series *RC* circuit has the following parameter values: $R = 330\ \Omega$, $X_C = 460\ \Omega$, and $I = 2\ A$. Determine the true power, the reactive power, and the apparent power.

9 BASIC APPLICATIONS

RC circuits are found in a variety of applications, often as part of a more complex circuit. Three applications are phase shift oscillators, frequency-selective circuits (filters), and ac coupling.

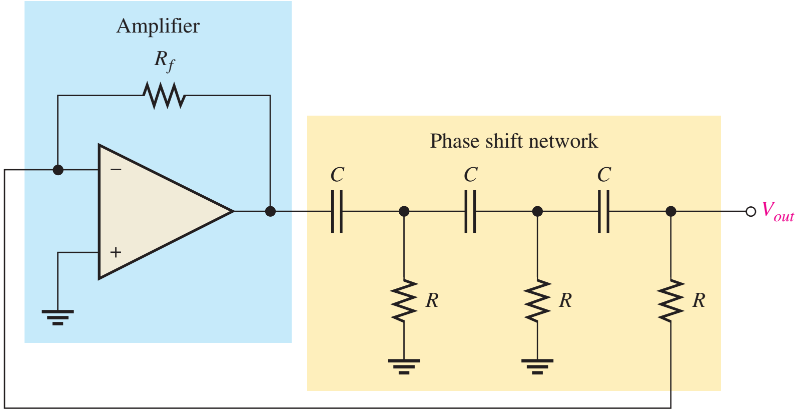
After completing this section, you should be able to

- **Discuss some basic *RC* applications**
  - Discuss how the *RC* circuit is used as an oscillator
  - Discuss how the *RC* circuit operates as a filter
  - Discuss ac coupling

The Phase Shift Oscillator

As you know, a series *RC* circuit will shift the phase of the output voltage by an amount that depends on the values of *R* and *C* and the frequency of the signal. This ability to shift phase depending on frequency is vital in certain feedback oscillator circuits. An **oscillator** is a circuit that generates a periodic waveform and is an important circuit for many electronic systems. You will study oscillators in a devices course, so the focus here is on the application of *RC* circuits for shifting phase. The requirement is that a fraction of the output of the oscillator is returned to the input (called "feedback") in the proper phase to reinforce the input and sustain oscillations. Generally, the requirement is to feed back the signal with a total of 180° of phase shift.

A single *RC* circuit is limited to phase shifts that are smaller than 90°. The basic *RC* lag circuit discussed in Section 4 can be "stacked" to form a complex *RC* network as shown in Figure 62, which shows a specific circuit called a phase-shift oscillator. The phase shift oscillator typically uses three equal-component *RC* circuits that produce the required 180° phase shift at a certain frequency, which will be the frequency at which the oscillator



▲ **FIGURE 62**

Phase shift oscillator.

works. The output of the amplifier is phase shifted by the *RC* network and returned to the input of the amplifier, which provides sufficient gain to maintain oscillations.

The process of putting several *RC* circuits together results in a loading effect, so the overall phase shift is not the same as simply adding the phase shifts of the individual *RC* circuits. With equal components, the frequency at which a 180° phase shift occurs is given by the equation

$$f_r = \frac{1}{2\pi\sqrt{6}RC}$$

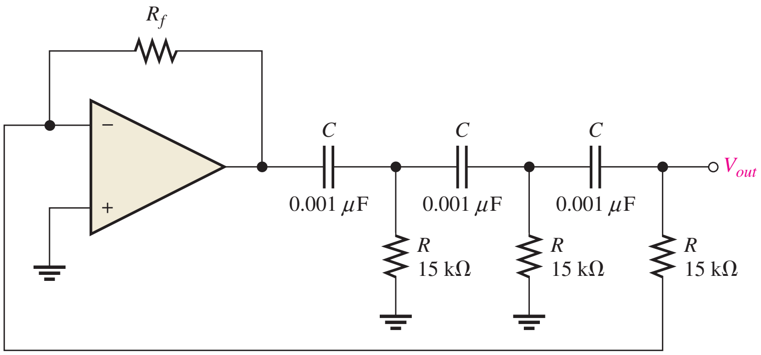
Equation 33

It also turns out that the *RC* network attenuates (reduces) the signal from the amplifier by a factor of 29; the amplifier must make up for this attenuation by having a gain of −29 (the minus sign takes into account the phase shift).

EXAMPLE 25

In Figure 63, calculate the output frequency.

▶ **FIGURE 63**



Solution

$$f_r = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(15\text{ k}\Omega)(0.001\ \mu\text{F})} = 4.33\text{ kHz}$$

Related Problem If all of the capacitors are changed to 0.0027 μF, what is the oscillator frequency?

The *RC* Circuit as a Filter

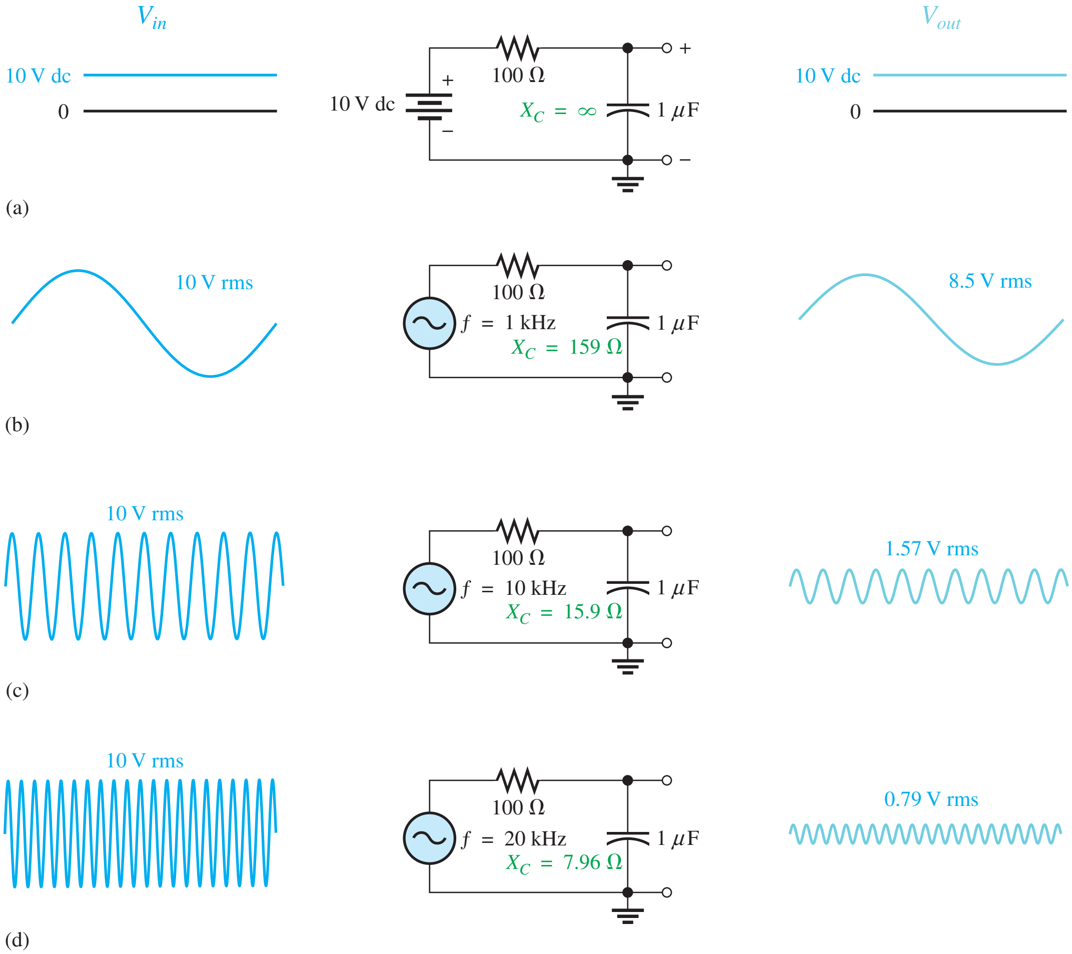
Filters are frequency-selective circuits that permit signals of certain frequencies to pass from the input to the output while blocking all others. That is, all frequencies but the selected ones are filtered out. Filters are introduced here as an application example.

Series *RC* circuits exhibit a frequency-selective characteristic and therefore act as basic filters. There are two types. The first one that we examine, called a **low-pass filter**, is realized by taking the output across the capacitor, just as in a lag circuit. The second type, called a **high-pass filter**, is implemented by taking the output across the resistor, as in a lead circuit.

Low-Pass Filter You have already seen what happens to the output magnitude and phase angle in a lag circuit. In terms of its filtering action, we are interested primarily in the variation of the output magnitude with frequency.

Figure 64 shows the filtering action of a series *RC* circuit using specific values for illustration. In part (a) of the figure, the input is zero frequency (dc). Since the capacitor blocks constant direct current, the output voltage equals the full value of the input voltage because there is no voltage dropped across *R*. Therefore, the circuit passes all of the input voltage to the output (10 V in, 10 V out).

In Figure 64(b), the frequency of the input voltage has been increased to 1 kHz, causing the capacitive reactance to decrease to 159 Ω. For an input voltage of 10 V rms, the



V_{in}
V_{out}
10 V dc
0
100 Ω
10 V dc
+
−
$X_C = \infty$
1 μF
+
−
10 V dc
0
(a)
10 V rms
100 Ω
$f = 1$ kHz
$X_C = 159\ \Omega$
1 μF
8.5 V rms
(b)
10 V rms
100 Ω
$f = 10$ kHz
$X_C = 15.9\ \Omega$
1 μF
1.57 V rms
(c)
10 V rms
100 Ω
$f = 20$ kHz
$X_C = 7.96\ \Omega$
1 μF
0.79 V rms
(d)

▲ FIGURE 64

Low-pass filtering action (phase shifts are not indicated).

output voltage is approximately 8.5 V rms, which can be calculated using the voltage-divider approach or Ohm's law.

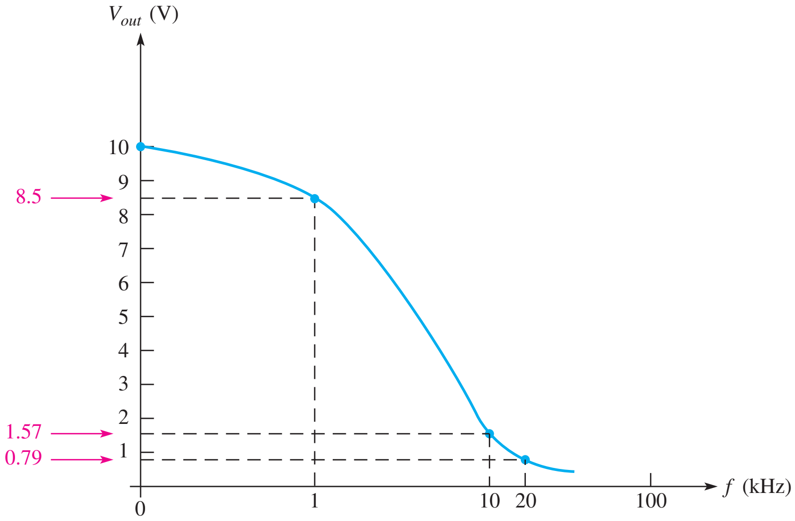
In Figure 64(c), the input frequency has been increased to 10 kHz, causing the capacitive reactance to decrease further to 15.9 Ω. For a constant input voltage of 10 V rms, the output voltage is now 1.57 V rms.

As the input frequency is increased further, the output voltage continues to decrease and approaches zero as the frequency becomes very high, as shown in Figure 64(d).

A description of the circuit action is as follows: As the frequency of the input increases, the capacitive reactance decreases. Because the resistance is constant and the capacitive reactance decreases, the voltage across the capacitor (output voltage) also decreases according to the voltage-divider principle. The input frequency can be increased until it reaches a value at which the reactance is so small compared to the resistance that the output voltage can be neglected because it is very small compared to the input voltage. At this value of frequency, the circuit is essentially completely blocking the input signal.

As shown in Figure 64, the circuit passes dc (zero frequency) completely. As the frequency of the input increases, less of the input voltage is passed through to the output; that is, the output voltage decreases as the frequency increases. It is apparent that the lower frequencies pass through the circuit much better than the higher frequencies. This *RC* circuit is therefore a very basic form of low-pass filter.

The **frequency response** of the low-pass filter circuit in Figure 64 is shown in Figure 65 with a graph of output voltage magnitude versus frequency. This graph, called a *response curve,* indicates that the output decreases as the frequency increases.



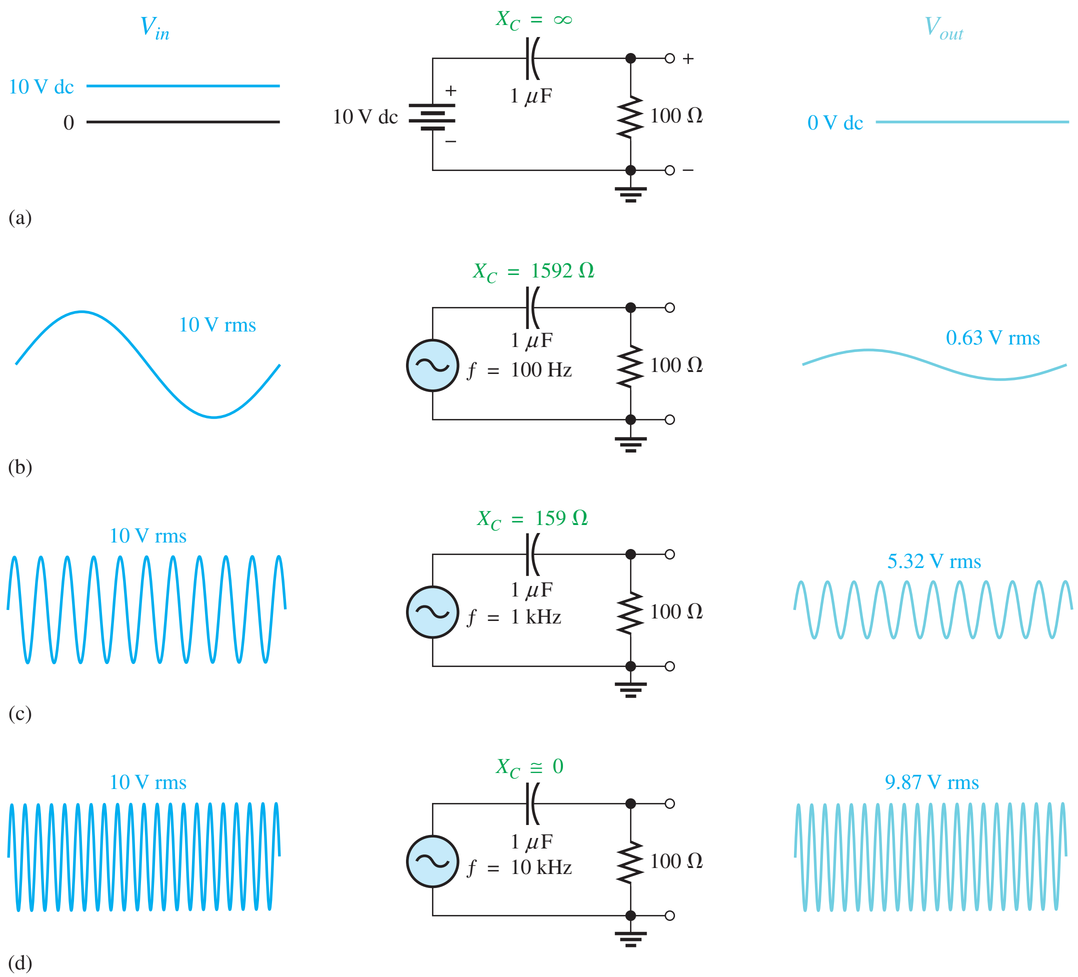
▲ **FIGURE 65**

Frequency response curve for the low-pass filter in Figure 64.

High-Pass Filter Figure 66 illustrates high-pass filtering action, where the output is taken across the resistor, just as in a lead circuit. When the input voltage is dc (zero frequency) in part (a), the output is zero volts because the capacitor blocks direct current; therefore, no voltage is developed across *R*.

In Figure 66(b), the frequency of the input signal has been increased to 100 Hz with an rms value of 10 V. The output voltage is 0.63 V rms. Thus, only a small percentage of the input voltage appears on the output at this frequency.

In Figure 66(c), the input frequency is increased further to 1 kHz, causing more voltage to be developed across the resistor because of the further decrease in the capacitive reactance. The output voltage at this frequency is 5.32 V rms. As you can see, the output



V_{in}
$X_C = \infty$
V_{out}
10 V dc
0
10 V dc
+
−
1 μF
100 Ω
+
−
0 V dc
(a)
$X_C = 1592\ \Omega$
10 V rms
1 μF
f = 100 Hz
100 Ω
0.63 V rms
(b)
$X_C = 159\ \Omega$
10 V rms
1 μF
f = 1 kHz
100 Ω
5.32 V rms
(c)
$X_C \cong 0$
10 V rms
1 μF
f = 10 kHz
100 Ω
9.87 V rms
(d)

▲ **FIGURE 66**

High-pass filtering action (phase shifts are not indicated).

voltage increases as the frequency increases. A value of frequency is reached at which the reactance is negligible compared to the resistance, and most of the input voltage appears across the resistor, as shown in Figure 66(d).

As illustrated, this circuit tends to prevent lower frequencies from appearing on the output but allows higher frequencies to pass through from input to output. Therefore, this *RC* circuit is a basic form of high-pass filter.

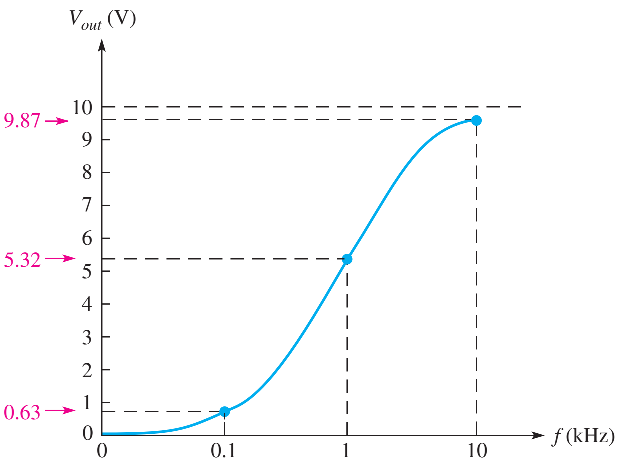
The frequency response of the high-pass filter circuit in Figure 66 is shown in Figure 67 with a graph of output voltage magnitude versus frequency. This response curve shows that the output increases as the frequency increases and then levels off and approaches the value of the input voltage.

The Cutoff Frequency and the Bandwidth of a Filter The frequency at which the capacitive reactance equals the resistance in a low-pass or high-pass *RC* filter is called the **cutoff frequency** and is designated f_c. This condition is expressed as $1/(2\pi f_c C) = R$. Solving for f_c results in the following formula:

Equation 34

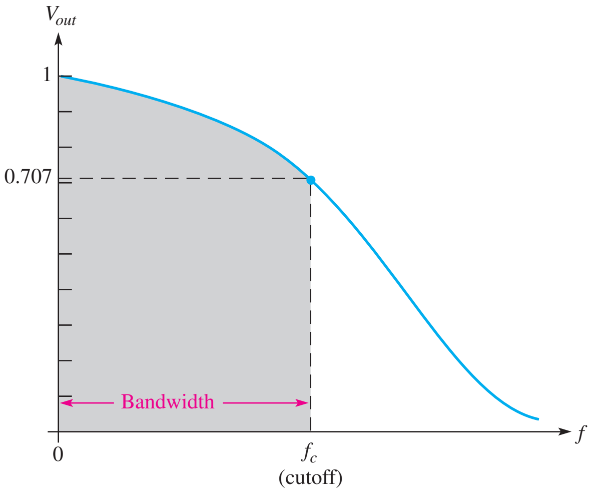
$$f_c = \frac{1}{2\pi RC}$$

At f_c, the output voltage of the filter is 70.7% of its maximum value. It is standard practice to consider the cutoff frequency as the limit of a filter's performance in terms of



▲ **FIGURE 67**

Frequency response curve for the high-pass filter in Figure 66.



▲ **FIGURE 68**

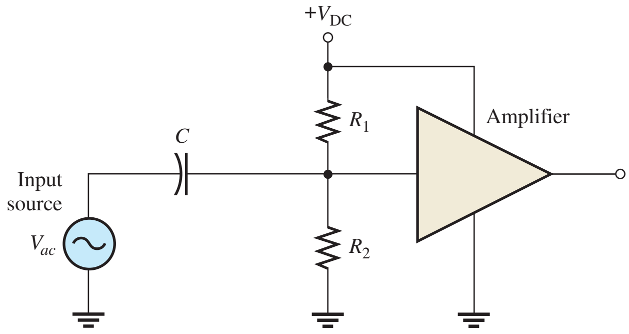
Normalized general response curve of a low-pass filter showing the cutoff frequency and the bandwidth.

passing or rejecting frequencies. For example, in a high-pass filter, all frequencies above f_c are considered to be passed by the filter, and all those below f_c are considered to be rejected. The reverse is true for a low-pass filter.

The range of frequencies that is considered to be passed by a filter is called the **bandwidth**. Figure 68 illustrates the bandwidth and the cutoff frequency for a low-pass filter.

Coupling an AC Signal into a DC Bias Circuit

Figure 69 shows an *RC* circuit that is used to create a dc voltage level with an ac voltage superimposed on it. This type of circuit is commonly found in amplifiers in which the dc voltage is required to **bias** the amplifier to the proper operating point and the signal voltage to be amplified is coupled through a capacitor and superimposed on the dc level. The capacitor prevents the low internal resistance of the signal source from affecting the dc bias voltage.

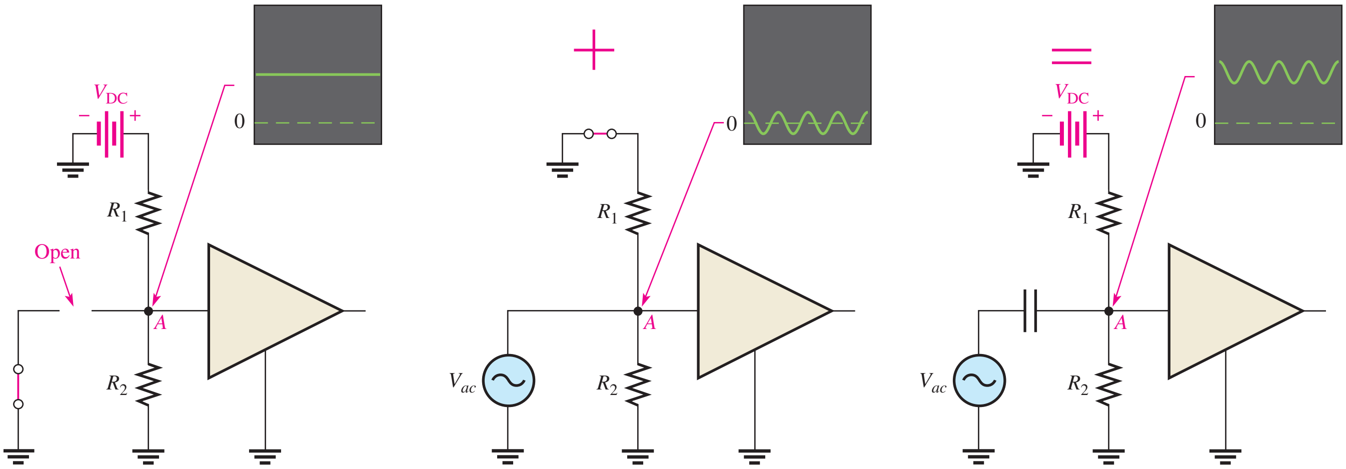


◀ **FIGURE 69**

Amplifier bias and signal-coupling circuit.

In this type of application, a relatively large value of capacitance is selected so that for the frequencies to be amplified, the reactance is very small compared to the resistance of the bias network. When the reactance is very small (ideally zero), there is practically no phase shift or signal voltage dropped across the capacitor. Therefore, all of the signal voltage passes from the source to the input of the amplifier.

Figure 70 illustrates the application of the superposition principle to the circuit in Figure 69. In part (a), the ac source has been effectively removed from the circuit and replaced with a short to represent its ideal internal resistance (actual generators typically have 50 Ω or 600 Ω of internal resistance). Since *C* is open to dc, the voltage at point *A* is determined by the voltage-divider action of R_1 and R_2 and the dc voltage source.



(a) dc equivalent: ac source replaced by short. *C* is open to dc. R_1 and R_2 act as dc voltage divider.

(b) ac equivalent: dc source is replaced by short. *C* is short to ac. All of V_{ac} is coupled to point *A*.

(c) dc + ac: Voltages are superimposed at point *A*.

▲ FIGURE 70

The superposition of dc and ac voltages in an *RC* bias and coupling circuit.

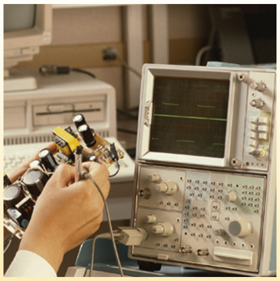
In Figure 70(b), the dc source has been effectively removed from the circuit and replaced with a short to represent its ideal internal resistance. Since *C* appears as a short at the frequency of the ac, the signal voltage is coupled directly to point *A* and appears across the parallel combination of R_1 and R_2.

Figure 70(c) illustrates that the combined effect of the superposition of the dc and the ac voltages results in the signal voltage "riding" on the dc level.

SECTION 9 CHECKUP

1. How much phase shift in produced by the *RC* circuit in a phase shift oscillator?
2. When an *RC* circuit is used as a low-pass filter, across which component is the output taken?

10 TROUBLESHOOTING



Typical component failures or degradation have an effect on the frequency response of basic *RC* circuits.

After completing this section, you should be able to

- **Troubleshoot *RC* circuits**
  - Find an open resistor or open capacitor
  - Find a shorted capacitor
  - Find a leaky capacitor

Effect of an Open Resistor It is easy to see how an open resistor affects the operation of a basic series *RC* circuit, as shown in Figure 71. Obviously, there is no path for current, so the capacitor voltage remains at zero; thus, the total voltage, V_s, appears across the open resistor.

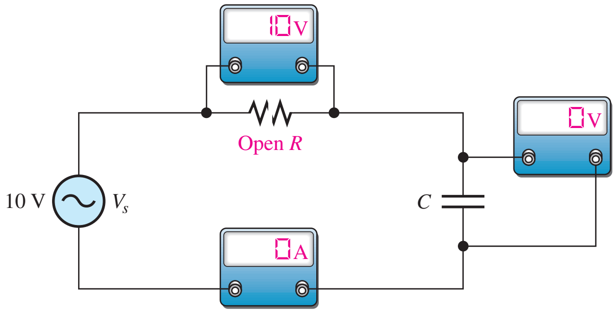


FIGURE 71

Effect of an open resistor.

Effect of an Open Capacitor When the capacitor is open, there is no current; thus, the resistor voltage remains at zero. The total source voltage is across the open capacitor, as shown in Figure 72.

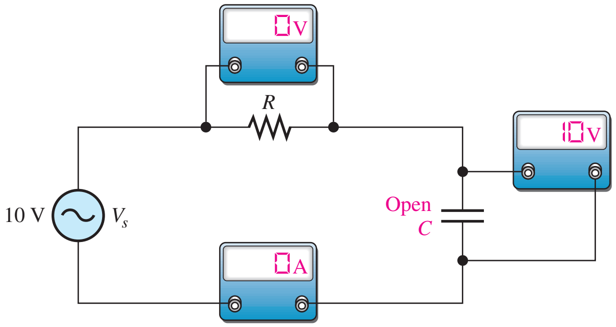


FIGURE 72

Effect of an open capacitor.

Effect of a Shorted Capacitor Capacitors rarely short; but when a capacitor does short out, the voltage across it is zero, the current equals V_s/R, and the total voltage appears across the resistor, as shown in Figure 73.

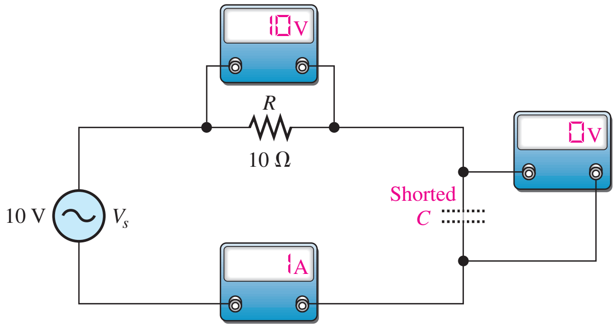
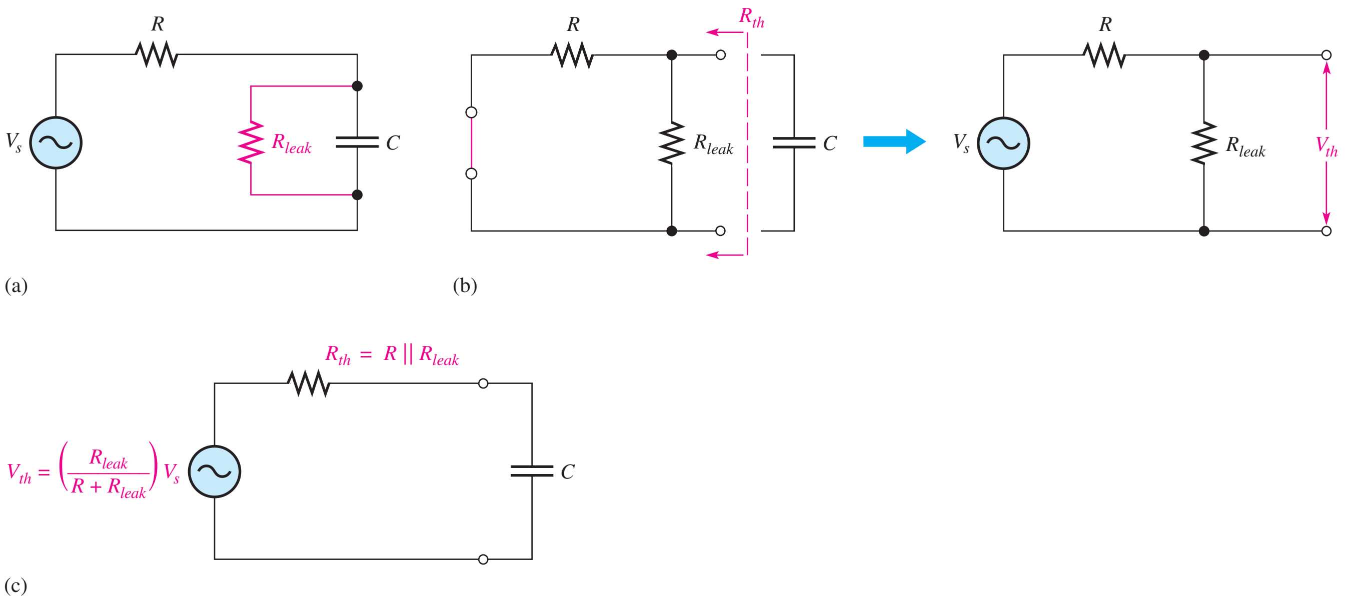


FIGURE 73

Effect of a shorted capacitor.

Effect of a Leaky Capacitor When a large electrolytic capacitor exhibits a high leakage current, the leakage resistance effectively appears in parallel with the capacitor, as shown in Figure 74(a). When the leakage resistance is comparable in value to the circuit resistance, *R*, the circuit response is drastically affected. The circuit, looking from the capacitor toward the source, can be thevenized, as shown in Figure 74(b). The Thevenin


R
V_s
R_{leak}
C
(a)
R
R_{th}
R_{leak}
C
(b)
R
V_s
R_{leak}
V_{th}
$R_{th} = R \parallel R_{leak}$
$V_{th} = \left(\frac{R_{leak}}{R + R_{leak}}\right) V_s$
C
(c)

▲ FIGURE 74

Effect of a leaky capacitor.

equivalent resistance is R in parallel with R_{leak} (the source appears as a short), and the Thevenin equivalent voltage is determined by the voltage-divider action of R and R_{leak}.

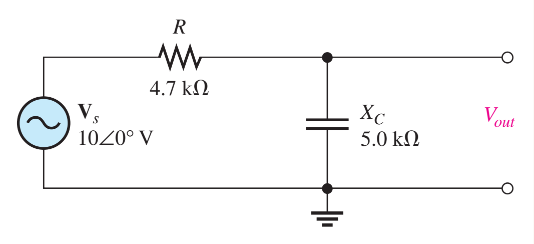
$$R_{th} = R \| R_{leak} = \frac{RR_{leak}}{R + R_{leak}}$$

$$V_{th} = \left(\frac{R_{leak}}{R + R_{leak}}\right)V_s$$

As you can see, the voltage across the capacitor is reduced since $V_{th} < V_s$. Also, the circuit time constant is reduced, and the current is increased. The Thevenin equivalent circuit is shown in Figure 74(c).

EXAMPLE 26

Assume that the capacitor in Figure 75 is degraded to a point where its leakage resistance is 10 kΩ. Determine the output voltage under the degraded condition.


R
4.7 kΩ
$\mathbf{V}_s$
10∠0° V
X_C
5.0 kΩ
V_{out}

▲ FIGURE 75

Solution The effective circuit resistance is

$$R_{th} = \frac{RR_{leak}}{R + R_{leak}} = \frac{(4.7\text{ k}\Omega)(10\text{ k}\Omega)}{14.7\text{ k}\Omega} = 3.20\text{ k}\Omega$$

To determine the output voltage, find the Thevenin equivalent voltage.

$$V_{th} = \left(\frac{R_{leak}}{R + R_{leak}}\right)V_s = \left(\frac{10\text{ k}\Omega}{14.7\text{ k}\Omega}\right)10\text{ V} = 6.80\text{ V}$$

Then,

$$V_{out} = \left(\frac{X_C}{\sqrt{R_{th}^2 + X_C^2}}\right)V_{th} = \left(\frac{5.0\text{ k}\Omega}{\sqrt{(3.2\text{ k}\Omega)^2 + (5.0\text{ k}\Omega)^2}}\right)6.80\text{ V} = \mathbf{5.73\text{ V}}$$

Related Problem What would the output voltage be if the capacitor were not leaky?

Other Troubleshooting Considerations

So far, you have learned about specific component failures and the associated voltage measurements. Many times, however, the failure of a circuit to work properly is not the result of a faulty component. A loose wire, a bad contact, or a poor solder joint can cause an open circuit. A short can be caused by a wire clipping or solder splash. Things as simple as not plugging in a power supply or a function generator happen more often than you might think. Wrong values in a circuit (such as an incorrect resistor value), the function generator set at the wrong frequency, or the wrong output connected to the circuit can cause improper operation.

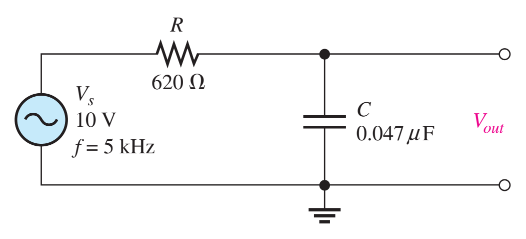
When you have problems with a circuit, always check to make sure that the instruments are properly connected to the circuits and to a power outlet. Also, look for obvious things such as a broken or loose contact, a connector that is not completely plugged in, or a piece of wire or a solder bridge that could be shorting something out.

The point is that you should consider all possibilities, not just faulty components, when a circuit is not working properly. The following example illustrates this approach with a simple circuit using the APM (analysis, planning, and measurement) method.

EXAMPLE 27

The circuit represented by the schematic in Figure 76 has no output voltage, which is the voltage across the capacitor. You expect to see about 7.4 V at the output. The circuit is physically constructed on a protoboard. Use your troubleshooting skills to find the problem.

▶ FIGURE 76



Solution Apply the APM method to this troubleshooting problem.

Analysis: First think of the possible causes for the circuit to have no output voltage.

1. There is no source voltage or the frequency is so high that the capacitive reactance is almost zero.

2. There is a short between the output terminals. Either the capacitor could be internally shorted, or there could be some physical short in the circuit.

3. There is an open between the source and the output. This would prevent current and thus cause the output voltage to be zero. The resistor could be open, or the conductive path could be open due to a broken or loose connecting wire or a bad protoboard contact.

4. There is an incorrect component value. The resistor could be so large that the current and, therefore, the output voltage are negligible. The capacitor could be so large that its reactance at the input frequency is near zero.

Planning: You decide to make some visual checks for problems such as the function generator power cord not plugged in or the frequency set at an incorrect value. Also, broken leads, shorted leads, as well as an incorrect resistor color code or capacitor label often can be found visually. If nothing is discovered after a visual check, then you will make voltage measurements to track down the cause of the problem. You decide to use a digital oscilloscope and a DMM to make the measurements.

Measurement: Assume that you find that the function generator is plugged in and the frequency setting appears to be correct. Also, you find no visible opens or shorts during your visual check, and the component values are correct.

The first step in the measurement process is to check the voltage from the source with the scope. Assume a 10 V rms sine wave with a frequency of 5 kHz is observed at the circuit input as shown in Figure 77(a). The correct voltage is present, so *the first possible cause has been eliminated.*

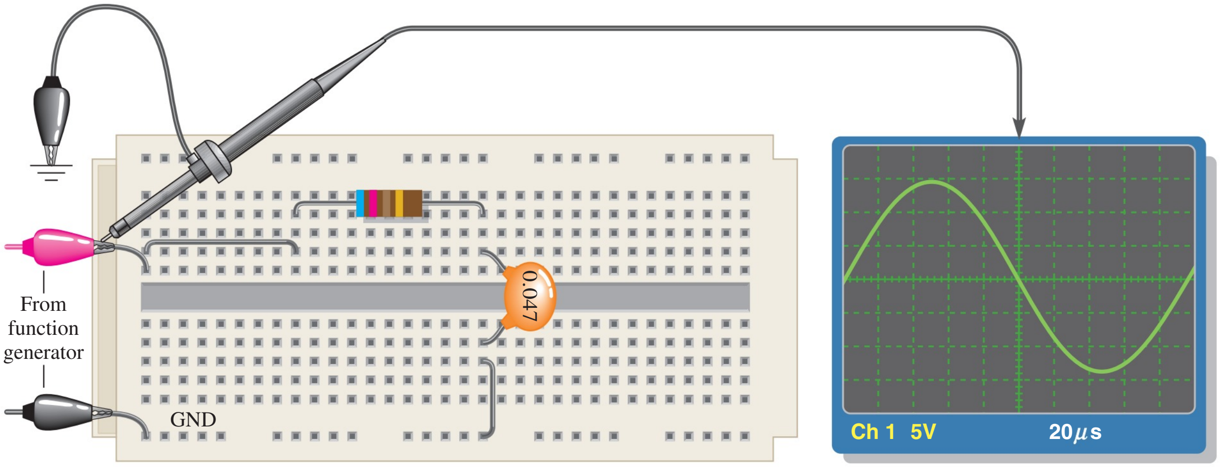
Next, check for a shorted capacitor by disconnecting the source and placing a DMM (set on the ohmmeter function) across the capacitor. If the capacitor is good, an open will be indicated by an OL (overload) in the meter display after a short charging time. Assume the capacitor checks okay, as shown in Figure 77(b). *The second possible cause has been eliminated.*

Since the voltage has been “lost” somewhere between the input and the output, you must now look for the voltage. Reconnect the source and measure the voltage across the resistor with the DMM (set on the voltmeter function) from one resistor lead to the other. The voltage across the resistor is zero. This means there is no current, which indicates an open somewhere in the circuit.

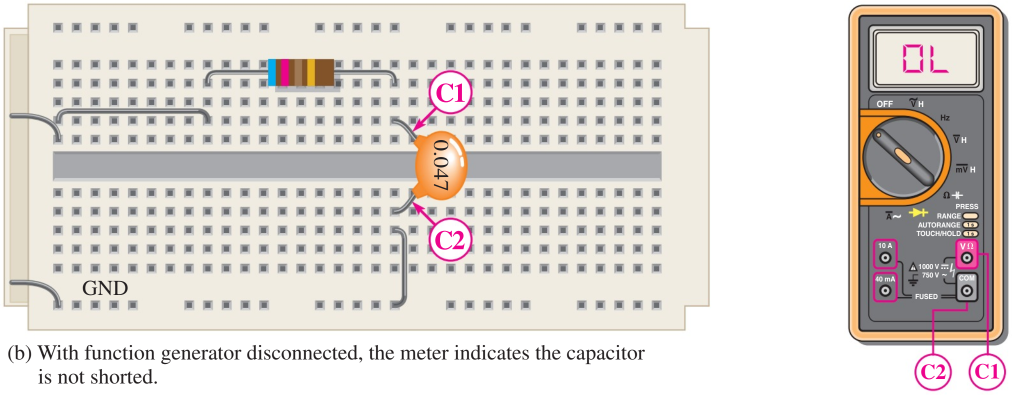
Now, begin tracing the circuit back toward the source looking for the voltage (you could also start from the source and work forward). You can use either the scope or the DMM but decide to use the multimeter with one lead connected to ground and the other used to probe the circuit. As shown in Figure 77(c), the voltage on the right lead of the resistor, point ①, reads zero. Since you already have measured zero voltage across the resistor, the voltage on the left resistor lead at point ② must be zero as the meter indicates. Next, moving the meter probe to point ③, you read 10 V. You have found the voltage! Since there is zero volts on the left resistor lead, and there is 10 V at point ③, one of the two contacts in the protoboard hole into which the wire leads are inserted is bad. It could be that the small contacts were pushed in too far and were bent or broken so that the circuit lead does not make contact.

Move either or both the resistor lead and the wire to another hole in the same row. Assume that when the resistor lead is moved to the hole just above, you have voltage at the output of the circuit (across the capacitor).

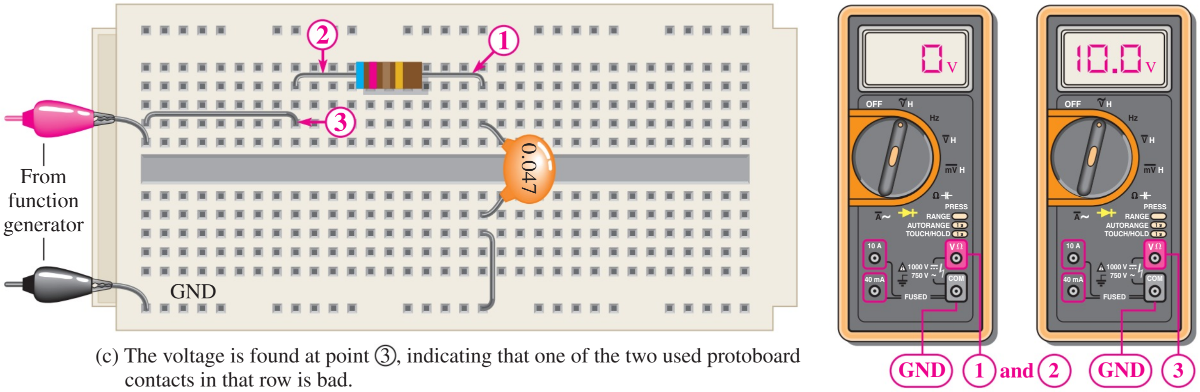
Related Problem Suppose you had measured 10 V across the resistor before the capacitor was checked. What would this have indicated?


From function generator
GND
0.047
Ch 1 5V
20μs

(a) Scope shows the correct voltage at the input.


C1
C2
0.047
GND
OL
OFF
Hz
RANGE
AUTORANGE
TOUCH/HOLD
10 A
40 mA
VΩ
COM
FUSED
C2
C1

(b) With function generator disconnected, the meter indicates the capacitor is not shorted.

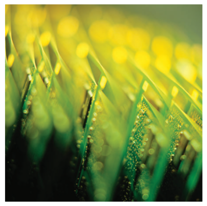

2
1
3
0.047
From function generator
GND
0 V
10.0 V
GND
1 and 2
GND
3

(c) The voltage is found at point ③, indicating that one of the two used protoboard contacts in that row is bad.

▲ **FIGURE 77**

SECTION 10 CHECKUP

1. Describe the effect of a leaky capacitor on the response of an *RC* circuit.
2. In a series *RC* circuit, if all of the applied voltage appears across the capacitor, what is the problem?
3. What faults can cause 0 V across a capacitor in a series *RC* circuit if the source is functioning properly?



Application Activity

In this application, the output voltage and phase lag of an amplifier's input circuit are examined to determine how they change with frequency. If too much voltage is dropped across the coupling capacitor, the overall performance of the amplifier is adversely affected.

The coupling capacitor (C_1) in Figure 78 passes the input signal voltage to the input of the amplifier (point *A* to point *B*) without affecting the dc level at point *B* produced by the resistive voltage divider (R_1 and R_2). If the input frequency is high enough so that the reactance of the coupling capacitor is negligibly small, essentially no ac signal voltage is dropped across the capacitor. As the signal frequency is reduced, the capacitive reactance increases and more of the signal voltage is dropped across the capacitor. This lowers the overall voltage gain of the amplifier and thus degrades its performance.

The amount of signal voltage that is coupled from the input source (point *A*) to the amplifier input (point *B*) is determined by the values of the capacitor and the dc bias resistors (assuming the amplifier has no loading effect) in Figure 78. These components actually form a high-pass *RC* filter, as shown in Figure 79. The voltage-divider bias resistors are effectively in parallel with each other as far as the ac source is concerned because the power supply has zero internal resistance. The lower end of

FIGURE 78

A capacitively coupled amplifier.

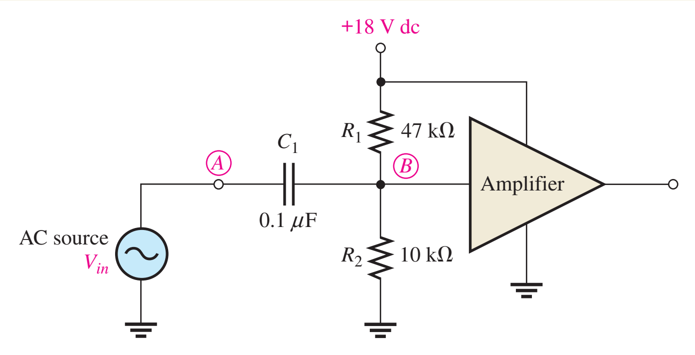
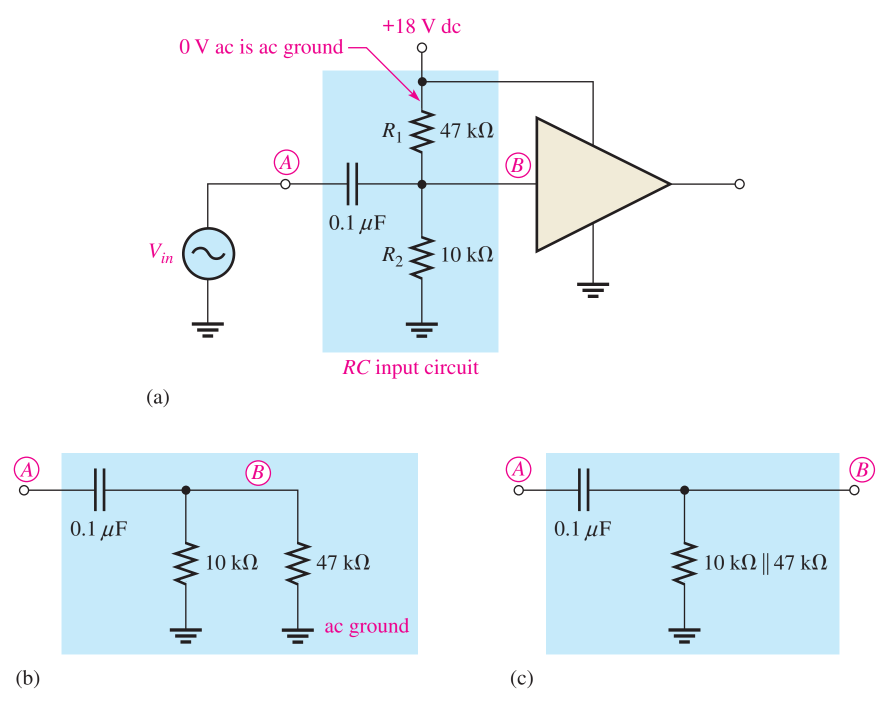
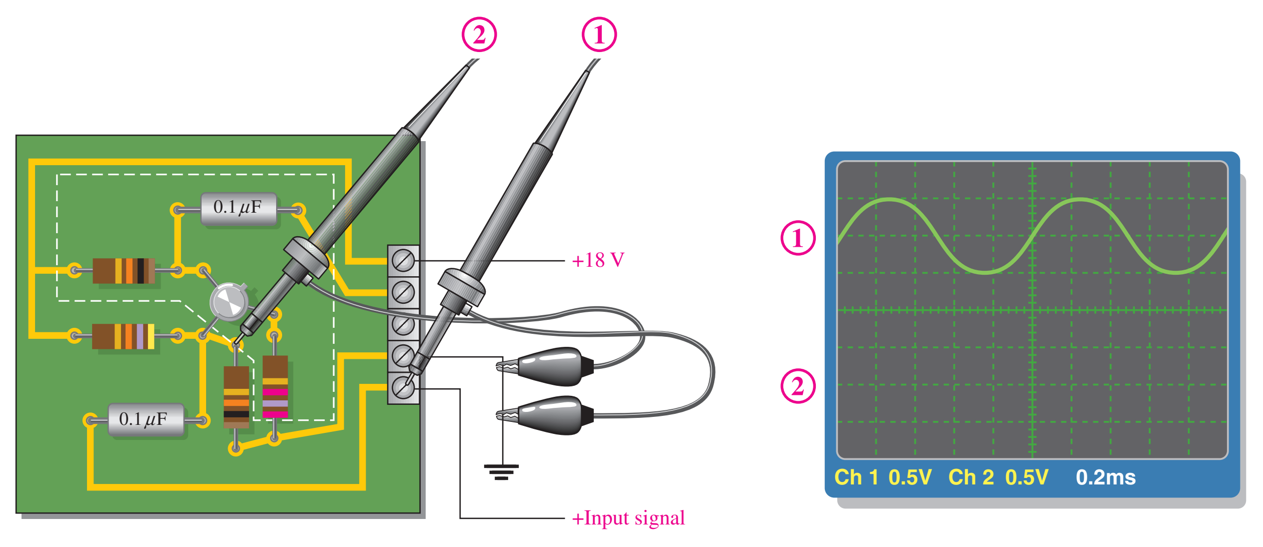


FIGURE 79

The *RC* input circuit acts effectively like a high-pass *RC* filter.





▲ FIGURE 80

Measuring the input circuit response at frequency f_1. Circled numbers relate scope inputs to the probes. The channel 1 waveform is shown.

R_2 goes to ground and the upper end of R_1 goes to the dc supply voltage as shown in Figure 79(a). Since there is no ac voltage at the +18 V dc terminal, the upper end of R_1 is at 0 V ac, which is referred to as *ac ground.* The development of the circuit into an effective high-pass *RC* filter is shown in parts (b) and (c).

The Amplifier Input Circuit

1. Determine the value of the equivalent resistance of the input circuit. Assume the amplifier (shown inside the white dashed lines in Figure 80) has no loading effect on the input circuit.

The Response at Frequency f_1

Refer to Figure 80. The input signal voltage is applied to the amplifier circuit board and displayed on channel 1 of the oscilloscope, and channel 2 is connected to a point on the circuit board.

2. Determine to what point on the circuit the channel 2 probe is connected, the frequency, and the voltage that should be displayed.

The Response at Frequency f_2

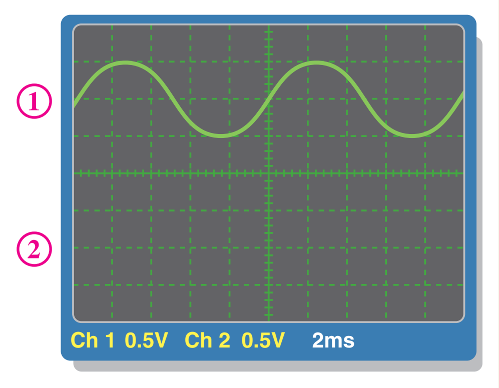
Refer to Figure 81 and the circuit board in Figure 80. The input signal voltage displayed on channel 1 of the oscilloscope is applied to the amplifier circuit board.

3. Determine the frequency and the voltage that should be displayed on channel 2.
4. State the difference between the channel 2 waveforms determined for f_1 and f_2. Explain the reason for the difference.

The Response at Frequency f_3

Refer to Figure 82 and the circuit board in Figure 80. The input signal voltage displayed on channel 1 of the oscilloscope is applied to the amplifier circuit board.

5. Determine the frequency and the voltage that should be displayed on channel 2.
6. State the difference between the channel 2 waveforms determined for f_2 and f_3. Explain the reason for the difference.

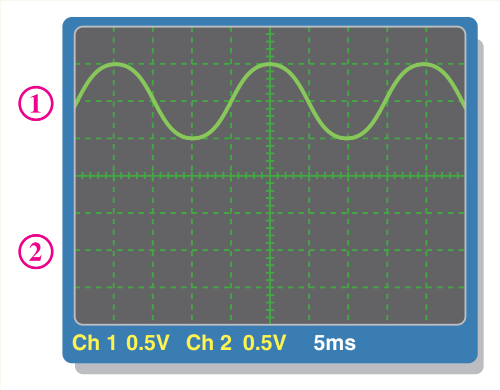


▲ FIGURE 81

Measuring the input circuit response at frequency f_2. The channel 1 waveform is shown.

▶ **FIGURE 82**

Measuring the input circuit response at frequency f_3. The channel 1 waveform is shown.



Response Curve for the Amplifier Input Circuit

7. Determine the frequency at which the signal voltage at point *B* in Figure 78 is 70.7% of its maximum value.
8. Plot the response curve using this voltage value and the values at frequencies f_1, f_2, and f_3.
9. How does this curve show that the input circuit acts as a high-pass filter?
10. What can you do to the circuit to lower the frequency at which the voltage is 70.7% of maximum without affecting the dc bias voltage?

Review

11. Explain the effect on the response of the amplifier input circuit of reducing the value of the coupling capacitor.
12. What is the voltage at point *B* in Figure 78 if the coupling capacitor opens when the ac input signal is 10 mV rms?
13. What is the voltage at point *B* in Figure 78 if resistor R_1 is open when the ac input signal is 10 mV rms?

Multisim Analysis

Using Multisim, connect the equivalent circuit in Figure 79(b).

- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 80. Measure the voltage at point *B* with the oscilloscope and compare the result from Activity 2.
- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 81. Measure the voltage at point *B* with the oscilloscope and compare the result from Activity 3.
- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 82. Measure the voltage at point *B* with the oscilloscope and compare the result from Activity 5.

SUMMARY

- A complex number represents a phasor quantity.
- The rectangular form of a complex number consists of a real part and a *j* part of the form $A + jB$.
- The polar form of a complex number consists of a magnitude and an angle of the form $C\angle \pm\theta$.
- Complex numbers can be added, subtracted, multiplied, and divided.
- When a sinusoidal voltage is applied to an *RC* circuit, the current and all the voltage drops are also sine waves.
- Total current in a series or parallel *RC* circuit always leads the source voltage.
- The resistor voltage is always in phase with the current.
- The capacitor voltage always lags the current by 90°.
- In a lag circuit, the output voltage lags the input voltage in phase.

- In a lead circuit, the output voltage leads the input voltage.
- In an *RC* circuit, the impedance is determined by both the resistance and the capacitive reactance combined.
- Impedance is expressed in units of ohms.
- The circuit phase angle is the angle between the total current and the applied (source) voltage.
- The impedance of a series *RC* circuit varies inversely with frequency.
- The phase angle (θ) of a series *RC* circuit varies inversely with frequency.
- For each parallel *RC* circuit, there is an equivalent series circuit for any given frequency.
- For each series *RC* circuit, there is an equivalent parallel circuit for any given frequency.
- The impedance of a circuit can be determined by measuring the applied voltage and the total current and then applying Ohm's law.
- In an *RC* circuit, part of the power is resistive and part reactive.
- The phasor combination of resistive power (true power) and reactive power is called *apparent power.*
- Apparent power is expressed in volt-amperes (VA).
- The power factor (PF) indicates how much of the apparent power is true power.
- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit.
- A filter passes certain frequencies and rejects others.
- A phase shift oscillator uses an *RC* network to produce a 180° phase shift.

KEY TERMS

Admittance (Y) A measure of the ability of a reactive circuit to permit current; the reciprocal of impedance. The unit is the siemens (S).

Apparent power (P_a) The phasor combination of resistive power (true power) and reactive power. The unit is the volt-ampere (VA).

Bandwidth The range of frequencies that is considered to be passed by a filter.

Capacitive susceptance (B_C) The ability of a capacitor to permit current: the reciprocal of capacitive reactance. The unit is the siemens (S).

Complex plane An area consisting of four quadrants on which a quantity containing both magnitude and direction can be represented.

Cutoff frequency The frequency at which the output voltage of a filter is 70.7% of the maximum output voltage.

Filter A type of circuit that passes certain frequencies and rejects all others.

Frequency response In electric circuits, the variation in the output voltage (or current) over a specified range of frequencies.

Imaginary number A number that exists on the vertical axis of the complex plane.

Impedance The total opposition to sinusoidal current expressed in ohms.

Polar form One form of a complex number made up of a magnitude and an angle.

Power factor The relationship between volt-amperes and true power or watts. Volt-amperes multiplied by the power factor equals true power.

***RC* lag circuit** A phase shift circuit in which the output voltage, taken across the capacitor, lags the input voltage by a specified angle.

***RC* lead circuit** A phase shift circuit in which the output voltage, taken across the resistor, leads the input voltage by a specified angle.

Real number A number that exists on the horizontal axis of the complex plane.

Rectangular form One form of a complex number made up of a real part and an imaginary part.

FORMULAS

Complex Numbers

1 $$C = \sqrt{A^2 + B^2}$$

2 $$\theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$

3 $$\pm A \pm jB = C\angle \pm\theta$$

4 $$A = C\cos\theta$$

5 $$B = C\sin\theta$$

6 $$C\angle\theta = A + jB$$

Series *RC* Circuits

7 $$\mathbf{Z} = R - jX_C$$

8 $$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

9 $$\mathbf{V} = \mathbf{IZ}$$

10 $$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$

11 $$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

12 $$\mathbf{V}_s = V_R - jV_C$$

13 $$\mathbf{V}_s = \sqrt{V_R^2 + V_C^2}\angle -\tan^{-1}\left(\frac{V_C}{V_R}\right)$$

Lag Circuit

14 $$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

15 $$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

Lead Circuit

16 $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

17 $$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

Parallel *RC* Circuits

18 $$\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$

19 $$\mathbf{Y} = G + jB_C$$

20 $$\mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}}$$

21 $$\mathbf{I} = \mathbf{VY}$$

22 $$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$$

23 $$\mathbf{I}_{tot} = I_R + jI_C$$

24 $$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

25 $$R_{\text{eq}} = Z\cos\theta$$

26 $X_{C(\text{eq})} = Z\sin\theta$

27 $\theta = \left(\frac{\Delta t}{T}\right)360^\circ$

Power in *RC* Circuits

28 $P_{\text{true}} = I^2R$

29 $P_r = I^2X_C$

30 $P_a = I^2Z$

31 $P_{\text{true}} = VI\cos\theta$

32 $PF = \cos\theta$

Applications

33 $f_r = \frac{1}{2\pi\sqrt{6}RC}$

34 $f_c = \frac{1}{2\pi RC}$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The imaginary axis and real axis are parts of the complex plane.
2. Two forms of complex numbers are rectangular and circular.
3. You cannot multiply or divide complex numbers.
4. The impedance of a series *RC* circuit can be expressed as a complex quantity.
5. Phasors can be used to represent complex quantities.
6. Voltage leads the current in terms of phase in a series *RC* circuit.
7. The impedance in a reactive circuit is dependent on frequency.
8. In a series *RC* lag circuit, the output is taken across the resistor.
9. Admittance is the reciprocal of impedance.
10. Power factor is determined by the phase angle between voltage and current.

SELF-TEST

Answers are at the end of the chapter.

1. A positive angle of 20° is equivalent to a negative angle of
   (a) -160° **(b)** -340° **(c)** -70° **(d)** -20°
2. In the complex plane, the number $3 + j4$ is located in the
   (a) first quadrant **(b)** second quadrant **(c)** third quadrant **(d)** fourth quadrant
3. In the complex plane, $12 - j6$ is located in the
   (a) first quadrant **(b)** second quadrant **(c)** third quadrant **(d)** fourth quadrant
4. The complex number $5 + j5$ is equivalent to
   (a) $5\angle 45^\circ$ **(b)** $25\angle 0^\circ$ **(c)** $7.07\angle 45^\circ$ **(d)** $7.07\angle 135^\circ$
5. The complex number $35\angle 60^\circ$ is equivalent to
   (a) $35 + j35$ **(b)** $35 + j60$ **(c)** $17.5 + j30.3$ **(d)** $30.3 + j17.5$
6. $(4 + j7) + (-2 + j9)$ is equal to
   (a) $2 + j16$ **(b)** $11 + j11$ **(c)** $-2 + j16$ **(d)** $2 - j2$
7. $(16 - j8) - (12 + j5)$ is equal to
   (a) $28 - j13$ **(b)** $4 - j13$ **(c)** $4 - j3$ **(d)** $-4 + j13$
8. $(5\angle 45^\circ)(2\angle 20^\circ)$ is equal to
   (a) $7\angle 65^\circ$ **(b)** $10\angle 25^\circ$ **(c)** $10\angle 65^\circ$ **(d)** $7\angle 25^\circ$

9. $(50\angle 10^\circ)/(25\angle 30^\circ)$ is equal to

(a) $25\angle 40^\circ$ **(b)** $2\angle 40^\circ$ **(c)** $25\angle -20^\circ$ **(d)** $2\angle -20^\circ$

10. In a series *RC* circuit, the voltage across the resistance is

(a) in phase with the source voltage **(b)** lagging the source voltage by 90°
(c) in phase with the current **(d)** lagging the current by 90°

11. In a series *RC* circuit, the voltage across the capacitor is

(a) in phase with the source voltage **(b)** lagging the resistor voltage by 90°
(c) in phase with the current **(d)** lagging the source voltage by 90°

12. When the frequency of the voltage applied to a series *RC* circuit is increased, the impedance

(a) increases **(b)** decreases **(c)** remains the same **(d)** doubles

13. When the frequency of the voltage applied to a series *RC* circuit is decreased, the phase angle

(a) increases **(b)** decreases **(c)** remains the same **(d)** becomes erratic

14. In a series *RC* circuit when the frequency and the resistance are doubled, the impedance

(a) doubles **(b)** is halved
(c) is quadrupled **(d)** cannot be determined without values

15. In a series *RC* circuit, 10 V rms is measured across the resistor and 10 V rms is also measured across the capacitor. The rms source voltage is

(a) 20 V **(b)** 14.14 V **(c)** 28.28 V **(d)** 10 V

16. The voltages in Question 15 are measured at a certain frequency. To make the resistor voltage greater than the capacitor voltage, the frequency

(a) must be increased **(b)** must be decreased
(c) is held constant **(d)** has no effect

17. When $R = X_C$, the phase angle is

(a) 0° **(b)** +90° **(c)** −90° **(d)** 45°

18. To decrease the phase angle below 45°, the following condition must exist:

(a) $R = X_C$ **(b)** $R < X_C$ **(c)** $R > X_C$ **(d)** $R = 10X_C$

19. When the frequency of the source voltage is increased, the impedance of a parallel *RC* circuit

(a) increases **(b)** decreases **(c)** does not change

20. In a parallel *RC* circuit, there is 1 A rms through the resistive branch and 1 A rms through the capacitive branch. The total rms current is

(a) 1 A **(b)** 2 A **(c)** 2.28 A **(d)** 1.414 A

21. A power factor of 1 indicates that the circuit phase angle is

(a) 90° **(b)** 45° **(c)** 180° **(d)** 0°

22. For a certain load, the true power is 100 W and the reactive power is 100 VAR. The apparent power is

(a) 200 VA **(b)** 100 VA **(c)** 141.4 VA **(d)** 141.4 W

23. Energy sources are normally rated in

(a) watts **(b)** volt-amperes **(c)** volt-amperes reactive **(d)** none of these

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 86.

1. If *C* opens, the voltage across it

(a) increases **(b)** decreases **(c)** stays the same

2. If *R* opens, the voltage across *C*

(a) increases **(b)** decreases **(c)** stays the same

3. If the frequency is increased, the voltage across R
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 87.

4. If R_1 opens, the voltage across R_2
   (a) increases **(b)** decreases **(c)** stays the same
5. If C_2 is increased to 0.47 μF, the voltage across it
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 93.

6. If R becomes open, the voltage across the capacitor
   (a) increases **(b)** decreases **(c)** stays the same
7. If the source voltage increases, X_C
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 98.

8. If R_2 opens, the voltage from the top of R_2 to ground
   (a) increases **(b)** decreases **(c)** stays the same
9. If C_2 shorts out, the voltage across C_1
   (a) increases **(b)** decreases **(c)** stays the same
10. If the frequency of the source voltage is increased, the current through the resistors
   (a) increases **(b)** decreases **(c)** stays the same
11. If the frequency of the source voltage is decreased, the current through the capacitors
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 103.

12. If C_3 opens, the voltage from point B to ground
   (a) increases **(b)** decreases **(c)** stays the same
13. If C_2 opens, the voltage from point B to ground
   (a) increases **(b)** decreases **(c)** stays the same
14. If a short develops from point C to ground, the voltage from point A to ground
   (a) increases **(b)** decreases **(c)** stays the same
15. If capacitor C_3 opens, the voltage from B to D
   (a) increases **(b)** decreases **(c)** stays the same
16. If the source frequency increases, the voltage from point C to ground
   (a) increases **(b)** decreases **(c)** stays the same
17. If the source frequency increases, the current from the source
   (a) increases **(b)** decreases **(c)** stays the same
18. If R_2 shorts out, the voltage across C_1
   (a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Complex Number System

1. What are the two characteristics of a quantity indicated by a complex number?
2. Locate the following numbers on the complex plane:
   (a) $+6$ **(b)** -2 **(c)** $+j3$ **(d)** $-j8$

3. Locate the points represented by each of the following coordinates on the complex plane:

(a) $3, j5$ **(b)** $-7, j1$ **(c)** $-10, -j10$

*__4.__ Determine the coordinates of each point having the same magnitude but located 180° away from each point in Problem 3.

*__5.__ Determine the coordinates of each point having the same magnitude but located 90° away from those in Problem 3.

6. Points on the complex plane are described below. Express each point as a complex number in rectangular form:

(a) 3 units to the right of the origin on the real axis, and up 5 units on the *j* axis.

(b) 2 units to the left of the origin on the real axis, and 1.5 units up on the *j* axis.

(c) 10 units to the left of the origin on the real axis, and down 14 units on the $-j$ axis.

7. What is the value of the hypotenuse of a right triangle whose sides are 10 and 15?

8. Convert each of the following rectangular numbers to polar form:

(a) $40 - j40$ **(b)** $50 - j200$ **(c)** $35 - j20$ **(d)** $98 + j45$

9. Convert each of the following polar numbers to rectangular form:

(a) $1000\angle -50^\circ$ **(b)** $15\angle 160^\circ$ **(c)** $25\angle -135^\circ$ **(d)** $3\angle 180^\circ$

10. Express each of the following polar numbers using a negative angle to replace the positive angle:

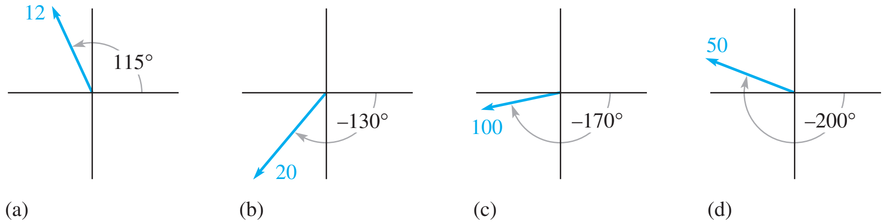
(a) $10\angle 120^\circ$ **(b)** $32\angle 85^\circ$ **(c)** $5\angle 310^\circ$

11. Identify the quadrant in which each point in Problem 8 is located.

12. Identify the quadrant in which each point in Problem 10 is located.

13. Write the polar expressions using positive angles for each phasor in Figure 83.

▶ **FIGURE 83**



14. Add the following sets of complex numbers:

(a) $9 + j3$ and $5 + j8$ **(b)** $3.5 - j4$ and $2.2 + j6$

(c) $-18 + j23$ and $30 - j15$ **(d)** $12\angle 45^\circ$ and $20\angle 32^\circ$

(e) $3.8\angle 75^\circ$ and $1 + j1.8$ **(f)** $50 - j39$ and $60\angle -30^\circ$

15. Perform the following subtractions:

(a) $(2.5 + j1.2) - (1.4 + j0.5)$ **(b)** $(-45 - j23) - (36 + j12)$

(c) $(8 - j4) - 3\angle 25^\circ$ **(d)** $48\angle 135^\circ - 33\angle -60^\circ$

16. Multiply the following numbers:

(a) $4.5\angle 48^\circ$ and $3.2\angle 90^\circ$ **(b)** $120\angle -220^\circ$ and $95\angle 200^\circ$

(c) $-3\angle 150^\circ$ and $4 - j3$ **(d)** $67 + j84$ and $102\angle 40^\circ$

(e) $15 - j10$ and $-25 - j30$ **(f)** $0.8 + j0.5$ and $1.2 - j1.5$

17. Perform the following divisions:

(a) $\dfrac{8\angle 50^\circ}{2.5\angle 39^\circ}$ **(b)** $\dfrac{63\angle -91^\circ}{9\angle 10^\circ}$ **(c)** $\dfrac{28\angle 30^\circ}{14 - j12}$ **(d)** $\dfrac{40 - j30}{16 + j8}$

18. Perform the following operations:

(a) $\dfrac{2.5\angle 65^\circ - 1.8\angle -23^\circ}{1.2\angle 37^\circ}$ **(b)** $\dfrac{(100\angle 15^\circ)(85 - j150)}{25 + j45}$

(c) $\dfrac{(250\angle 90^\circ + 175\angle 75^\circ)(50 - j100)}{(125 + j90)(35\angle 50^\circ)}$ **(d)** $\dfrac{(1.5)^2(3.8)}{1.1} + j\left(\dfrac{8}{4} - j\dfrac{4}{2}\right)$

PART 1: SERIES CIRCUITS

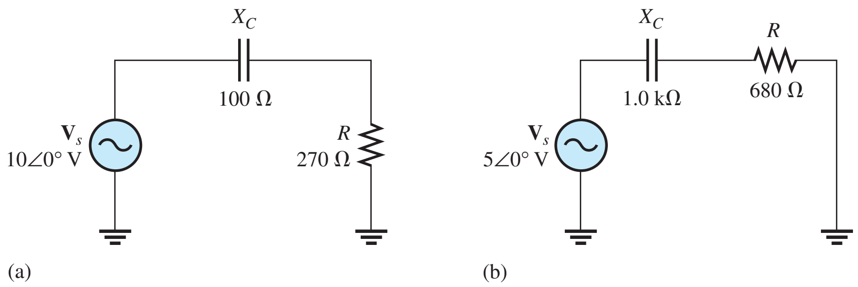
SECTION 2 Sinusoidal Response of Series *RC* Circuits

19. An 8 kHz sinusoidal voltage is applied to a series *RC* circuit. What is the frequency of the voltage across the resistor? Across the capacitor?

20. What is the wave shape of the current in the circuit of Problem 19?

SECTION 3 Impedance of Series *RC* Circuits

21. Express the total impedance of each circuit in Figure 84 in both polar and rectangular forms.

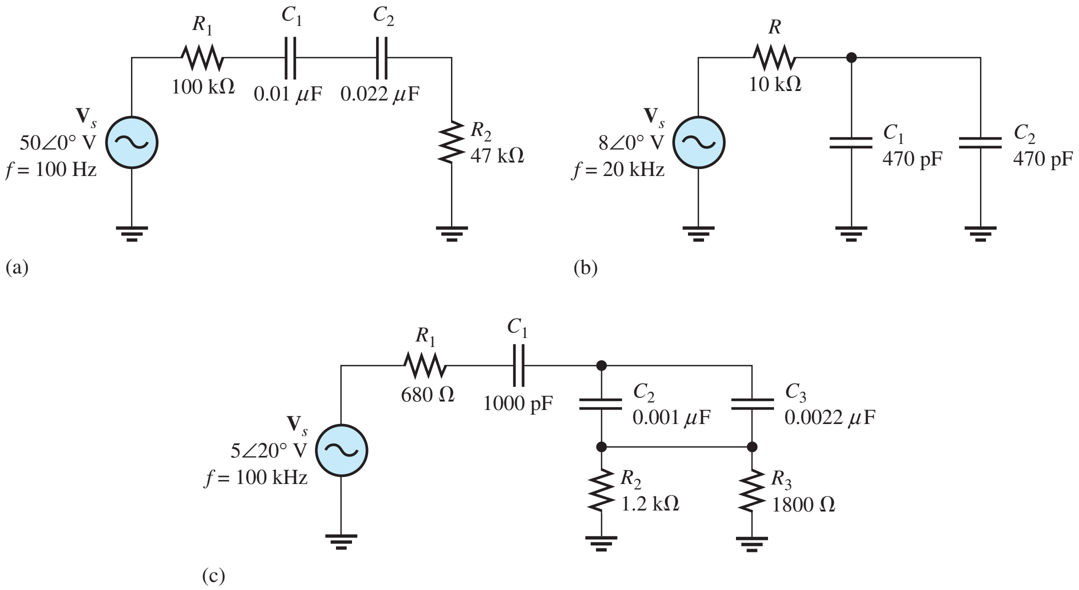


X_C
100 Ω
$\mathbf{V}_s$
10∠0° V
R
270 Ω
X_C
1.0 kΩ
R
680 Ω
$\mathbf{V}_s$
5∠0° V

(a) (b)

▲ FIGURE 84

22. Determine the impedance magnitude and phase angle in each circuit in Figure 85.



R_1
100 kΩ
C_1
0.01 μF
C_2
0.022 μF
$\mathbf{V}_s$
50∠0° V
f = 100 Hz
R_2
47 kΩ
R
10 kΩ
$\mathbf{V}_s$
8∠0° V
f = 20 kHz
C_1
470 pF
C_2
470 pF
R_1
680 Ω
C_1
1000 pF
C_2
0.001 μF
C_3
0.0022 μF
$\mathbf{V}_s$
5∠20° V
f = 100 kHz
R_2
1.2 kΩ
R_3
1800 Ω

(a) (b) (c)

▲ FIGURE 85

23. For the circuit of Figure 86, determine the impedance expressed in rectangular form for each of the following frequencies:

(a) 100 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 2.5 kHz

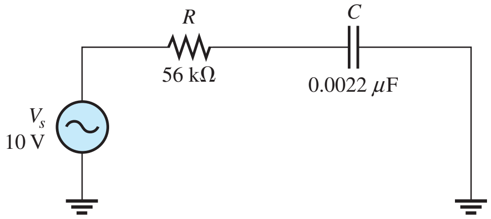
24. Repeat Problem 23 for $C = 0.0047\ \mu\text{F}$.

25. Determine the values of R and X_C in a series *RC* circuit for the following values of total impedance:

(a) $\mathbf{Z} = 33\ \Omega - j50\ \Omega$ **(b)** $\mathbf{Z} = 300\angle -25^\circ\ \Omega$

(c) $\mathbf{Z} = 1.8\angle -67.2^\circ\ \text{k}\Omega$ **(d)** $\mathbf{Z} = 789\angle -45^\circ\ \Omega$

▶ FIGURE 86



R
56 kΩ
C
0.0022 μF
V_s
10 V

SECTION 4 Analysis of Series *RC* Circuits

26. Express the current in polar form for each circuit of Figure 84.

27. Express the current in each circuit of Figure 84 in rectangular form.

28. Calculate the total current in each circuit of Figure 85 and express in polar form.

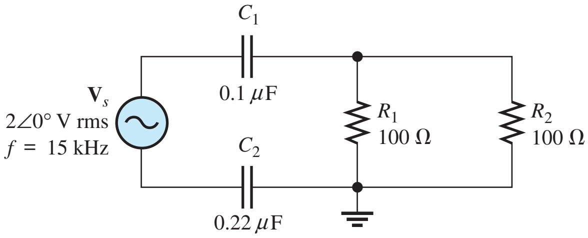
29. Express the current in each circuit of Figure 85 in rectangular form.

30. Determine the phase angle between the applied voltage and the current for each circuit in Figure 85.

31. Repeat Problem 30 for the circuit in Figure 86, using $f = 5$ kHz.

32. For the circuit in Figure 87, draw the phasor diagram showing all voltages and the total current. Indicate the phase angles.

▶ FIGURE 87

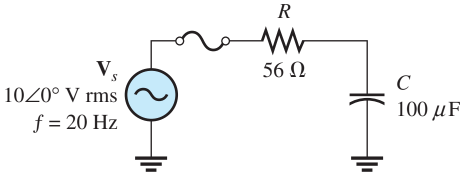


C_1
0.1 μF
$\mathbf{V}_s$
2∠0° V rms
f = 15 kHz
C_2
0.22 μF
R_1
100 Ω
R_2
100 Ω

33. For the circuit in Figure 88, determine the following in polar form:

(a) $\mathbf{Z}$ **(b)** $\mathbf{I}_{tot}$ **(c)** $\mathbf{V}_R$ **(d)** $\mathbf{V}_C$

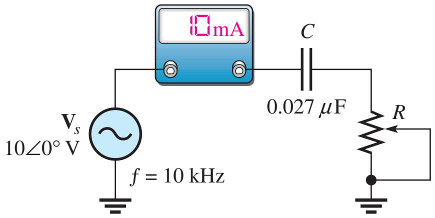
▶ FIGURE 88



R
56 Ω
$\mathbf{V}_s$
10∠0° V rms
f = 20 Hz
C
100 μF

*__34.__ To what value must the rheostat be set in Figure 89 to make the total current 10 mA? What is the resulting angle?

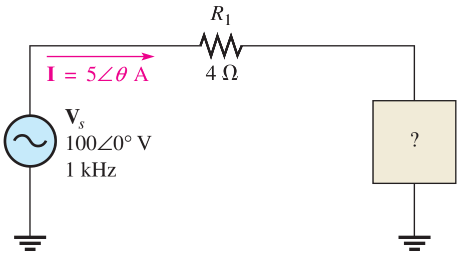
▶ FIGURE 89



10mA
C
0.027 μF
R
$\mathbf{V}_s$
10∠0° V
f = 10 kHz

*__35.__ Determine the series element or elements that must be installed in the block of Figure 90 to meet the following requirements: $P_{\text{true}} = 400$ W and there is a leading power factor (I_{tot} leads V_s).

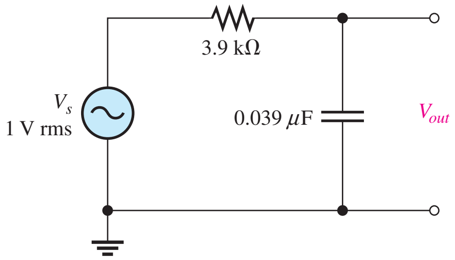
▶ FIGURE 90



36. For the lag circuit in Figure 91, determine the phase shift between the input voltage and the output voltage for each of the following frequencies:

(a) 1 Hz **(b)** 100 Hz **(c)** 1 kHz **(d)** 10 kHz

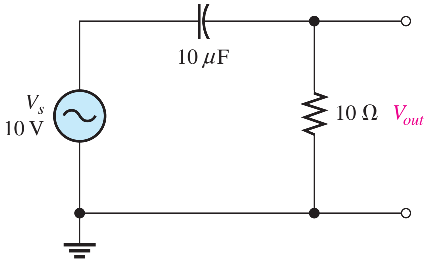
▶ FIGURE 91



37. The lag circuit in Figure 91 also acts as a low-pass filter. Draw a response curve for this circuit by plotting the output voltage versus frequency for 0 Hz to 10 kHz in 1 kHz increments.

38. Repeat Problem 36 for the lead circuit in Figure 92.

▶ FIGURE 92



39. Plot the frequency response curve of the output amplitude for the lead circuit in Figure 92 for a frequency range of 0 Hz to 10 kHz in 1 kHz increments.

40. Draw the voltage phasor diagram for the circuit in Figure 91 for a frequency of 5 kHz with $V_s = 1$ V rms.

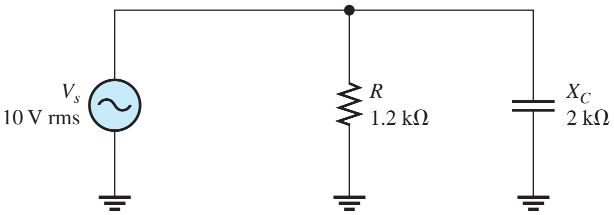
41. Repeat Problem 40 for the circuit in Figure 92. $V_s = 10$ V rms and $f = 1$ kHz.

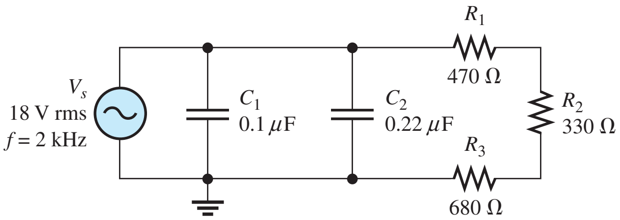
PART 2: PARALLEL CIRCUITS

SECTION 5 Impedance and Admittance of Parallel *RC* Circuits

42. Determine the impedance and express it in polar form for the circuit in Figure 93.

▶ FIGURE 93




V_s
18 V rms
f = 2 kHz
C_1
0.1 μF
C_2
0.22 μF
R_1
470 Ω
R_2
330 Ω
R_3
680 Ω

▲ FIGURE 94

43. Determine the impedance magnitude and phase angle in Figure 94.

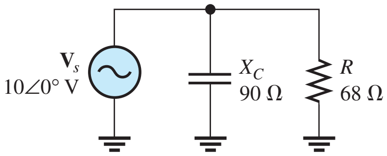
44. Repeat Problem 43 for the following frequencies:

(a) 1.5 kHz **(b)** 3 kHz **(c)** 5 kHz **(d)** 10 kHz

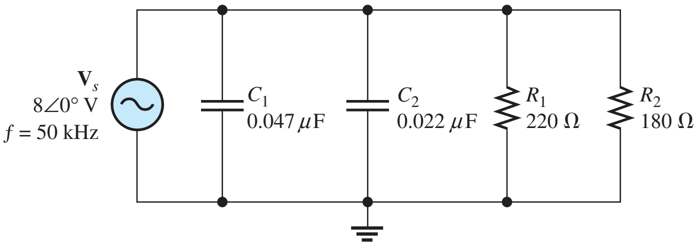
SECTION 6 Analysis of Parallel *RC* Circuits

45. For the circuit in Figure 95, find all the currents and voltages in polar form.

▶ FIGURE 95


$\mathbf{V}_s$
10∠0° V
X_C
90 Ω
R
68 Ω

46. For the parallel circuit in Figure 96, find the magnitude of each branch current and the total current. What is the phase angle between the applied voltage and the total current?

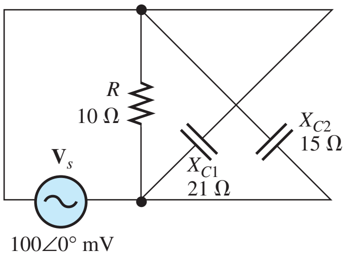

$\mathbf{V}_s$
8∠0° V
f = 50 kHz
C_1
0.047 μF
C_2
0.022 μF
R_1
220 Ω
R_2
180 Ω

▲ FIGURE 96

47. For the circuit in Figure 97, determine the following:

(a) $\mathbf{Z}$ **(b)** $\mathbf{I}_R$ **(c)** $\mathbf{I}_{C(tot)}$ **(d)** $\mathbf{I}_{tot}$ **(e)** θ

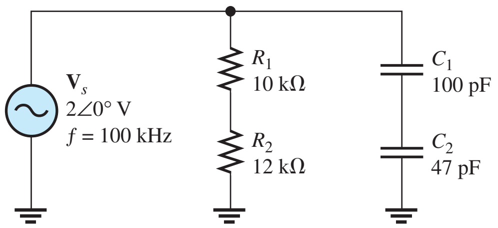
▶ FIGURE 97


R
10 Ω
$\mathbf{V}_s$
100∠0° mV
X_{C1}
21 Ω
X_{C2}
15 Ω

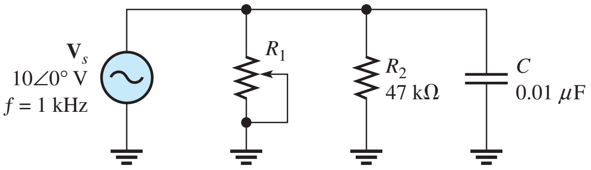
48. Repeat Problem 47 for $R = 5.6\text{ k}\Omega$, $C_1 = 0.047\ \mu\text{F}$, $C_2 = 0.022\ \mu\text{F}$, and $f = 500\text{ Hz}$.

*__49.__ Convert the circuit in Figure 98 to an equivalent series form.

▶ FIGURE 98


V_s
2∠0° V
f = 100 kHz
R_1
10 kΩ
R_2
12 kΩ
C_1
100 pF
C_2
47 pF

*__50.__ Determine the value to which R_1 must be adjusted to get a phase angle of 30° between the source voltage and the total current in Figure 99.


V_s
10∠0° V
f = 1 kHz
R_1
R_2
47 kΩ
C
0.01 μF

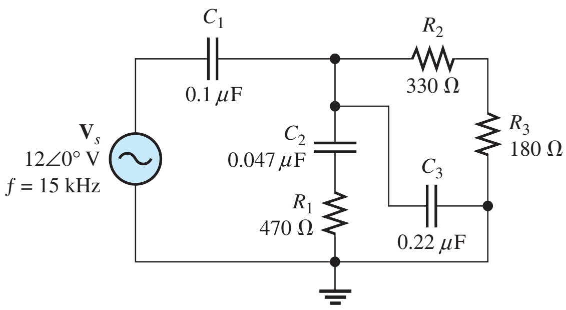
▲ FIGURE 99

PART 3: SERIES-PARALLEL CIRCUITS

SECTION 7 Analysis of Series-Parallel *RC* Circuits

51. Determine the voltages in polar form across each element in Figure 100. Draw the voltage phasor diagram.

▶ FIGURE 100


C_1
0.1 μF
R_2
330 Ω
R_3
180 Ω
V_s
12∠0° V
f = 15 kHz
C_2
0.047 μF
C_3
R_1
470 Ω
0.22 μF

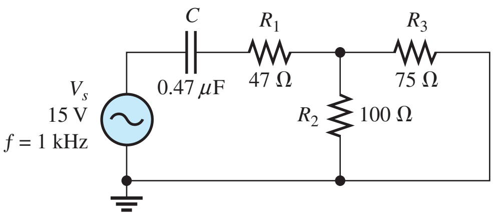
52. Is the circuit in Figure 100 predominantly resistive or predominantly capacitive?

53. Find the current through each branch and the total current in Figure 100. Express the currents in polar form. Draw the current phasor diagram.

54. For the circuit in Figure 101, determine the following:

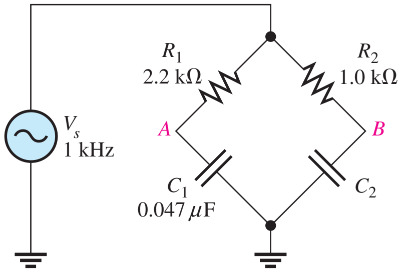
(a) $\mathbf{I}_{tot}$ **(b)** θ **(c)** $\mathbf{V}_{R1}$ **(d)** $\mathbf{V}_{R2}$ **(e)** $\mathbf{V}_{R3}$ **(f)** $\mathbf{V}_C$

▶ FIGURE 101


C
R_1
R_3
0.47 μF
47 Ω
75 Ω
V_s
15 V
f = 1 kHz
R_2
100 Ω

*__55.__ Determine the value of C_2 in Figure 102 when $V_A = V_B$.

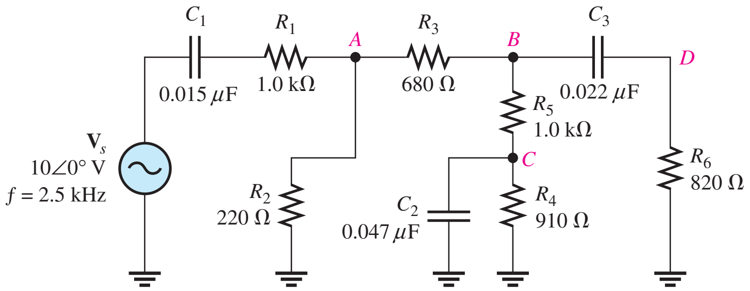
▶ FIGURE 102


R_1
2.2 kΩ
R_2
1.0 kΩ
V_s
1 kHz
A
B
C_1
0.047 μF
C_2

*__56.__ Determine the voltage and its phase angle at each point labeled in Figure 103.

*__57.__ Find the current through each component in Figure 103.

*__58.__ Draw the voltage and current phasor diagram for Figure 103.


C_1
0.015 μF
R_1
1.0 kΩ
A
R_3
680 Ω
B
C_3
0.022 μF
D
R_5
1.0 kΩ
$\mathbf{V}_s$
10∠0° V
f = 2.5 kHz
C
R_6
820 Ω
R_2
220 Ω
C_2
0.047 μF
R_4
910 Ω

▲ FIGURE 103

PART 4: SPECIAL TOPICS

SECTION 8 Power in *RC* Circuits

59. In a certain series *RC* circuit, the true power is 2 W, and the reactive power is 3.5 VAR. Determine the apparent power.

60. In Figure 88, what is the true power and the reactive power?

61. What is the power factor for the circuit of Figure 98?

62. Determine P_{true}, P_r, P_a, and *PF* for the circuit in Figure 101. Draw the power triangle.

*__63.__ A single 240 V, 60 Hz source drives two loads. Load *A* has an impedance of 50 Ω and a power factor of 0.85. Load *B* has an impedance of 72 Ω and a power factor of 0.95.

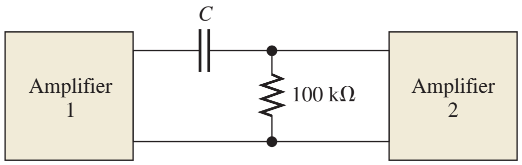
- **(a)** How much current does each load draw?
- **(b)** What is the reactive power in each load?
- **(c)** What is the true power in each load?
- **(d)** What is the apparent power in each load?
- **(e)** Which load has more voltage drop along the lines connecting it to the source?

SECTION 9 Basic Applications

64. Calculate the frequency of oscillation for the circuit in Figure 62 if all *C*s are 0.0022 μF and all *R*s are 10 kΩ.

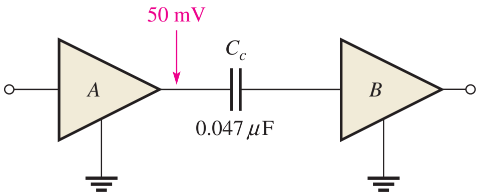
*__65.__ What value of coupling capacitor is required in Figure 104 so that the signal voltage at the input of amplifier 2 is at least 70.7% of the signal voltage at the output of amplifier 1 when the frequency is 20 Hz?

▶ FIGURE 104


C
Amplifier
1
100 kΩ
Amplifier
2

66. The rms value of the signal voltage out of amplifier A in Figure 105 is 50 mV. If the input resistance to amplifier B is $10\ \text{k}\Omega$, how much of the signal is lost due to the coupling capacitor when the frequency is 3 kHz?

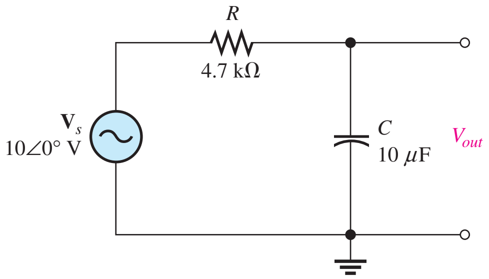
▶ FIGURE 105


50 mV
C_c
A
B
$0.047\ \mu\text{F}$

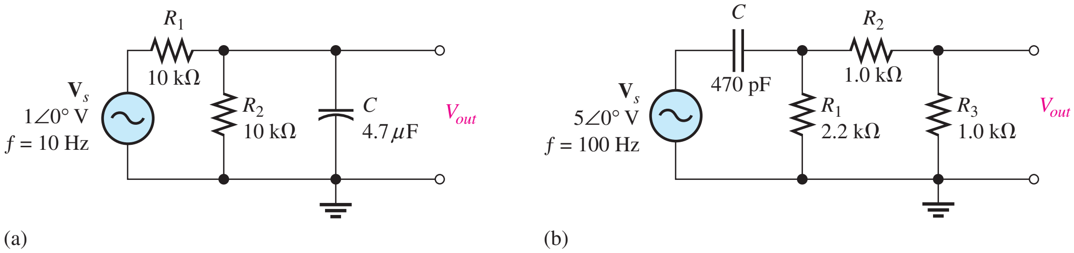
SECTION 10 Troubleshooting

67. Assume that the capacitor in Figure 106 is excessively leaky. Show how this degradation affects the output voltage and phase angle, assuming that the leakage resistance is $5\ \text{k}\Omega$ and the frequency is 10 Hz.

▶ FIGURE 106


R
$4.7\ \text{k}\Omega$
$\mathbf{V}_s$
$10\angle 0°\ \text{V}$
C
$10\ \mu\text{F}$
V_{out}

*__68.__ Each of the capacitors in Figure 107 has developed a leakage resistance of $2\ \text{k}\Omega$. Determine the output voltages under this condition for each circuit.


R_1
$10\ \text{k}\Omega$
$\mathbf{V}_s$
$1\angle 0°\ \text{V}$
$f = 10\ \text{Hz}$
R_2
$10\ \text{k}\Omega$
C
$4.7\ \mu\text{F}$
V_{out}
C
$470\ \text{pF}$
R_2
$1.0\ \text{k}\Omega$
$\mathbf{V}_s$
$5\angle 0°\ \text{V}$
$f = 100\ \text{Hz}$
R_1
$2.2\ \text{k}\Omega$
R_3
$1.0\ \text{k}\Omega$
V_{out}

(a)

(b)

▲ FIGURE 107

69. Determine the output voltage for the circuit in Figure 107(a) for each of the following failure modes, and compare it to the correct output:

(a) R_1 open **(b)** R_2 open **(c)** C open **(d)** C shorted

70. Determine the output voltage for the circuit in Figure 107(b) for each of the following failure modes, and compare it to the correct output:

(a) C open **(b)** C shorted **(c)** R_1 open **(d)** R_2 open **(e)** R_3 open

Multisim Troubleshooting and Analysis

These problems require Multisim.

71. Open file P15-71 and determine if there is a fault. If so, find the fault.

72. Open file P15-72 and determine if there is a fault. If so, find the fault.

73. Open file P15-73 and determine if there is a fault. If so, find the fault.

74. Open file P15-74 and determine if there is a fault. If so, find the fault.

75. Open file P15-75 and determine if there is a fault. If so, find the fault.

76. Open file P15-76 and determine if there is a fault. If so, find the fault.

77. Open file P15-77 and determine the frequency response for the filter.

78. Open file P15-78 and determine the frequency response for the filter.

ANSWERS

SECTION CHECKUPS

SECTION 1 The Complex Number System

1. $2.828\angle 45°$; first
2. $3.54 - j3.54$, fourth
3. $4 + j1$
4. $3 + j7$
5. $16\angle 110°$
6. $5\angle 15°$

SECTION 2 Sinusoidal Response of Series *RC* Circuits

1. The voltage frequency is 60 Hz. The current frequency is 60 Hz.
2. The capacitive reactance causes the phase shift.
3. The phase angle is closer to 0°.

SECTION 3 Impedance of Series *RC* Circuits

1. $R = 150\ \Omega$; $X_C = 220\ \Omega$
2. $\mathbf{Z} = 33\ \text{k}\Omega - j50\ \text{k}\Omega$
3. $Z = \sqrt{R^2 + X_C^2} = 59.9\ \text{k}\Omega$; $\theta = -\tan^{-1}(X_C/R) = -56.6°$

SECTION 4 Analysis of Series *RC* Circuits

1. $V_s = \sqrt{V_R^2 + V_C^2} = 7.21\ \text{V}$
2. $\theta = -\tan^{-1}(X_C/R) = -56.3°$
3. $\theta = 90°$
4. When f increases, X_C decreases, Z decreases, and θ decreases.
5. $\phi = -90° + \tan^{-1}(X_C/R) = -62.8°$
6. $V_{out} = (R/\sqrt{R^2 + X_C^2})V_{in} = 8.90\ \text{V rms}$

SECTION 5 Impedance and Admittance of Parallel *RC* Circuits

1. Conductance is the reciprocal of resistance, capacitive susceptance is the reciprocal of capacitive reactance, and admittance is the reciprocal of impedance.
2. $Y = 1/Z = 1/100\ \Omega = 10\ \text{mS}$
3. $\mathbf{Y} = 1/\mathbf{Z} = 25.1\angle 32.1°\ \mu\text{S}$
4. $\mathbf{Z} = 39.8\angle -32.1°\ \text{k}\Omega$

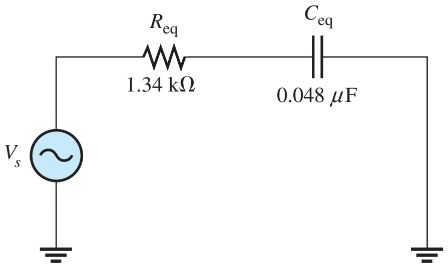
SECTION 6 Analysis of Parallel *RC* Circuits

1. $I_{tot} = V_sY = 21\ \text{mA}$
2. $I_{tot} = \sqrt{I_R^2 + I_C^2} = 18\ \text{mA}$; $\theta = \tan^{-1}(I_C/I_R) = 56.3°$; θ is with respect to applied voltage.
3. $\theta = 90°$

SECTION 7 Analysis of Series-Parallel *RC* Circuits

1. See Figure 108.
2. $\mathbf{Z}_{tot} = \mathbf{V}_s/\mathbf{I}_{tot} = 36.9\angle -51.6°\ \Omega$

▶ FIGURE 108



SECTION 8 Power in *RC* Circuits

1. Power dissipation is due to resistance.
2. $PF = \cos\theta = 0.707$
3. $P_{\text{true}} = I^2R = 1.32\ \text{kW}; P_r = I^2X_C = 1.84\ \text{kVAR}; P_a = I^2Z = 2.26\ \text{kVA}$

SECTION 9 Basic Applications

1. 180°
2. The output is across the capacitor.

SECTION 10 Troubleshooting

1. The leakage resistance acts in parallel with *C*, which alters the circuit time constant.
2. The capacitor is open.
3. An open series resistor or the capacitor shorted will result in 0 V across the capacitor.

RELATED PROBLEMS FOR EXAMPLES

1 **(a)** 1st **(b)** 4th **(c)** 3rd **(d)** 2nd

2 $29.2\angle 52°$

3 $70.1 - j34.2$

4 $-1 - j8$

5 $-13.5 - j4.5$

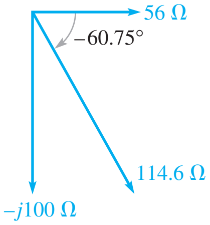
6 $1500\angle -50°$

7 $4\angle -42°$

8 $114.612390255\ \angle -60.75\ldots$

See Figure 109.

▶ FIGURE 109



9 $\mathbf{V}_s = 2.56\angle -38.5°\ \text{V}$

10 $\mathbf{I} = 3.80\angle 33.4°\ \text{mA}$

11 $Z = 15.9\ \text{k}\Omega, \theta = -86.4^\circ$

12 The phase lag increases.

13 The output voltage decreases.

14 The phase lead decreases.

15 The output voltage increases.

16 $\mathbf{Z} = 24.3\angle -76.0^\circ\ \Omega$

17 $\mathbf{Y} = 4.60\angle 48.8^\circ\ \text{mS}$

18 $\mathbf{I} = 6.16\angle 42.4^\circ\ \text{mA}$

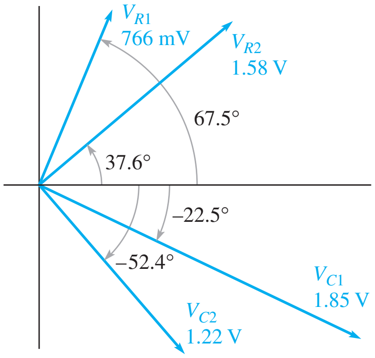
19 $\mathbf{I}_{tot} = 117\angle 31.0^\circ\ \text{mA}$

20 $R_{\text{eq}} = 8.99\ \text{k}\Omega, X_{C(\text{eq})} = 4.38\ \text{k}\Omega$

21 $\mathbf{V}_1 = 7.05\angle 8.9^\circ\ \text{V}, \mathbf{V}_2 = 3.21\angle -18.9^\circ\ \text{V}$

22 $\mathbf{V}_{R1} = 766\angle 67.5^\circ\ \text{mV}; \mathbf{V}_{C1} = 1.85\angle -22.5^\circ\ \text{V}; \mathbf{V}_{R2} = 1.58\angle 37.6^\circ\ \text{V};$ $\mathbf{V}_{C2} = 1.22\angle -52.4^\circ\ \text{V}$; See Figure 110.

FIGURE 110



23 $PF = 0.146$

24 $P_{\text{true}} = 213\ \text{mW}$

25 1.60 kHz

26 $V_{out} = 7.29\ \text{V}$

27 Resistor open

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** T
6. F **7.** T **8.** F **9.** T **10.** T

SELF-TEST

1. (b) **2.** (a) **3.** (d) **4.** (c) **5.** (c) **6.** (a) **7.** (b)
8. (c) **9.** (d) **10.** (c) **11.** (b) **12.** (b) **13.** (a) **14.** (d)
15. (b) **16.** (a) **17.** (d) **18.** (c) **19.** (b) **20.** (d) **21.** (d)
22. (c) **23.** (b)

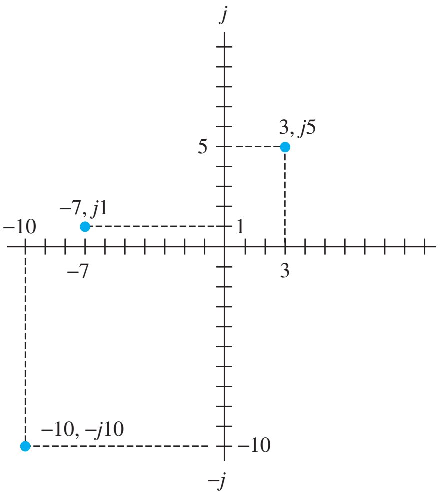
CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (b) **6.** (c) **7.** (c) **8.** (a)
9. (a) **10.** (c) **11.** (b) **12.** (a) **13.** (a) **14.** (b) **15.** (a) **16.** (b)
17. (a) **18.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. Magnitude, angle

3. See Figure 111.



▲ **FIGURE 111**

5. **(a)** $-5, +j3$ and $5, -j3$
(b) $-1, -j7$ and $1, +j7$
(c) $-10, +j10$ and $10, -j10$

7. 18.0

9. **(a)** $643 - j766$ **(b)** $-14.1 + j5.13$
(c) $-17.7 - j17.7$ **(d)** $-3 + j0$

11. **(a)** Fourth **(b)** Fourth **(c)** Fourth **(d)** First

13. **(a)** $12\angle 115^\circ$ **(b)** $20\angle 230^\circ$
(c) $100\angle 190^\circ$ **(d)** $50\angle 160^\circ$

15. **(a)** $1.1 + j0.7$ **(b)** $-81 - j35$
(c) $5.28 - j5.27$ **(d)** $-50.4 + j62.5$

17. **(a)** $3.2\angle 11^\circ$ **(b)** $7\angle -101^\circ$
(c) $1.52\angle 70.6^\circ$ **(d)** $2.79\angle -63.5^\circ$

19. 8 kHz, 8 kHz

21. **(a)** $270\ \Omega - j100\ \Omega$, $288\angle -20.3^\circ\ \Omega$
(b) $680\ \Omega - j1000\ \Omega$, $1.21\angle -55.8^\circ$ kΩ

23. **(a)** 56 k$\Omega - j723$ kΩ
(b) 56 k$\Omega - j145$ kΩ
(c) 56 k$\Omega - j72.3$ kΩ
(d) 56 k$\Omega - j28.9$ kΩ

25. **(a)** $R = 33\ \Omega, X_C = 50\ \Omega$
(b) $R = 272\ \Omega, X_C = 127\ \Omega$
(c) $R = 698\ \Omega, X_C = 1.66$ kΩ
(d) $R = 558\ \Omega, X_C = 558\ \Omega$

27. **(a)** 32.5 mA $+ j12.0$ mA
(b) 2.32 mA $+ j3.42$ mA

29. **(a)** $98.3\ \mu$A $+ j154\ \mu$A
(b) $466\ \mu$A $+ j395\ \mu$A
(c) 0.472 mA $+ j1.92$ mA

31. -14.5°

33. **(a)** $97.3\angle -54.9^\circ\ \Omega$ **(b)** $103\angle 54.9^\circ$ mA
(c) $5.76\angle 54.9^\circ$ V **(d)** $8.18\angle -35.1^\circ$ V

35. $R_X = 12\ \Omega$, $C_X = 13.3\ \mu$F in series.

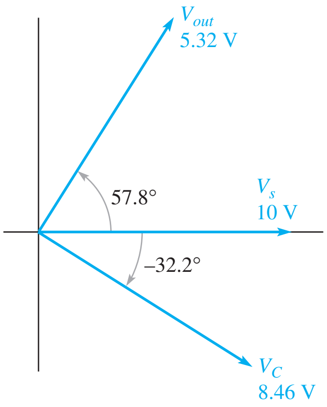
37.

0 Hz	1 V
1 kHz	723 mV
2 kHz	464 mV
3 kHz	329 mV
4 kHz	253 mV
5 kHz	205 mV
6 kHz	172 mV
7 kHz	148 mV
8 kHz	130 mV
9 kHz	115 mV
10 kHz	104 mV

39.

0 Hz	0 V
1 kHz	5.32 V
2 kHz	7.82 V
3 kHz	8.83 V
4 kHz	9.29 V
5 kHz	9.53 V
6 kHz	9.66 V
7 kHz	9.76 V
8 kHz	9.80 V
9 kHz	9.84 V
10 kHz	9.87 V

41. See Figure 112.



▲ **FIGURE 112**

43. 245 Ω, −80.5°

45. $\mathbf{V}_C = \mathbf{V}_R = 10\angle 0^\circ$ V

$\mathbf{I}_{tot} = 184\angle 37.1^\circ$ mA

$\mathbf{I}_R = 147\angle 0^\circ$ mA

$\mathbf{I}_C = 111\angle 90^\circ$ mA

47. **(a)** $6.59\angle -48.8^\circ$ Ω **(b)** $10\angle 0^\circ$ mA

(c) $11.4\angle 90^\circ$ mA **(d)** $15.2\angle 48.8^\circ$ mA

(e) −48.8° (I_{tot} leading V_s)

49. 18.4 kΩ resistor in series with 196 pF capacitor.

51. $\mathbf{V}_{C1} = 8.42\angle -2.9^\circ$ V, $\mathbf{V}_{C2} = 1.58\angle -57.5^\circ$ V

$\mathbf{V}_{C3} = 3.65\angle 6.8^\circ$ V, $\mathbf{V}_{R1} = 3.29\angle 32.5^\circ$ V

$\mathbf{V}_{R2} = 2.36\angle 6.8^\circ$ V, $\mathbf{V}_{R3} = 1.29\angle 6.8^\circ$ V

53. $\mathbf{I}_{tot} = 79.5\angle 87.1^\circ$ mA, $\mathbf{I}_{C2R1} = 6.99\angle 32.5^\circ$ mA

$\mathbf{I}_{C3} = 75.7\angle 96.8^\circ$ mA, $\mathbf{I}_{R2R3} = 7.16\angle 6.8^\circ$ mA

55. 0.103 μF

57. $\mathbf{I}_{C1} = \mathbf{I}_{R1} = 2.27\angle 74.5^\circ$ mA

$\mathbf{I}_{R2} = 2.04\angle 72.0^\circ$ mA

$\mathbf{I}_{R3} = 246\angle 84.3^\circ$ μA

$\mathbf{I}_{R4} = 149\angle 41.2^\circ$ μA

$\mathbf{I}_{R5} = 180\angle 75.1^\circ$ μA

$\mathbf{I}_{R6} = \mathbf{I}_{C3} = 101\angle 135^\circ$ μA

$\mathbf{I}_{C2} = 101\angle 131^\circ$ μA

59. 4.03 VA

61. 0.914

63. **(a)** $I_{LA} = 4.8$ A, $I_{LB} = 3.33$ A

(b) $P_{rA} = 606$ VAR, $P_{rB} = 250$ VAR

(c) $P_{\text{true}A} = 979$ W, $P_{\text{true}B} = 759$ W

(d) $P_{aA} = 1151$ VA, $P_{aB} = 799$ VA

(e) Load *A*

65. 0.0796 μF

67. Reduces V_{out} to 2.83 V and θ to −56.7°

69. **(a)** No output voltage **(b)** $320\angle -71.3^\circ$ mV

(c) $500\angle 0^\circ$ mV **(d)** 0 V

71. No fault

73. R_1 open

75. No fault

77. 48.4 Hz

RL CIRCUITS

CHAPTER OUTLINE

PART 1: SERIES CIRCUITS

1 Sinusoidal Response of Series *RL* Circuits
2 Impedance of Series *RL* Circuits
3 Analysis of Series *RL* Circuits

PART 2: PARALLEL CIRCUITS

4 Impedance and Admittance of Parallel *RL* Circuits
5 Analysis of Parallel *RL* Circuits

PART 3: SERIES-PARALLEL CIRCUITS

6 Analysis of Series-Parallel *RL* Circuits

PART 4: SPECIAL TOPICS

7 Power in *RL* Circuits
8 Basic Applications
9 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

PART 1: SERIES CIRCUITS

- Describe the relationship between current and voltage in a series *RL* circuit
- Determine the impedance of a series *RL* circuit
- Analyze a series *RL* circuit

PART 2: PARALLEL CIRCUITS

- Determine impedance and admittance in a parallel *RL* circuit
- Analyze a parallel *RL* circuit

PART 3: SERIES-PARALLEL CIRCUITS

- Analyze series-parallel *RL* circuits

PART 4: SPECIAL TOPICS

- Determine power in *RL* circuits
- Describe two examples of *RL* circuit applications
- Troubleshoot *RL* circuits

KEY TERMS

- *RL* lead circuit
- *RL* lag circuit
- Inductive susceptance (B_L)

APPLICATION ACTIVITY PREVIEW

In the application activity, you will use your knowledge of *RL* circuits to determine, based on parameter measurements, the type of filter circuits that are encapsulated in sealed modules and their component values.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter you will study series and parallel *RL* circuits. The analyses of *RL* and *RC* circuits are similar. The major difference is that the phase responses are opposite; inductive reactance increases with frequency, while capacitive reactance decreases with frequency.

An *RL* circuit contains both resistance and inductance. In this chapter, basic series and parallel *RL* circuits and their responses to sinusoidal ac voltages are presented. Series-parallel combinations are also analyzed. True, reactive, and apparent power in *RL* circuits are discussed and some basic *RL* circuit applications are introduced. Applications of *RL* circuits include filters and switching regulators. Troubleshooting is also covered in this chapter.

From Chapter 16 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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Part 1

SERIES CIRCUITS

1 SINUSOIDAL RESPONSE OF SERIES *RL* CIRCUITS

As with the *RC* circuit, all currents and voltages in a series *RL* circuit are sinusoidal when the input voltage is sinusoidal. The inductance causes a phase shift between the voltage and the current that depends on the relative values of the resistance and the inductive reactance.

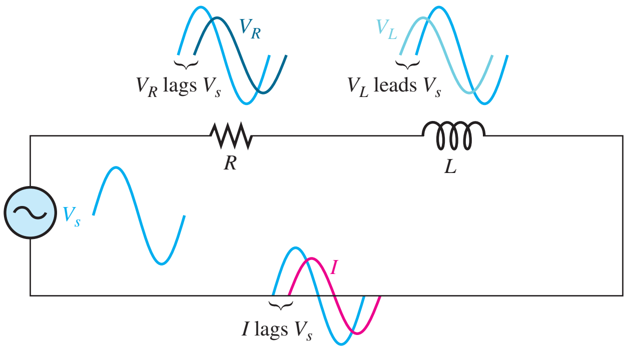
After completing this section, you should be able to

- **Describe the relationship between current and voltage in a series *RL* circuit**
    - Discuss voltage and current waveforms
    - Discuss phase shift

In an *RL* circuit, the resistor voltage and the current lag the source voltage. The inductor voltage leads the source voltage. Ideally, the phase angle between the current and the inductor voltage is always 90°. These generalized phase relationships are indicated in Figure 1. Notice how they differ from those of the *RC* circuit.

The amplitudes and the phase relationships of the voltages and current depend on the values of the resistance and the **inductive reactance**. When a circuit is purely inductive, the phase angle between the applied voltage and the total current is 90°, with the current lagging the voltage. When there is a combination of both resistance and inductive reactance in a circuit, the phase angle is somewhere between 0° and 90°, depending on the relative values of the resistance and the inductive reactance.

Recall that practical inductors have winding resistance, capacitance between windings, and other factors that prevent an inductor from behaving as an ideal component. In practical



▲ FIGURE 1

Illustration of sinusoidal response with general phase relationships of V_R, V_L, and I relative to the source voltage. V_R and I are in phase, while V_R and V_L are 90° out of phase with each other.

circuits, these effects can be significant; however, for the purpose of isolating the inductive effects, we will treat inductors in this chapter as ideal (except in the Application Activity).

SECTION 1 CHECKUP
Answers are at the end of the chapter.

1. A 1 kHz sinusoidal voltage is applied to an *RL* circuit. What is the frequency of the resulting current?
2. When the resistance in an *RL* circuit is greater than the inductive reactance, is the phase angle between the applied voltage and the total current closer to 0° or to 90°?

2 IMPEDANCE OF SERIES *RL* CIRCUITS

The impedance of a series *RL* circuit consists of resistance and inductive reactance and is the total opposition to sinusoidal current. Its unit is the ohm. The impedance also results in a phase difference between the total current and the source voltage. Therefore, the impedance consists of a magnitude component and a phase angle component and is represented as a phasor quantity.

After completing this section, you should be able to

- **Determine the impedance of a series *RL* circuit**
    - Express inductive reactance in complex form
    - Express total impedance in complex form
    - Calculate impedance magnitude and the phase angle

The impedance of a series *RL* circuit is determined by the resistance and the inductive reactance. Inductive reactance is expressed as a phasor quantity in rectangular form as

$$\mathbf{X}_L = jX_L$$

In the series *RL* circuit of Figure 2, the total impedance is the phasor sum of *R* and jX_L and is expressed as

$$\mathbf{Z} = R + jX_L$$

Equation 1

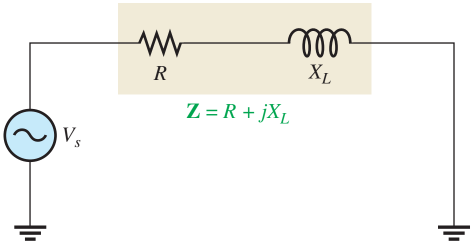
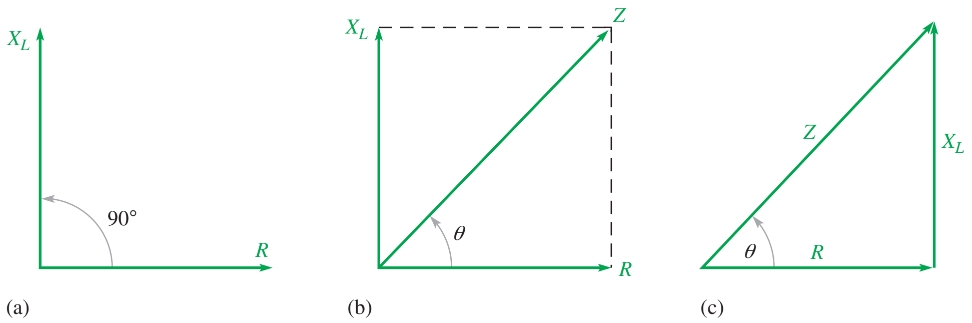


FIGURE 2

Impedance in a series *RL* circuit.

In ac analysis, both *R* and X_L are as shown in the phasor diagram of Figure 3(a), with X_L appearing at a +90° angle with respect to *R*. This relationship comes from the fact that the inductor voltage leads the current, and thus the resistor voltage, by 90°. Since **Z** is the phasor sum of *R* and jX_L, its phasor representation is as shown in Figure 3(b). A repositioning of the phasors, as shown in part (c), forms a right triangle called the *impedance triangle.* The length of each phasor represents the magnitude of the quantity, and θ is the phase angle between the applied voltage and the current in the *RL* circuit.



▲ FIGURE 3

Development of the impedance triangle for a series *RL* circuit.

The impedance magnitude of the series *RL* circuit can be expressed in terms of the resistance and reactance as

$$Z = \sqrt{R^2 + X_L^2}$$

The magnitude of the impedance is expressed in ohms.

The phase angle, θ, is expressed as

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

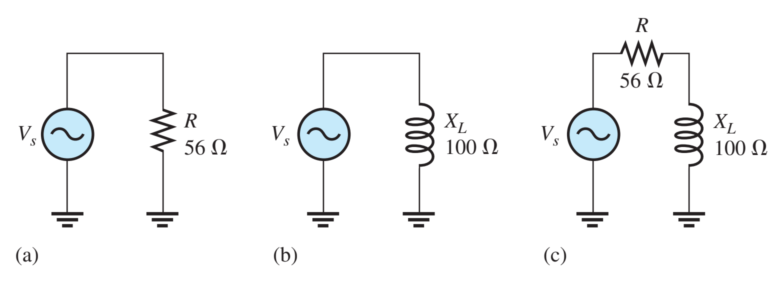
Combining the magnitude and the angle, the impedance can be expressed in polar form as

Equation 2

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

EXAMPLE 1

For each circuit in Figure 4, write the phasor expression for the impedance in both rectangular and polar forms.



▲ FIGURE 4

Solution

For the circuit in Figure 4(a), the impedance is

$$\mathbf{Z} = R + j0 = R = \mathbf{56\ \Omega} \qquad \text{in rectangular form } (X_L = 0)$$
$$\mathbf{Z} = R\angle 0° = \mathbf{56\angle 0°\ \Omega} \qquad \text{in polar form}$$

The impedance is simply equal to the resistance, and the phase angle is zero because pure resistance does not introduce a phase shift.

For the circuit in Figure 4(b), the impedance is

$$\mathbf{Z} = 0 + jX_L = \mathbf{j100\ \Omega} \qquad \text{in rectangular form } (R = 0)$$
$$\mathbf{Z} = X_L\angle 90° = \mathbf{100\angle 90°\ \Omega} \qquad \text{in polar form}$$

The impedance equals the inductive reactance in this case, and the phase angle is +90° because the inductance causes the current to lag the voltage by 90°.

For the circuit in Figure 4(c), the impedance in rectangular form is

$$\mathbf{Z} = R + jX_L = \mathbf{56\ \Omega + j100\ \Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$
$$= \sqrt{(56\ \Omega)^2 + (100\ \Omega)^2}\angle\tan^{-1}\left(\frac{100\ \Omega}{56\ \Omega}\right) = \mathbf{115\angle 60.8°\ \Omega}$$

In this case, the impedance is the phasor sum of the resistance and the inductive reactance. The phase angle is fixed by the relative values of X_L and R.

*Related Problem** In a series *RL* circuit, $R = 1.8\ \text{k}\Omega$ and $X_L = 950\ \Omega$. Express the impedance in both rectangular and polar forms.

---

*Answers are at the end of the chapter.

SECTION 2 CHECKUP

1. The impedance of a certain *RL* circuit is $150\ \Omega + j220\ \Omega$. What is the value of the resistance? The inductive reactance?
2. A series *RL* circuit has a total resistance of 33 kΩ and an inductive reactance of 50 kΩ. Write the phasor expression for the impedance in rectangular form. Convert the impedance to polar form.

3 ANALYSIS OF SERIES *RL* CIRCUITS

In this section, Ohm's law and Kirchhoff's voltage law are used in the analysis of series *RL* circuits to determine voltage, current, and impedance. Also, *RL* lead and lag circuits are examined.

After completing this section, you should be able to

- **Analyze a series *RL* circuit**
  - Apply Ohm's law and Kirchhoff's voltage law to series *RL* circuits
  - Express the voltages and current as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Discuss and analyze the *RL* lead circuit
  - Discuss and analyze the *RL* lag circuit

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

Ohm's Law

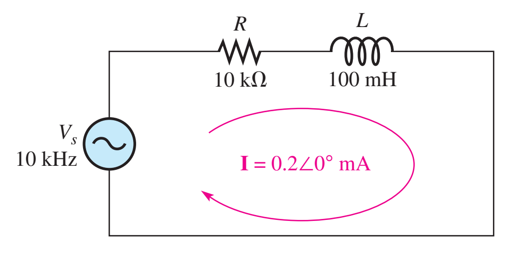
The application of Ohm's law to series *RL* circuits involves the use of the phasor quantities of **Z**, **V**, and **I**. The three equivalent forms of Ohm's law apply to *RL* circuits:

$$\mathbf{V} = \mathbf{IZ} \qquad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \qquad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Recall that since Ohm's law calculations involve multiplication and division operations, you should express the voltage, current, and impedance in polar form.

EXAMPLE 2

The current in Figure 5 is expressed in polar form as $\mathbf{I} = 0.2\angle 0^\circ$ mA. Determine the source voltage expressed in polar form, and draw a phasor diagram showing the relationship between the source voltage and the current.



▲ FIGURE 5

Solution The magnitude of the inductive reactance is

$$X_L = 2\pi fL = 2\pi(10\,\text{kHz})(100\,\text{mH}) = 6.28\,\text{k}\Omega$$

The impedance in rectangular form is

$$\mathbf{Z} = R + jX_L = 10\,\text{k}\Omega + j6.28\,\text{k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

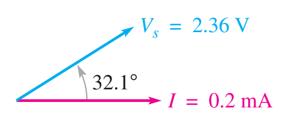
$$= \sqrt{(10\,\text{k}\Omega)^2 + (6.28\,\text{k}\Omega)^2}\angle\tan^{-1}\left(\frac{6.28\,\text{k}\Omega}{10\,\text{k}\Omega}\right) = \mathbf{11.8\angle 32.1^\circ\,k\Omega}$$

Use Ohm's law to determine the source voltage.

$$\mathbf{V}_s = \mathbf{IZ} = (0.2\angle 0^\circ\,\text{mA})(11.8\angle 32.1^\circ\,\text{k}\Omega) = \mathbf{2.36\angle 32.1^\circ\,V}$$

The magnitude of the source voltage is 2.36 V at an angle of 32.1° with respect to the current; that is, the voltage leads the current by 32.1°, as shown in the phasor diagram of Figure 6.

▶ FIGURE 6

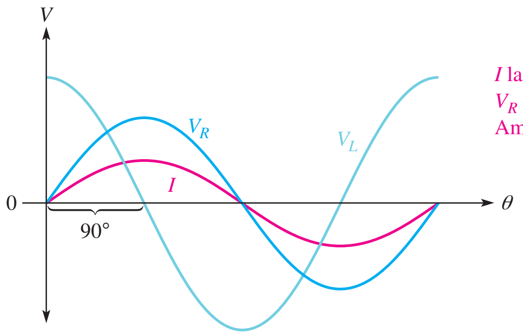


Related Problem If the source voltage in Figure 5 were $5\angle 0°$ V, what would be the current expressed in polar form?

Use Multisim files E16-02A and E16-02B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Current and Voltages

In a series *RL* circuit, the current is the same through both the resistor and the inductor. Thus, the resistor voltage is in phase with the current, and the inductor voltage leads the current by 90°. Therefore, there is a phase difference of 90° between the resistor voltage, V_R, and the inductor voltage, V_L, as shown in the waveform diagram of Figure 7.



I lags V_L by 90°.
V_R and I are in phase.
Amplitudes depend on circuit.

FIGURE 7

Phase relation of voltages and current in a series *RL* circuit.

From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage. However, since V_R and V_L are not in phase with each other, they must be added as phasor quantities with V_L leading V_R by 90°, as shown in Figure 8(a). As shown in part (b), $\mathbf{V}_s$ is the phasor sum of V_R and V_L.

$$\mathbf{V}_s = V_R + jV_L \qquad \text{Equation 3}$$

This equation can be expressed in polar form as

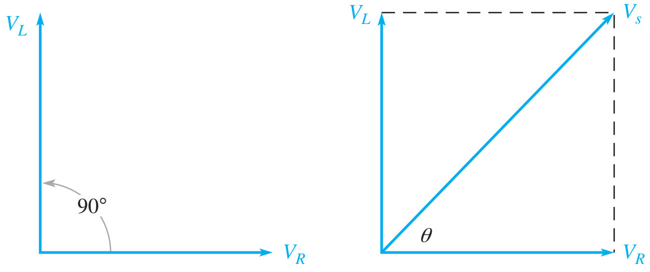
$$\mathbf{V}_s = \sqrt{V_R^2 + V_L^2}\angle \tan^{-1}\left(\frac{V_L}{V_R}\right) \qquad \text{Equation 4}$$

where the magnitude of the source voltage is

$$V_s = \sqrt{V_R^2 + V_L^2}$$

and the phase angle between the resistor voltage and the source voltage is

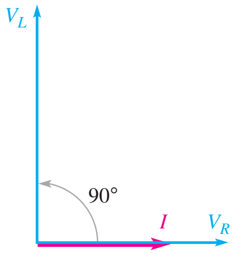
$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$



(a) (b)

FIGURE 8

Voltage phasor diagram for a series *RL* circuit.



▲ **FIGURE 9**

Voltage and current phasor diagram for the waveforms in Figure 7.

Since the resistor voltage and the current are in phase, θ is also the phase angle between the source voltage and the current. Figure 9 shows a voltage and current phasor diagram that represents the waveform diagram of Figure 7.

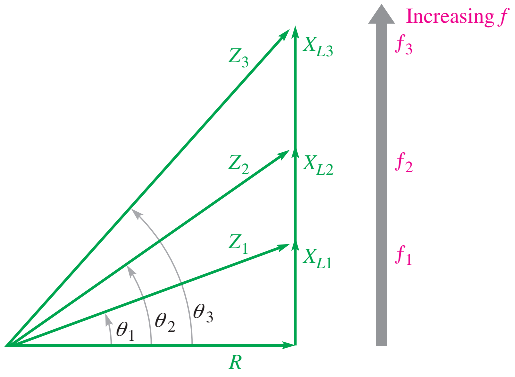
Variation of Impedance and Phase Angle with Frequency

The impedance triangle is useful in visualizing how the frequency of the applied voltage affects the *RL* circuit response. As you know, inductive reactance varies directly with frequency. When X_L increases, the magnitude of the total impedance also increases; and when X_L decreases, the magnitude of the total impedance decreases. Thus, Z is directly dependent on frequency. The phase angle θ also varies directly with frequency because $\theta = \tan^{-1}(X_L/R)$. As X_L increases with frequency, so does θ, and vice versa.

The impedance triangle is used in Figure 10 to illustrate the variations in X_L, Z, and θ as the frequency changes. Of course, R remains constant. The main point is that because X_L varies directly with the frequency, so also do the magnitude of the total impedance and the phase angle. Example 3 illustrates this.

▶ **FIGURE 10**

As the frequency increases, X_L increases, Z increases, and θ increases. Each value of frequency can be visualized as forming a different impedance triangle.

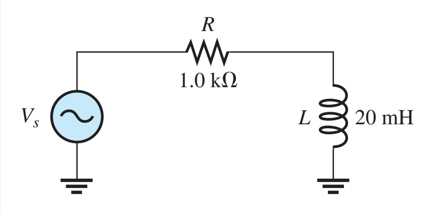


EXAMPLE 3

For the series *RL* circuit in Figure 11, determine the magnitude of the total impedance and the phase angle for each of the following frequencies:

(a) 10 kHz **(b)** 20 kHz **(c)** 30 kHz

▶ **FIGURE 11**



Solution **(a)** For $f = 10\text{ kHz}$,

$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(20\text{ mH}) = 1.26\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.26\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{1.26\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.61\angle 51.6^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{1.61\text{ k}\Omega}$ and $\theta = \mathbf{51.6^\circ}$.

(b) For $f = 20$ kHz,

$$X_L = 2\pi(20\text{ kHz})(20\text{ mH}) = 2.51\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (2.51\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2.51\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 2.70\angle 68.3^\circ\text{ k}\Omega$$

Thus, $Z =$ **2.70 kΩ** and $\theta =$ **68.3°**.

(c) For $f = 30$ kHz,

$$X_L = 2\pi(30\text{ kHz})(20\text{ mH}) = 3.77\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (3.77\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{3.77\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = \mathbf{3.90\angle 75.1^\circ\text{ k}\Omega}$$

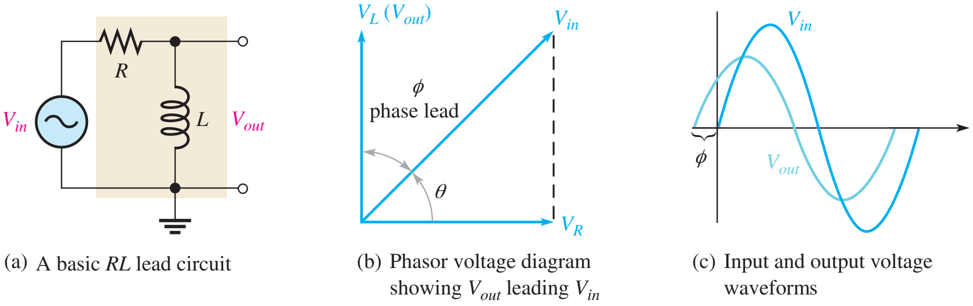
Thus, $Z =$ **3.90 kΩ** and $\theta =$ **75.1°**.

Notice that as the frequency increases, X_L, Z, and θ also increase.

Related Problem Determine Z and θ in Figure 11 if f is 100 kHz.

The *RL* Lead Circuit

An ***RL* lead circuit** is a phase shift circuit in which the output voltage leads the input voltage by a specified amount. Figure 12(a) shows a series *RL* circuit with the output voltage taken across the inductor. Note that in the *RC* lead circuit, the output was taken across the resistor. The source voltage is the input, V_{in}. As you know, θ is the angle between the current and the input voltage; it is also the angle between the resistor voltage and the input voltage because V_R and I are in phase.



(a) A basic *RL* lead circuit

(b) Phasor voltage diagram showing V_{out} leading V_{in}

(c) Input and output voltage waveforms

▲ **FIGURE 12**

The *RL* lead circuit ($V_{out} = V_L$).

Since V_L leads V_R by 90°, the phase angle between the inductor voltage and the input voltage is the difference between 90° and θ, as shown in Figure 12(b). The inductor voltage is the output; it leads the input, thus creating a basic lead circuit.

The input and output voltage waveforms of the lead circuit are shown in Figure 12(c). The amount of phase difference, designated ϕ, between the input and the output is dependent on the relative values of the inductive reactance and the resistance, as is the magnitude of the output voltage.

Phase Difference Between Input and Output The angle between V_{out} and V_{in} is designated ϕ (phi) and is developed as follows. The polar expressions for the input voltage and the current are $V_{in}\angle 0^\circ$ and $I\angle -\theta$, respectively. The output voltage in polar form is

$$\mathbf{V}_{out} = (I\angle -\theta)(X_L\angle 90^\circ) = IX_L\angle(90^\circ - \theta)$$

This expression shows that the output voltage is at an angle of 90° − θ with respect to the input voltage. Since $\theta = \tan^{-1}(X_L/R)$, the angle ϕ between the input and output is

$$\phi = 90° - \tan^{-1}\left(\frac{X_L}{R}\right)$$

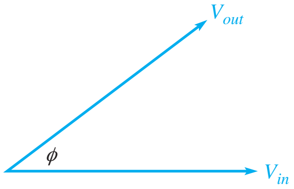
Equivalently, this angle can be expressed as

Equation 5

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

The angle ϕ between the output and input is always positive, indicating that the output voltage leads the input voltage, as indicated in Figure 13.

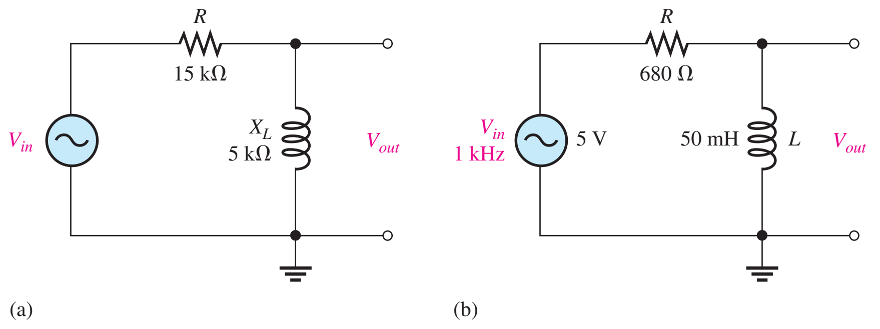
FIGURE 13



EXAMPLE 4

Determine the amount of phase lead from input to output in each lead circuit in Figure 14.

FIGURE 14



Solution For the lead circuit in Figure 14(a),

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{15\text{ k}\Omega}{5\text{ k}\Omega}\right) = \mathbf{71.6°}$$

The output leads the input by 71.6°.

For the lead circuit in Figure 14(b), first determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(50\text{ mH}) = 314\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{680\ \Omega}{314\ \Omega}\right) = \mathbf{65.2°}$$

The output leads the input by 65.2°.

Related Problem In a certain lead circuit, $R = 2.2\text{ k}\Omega$ and $X_L = 1\text{ k}\Omega$. What is the phase lead?

Use Multisim files E16-04A, E16-04B, and E16-04C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage To evaluate the output voltage in terms of its magnitude, visualize the *RL* lead circuit as a voltage divider. A portion of the total input voltage is dropped across the resistor and a portion across the inductor. Because the output voltage is the voltage across the inductor, it can be calculated using either Ohm's law ($V_{out} = IX_L$) or the voltage-divider formula.

$$V_{out} = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

Equation 6

The phasor expression for the output voltage of an *RL* lead circuit is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 5

For the lead circuit in Figure 14(b) (Example 4), determine the output voltage in phasor form when the input voltage has an rms value of 5 V. Draw the input and output voltage waveforms showing their peak values. The inductive reactance X_L (314 Ω) and ϕ (65.2°) were found in Example 4.

Solution The output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}\angle\phi$$

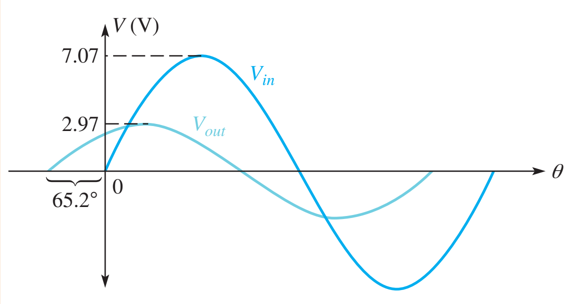
$$= \left(\frac{314\ \Omega}{\sqrt{(680\ \Omega)^2 + (314\ \Omega)^2}}\right)5\angle 65.2^\circ\ \text{V} = \mathbf{2.10\angle 65.2^\circ\ V}$$

The peak values of voltage are

$$V_{in(p)} = 1.414V_{in(rms)} = 1.414(5\ \text{V}) = 7.07\ \text{V}$$
$$V_{out(p)} = 1.414V_{out(rms)} = 1.414(2.10\ \text{V}) = 2.97\ \text{V}$$

The waveforms with their peak values are shown in Figure 15. Notice that the output voltage leads the input voltage by 65.2°.

▶ FIGURE 15

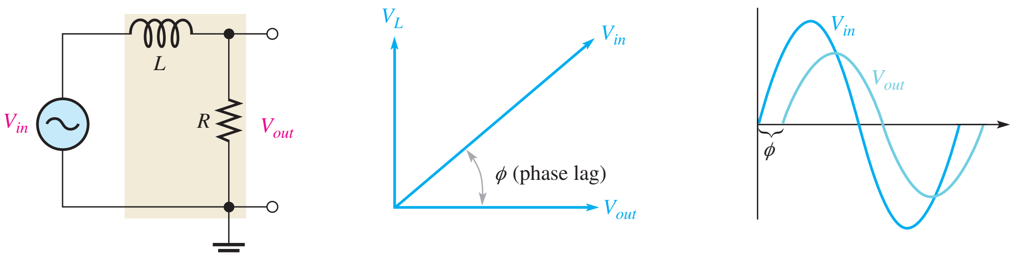


Related Problem In a lead circuit, does the output voltage increase or decrease when the frequency increases?

Use Multisim files E16-05A and E16-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

The *RL* Lag Circuit

An ***RL* lag circuit** is a phase shift circuit in which the output voltage lags the input voltage by a specified amount. When the output of a series *RL* circuit is taken across the resistor rather than the inductor, as shown in Figure 16(a), it becomes a lag circuit.



(a) A basic *RL* lag circuit

(b) Phasor voltage diagram showing phase lag between V_{in} and V_{out}

(c) Input and output waveforms

▲ **FIGURE 16**

The *RL* lag circuit ($V_{out} = V_R$).

Phase Difference Between Input and Output In a series *RL* circuit, the current lags the input voltage. Since the output voltage is taken across the resistor, the output lags the input, as indicated by the phasor diagram in Figure 16(b). The waveforms are shown in Figure 16(c).

As in the lead circuit, the amount of phase difference between the input and output and the magnitude of the output voltage in the lag circuit are dependent on the relative values of the resistance and the inductive reactance. When the input voltage is assigned a reference angle of 0°, the angle of the output voltage (ϕ) with respect to the input voltage equals θ because the resistor voltage (output) and the current are in phase with each other. The expression for the angle between the input voltage and the output voltage is

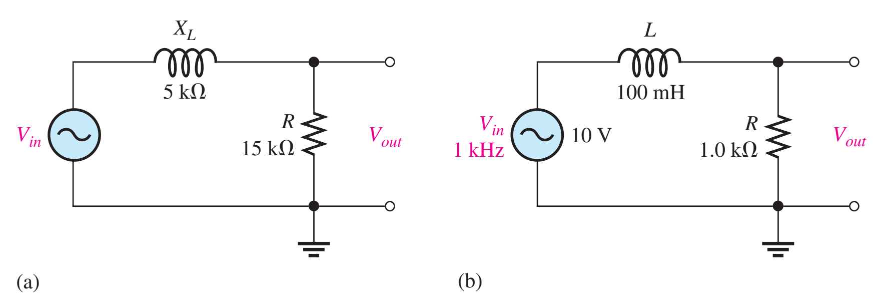
Equation 7

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

This angle is negative because the output lags the input.

EXAMPLE 6

Calculate the output phase angle for each circuit in Figure 17.



▲ **FIGURE 17**

Solution For the lag circuit in Figure 17(a).

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right) = -\tan^{-1}\left(\frac{5\text{ k}\Omega}{15\text{ k}\Omega}\right) = \mathbf{-18.4^\circ}$$

The output lags the input by 18.4°.

For the lag circuit in Figure 17(b), first determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right) = -\tan^{-1}\left(\frac{628\ \Omega}{1.0\text{ k}\Omega}\right) = \mathbf{-32.1^\circ}$$

The output lags the input by 32.1°.

Related Problem In a certain lag circuit, $R = 5.6\text{ k}\Omega$ and $X_L = 3.5\text{ k}\Omega$. Determine the phase angle.

Use Multisim files E16-06A, E16-06B, and E16-06C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage Since the output voltage of an *RL* lag circuit is taken across the resistor, the magnitude can be calculated using either Ohm's law ($V_{out} = IR$) or the voltage-divider formula.

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

Equation 8

The expression for the output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 7

The input voltage in Figure 17(b) (Example 6) has an rms value of 10 V. Determine the phasor expression for the output voltage. Draw the waveform relationships for the input and output voltages. The phase angle (−32.1°) and X_L (628 Ω) were found in Example 6.

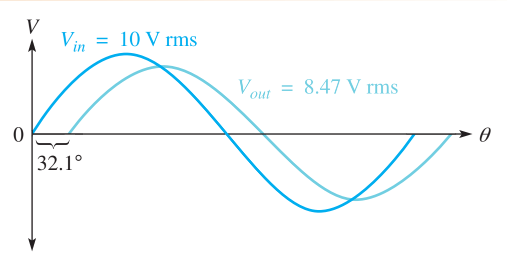
Solution The phasor expression for the output voltage is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.0\text{ k}\Omega}{1181\ \Omega}\right)10\angle -32.1^\circ\text{ V} = \mathbf{8.47\angle -32.1^\circ\ V\ rms}$$

The waveforms are shown in Figure 18.

▶ FIGURE 18



Related Problem In a lag circuit, $R = 4.7\text{ k}\Omega$ and $X_L = 6\text{ k}\Omega$. If the rms input voltage is 20 V, what is the output voltage?

Use Multisim files E16-07A and E16-07B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 3 CHECKUP

1. In a certain series *RL* circuit, $V_R = 2$ V and $V_L = 3$ V. What is the magnitude of the source voltage?
2. In Question 1, what is the phase angle between the source voltage and the current?
3. When the frequency of the applied voltage in a series *RL* circuit is increased, what happens to the inductive reactance? What happens to the magnitude of the total impedance? What happens to the phase angle?
4. A certain *RL* lead circuit consists of a 3.3 kΩ resistor and a 15 mH inductor. Determine the phase shift between input and output at a frequency of 5 kHz.
5. An *RL* lag circuit has the same component values as the lead circuit in Question 4. What is the magnitude of the output voltage at 5 kHz when the input is 10 V rms?

PARALLEL CIRCUITS

Part 2

4 IMPEDANCE AND ADMITTANCE OF PARALLEL *RL* CIRCUITS

In this section, you will learn how to determine the impedance and phase angle of a parallel *RL* circuit. The impedance consists of a magnitude component and a phase angle component. Also, inductive susceptance and admittance of a parallel *RL* circuit are introduced.

After completing this section, you should be able to

- **Determine impedance and admittance in a parallel *RL* circuit**
  - Express total impedance in complex form
  - Define and calculate *inductive susceptance* and *admittance*

Figure 19 shows a basic parallel *RL* circuit connected to an ac voltage source.

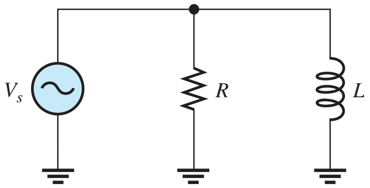


FIGURE 19

Parallel *RL* circuit.

The expression for the total impedance of a two-component parallel *RL* circuit is developed as follows, using the product-over-sum rule.

$$\mathbf{Z} = \frac{(R\angle 0^\circ)(X_L\angle 90^\circ)}{R + jX_L} = \frac{RX_L\angle(0^\circ + 90^\circ)}{\sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)}$$

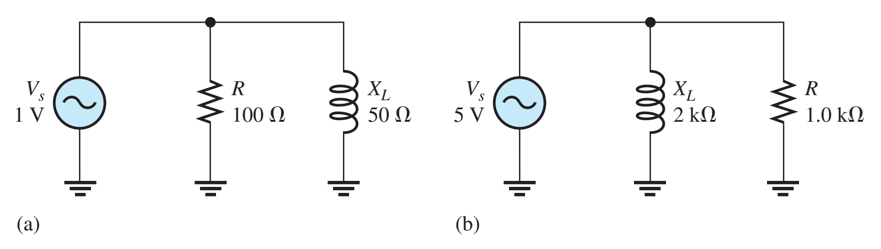
$$= \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\left(90^\circ - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

Equivalently, this equation can be expressed as

$$\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$$

Equation 9

EXAMPLE 8 For each circuit in Figure 20, determine the magnitude and phase angle of the total impedance.



▲ FIGURE 20

Solution For the circuit in Figure 20(a), the total impedance is

$$\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$$

$$= \left(\frac{(100\ \Omega)(50\ \Omega)}{\sqrt{(100\ \Omega)^2 + (50\ \Omega)^2}}\right)\angle\tan^{-1}\left(\frac{100\ \Omega}{50\ \Omega}\right) = \mathbf{44.7\angle 63.4^\circ\ \Omega}$$

Thus, $Z = 44.7\ \Omega$ and $\theta = 63.4^\circ$.

For the circuit in Figure 20(b), the total impedance is

$$\mathbf{Z} = \left(\frac{(1.0\ \text{k}\Omega)(2\ \text{k}\Omega)}{\sqrt{(1.0\ \text{k}\Omega)^2 + (2\ \text{k}\Omega)^2}}\right)\angle\tan^{-1}\left(\frac{1.0\ \text{k}\Omega}{2\ \text{k}\Omega}\right) = \mathbf{894\angle 26.6^\circ\ \Omega}$$

Thus, $Z = 894\ \Omega$ and $\theta = 26.6^\circ$.

Notice that the positive angle indicates that the voltage leads the current, in contrast to the *RC* case where the voltage lags the current.

Related Problem In a parallel circuit, $R = 10\ \text{k}\Omega$ and $X_L = 14\ \text{k}\Omega$. Determine the total impedance in polar form.

Conductance, Susceptance, and Admittance

Conductance (G) is the reciprocal of resistance, susceptance (B) is the reciprocal of reactance, and admittance (Y) is the reciprocal of impedance.

For parallel *RL* circuits, the phasor expression for **inductive susceptance (B_L)** is

$$\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$$

and the phasor expression for **admittance** is

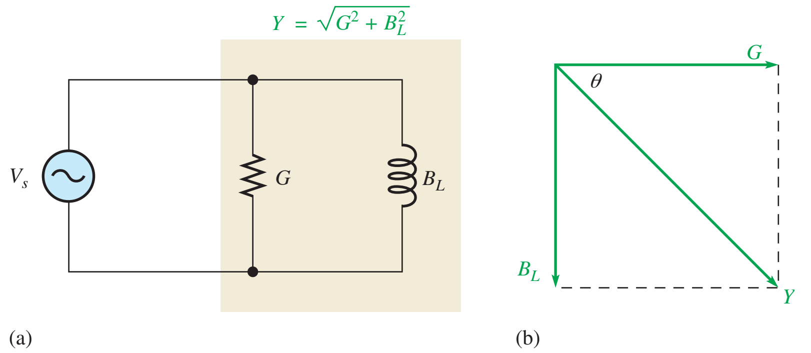
$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta$$

For the basic parallel *RL* circuit shown in Figure 21(a), the total admittance is the phasor sum of the conductance and the inductive susceptance, as shown in part (b).

Equation 10

$$\mathbf{Y} = G - jB_L$$

As with the *RC* circuit, the unit for conductance (G), inductive susceptance (B_L), and admittance (Y) is the siemens (S).



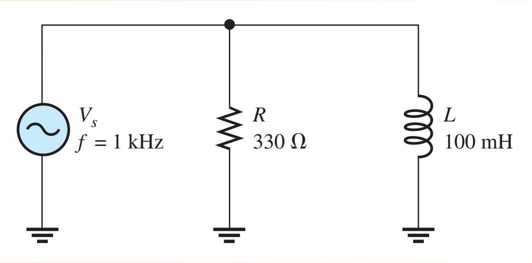
▲ **FIGURE 21**

Admittance in a parallel *RL* circuit.

EXAMPLE 9

Determine the total admittance and then convert it to total impedance in Figure 22. Draw the admittance phasor diagram.

▶ **FIGURE 22**



Solution First, determine the conductance magnitude. $R = 330\ \Omega$; thus,

$$G = \frac{1}{R} = \frac{1}{330\ \Omega} = 3.03\text{ mS}$$

Then, determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1000\text{ Hz})(100\text{ mH}) = 628\ \Omega$$

The inductive susceptance magnitude is

$$B_L = \frac{1}{X_L} = \frac{1}{628\ \Omega} = 1.59\text{ mS}$$

The total admittance is

$$\mathbf{Y}_{tot} = G - jB_L = 3.03\text{ mS} - j1.59\text{ mS}$$

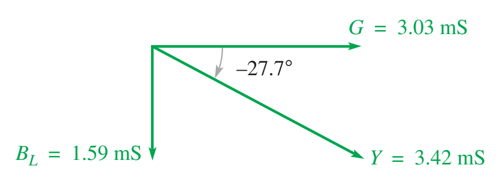
which can be expressed in polar form as

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_L^2}\angle -\tan^{-1}\left(\frac{B_L}{G}\right)$$

$$= \sqrt{(3.03\text{ mS})^2 + (1.59\text{ mS})^2}\angle -\tan^{-1}\left(\frac{1.59\text{ mS}}{3.03\text{ mS}}\right) = \mathbf{3.42\angle -27.7^\circ\ mS}$$

The admittance phasor diagram is shown in Figure 23.

► FIGURE 23



Convert total admittance to total impedance as follows:

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}_{tot}} = \frac{1}{3.42\angle -27.7° \text{ mS}} = \mathbf{292\angle 27.7°\ \Omega}$$

The positive phase angle indicates that the voltage leads the current.

Related Problem What is the total admittance of the circuit in Figure 22 if *f* is increased to 2 kHz?

SECTION 4 CHECKUP

1. If $Z = 500\ \Omega$, what is the value of the magnitude of the admittance Y?
2. In a certain parallel *RL* circuit, $R = 470\ \Omega$ and $X_L = 750\ \Omega$. Determine Y.
3. In the circuit of Question 2, does the total current lead or lag the applied voltage? By what phase angle?

5 ANALYSIS OF PARALLEL *RL* CIRCUITS

Ohm's law and Kirchhoff's current law are used in the analysis of *RL* circuits. Current and voltage relationships in a parallel *RL* circuit are examined.

After completing this section, you should be able to

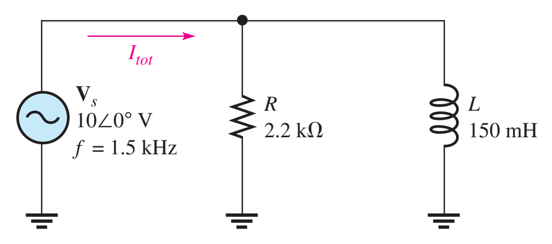
- **Analyze a parallel *RL* circuit**
  - Apply Ohm's law and Kirchhoff's current law to parallel *RL* circuits
  - Express the voltages and currents as phasor quantities

The following example applies Ohm's law to the analysis of a parallel *RL* circuit.

EXAMPLE 10

Determine the total current and the phase angle in the circuit of Figure 24. Draw a phasor diagram showing the relationship of $\mathbf{V}_s$ and $\mathbf{I}_{tot}$.

► FIGURE 24



Solution The inductive reactance is

$$X_L = 2\pi fL = 2\pi(1.5\text{ kHz})(150\text{ mH}) = 1.41\text{ k}\Omega$$

The inductive susceptance magnitude is

$$B_L = \frac{1}{X_L} = \frac{1}{1.41\text{ k}\Omega} = 709\ \mu\text{S}$$

The conductance magnitude is

$$G = \frac{1}{R} = \frac{1}{2.2\text{ k}\Omega} = 455\ \mu\text{S}$$

The total admittance is

$$\mathbf{Y}_{tot} = G - jB_L = 455\ \mu\text{S} - j709\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_L^2}\angle -\tan^{-1}\left(\frac{B_L}{G}\right)$$

$$= \sqrt{(455\ \mu\text{S})^2 + (709\ \mu\text{S})^2}\angle -\tan^{-1}\left(\frac{709\ \mu\text{S}}{455\ \mu\text{S}}\right) = 842\angle -57.3^\circ\ \mu\text{S}$$

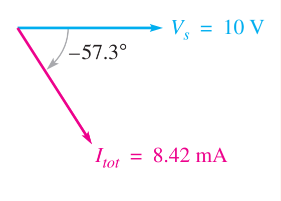
The phase angle is −57.3°.

Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \mathbf{V}_s\mathbf{Y}_{tot} = (10\angle 0^\circ\text{ V})(842\angle -57.3^\circ\ \mu\text{S}) = \mathbf{8.42\angle -57.3^\circ\ mA}$$

The magnitude of the total current is 8.42 mA, and it lags the applied voltage by **57.3**°, as indicated by the negative angle associated with it. The phasor diagram in Figure 25 shows these relationships.

▶ FIGURE 25



Related Problem Determine the current in polar form if *f* is reduced to 800 Hz in Figure 24.

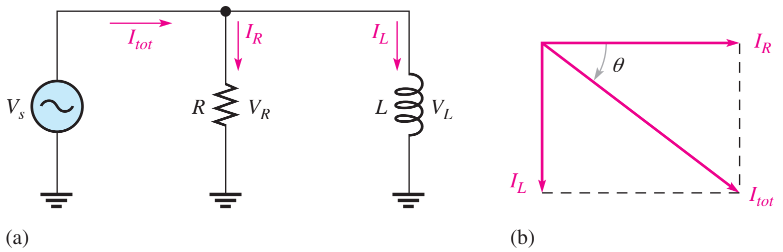
Use Multisim files E16-10A and E16-10B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Currents and Voltages

Figure 26(a) shows all the currents in a basic parallel *RL* circuit. The total current, I_{tot}, divides at the junction into the two branch currents, I_R and I_L. The applied voltage, V_s, appears across both the resistive and the inductive branches, so V_s, V_R, and V_L are all in phase and of the same magnitude.

The current through the resistor is in phase with the voltage. The current through the inductor lags the voltage and the resistor current by 90°. By Kirchhoff's current law, the total current is the phasor sum of the two branch currents, as shown by the phasor diagram in Figure 26(b). The total current is expressed as

$$\mathbf{I}_{tot} = I_R - jI_L \qquad \textbf{Equation 11}$$



▲ FIGURE 26

Currents in a parallel *RL* circuit. The current directions shown in part (a) are instantaneous and, of course, reverse when the source voltage reverses during each cycle.

This equation can be expressed in polar form as

Equation 12

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

where the magnitude of the total current is

$$I_{tot} = \sqrt{I_R^2 + I_L^2}$$

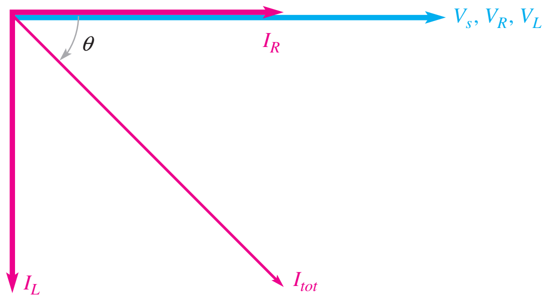
and the phase angle between the resistor current and the total current is

$$\theta = -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

Since the resistor current and the applied voltage are in phase, θ also represents the phase angle between the total current and the applied voltage. Figure 27 shows a complete current and voltage phasor diagram.

▶ FIGURE 27

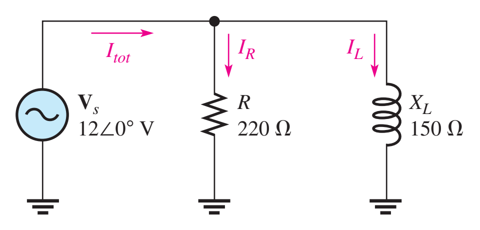
Current and voltage phasor diagram for a parallel *RL* circuit (amplitudes are arbitrary).



EXAMPLE 11

Determine the value of each current in Figure 28, and describe the phase relationship of each with the applied voltage. Draw the current phasor diagram.

▶ FIGURE 28



Solution The resistor current, the inductor current, and the total current are expressed as follows:

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{12\angle 0^\circ\ \text{V}}{220\angle 0^\circ\ \Omega} = \mathbf{54.5\angle 0^\circ\ mA}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{X}_L} = \frac{12\angle 0^\circ\ \text{V}}{150\angle 90^\circ\ \Omega} = \mathbf{80\angle -90^\circ\ mA}$$

$$\mathbf{I}_{tot} = I_R - jI_L = 54.5\text{ mA} - j80\text{ mA}$$

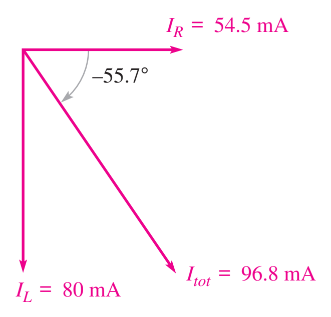
Converting $\mathbf{I}_{tot}$ to polar form yields

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

$$= \sqrt{(54.5\text{ mA})^2 + (80\text{ mA})^2}\angle -\tan^{-1}\left(\frac{80\text{ mA}}{54.5\text{ mA}}\right) = \mathbf{96.8\angle -55.7^\circ\ mA}$$

As the results show, the resistor current is 54.5 mA and is in phase with the applied voltage. The inductor current is 80 mA and lags the applied voltage by 90°. The total current is 96.8 mA and lags the voltage by 55.7°. The phasor diagram in Figure 29 shows these relationships.

▶ FIGURE 29



Related Problem Find the magnitude of $\mathbf{I}_{tot}$ and the circuit phase angle if $X_L = 300\ \Omega$ in Figure 28.

SECTION 5 CHECKUP

1. The admittance of an *RL* circuit is 4 mS, and the applied voltage is 8 V. What is the total current?
2. In a certain parallel *RL* circuit, the resistor current is 12 mA, and the inductor current is 20 mA. Determine the magnitude and phase angle of the total current. This phase angle is measured with respect to what?
3. What is the phase angle between the inductor current and the applied voltage in a parallel *RL* circuit?

Part 3

SERIES-PARALLEL CIRCUITS

6 ANALYSIS OF SERIES-PARALLEL *RL* CIRCUITS

The concepts studied with respect to series and parallel circuits are used to analyze circuits with combinations of both series and parallel *R* and *L* components.

After completing this section, you should be able to

- **Analyze series-parallel *RL* circuits**
    - Determine total impedance
    - Calculate currents and voltages

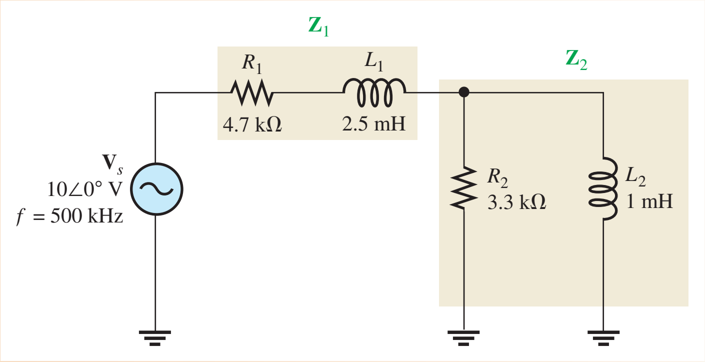
The impedance of series components is most easily expressed in rectangular form and that the impedance of parallel components is best found by using polar form. The steps for analyzing a circuit with a series and a parallel component are illustrated in Example 12. First express the impedance of the series part of the circuit in rectangular form and the impedance of the parallel part in polar form. Next, convert the impedance of the parallel part to rectangular form and add it to the impedance of the series part. Once you determine the rectangular form of the total impedance, you can convert it to polar form in order to see the magnitude and phase angle and to calculate the current.

EXAMPLE 12

In the circuit of Figure 30, determine the following values:

(a) $\mathbf{Z}_{tot}$ **(b)** $\mathbf{I}_{tot}$ **(c)** θ

FIGURE 30



Solution **(a)** First, calculate the magnitudes of inductive reactance.

$$X_{L1} = 2\pi f L_1 = 2\pi(500\text{ kHz})(2.5\text{ mH}) = 7.85\text{ k}\Omega$$
$$X_{L2} = 2\pi f L_2 = 2\pi(500\text{ kHz})(1\text{mH}) = 3.14\text{ k}\Omega$$

One approach is to find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance. The impedance of the series combination of R_1 and L_1 is

$$\mathbf{Z}_1 = R_1 + jX_{L1} = 4.7\text{ k}\Omega + j7.85\text{ k}\Omega$$

To determine the impedance of the parallel portion, first determine the admittance of the parallel combination of R_2 and L_2.

$$G_2 = \frac{1}{R_2} = \frac{1}{3.3\text{ k}\Omega} = 303\ \mu\text{S}$$

$$B_{L2} = \frac{1}{X_{L2}} = \frac{1}{3.14\text{ k}\Omega} = 318\ \mu\text{S}$$

$$\mathbf{Y}_2 = G_2 - jB_L = 303\ \mu\text{S} - j318\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_2 = \sqrt{G_2^2 + B_{L2}^2}\angle -\tan^{-1}\left(\frac{B_{L2}}{G_2}\right)$$

$$= \sqrt{(303\ \mu\text{S})^2 + (318\ \mu\text{S})^2}\angle -\tan^{-1}\left(\frac{318\ \mu\text{S}}{303\ \mu\text{S}}\right) = 439\angle -46.4^\circ\ \mu\text{S}$$

Then, the impedance of the parallel portion is

$$\mathbf{Z}_2 = \frac{1}{\mathbf{Y}_2} = \frac{1}{439\angle -46.4^\circ\ \mu\text{S}} = 2.28\angle 46.4^\circ\text{ k}\Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta + jZ_2\sin\theta$$

$$= (2.28\text{ k}\Omega)\cos(46.4^\circ) + j(2.28\text{ k}\Omega)\sin(46.4^\circ) = 1.57\text{ k}\Omega + j1.65\text{ k}\Omega$$

The series portion and the parallel portion are in series with each other. Combine $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get the total impedance.

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$= (4.7\text{ k}\Omega + j7.85\text{ k}\Omega) + (1.57\text{ k}\Omega + j1.65\text{ k}\Omega) = 6.27\text{ k}\Omega + j9.50\text{ k}\Omega$$

Expressing $\mathbf{Z}_{tot}$ in polar form yields

$$\mathbf{Z}_{tot} = \sqrt{Z_1^2 + Z_2^2}\angle\tan^{-1}\left(\frac{Z_2}{Z_1}\right)$$

$$= \sqrt{(6.27\text{ k}\Omega)^2 + (9.50\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{9.50\text{ k}\Omega}{6.27\text{ k}\Omega}\right) = \mathbf{11.4\angle 56.6^\circ\ k\Omega}$$

(b) Use Ohm's law to find the total current.

$$\mathbf{I}_{tot} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{10\angle 0^\circ\text{ V}}{11.4\angle 56.6^\circ\text{ k}\Omega} = \mathbf{877\angle -56.6^\circ\ \mu A}$$

(c) The total current lags the applied voltage by **56.6°**.

Related Problem **(a)** Determine the voltage across the series part of the circuit in Figure 30.

(b) Determine the voltage across the parallel part of the circuit in Figure 30.

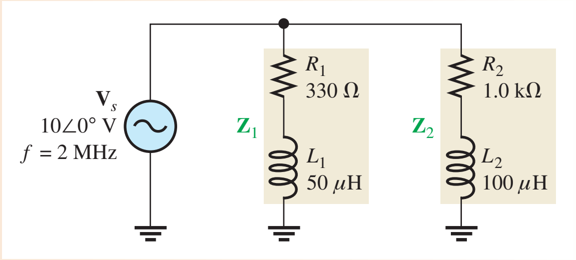
Use Multisim files E16-12A and E16-12B to verify the calculated results in the (b) part of this example and to confirm your calculation for the related problem, part (b).

Example 13 shows two sets of series components in parallel. The approach is to first express each branch impedance in rectangular form and then convert each of these impedances to polar form. Next, calculate each branch current using polar notation. Once you know the branch currents, you can find the total current by adding the two branch currents in rectangular form. In this particular case, the total impedance is not required.

EXAMPLE 13

Determine the voltage across each component in Figure 31. Draw a voltage phasor diagram and a current phasor diagram.

▶ FIGURE 31



Solution First, calculate X_{L1} and X_{L2}.

$$X_{L1} = 2\pi f L_1 = 2\pi(2\text{ MHz})(50\ \mu\text{H}) = 628\ \Omega$$
$$X_{L2} = 2\pi f L_2 = 2\pi(2\text{ MHz})(100\ \mu\text{H}) = 1.26\text{ k}\Omega$$

Next, determine the impedance of each branch.

$$\mathbf{Z}_1 = R_1 + jX_{L1} = 330\ \Omega + j628\ \Omega$$
$$\mathbf{Z}_2 = R_2 + jX_{L2} = 1.0\text{ k}\Omega + j1.26\text{ k}\Omega$$

Convert these impedances to polar form.

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_{L1}^2}\angle\tan^{-1}\left(\frac{X_{L1}}{R_1}\right)$$
$$= \sqrt{(330\ \Omega)^2 + (628\ \Omega)^2}\angle\tan^{-1}\left(\frac{628\ \Omega}{330\ \Omega}\right) = 709\angle 62.3°\ \Omega$$
$$\mathbf{Z}_2 = \sqrt{R_2^2 + X_{L2}^2}\angle\tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$
$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.26\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{1.26\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.61\angle 51.6°\text{ k}\Omega$$

Calculate each branch current.

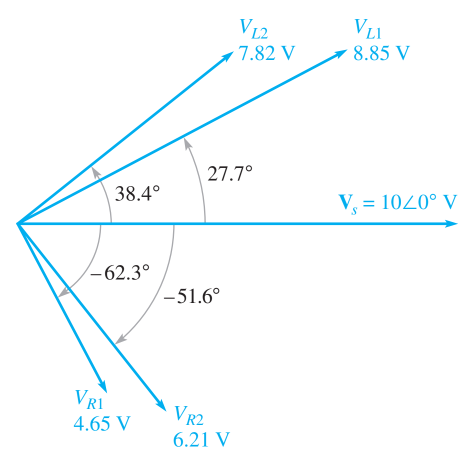
$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{Z}_1} = \frac{10\angle 0°\text{ V}}{709\angle 62.3°\ \Omega} = 14.1\angle -62.3°\text{ mA}$$
$$\mathbf{I}_2 = \frac{\mathbf{V}_s}{\mathbf{Z}_2} = \frac{10\angle 0°\text{ V}}{1.61\angle 51.6°\text{ k}\Omega} = 6.21\angle -51.6°\text{ mA}$$

Now, use Ohm's law to get the voltage across each element.

$$\mathbf{V}_{R1} = \mathbf{I}_1\mathbf{R}_1 = (14.1\angle -62.3°\text{ mA})(330\angle 0°\ \Omega) = \mathbf{4.65\angle -62.3°\ V}$$
$$\mathbf{V}_{L1} = \mathbf{I}_1\mathbf{X}_{L1} = (14.1\angle -62.3°\text{ mA})(628\angle 90°\ \Omega) = \mathbf{8.85\angle 27.7°\ V}$$
$$\mathbf{V}_{R2} = \mathbf{I}_1\mathbf{R}_2 = (6.21\angle -51.6°\text{ mA})(1\angle 0°\text{ k}\Omega) = \mathbf{6.21\angle -51.6°\ V}$$
$$\mathbf{V}_{L2} = \mathbf{I}_2\mathbf{X}_{L2} = (6.21\angle -51.6°\text{ mA})(1.26\angle 90°\text{ k}\Omega) = \mathbf{7.82\angle 38.4°\ V}$$

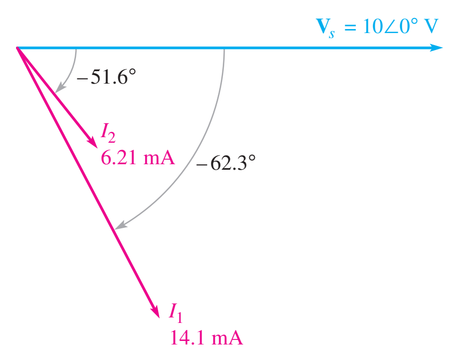
The voltage phasor diagram is shown in Figure 32, and the current phasor diagram is shown in Figure 33.

▶ **FIGURE 32**



V_{L2}
7.82 V
V_{L1}
8.85 V
27.7°
38.4°
$\mathbf{V}_s = 10\angle 0°$ V
−62.3°
−51.6°
V_{R1}
4.65 V
V_{R2}
6.21 V

▶ **FIGURE 33**



$\mathbf{V}_s = 10\angle 0°$ V
−51.6°
I_2
6.21 mA
−62.3°
I_1
14.1 mA

Related Problem What is the total current in polar form in Figure 31?

Use Multisim files E16-13A through E16-13E to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 6 CHECKUP

1. What is the total impedance in polar form of the circuit in Figure 31?
2. Determine the total current in rectangular form for the circuit in Figure 31.

Part 4

SPECIAL TOPICS

7 POWER IN *RL* CIRCUITS

In a purely resistive ac circuit, all of the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely inductive ac circuit, all of the energy delivered by the source is stored by the inductor in its magnetic field during a portion of the voltage cycle and then returned to the source during another portion of the cycle so that there is no net energy conversion to heat. When there is both resistance and inductance, some of the energy is alternately stored and returned by the inductance and some is dissipated by the resistance. The amount of energy converted to heat is determined by the relative values of the resistance and the inductive reactance.

After completing this section, you should be able to

- **Determine power in *RL* circuits**
    - Explain true and reactive power
    - Draw the power triangle
    - Explain power factor correction

When the resistance in a series *RL* circuit is greater than the inductive reactance, more of the total energy delivered by the source is converted to heat by the resistance than is stored by the inductor. Likewise, when the reactance is greater than the resistance, more of the total energy is stored and returned than is converted to heat.

As you know, the power dissipation in a resistance is called the *true power.* The power in an inductor is reactive power and is expressed as

Equation 13

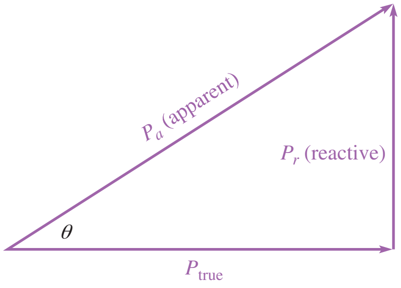
$$P_r = I^2X_L$$

The Power Triangle for *RL* Circuits

The generalized power triangle for a series *RL* circuit is shown in Figure 34. The **apparent power, P_a**, is the resultant of the average power, P_{true}, and the reactive power, P_r.

▶ FIGURE 34

Power triangle for an *RL* circuit.

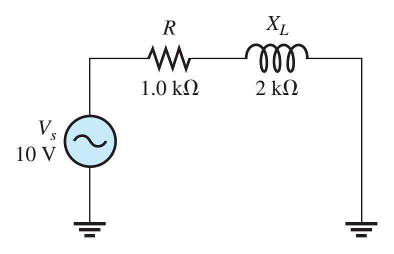


Recall that the **power factor** equals the cosine of θ ($PF = \cos\theta$). As the phase angle between the applied voltage and the total current increases, the power factor decreases, indicating an increasingly reactive circuit. A smaller power factor indicates less true power and more reactive power.

EXAMPLE 14

Determine the power factor, the true power, the reactive power, and the apparent power in the circuit in Figure 35.

▶ **FIGURE 35**



Solution The total impedance of the circuit in rectangular form is

$$\mathbf{Z} = R + jX_L = 1.0\text{ k}\Omega + j2\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R_2^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (2\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 2.24\angle 63.4^\circ\text{ k}\Omega$$

The current magnitude is

$$I = \frac{V_s}{Z} = \frac{10\text{ V}}{2.24\text{ k}\Omega} = 4.46\text{ mA}$$

The phase angle, indicated in the expression for **Z**, is

$$\theta = 63.4^\circ$$

The power factor is, therefore,

$$PF = \cos\theta = \cos(63.4^\circ) = \mathbf{0.448}$$

The true power is

$$P_{\text{true}} = V_sI\cos\theta = (10\text{ V})(4.46\text{ mA})(0.448) = \mathbf{20\text{ mW}}$$

The reactive power is

$$P_r = I^2X_L = (4.46\text{ mA})^2(2\text{ k}\Omega) = \mathbf{39.8\text{ mVAR}}$$

The apparent power is

$$P_a = I^2Z = (4.46\text{ mA})^2(2.24\text{ k}\Omega) = \mathbf{44.6\text{ mVA}}$$

Related Problem If the frequency in Figure 35 is increased, what happens to P_{true}, P_r, and P_a?

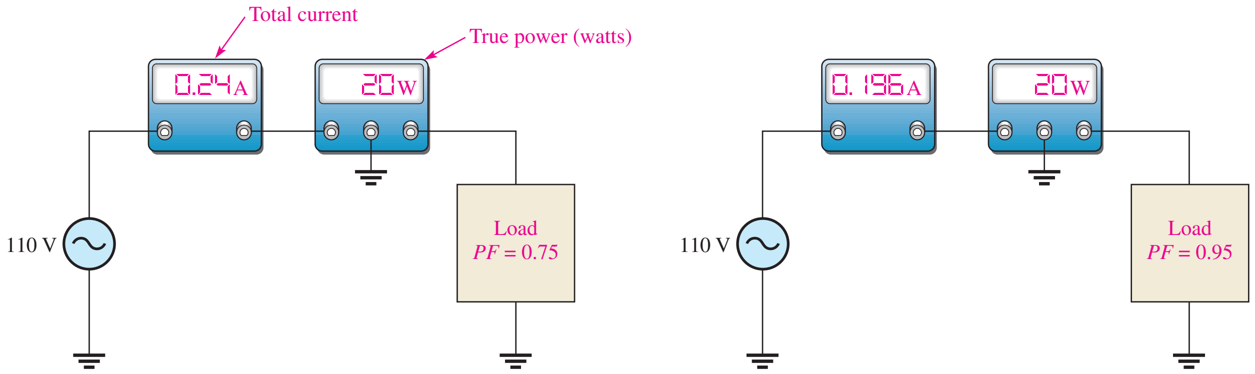
Significance of the Power Factor

The power factor (PF) is important in determining how much useful power (true power) is transferred to a load. The highest power factor is 1, which indicates that all of the current to a load is in phase with the voltage (resistive). When the power factor is 0, all of the current to a load is 90° out of phase with the voltage (reactive).

Generally, a power factor as close to 1 as possible is desirable because then most of the power transferred from the source to the load is the useful or true power. True power goes only one way—from source to load—and performs work on the load in terms of energy dissipation. Reactive power simply goes back and forth between the source and the load with no net work being done. Energy must be used in order for work to be done.

Many practical loads have inductance as a result of their particular function, and it is essential for their proper operation. Examples are transformers, electric motors, and speakers, to name a few. Therefore, inductive (and capacitive) loads are important considerations.

To see the effect of the power factor on system requirements, refer to Figure 36. This figure shows a representation of a typical inductive load consisting effectively of inductance and resistance in parallel. Part (a) shows a load with a relatively low power factor (0.75), and part (b) shows a load with a relatively high power factor (0.95). Both loads dissipate equal amounts of power as indicated by the wattmeters. Thus, an equal amount of work is done on both loads.



(a) A lower power factor means more total current for a given power dissipation (watts). A larger source VA rating is required to deliver the true power (watts).

(b) A higher power factor means less total current for a given power dissipation. A smaller source can deliver the same true power (watts).

▲ **FIGURE 36**

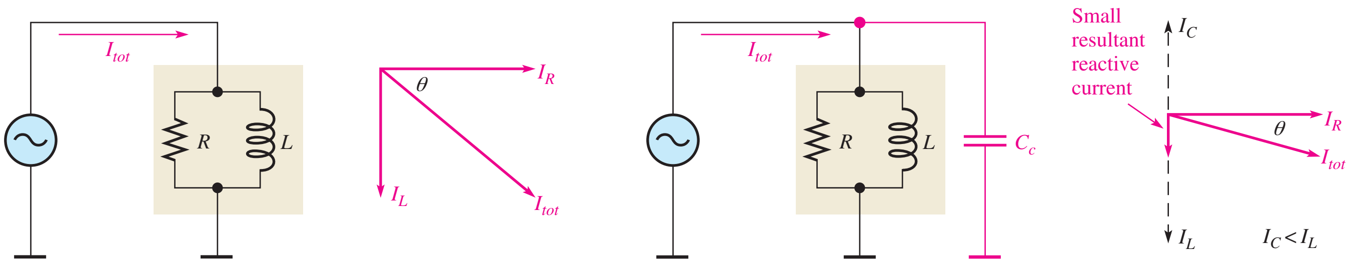
Illustration of the effect of the power factor on system requirements such as source rating (VA) and conductor size.

Although both loads are equivalent in terms of the amount of work done (true power), the low power factor load in Figure 36(a) draws more current from the source than does the high power factor load in Figure 36(b), as indicated by the ammeters. Therefore, the source in part (a) must have a higher VA rating than the one in part (b). Also, the lines connecting the source to the load in part (a) must be a larger wire gauge than those in part (b), a condition that becomes significant when very long transmission lines are required, such as in power distribution.

Figure 36 has demonstrated that a higher power factor is an advantage in delivering power more efficiently to a load.

Power Factor Correction

The power factor of an inductive load can be increased by the addition of a capacitor in parallel, as shown in Figure 37. The capacitor compensates for the phase lag of the total current by creating a capacitive component of current that is 180° out of phase with the inductive component. This has a canceling effect and reduces the phase angle and the total current and increases the power factor, as illustrated in the figure.



(a) Total current is the resultant of I_R and I_L.

(b) I_C subtracts from I_L, leaving only a small reactive current, thus decreasing I_{tot} and the phase angle.

FIGURE 37

Example of how the power factor can be increased by the addition of a compensating capacitor (C_c).

SECTION 7 CHECKUP

1. To which component in an *RL* circuit is the power dissipation due?
2. Calculate the power factor when $\theta = 50°$.
3. A certain *RL* circuit consists of a 470 Ω resistor and an inductive reactance of 620 Ω at the operating frequency. Determine P_{true}, P_r, and P_a when $I = 100$ mA.

8 BASIC APPLICATIONS

Two applications of *RL* circuits are covered in this section. The first application is a basic frequency selective (filter) circuit. The second application is the switching regulator, a widely used circuit in power supplies because of its high efficiency. The switching regulator uses other components, but the *RL* circuit is emphasized.

After completing this section, you should be able to

- **Describe two examples of *RL* circuit applications**
  - Discuss how the *RL* circuit operates as a filter
  - Discuss the advantage of an inductor in a switching regulator

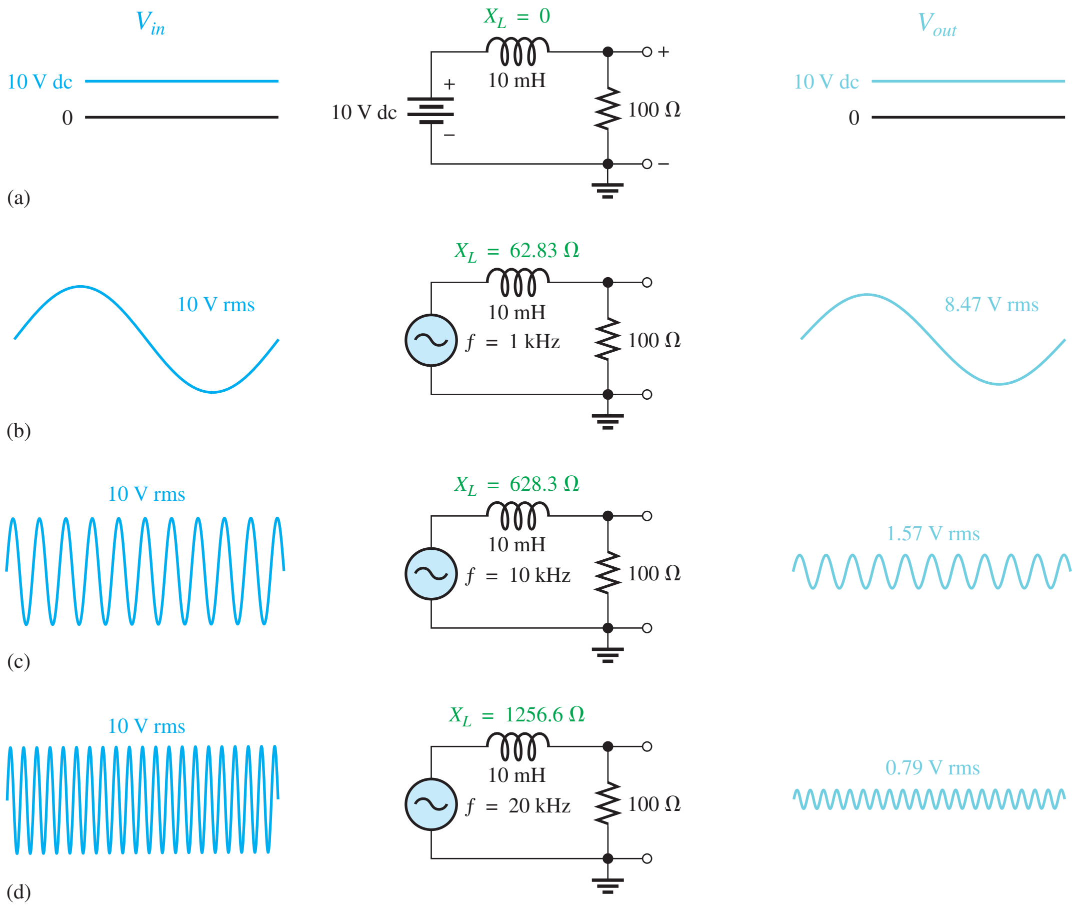
The *RL* Circuit as a Filter

As with *RC* circuits, series *RL* circuits also exhibit a frequency-selective characteristic and therefore act as basic filters.

Low-Pass Filter You have seen what happens to the output magnitude and phase angle in a lag circuit. In terms of the filtering action, the variation of the magnitude of the output voltage as a function of frequency is important.

Figure 38 shows the filtering action of a series *RL* circuit using specific values for purposes of illustration. In part (a) of the figure, the input is zero frequency (dc). Since the inductor ideally acts as a short to constant direct current, the output voltage equals the full value of the input voltage (neglecting the winding resistance). Therefore, the circuit passes all of the input voltage to the output (10 V in, 10 V out).

In Figure 38(b), the frequency of the input voltage has been increased to 1 kHz, causing the inductive reactance to increase to 62.83 Ω. For an input voltage of 10 V rms, the output voltage is approximately 8.47 V rms, which can be calculated using the voltage-divider approach or Ohm's law.



▲ **FIGURE 38**

Low-pass filtering action of an *RL* circuit (phase shift from input to output is not indicated).

In Figure 38(c), the input frequency has been increased to 10 kHz, causing the inductive reactance to increase further to 628.3 Ω. For a constant input voltage of 10 V rms, the output voltage is now 1.57 V rms.

As the input frequency is increased further, the output voltage continues to decrease and approaches zero as the frequency becomes very high, as shown in Figure 38(d) for $f = 20$ kHz.

A description of the circuit action is as follows: As the frequency of the input increases, the inductive reactance increases. Because the resistance is constant and the inductive reactance increases, the voltage across the inductor increases and that across the resistor (output voltage) decreases. The input frequency can be increased until it reaches a value at which the reactance is so large compared to the resistance that the output voltage can be neglected because it becomes very small compared to the input voltage.

As shown in Figure 38, the circuit passes dc (zero frequency) completely. As the frequency of the input increases, less of the input voltage is passed through to the output. That is, the output voltage decreases as the frequency increases. It is apparent that the lower frequencies pass through the circuit much better than the higher frequencies. This *RL* circuit is therefore a basic form of low-pass filter.

Figure 39 shows a response curve for a low-pass filter.

High-Pass Filter Figure 40 illustrates high-pass filtering action, where the output is taken across the inductor. When the input voltage is dc (zero frequency) in part (a), the output is zero volts because the inductor ideally appears as a short across the output.

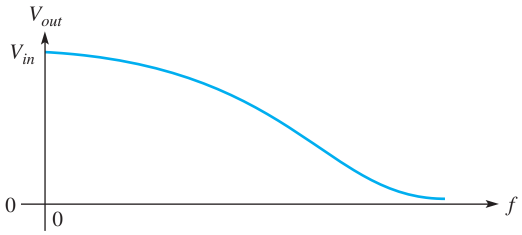


FIGURE 39

Low-pass filter response curve.

In Figure 40(b), the frequency of the input signal has been increased to 100 Hz with an rms value of 10 V. The output voltage is 0.63 V rms. Thus, only a small percentage of the input voltage appears at the output at this frequency.

In Figure 40(c), the input frequency is increased further to 1 kHz, causing more voltage to be developed as a result of the increase in the inductive reactance. The output voltage at this frequency is 5.32 V rms. As you can see, the output voltage increases as the frequency increases. A value of frequency is reached at which the reactance is very large compared to the resistance and most of the input voltage appears across the inductor, as shown in Figure 40(d).

This circuit tends to prevent lower frequency signals from appearing on the output but permits higher frequency signals to pass through from input to output; thus, it is a basic form of high-pass filter.

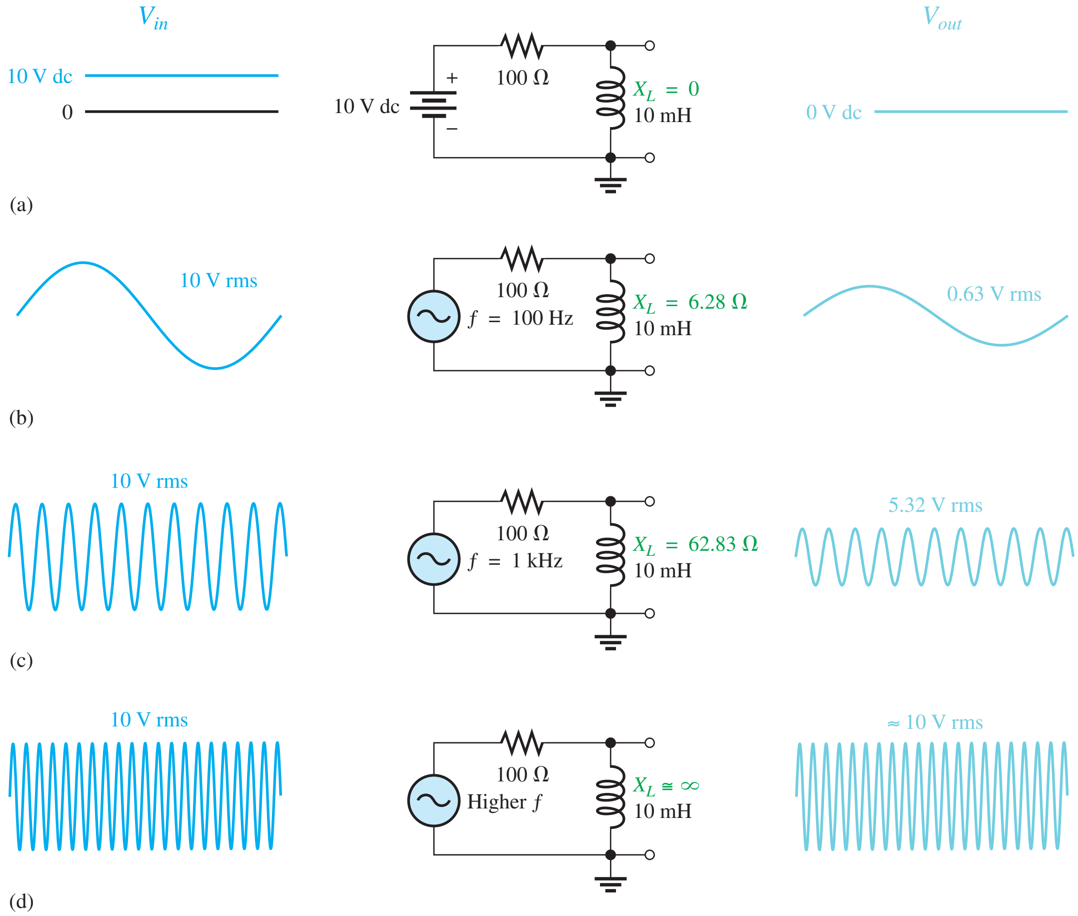

V_{in}
V_{out}
10 V dc
0
10 V dc
+
−
100 Ω
X_L = 0
10 mH
0 V dc
(a)
10 V rms
100 Ω
f = 100 Hz
X_L = 6.28 Ω
10 mH
0.63 V rms
(b)
10 V rms
100 Ω
f = 1 kHz
X_L = 62.83 Ω
10 mH
5.32 V rms
(c)
10 V rms
100 Ω
Higher f
X_L ≅ ∞
10 mH
≈ 10 V rms
(d)

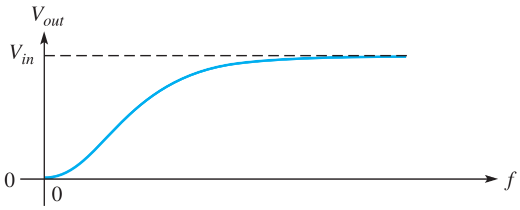
FIGURE 40

High-pass filtering action of an *RL* circuit (phase shift from input to output is not indicated).

The response curve in Figure 41 shows that the output voltage increases and then levels off as it approaches the value of the input voltage as the frequency increases.

FIGURE 41

High-pass filter response curve.



The Switching Regulator

In high-frequency switching power supplies, small inductors are used as an essential part of the filter section. A switching power supply is much more efficient at converting ac to dc than any other type of supply. For this reason it is widely used in computers and other electronic systems. A switching regulator precisely controls the dc voltage. One type of switching regulator is shown in Figure 42. It uses an electronic switch to change unregulated dc to high-frequency pulses. The output is the average value of the pulses. The pulse width is controlled by the pulse width modulator, which rapidly turns on and off a transistor switch and then is filtered by the filter section to produce regulated dc. (Ripple in the figure is exaggerated to show the cycle.) The pulse width modulator can increase pulse width if the output drops, or decrease it if the output rises, thus maintaining a constant average output for varying conditions.

FIGURE 42

Diagram showing an inductor in a switching regulator.

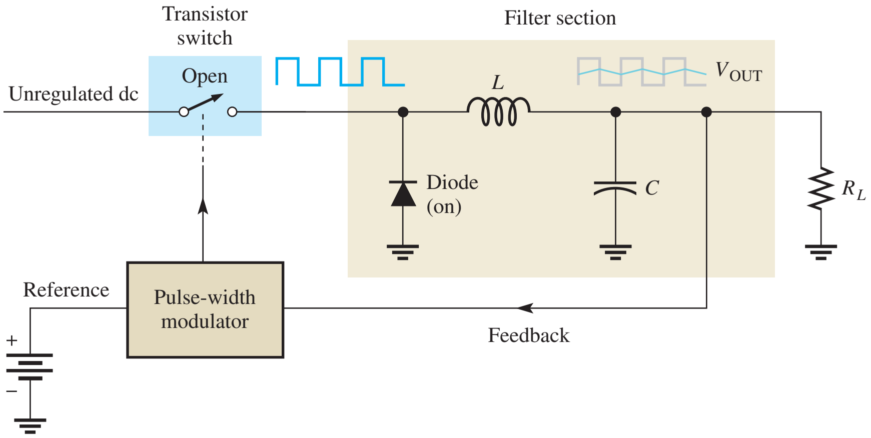
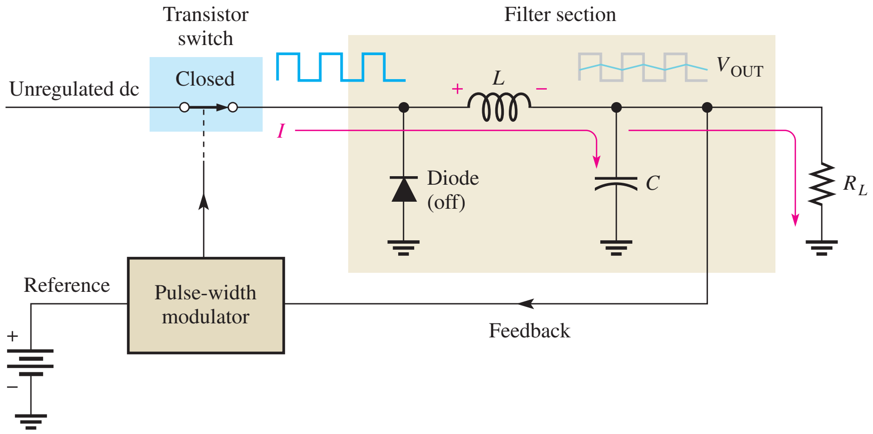
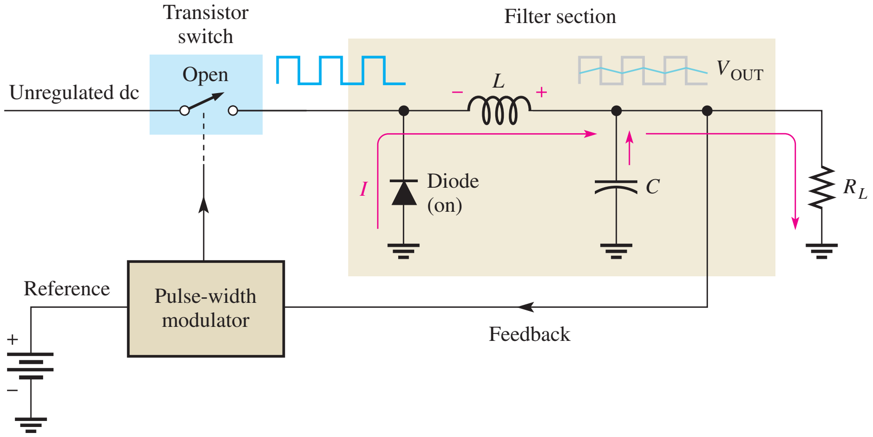


Figure 43 illustrates the basic filtering action. The filter consists of a diode, an inductor, and a capacitor. The diode is a one-way device for current that you will study in a devices course. In this application the diode acts as an on-off switch that allows current in only one direction.

An important component of the filter section is the inductor, which in this type of regulator will always have current in it. The average voltage and the load resistor determine the amount of current. Recall that Lenz's law states that an induced voltage is created across a coil that opposes a *change* in current. When the transistor switch is closed, the pulse is high and current is passed through the inductor and the load, as shown in Figure 43(a). The diode is off at this time. Notice that the inductor has an induced voltage across it that opposes a change in current. When the pulse goes low, as in Figure 43(b), the transistor is off and the inductor develops a voltage in the opposite direction than before. The diode acts as a closed switch, which provides a path for current. This action tends to keep the load current constant. The capacitor adds to this smoothing action by charging and discharging a small amount during the process.


Transistor switch
Closed
Unregulated dc
Filter section
+
L
−
V_{OUT}
I
Diode (off)
C
R_L
Reference
+
−
Pulse-width modulator
Feedback

(a) Pulse high


Transistor switch
Open
Unregulated dc
Filter section
−
L
+
V_{OUT}
I
Diode (on)
C
R_L
Reference
+
−
Pulse-width modulator
Feedback

(b) Pulse low

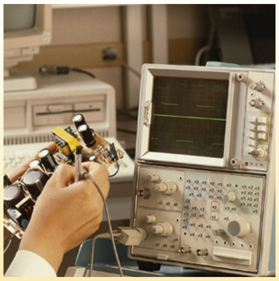
▲ **FIGURE 43**

Switching regulator action.

SECTION 8 CHECKUP

1. When an *RL* circuit is used as a low-pass filter, across which component is the output taken?
2. What is the major advantage of a switching regulator?
3. What happens to the pulse width of a switching regulator if the output voltage drops?

9 TROUBLESHOOTING



Typical component failures have an effect on the frequency response of basic *RL* circuits.

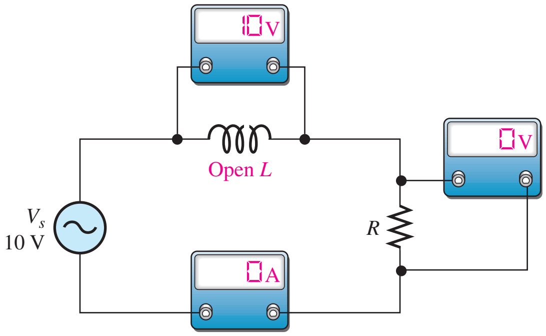
After completing this section, you should be able to

- **Troubleshoot *RL* circuits**
  - Find an open inductor
  - Find an open resistor
  - Find an open in a parallel circuit

Effect of an Open Inductor The most common failure mode for inductors occurs when the winding opens as a result of excessive current or a mechanical contact failure. It is easy to see how an open coil affects the operation of a basic series *RL* circuit, as shown in Figure 44. Obviously, there is no current path; therefore, the resistor voltage is zero, and the total applied voltage appears across the inductor. If you suspect an open coil, remove one or both leads from the circuit and check continuity with an ohmmeter.

▶ FIGURE 44

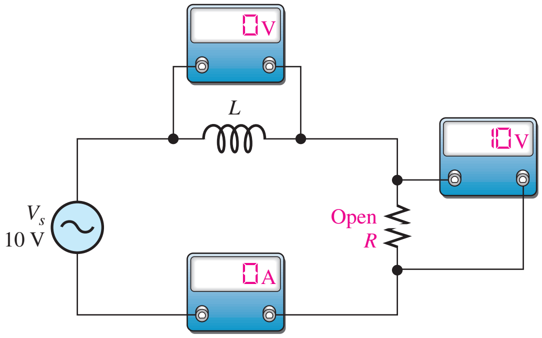
Effect of an open coil.



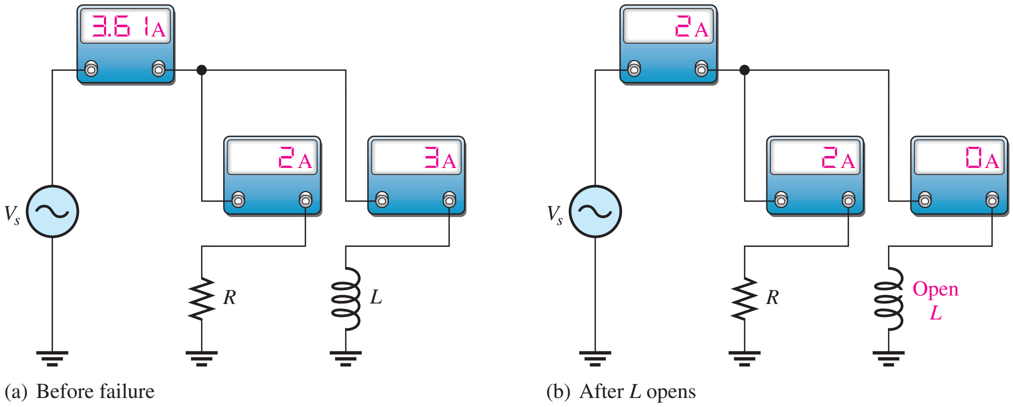
Effect of an Open Resistor When the resistor is open, there is no current and the inductor voltage is zero. The total input voltage is across the open resistor, as shown in Figure 45.

▶ FIGURE 45

Effect of an open resistor.



Open Components in Parallel Circuits In a parallel *RL* circuit, an open resistor or inductor will cause the total current to decrease because the total impedance will increase. Obviously, the branch with the open component will have zero current. Figure 46 illustrates these conditions.



(a) Before failure

(b) After *L* opens

▲ FIGURE 46

Effect of an open component in a parallel circuit with V_s constant.

Effect of an Inductor with Shorted Windings Although a rare occurrence, it is possible for some of the windings of coils to short together as a result of damaged insulation. This failure mode is much less likely than the open coil and is difficult to detect. Shorted windings may result in a reduction in inductance because the inductance of a coil is proportional to the square of the number of turns. A short between windings effectively reduces the number of turns, which may or may not have an adverse effect on the circuit depending on the number of turns that are shorted.

Other Troubleshooting Considerations

The failure of a circuit to work properly is not always the result of a faulty component. A loose wire, a bad contact, or a poor solder joint can cause an open circuit. A short can be caused by a wire clipping or solder splash. Things as simple as not plugging in a power supply or a function generator happen more often than you might think. Wrong values in a circuit (such as an incorrect resistor value), the function generator set at the wrong frequency, or the wrong output connected to the circuit can cause improper operation.

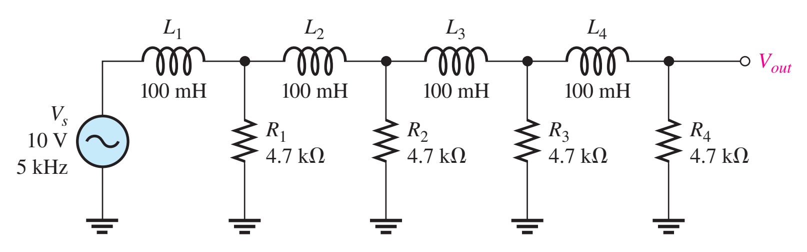
Always check to make sure that the instruments are properly connected to the circuits and to a power outlet. Also, look for obvious things such as a broken or loose contact, a connector that is not completely plugged in, or a piece of wire or a solder bridge that could be shorting something out.

The following example illustrates a troubleshooting approach to a circuit containing inductors and resistors using the APM (analysis, planning, and measurement) method and half-splitting.

EXAMPLE 15

The circuit represented by the schematic in Figure 47 has no output voltage. The circuit is physically constructed on a protoboard. Use your trouble- shooting skills to find the problem.

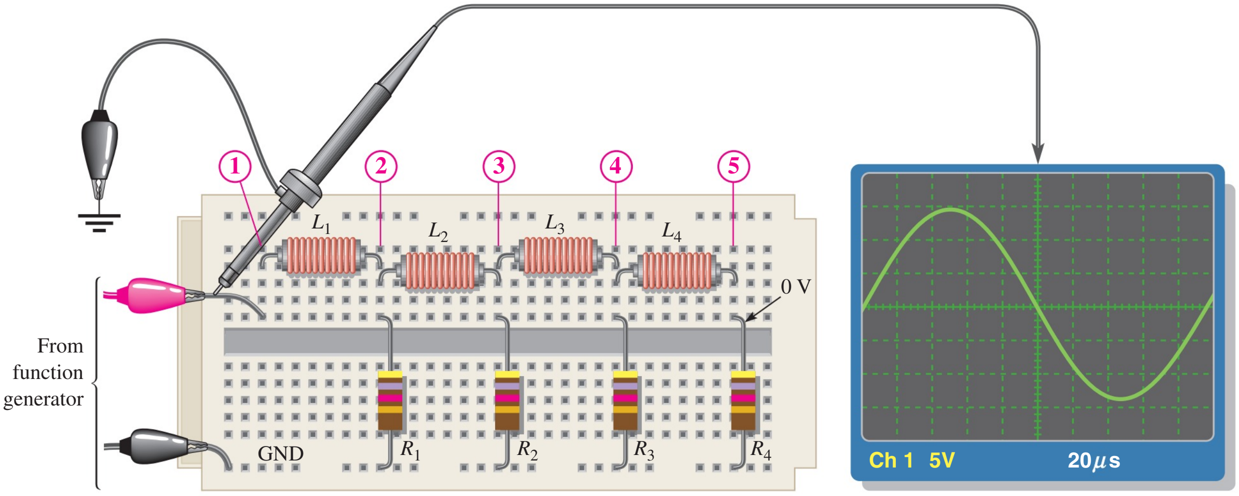
▶ FIGURE 47



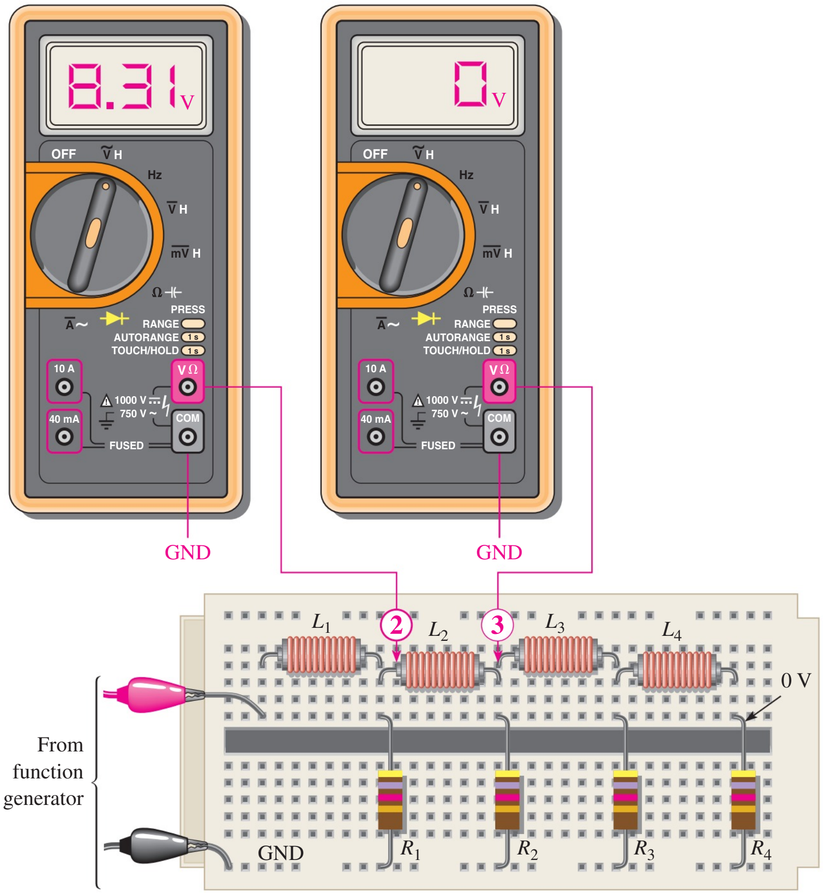
Solution Apply the APM method to this troubleshooting problem.

Analysis: First think of the possible causes for the circuit to have no output voltage.

1. There is no source voltage or the frequency is so high that the inductors appear to be open because their reactances are extremely high compared to the resistance values.
2. There is a short between one of the resistors and ground. Either a resistor could be shorted, or there could be some physical short. A shorted resistor is not a common fault.
3. There is an open between the source and the output. This would prevent current and thus cause the output voltage to be zero. An inductor could be open, or the conductive path could be open due to a broken or loose connecting wire or a bad protoboard contact.
4. There is an incorrect component value. A resistor could be so small that the voltage across it is negligible. An inductor could be so large that its reactance at the input frequency is extremely high.


①
②
③
④
⑤
L_1
L_2
L_3
L_4
0 V
From function generator
GND
R_1
R_2
R_3
R_4
Ch 1 5V
20μs

(a) Scope shows the correct voltage at the input.


8.31 V
0 V
OFF
Ṽ H
Hz
V̄ H
mV̄ H
Ω
PRESS
RANGE
AUTORANGE 1 s
TOUCH/HOLD 1 s
Ā~
10 A
40 mA
VΩ
COM
1000 V
750 V ~
FUSED
GND
GND
②
③
L_1
L_2
L_3
L_4
0 V
From function generator
GND
R_1
R_2
R_3
R_4

(b) A zero voltage at point ③ indicates the fault is between point ③ and the source. A reading of 8.31 V at point ② shows that L_2 is open.

▲ **FIGURE 48**

Planning: You decide to make some visual checks for problems such as the function generator power cord not plugged in or the frequency set at an incorrect value. Also, broken leads, shorted leads, as well as an incorrect resistor color code or inductor value often can be found visually. If nothing is discovered after a visual check, then you will make voltage measurements to track down the cause of the problem. You decide to use a digital oscilloscope and a DMM to make the measurements using the half-splitting technique to more quickly isolate the fault.

Measurement: Assume that you find that the function generator is plugged in and the frequency setting appears to be correct. Also, you find no visible opens or shorts during your visual check, and the component values are correct.

The first step in the measurement process is to check the voltage from the source with the scope. Assume a 10 V rms sine wave with a frequency of 5 kHz is observed at the circuit input as shown in Figure 48(a). The correct ac voltage is present, so *the first possible cause has been eliminated.*

Next, check for a short by disconnecting the source and placing the DMM (set on the ohmmeter function) across each resistor. If any resistor is shorted (unlikely), the meter will read zero or a very small resistance. Assuming the meter readings are okay, *the second possible cause has been eliminated.*

Since the voltage has been "lost" somewhere between the input and the output, you must now look for the voltage. You reconnect the source and, using the half-splitting approach, measure the voltage at point ③ (the middle of the circuit) with respect to ground. The DMM test lead is placed on the right lead of inductor L_2, as indicated in Figure 48(b). Assume the voltage at this point is zero. This tells you that the part of the circuit to the right of point ③ is probably okay and the fault is in the circuit between point ③ and the source.

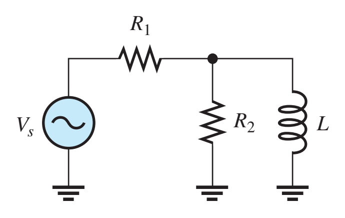
Now, you begin tracing the circuit back toward the source looking for the voltage (you could also start from the source and work forward). Placing the meter test lead on point ②, at the left lead of inductor L_2, results in a reading of 8.31 V as shown in Figure 48(b). This, of course, indicates that L_2 is open. Fortunately, in this case, a component, and not a contact on the board, is faulty. It is usually easier to replace a component than to repair a bad contact.

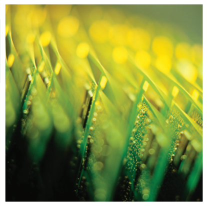
Related Problem Suppose you had measured 0 V at the left lead of L_2 and 10 V at the right lead of L_1. What would this have indicated?

SECTION 9 CHECKUP

1. Describe the effect of an inductor with shorted windings on the response of a series *RL* circuit.
2. In the circuit of Figure 49, indicate whether I_{tot}, V_{R1}, and V_{R2} increase or decrease as a result of *L* opening.

▶ FIGURE 49





Application Activity

Two sealed modules have been removed from a communications system that is being modified. Each module has three terminals and is labeled as an *RL* filter, but no specifications are given. You are asked to test the modules to determine the type of filters and the component values.

The sealed modules have three terminals labeled IN, GND, and OUT as shown in Figure 50. You will apply your knowledge of series *RL* circuits and some basic measurements to determine the internal circuit configuration and the component values.

Resistance Measurements of Module 1

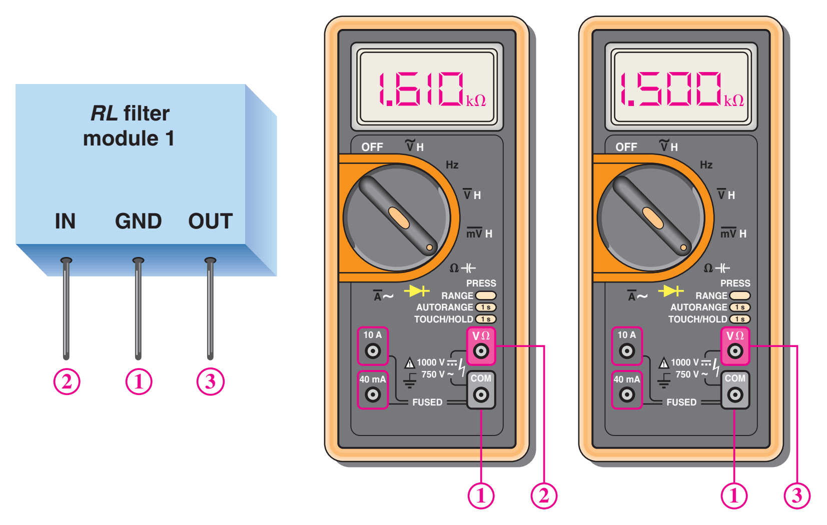
1. Determine the arrangement of the two components and the values of the resistor and winding resistance for module 1 indicated by the meter readings in Figure 50.

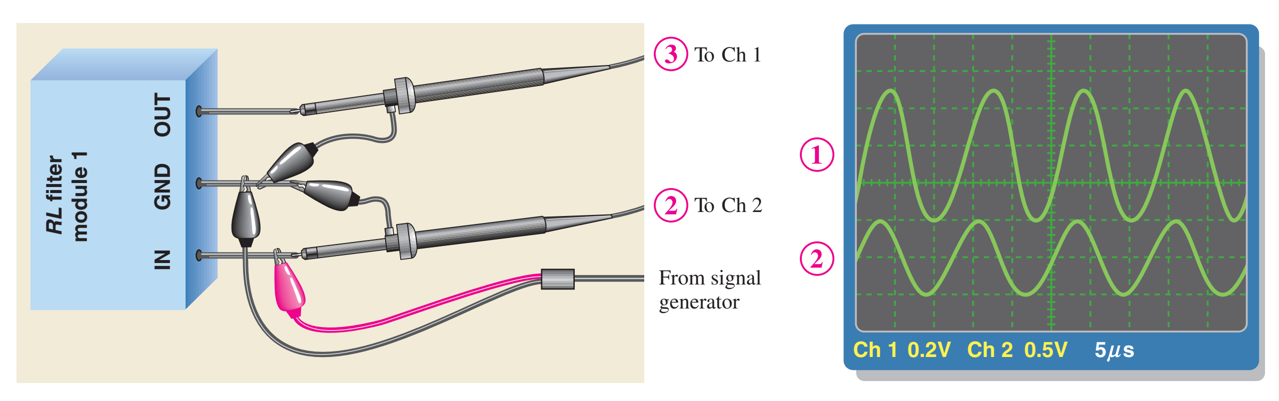
AC Measurements of Module 1

2. Determine the inductance value for module 1 indicated by the test setup in Figure 51.

▶ FIGURE 50

Resistance measurements of module 1.


RL filter
module 1
IN
GND
OUT
②
①
③
1.610 kΩ
1.500 kΩ
OFF
Hz
PRESS
RANGE
AUTORANGE
TOUCH/HOLD
10 A
40 mA
VΩ
COM
1000 V
750 V
FUSED
①
②
①
③


RL filter
module 1
IN
GND
OUT
③ To Ch 1
② To Ch 2
From signal generator
①
②
Ch 1 0.2V
Ch 2 0.5V
5μs

▲ FIGURE 51

AC measurements for module 1.

Resistance Measurements of Module 2

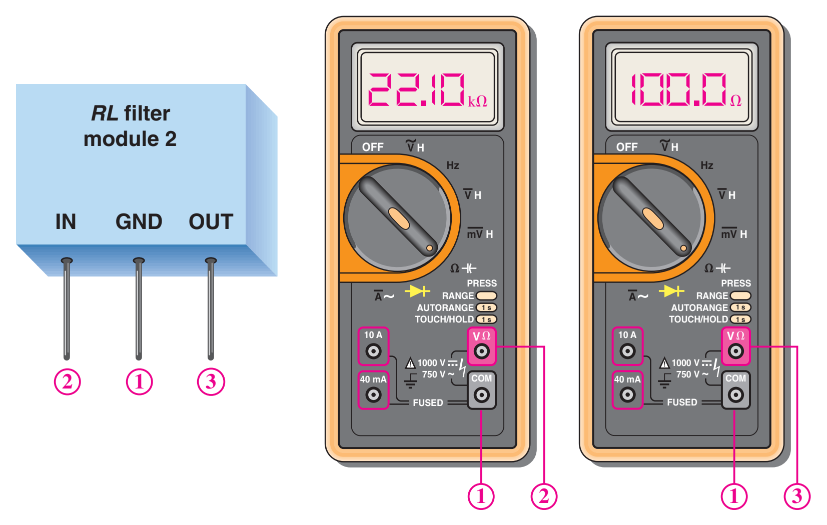
3. Determine the arrangement of the two components and the values of the resistor and the winding resistance for module 2 indicated by the meter readings in Figure 52.

AC Measurements of Module 2

4. Determine the inductance value for module 2 indicated by the test setup in Figure 53.

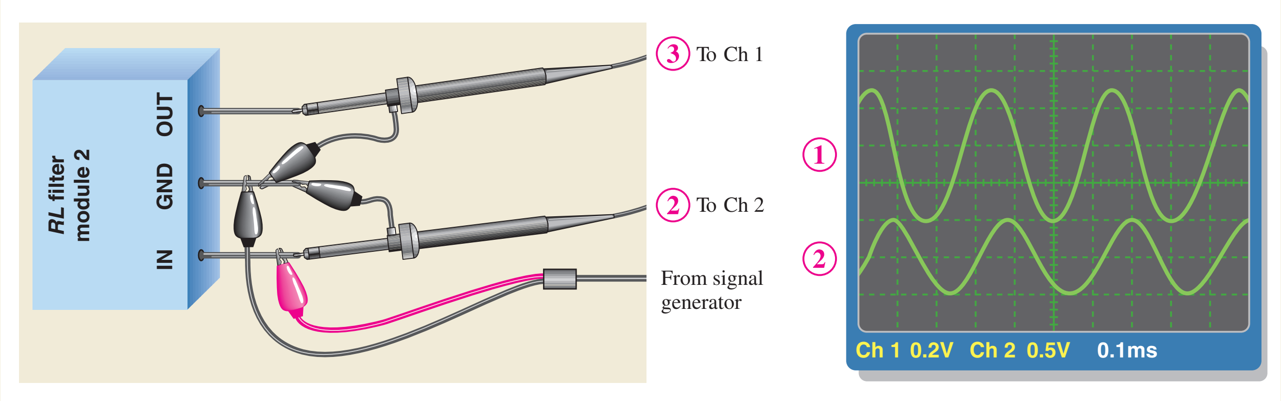
Review

5. If the inductor in module 1 were open, what would you measure on the output with the test setup of Figure 51?
6. If the inductor in module 2 were open, what would you measure on the output with the test setup of Figure 53?


RL filter module 2
IN
GND
OUT
②
①
③
22.10 kΩ
100.0 Ω
OFF
V H
Hz
mV H
Ω
PRESS
RANGE
AUTORANGE
TOUCH/HOLD
10 A
40 mA
VΩ
COM
1000 V
750 V ~
FUSED
①
②
①
③

▲ FIGURE 52

Resistance measurements of module 2.


RL filter module 2
OUT
GND
IN
③ To Ch 1
② To Ch 2
From signal generator
①
②
Ch 1 0.2V
Ch 2 0.5V
0.1ms

▲ FIGURE 53

AC measurements for module 2.

SUMMARY

- When a sinusoidal voltage is applied to an *RL* circuit, the current and all the voltage drops are also sine waves.
- Total current in a series or parallel *RL* circuit always lags the source voltage.
- The resistor voltage is always in phase with the current.
- In an ideal inductor, the voltage always leads the current by 90°.
- In a lag circuit, the output voltage lags the input voltage in phase.
- In a lead circuit, the output voltage leads the input voltage in phase.
- In an *RL* circuit, the impedance is determined by both the resistance and the inductive reactance combined.
- Impedance is expressed in units of ohms.
- The impedance of an *RL* circuit varies directly with frequency.
- The phase angle (θ) of a series *RL* circuit varies directly with frequency.
- You can determine the impedance of a circuit by measuring the applied voltage and the total current and then applying Ohm's law.
- In an *RL* circuit, part of the power is resistive and part reactive.
- The power factor indicates how much of the apparent power is true power.
- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit.
- A filter passes certain frequencies and rejects others.

KEY TERMS

Inductive susceptance (B_L) The ability of an inductor to permit current; the reciprocal of inductive reactance. The unit is the siemens (S).

***RL* lag circuit** A phase shift circuit in which the output voltage, taken across the resistor, lags the input voltage by a specified angle.

***RL* lead circuit** A phase shift circuit in which the output voltage, taken across the inductor, leads the input voltage by a specified angle.

FORMULAS

Series *RL* Circuits

1 $$\mathbf{Z} = R + jX_L$$

2 $$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

3 $$\mathbf{V}_s = V_R + jV_L$$

4 $$\mathbf{V}_s = \sqrt{V_R^2 + V_L^2}\angle\tan^{-1}\left(\frac{V_L}{V_R}\right)$$

Lead Circuit

5 $$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

6 $$V_{out} = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

Lag Circuit

7 $$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

8 $$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

Parallel *RL* Circuits

9 $$\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$$

10 $$\mathbf{Y} = G - jB_L$$

11 $$\mathbf{I}_{tot} = I_R - jI_L$$

12 $$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

Power in *RL* Circuits

13 $$P_r = I^2X_L$$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. If a sine wave voltage is applied to a series *RL* circuit, the current is also a sine wave.
2. The total impedance of a series *RL* circuit is the algebraic sum of the resistance and the inductive reactance.
3. Ohm's law cannot be applied to reactive circuits.
4. The impedance of an *RL* circuit can be expressed as a phasor quantity using complex numbers.
5. $R + X_L$ is a phasor sum.
6. Voltage leads the current in terms of phase in a series *RL* circuit.
7. The impedance in any *RL* circuit increases with frequency.
8. In a series *RL* lag circuit, the output is taken across the resistor.
9. Susceptance is the reciprocal of impedance.
10. Power factor is determined by the magnitudes of the voltage and current.

SELF-TEST

Answers are at the end of the chapter.

1. In a series *RL* circuit, the resistor voltage
   (a) leads the applied voltage **(b)** lags the applied voltage
   (c) is in phase with the applied voltage **(d)** is in phase with the current
   (e) answers (a) and (d) **(f)** answers (b) and (d)
2. When the frequency of the voltage applied to a series *RL* circuit is increased, the impedance
   (a) decreases **(b)** increases **(c)** does not change
3. When the frequency of the voltage applied to a series *RL* circuit is decreased, the phase angle
   (a) decreases **(b)** increases **(c)** does not change
4. If the frequency is doubled and the resistance is halved, the impedance of a series *RL* circuit
   (a) doubles **(b)** halves
   (c) remains constant **(d)** cannot be determined without values
5. To reduce the current in a series *RL* circuit, the frequency should be
   (a) increased **(b)** decreased **(c)** constant
6. In a series *RL* circuit, 10 V rms is measured across the resistor, and 10 V rms is measured across the inductor. The peak value of the source voltage is
   (a) 14.14 V **(b)** 28.28 V **(c)** 10 V **(d)** 20 V
7. The voltages in Problem 6 are measured at a certain frequency. To make the resistor voltage greater than the inductor voltage, the frequency is
   (a) increased **(b)** decreased **(c)** doubled **(d)** not a factor

8. When the resistor voltage in a series *RL* circuit becomes greater than the inductor voltage, the phase angle

   (a) increases **(b)** decreases **(c)** is not affected

9. When the frequency of the source voltage is increased, the impedance of a parallel *RL* circuit

   (a) increases **(b)** decreases **(c)** remains constant

10. In a parallel *RL* circuit, there are 2 A rms in the resistive branch and 2 A rms in the inductive branch. The total rms current is

    (a) 4 A **(b)** 5.656 A **(c)** 2 A **(d)** 2.828 A

11. You are observing two voltage waveforms on an oscilloscope. The time base (time/division) of the scope is adjusted so that one-half cycle of the waveforms covers the ten horizontal divisions. The positive-going zero crossing of one waveform is at the leftmost division, and the positive-going zero crossing of the other is three divisions to the right. The phase angle between these two waveforms is

    (a) 18° **(b)** 36° **(c)** 54° **(d)** 180°

12. Which of the following power factors results in less energy being converted to heat in an *RL* circuit?

    (a) 1 **(b)** 0.9 **(c)** 0.5 **(d)** 0.1

13. If a load is purely inductive and the reactive power is 10 VAR, the apparent power is

    (a) 0 VA **(b)** 10 VA **(c)** 14.14 VA **(d)** 3.16 VA

14. For a certain load, the true power is 10 W and the reactive power is 10 VAR. The apparent power is

    (a) 5 VA **(b)** 20 VA **(c)** 14.14 VA **(d)** 100 VA

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 56.

1. If L opens, the voltage across it

   (a) increases **(b)** decreases **(c)** stays the same

2. If R opens, the voltage across L

   (a) increases **(b)** decreases **(c)** stays the same

3. If the frequency is increased, the voltage across R

   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 63.

4. If L opens, the voltage across R

   (a) increases **(b)** decreases **(c)** stays the same

5. If f is increased, the current through R

   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 69.

6. If R_1 becomes open, the current through L_1

   (a) increases **(b)** decreases **(c)** stays the same

7. If L_2 opens, the voltage across R_2

   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 70.

8. If L_2 opens, the voltage from point B to ground

   (a) increases **(b)** decreases **(c)** stays the same

9. If L_1 opens, the voltage from point B to ground

   (a) increases **(b)** decreases **(c)** stays the same

10. If the frequency of the source voltage is increased, the current through R_1
   (a) increases **(b)** decreases **(c)** stays the same

11. If the frequency of the source voltage is decreased, the voltage from point A to ground
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 73.

12. If L_2 opens, the voltage across L_1
   (a) increases **(b)** decreases **(c)** stays the same

13. If R_1 opens, the output voltage
   (a) increases **(b)** decreases **(c)** stays the same

14. If R_3 becomes open, the output voltage
   (a) increases **(b)** decreases **(c)** stays the same

15. If a partial short develops in L_1, the source current
   (a) increases **(b)** decreases **(c)** stays the same

16. If the source frequency increases, the output voltage
   (a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

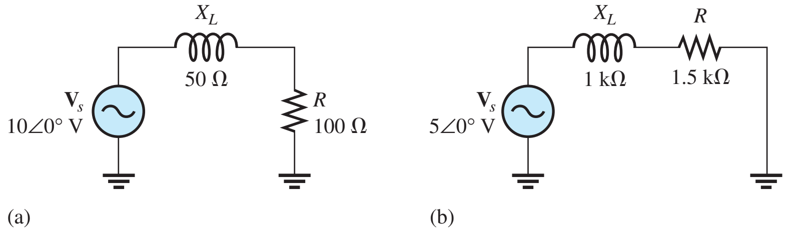
PART 1: SERIES CIRCUITS

SECTION 1 Sinusoidal Response of Series *RL* Circuits

1. A 15 kHz sinusoidal voltage is applied to a series *RL* circuit. What is the frequency of I, V_R, and V_L?
2. What are the wave shapes of I, V_R, and V_L in Problem 1?

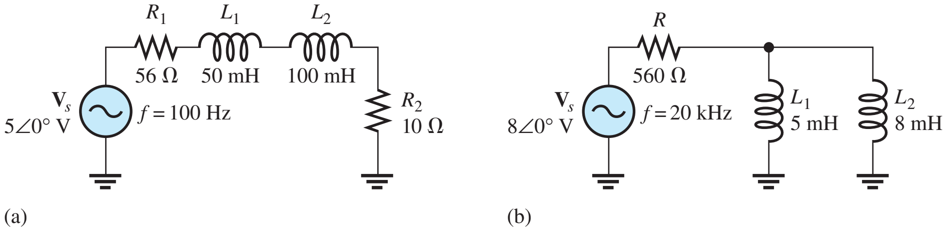
SECTION 2 Impedance of Series *RL* Circuits

3. Express the total impedance of each circuit in Figure 54 in both polar and rectangular forms.



▲ FIGURE 54

4. Determine the impedance magnitude and phase angle in each circuit in Figure 55. Draw the impedance diagrams.

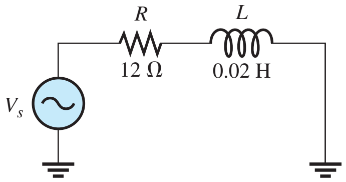


▲ FIGURE 55

5. In Figure 56, determine the impedance at each of the following frequencies:

(a) 100 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 2 kHz

▶ **FIGURE 56**



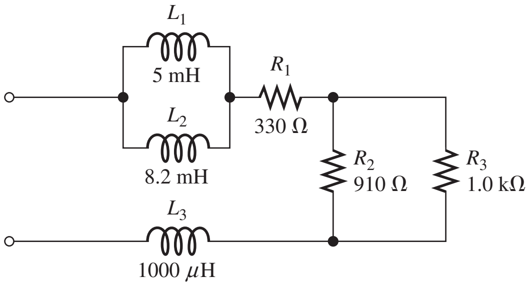
6. Determine the values of R and X_L in a series *RL* circuit for the following values of total impedance:

(a) $\mathbf{Z} = 20\,\Omega + j45\,\Omega$ **(b)** $\mathbf{Z} = 500\angle 35^\circ\,\Omega$

(c) $\mathbf{Z} = 2.5\angle 72.5^\circ\,\text{k}\Omega$ **(d)** $\mathbf{Z} = 998\angle 45^\circ\,\Omega$

7. Reduce the circuit in Figure 57 to a single resistance and inductance in series.

▶ **FIGURE 57**



SECTION 3 Analysis of Series *RL* Circuits

8. A 5 V, 10 kHz sinusoidal voltage is applied to the circuit in Figure 57. Calculate the voltage across the total resistance found in Problem 7.

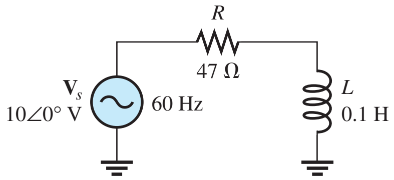
9. For the same applied voltage in Problem 8, determine the voltage across L_3 for the circuit in Figure 57.

10. Express the current in polar form for each circuit of Figure 54.

11. Calculate the total current in each circuit of Figure 55 and express in polar form.

12. Determine θ for the circuit in Figure 58.

▶ **FIGURE 58**



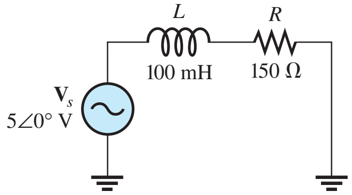
13. If the inductance in Figure 58 is doubled, does θ increase or decrease, and by how many degrees?

14. Draw the waveforms for $\mathbf{V}_s$, $\mathbf{V}_R$, and $\mathbf{V}_L$ in Figure 58. Show the proper phase relationships.

15. For the circuit in Figure 59, find $\mathbf{V}_R$ and $\mathbf{V}_L$ for each of the following frequencies:

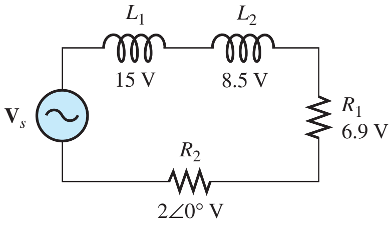
(a) 60 Hz **(b)** 200 Hz **(c)** 500 Hz **(d)** 1 kHz

▶ **FIGURE 59**



16. Determine the magnitude and phase angle of the source voltage in Figure 60.

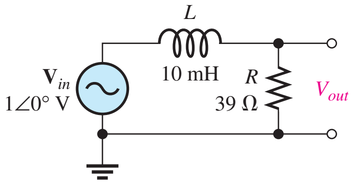
▶ **FIGURE 60**


L_1
15 V
L_2
8.5 V
$\mathbf{V}_s$
R_1
6.9 V
R_2
$2\angle 0°$ V

17. For the lag circuit in Figure 61, determine the phase lag of the output voltage with respect to the input for the following frequencies:

(a) 1 Hz **(b)** 100 Hz **(c)** 1 kHz **(d)** 10 kHz

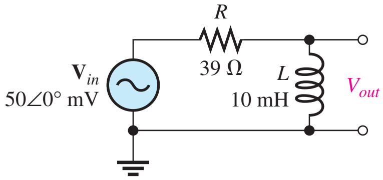
▶ **FIGURE 61**


L
10 mH
$\mathbf{V}_{in}$
$1\angle 0°$ V
R
39 Ω
V_{out}

18. Repeat Problem 17 for the lead circuit to find the phase lead in Figure 62.

19. Express $\mathbf{V}_{out}$ in Figure 62 in polar form for each frequency in Problem 17.

▶ **FIGURE 62**


R
39 Ω
$\mathbf{V}_{in}$
$50\angle 0°$ mV
L
10 mH
V_{out}

PART 2: PARALLEL CIRCUITS

SECTION 4 Impedance and Admittance of Parallel *RL* Circuits

20. What is the impedance in polar form for the circuit in Figure 63?

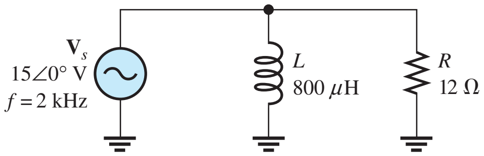
21. What is the impedance in rectanglar form for the circuit in Figure 63?

22. Repeat Problem 20 for the following frequencies:

(a) 1.5 kHz **(b)** 3 kHz **(c)** 5 kHz **(d)** 10 kHz

23. At what frequency does X_L equal R in Figure 63?

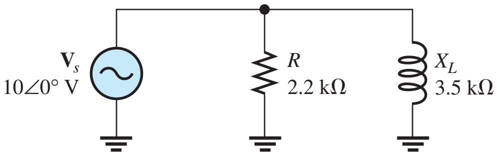
▶ **FIGURE 63**


$\mathbf{V}_s$
$15\angle 0°$ V
$f = 2$ kHz
L
800 μH
R
12 Ω

SECTION 5 Analysis of Parallel *RL* Circuits

24. Find the total current and each branch current in Figure 64.

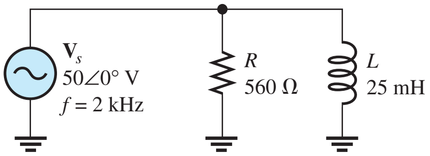
▶ FIGURE 64


$\mathbf{V}_s$
$10\angle 0°$ V
R
2.2 kΩ
X_L
3.5 kΩ

25. Determine the following quantities in Figure 65:

(a) $\mathbf{Z}$ **(b)** $\mathbf{I}_R$ **(c)** $\mathbf{I}_L$ **(d)** $\mathbf{I}_{tot}$ **(e)** θ

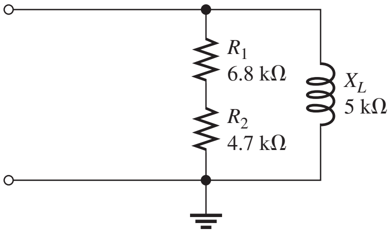
▶ FIGURE 65


$\mathbf{V}_s$
$50\angle 0°$ V
$f = 2$ kHz
R
560 Ω
L
25 mH

26. Repeat Problem 25 for $R = 56\ \Omega$ and $L = 330\ \mu$H.

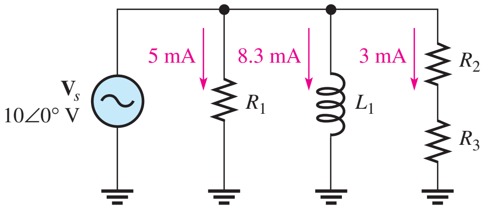
27. Convert the circuit in Figure 66 to an equivalent series form.

▶ FIGURE 66


R_1
6.8 kΩ
R_2
4.7 kΩ
X_L
5 kΩ

28. Find the magnitude and phase angle of the total current in Figure 67.

▶ FIGURE 67

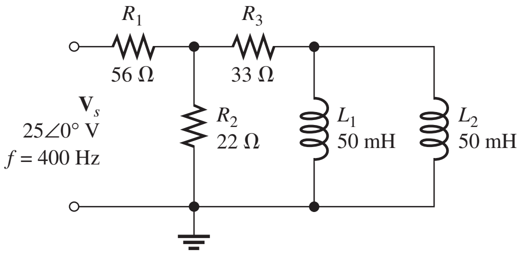

$\mathbf{V}_s$
$10\angle 0°$ V
5 mA
8.3 mA
3 mA
R_1
L_1
R_2
R_3

PART 3: SERIES-PARALLEL CIRCUITS

SECTION 6 Analysis of Series-Parallel *RL* Circuits

29. Determine the voltages in polar form across each element in Figure 68. Draw the voltage phasor diagram.

▶ FIGURE 68


R_1
56 Ω
R_3
33 Ω
$\mathbf{V}_s$
$25\angle 0°$ V
$f = 400$ Hz
R_2
22 Ω
L_1
50 mH
L_2
50 mH

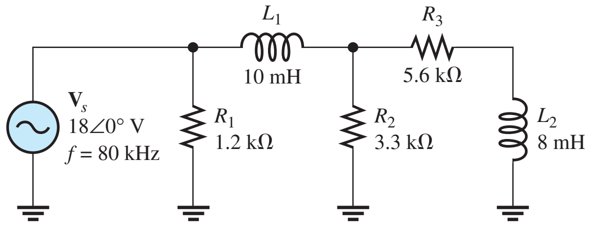
30. Is the circuit in Figure 68 predominantly resistive or predominantly inductive?

31. Find the current in each branch and the total current in Figure 68. Express the currents in polar form. Draw the current phasor diagram.

32. For the circuit in Figure 69, determine the following:

(a) $\mathbf{I}_{tot}$ **(b)** θ **(c)** $\mathbf{V}_{R1}$ **(d)** $\mathbf{V}_{R2}$ **(e)** $\mathbf{V}_{R3}$ **(f)** $\mathbf{V}_{L1}$ **(g)** $\mathbf{V}_{L2}$

▶ **FIGURE 69**



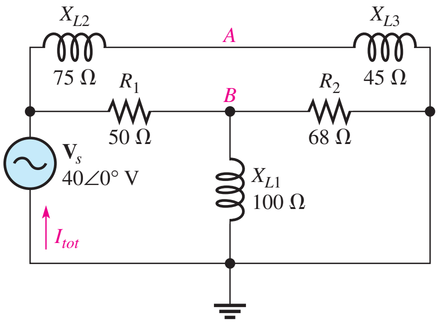
L_1
10 mH
R_3
5.6 kΩ
$\mathbf{V}_s$
18∠0° V
f = 80 kHz
R_1
1.2 kΩ
R_2
3.3 kΩ
L_2
8 mH

***33.** For the circuit in Figure 70, determine the following:

(a) $\mathbf{I}_{tot}$ **(b)** $\mathbf{V}_{L1}$ **(c)** $\mathbf{V}_{AB}$

***34.** Draw the phasor diagram of all voltages and currents in Figure 70.

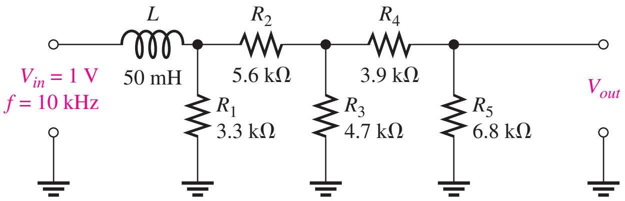
▶ **FIGURE 70**



X_{L2}
75 Ω
A
X_{L3}
45 Ω
R_1
50 Ω
B
R_2
68 Ω
$\mathbf{V}_s$
40∠0° V
X_{L1}
100 Ω
I_{tot}

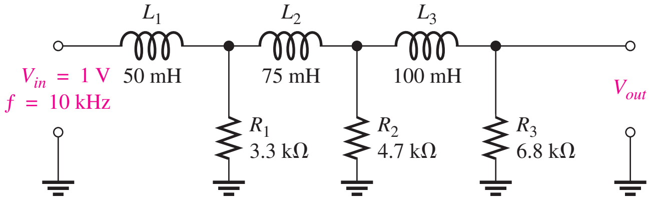
35. Determine the phase shift and attenuation (ratio of V_{out} to V_{in}) from the input to the output for the circuit in Figure 71.

▶ **FIGURE 71**



V_{in} = 1 V
f = 10 kHz
L
50 mH
R_2
5.6 kΩ
R_4
3.9 kΩ
R_1
3.3 kΩ
R_3
4.7 kΩ
R_5
6.8 kΩ
V_{out}

***36.** Determine the phase shift and attenuation from the input to the output for the ladder network in Figure 72.



V_{in} = 1 V
f = 10 kHz
L_1
50 mH
L_2
75 mH
L_3
100 mH
R_1
3.3 kΩ
R_2
4.7 kΩ
R_3
6.8 kΩ
V_{out}

▲ **FIGURE 72**

37. Design an ideal inductive switching circuit that will provide a momentary voltage of 2.5 kV from a 12 V dc source when a switch is thrown instantaneously from one position to another. The drain on the source must not exceed 1 A.

PART 4: SPECIAL TOPICS

SECTION 7 Power in *RL* Circuits

38. In a certain *RL* circuit, the true power is 100 mW, and the reactive power is 340 mVAR. What is the apparent power?

39. Determine the true power and the reactive power in Figure 58.

40. What is the power factor in Figure 64?

41. Determine P_{true}, P_r, P_a, and *PF* for the circuit in Figure 69. Sketch the power triangle.

***42.** Find the true power for the circuit in Figure 70.

SECTION 8 Basic Applications

43. Draw the response curve for the circuit in Figure 61. Show the output voltage versus frequency in 1 kHz increments from 0 Hz to 5 kHz.

44. Using the same procedure as in Problem 41, draw the response curve for Figure 62.

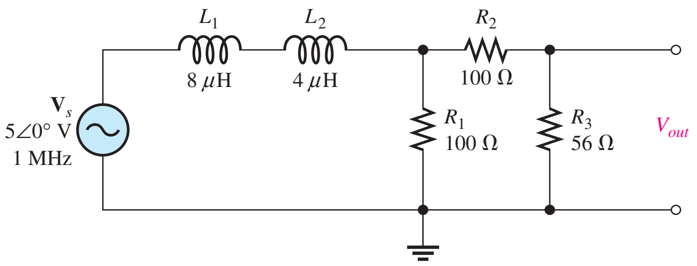
45. Draw the voltage phasor diagram for each circuit in Figures 61 and 62 for a frequency of 8 kHz.

SECTION 9 Troubleshooting

46. Determine the voltage across each component in Figure 73 if L_1 is open.

47. Determine the output voltage in Figure 73 for each of the following failure modes:

(a) L_1 open **(b)** L_2 open **(c)** R_1 open **(d)** a short across R_2



▲ FIGURE 73

Multisim Troubleshooting and Analysis

These problems require Multisim.

48. Open file P16-48 and determine if there is a fault. If so, find the fault.

49. Open file P16-49 and determine if there is a fault. If so, find the fault.

50. Open file P16-50 and determine if there is a fault. If so, find the fault.

51. Open file P16-51 and determine if there is a fault. If so, find the fault.

52. Open file P16-52 and determine if there is a fault. If so, find the fault.

53. Open file P16-53 and determine if there is a fault. If so, find the fault.

54. Open file P16-54 and determine the frequency response for the filter.

55. Open file P16-55 and determine the frequency response for the filter.

ANSWERS

SECTION CHECKUPS

SECTION 1 Sinusoidal Response of Series *RL* Circuits

1. The current frequency is 1 kHz.
2. The phase angle is closer to 0°.

SECTION 2 Impedance of Series *RL* Circuits

1. $R = 150\ \Omega$; $X_L = 220\ \Omega$
2. $\mathbf{Z} = R + jX_L = 33\text{ k}\Omega + j50\text{ k}\Omega$; $\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}(X_L/R) = 59.9\angle 56.6°\text{ k}\Omega$

SECTION 3 Analysis of Series *RL* Circuits

1. $V_s = \sqrt{V_R^2 + V_L^2} = 3.61\text{ V}$
2. $\theta = \tan^{-1}(V_L/V_R) = 56.3°$
3. When f increases, X_L increases, Z increases, and θ increases.
4. $\phi = 81.9°$
5. $V_{out} = 9.90\text{ V}$

SECTION 4 Impedance and Admittance of Parallel *RL* Circuits

1. $Y = \dfrac{1}{Z} = 2\text{ mS}$
2. $Y = \sqrt{G^2 + B_L^2} = 2.5\text{ mS}$
3. $\mathbf{I}$ lags V_s; $\theta = 32.1°$

SECTION 5 Analysis of Parallel *RL* Circuits

1. $I_{tot} = 32\text{ mA}$
2. $\mathbf{I}_{tot} = 23.3\angle -59.0°\text{ mA}$; θ is with respect to the input voltage.
3. $\theta = -90°$

SECTION 6 Analysis of Series-Parallel *RL* Circuits

1. $\mathbf{Z} = 494\angle 59.0°\ \Omega$
2. $I_{tot} = 10.4\text{ mA} - j17.3\text{ mA}$

SECTION 7 Power in *RL* Circuits

1. Power dissipation is due to resistance.
2. $PF = 0.643$
3. $P_{\text{true}} = 4.7\text{ W}$; $P_r = 6.2\text{ VAR}$; $P_a = 7.78\text{ VA}$

SECTION 8 Basic Applications

1. The output is across the resistor.
2. It is more efficient than other types.
3. It is adjusted by the pulse width modulator to be longer.

SECTION 9 Troubleshooting

1. Shorted windings reduce L and thereby reduce X_L at any given frequency.
2. I_{tot} decreases, V_{R1} decreases, V_{R2} increases.

RELATED PROBLEMS FOR EXAMPLES

1 $\mathbf{Z} = 1.8\text{ k}\Omega + j950\ \Omega$; $\mathbf{Z} = 2.04\angle 27.8^\circ\text{ k}\Omega$

2 $\mathbf{I} = 423\angle -32.1^\circ\ \mu\text{A}$

3 $Z = 12.6\text{ k}\Omega$; $\theta = 85.5^\circ$

4 $\phi = 65.6^\circ$

5 V_{out} increases.

6 $\phi = -32^\circ$

7 $V_{out} = 12.3\text{ V rms}$

8 $\mathbf{Z} = 8.14\angle 35.5^\circ\text{ k}\Omega$

9 $\mathbf{Y} = 3.03\text{ mS} - j0.796\text{ mS}$

10 $\mathbf{I} = 14.0\angle -71.1^\circ\text{ mA}$

11 $I_{tot} = 67.6\text{ mA}$; $\theta = 36.3^\circ$

12 **(a)** $\mathbf{V}_1 = 8.04\angle 2.52^\circ\text{ V}$ **(b)** $\mathbf{V}_2 = 2.00\angle -10.2^\circ\text{ V}$

13 $\mathbf{I}_{tot} = 20.2\angle -59.0^\circ\text{ mA}$

14 P_{true}, P_r, and P_a decrease.

15 Open connection between L_1 and L_2

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** F **6.** T **7.** T **8.** T **9.** T **10.** F

SELF-TEST

1. (f) **2.** (b) **3.** (a) **4.** (d) **5.** (a) **6.** (d) **7.** (b)
8. (b) **9.** (a) **10.** (d) **11.** (c) **12.** (d) **13.** (b) **14.** (c)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (c) **7.** (a) **8.** (c)
9. (a) **10.** (b) **11.** (c) **12.** (b) **13.** (a) **14.** (a) **15.** (a) **16.** (b)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. 15 kHz

3. **(a)** $100\ \Omega + j50\ \Omega$; $112– 26.6^\circ\ \Omega$

(b) $1.5\ \text{k}\Omega + j1\ \text{k}\Omega$; $1.80– 33.7^\circ\ \text{k}\Omega$

5. **(a)** $17.4– 46.4^\circ\ \Omega$ **(b)** $64.0– 79.2^\circ\ \Omega$

(c) $127– 84.6^\circ\ \Omega$ **(d)** $251– 87.3^\circ\ \Omega$

7. $806\ \Omega$, 4.11 mH

9. 0.370 V

11. **(a)** $43.5– -55^\circ$ mA **(b)** $11.8– -34.6^\circ$ mA

13. θ increases from 38.7° to 58.1°.

15. **(a)** $\mathbf{V}_R = 4.85– -14.1^\circ$ V

$\mathbf{V}_L = 1.22– 75.9^\circ$ V

(b) $\mathbf{V}_R = 3.83– -40.0^\circ$ V

$\mathbf{V}_L = 3.21– 50.0^\circ$ V

(c) $\mathbf{V}_R = 2.16– -64.5^\circ$ V

$\mathbf{V}_L = 4.51– 25.5^\circ$ V

(d) $\mathbf{V}_R = 1.16– -76.6^\circ$ V

$\mathbf{V}_L = 4.86– 13.4^\circ$ V

17. **(a)** -0.0923° **(b)** -9.15° **(c)** -58.2° **(d)** -86.4°

19. **(a)** $80.5– 90^\circ\ \mu$V **(b)** $805– 90^\circ\ \mu$V

(c) $7.95– 80.9^\circ$ mV **(d)** $42.5– 31.8^\circ$ mV

21. $4.99\ \Omega + j5.93\ \Omega$

23. 2.39 kHz

25. **(a)** $274– 60.7^\circ\ \Omega$ **(b)** $89.3– 0^\circ$ mA

(c) $159– -90^\circ$ mA **(d)** $182– -60.7^\circ$ mA

(e) 60.7° (I_{tot} lagging V_s)

27. $1.83\ \text{k}\Omega$ resistor in series with $4.21\ \text{k}\Omega$ inductive reactance

29. $\mathbf{V}_{R1} = 21.8– -3.89^\circ$ V

$\mathbf{V}_{R2} = 7.27– 9.61^\circ$ V

$\mathbf{V}_{R3} = 3.38– -53.3^\circ$ V

$\mathbf{V}_{L1} = \mathbf{V}_{L2} = 6.44– 37.3^\circ$ V

31. $\mathbf{I}_{R1} = I_T = 389– -3.89^\circ$ mA

$\mathbf{I}_{R2} = 330– 9.61^\circ$ mA

$\mathbf{I}_{R3} = 102– -53.3^\circ$ mA

$\mathbf{I}_{L1} = \mathbf{I}_{L2} = 51.3– -52.7^\circ$ mA

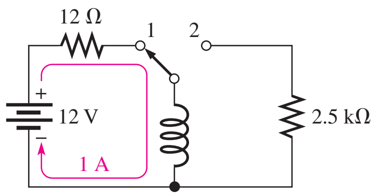
33. **(a)** $588– -50.5^\circ$ mA **(b)** $22.0– 16.1^\circ$ V

(c) $8.63– -135^\circ$ V

35. $\theta = 52.5^\circ$ (V_{out} lags V_{in}), 0.143

37. See Figure 74.

▶ FIGURE 74



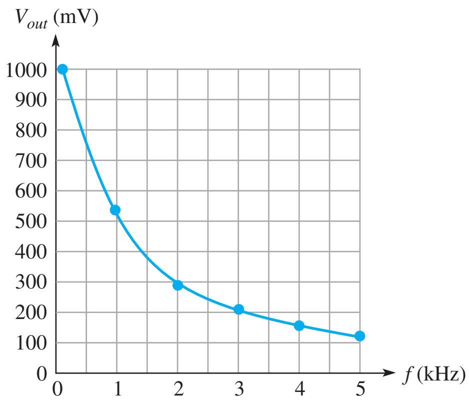
39. 1.29 W, 1.04 VAR

41. $P_{\text{true}} = 290$ mW; $P_r = 50.8$ mVAR;

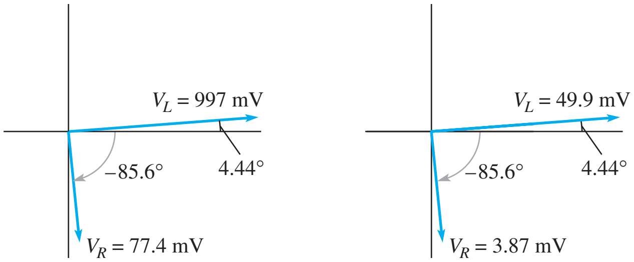
$P_a = 296$ mVA; $PF = 0.985$

43. Use the formula, $V_{out} = \left(\dfrac{R}{Z_{tot}}\right)V_{in}$. See Figure 75.

FREQUENCY (kHz)	X_L	Z_{tot}	V_{out}
0	$0\ \Omega$	$39.0\ \Omega$	1 V
1	$62.8\ \Omega$	$73.9\ \Omega$	528 mV
2	$126\ \Omega$	$132\ \Omega$	296 mV
3	$189\ \Omega$	$193\ \Omega$	203 mV
4	$251\ \Omega$	$254\ \Omega$	153 mV
5	$314\ \Omega$	$317\ \Omega$	123 mV



▲ FIGURE 75


V_L = 997 mV
−85.6°
4.44°
V_R = 77.4 mV
V_L = 49.9 mV
−85.6°
4.44°
V_R = 3.87 mV

▲ FIGURE 76

45. See Figure 76.

47. **(a)** 0 V **(b)** 0 V
(c) 1.62– −25.8° V **(d)** 2.15– −64.5° V

49. L_1 leaky

51. L_1 open

53. No fault

55. $f_c \approx$ 53.214 kHz

RLC CIRCUITS AND RESONANCE

CHAPTER OUTLINE

PART 1: SERIES CIRCUITS

1 Impedance of Series *RLC* Circuits
2 Analysis of Series *RLC* Circuits
3 Series Resonance

PART 2: PARALLEL CIRCUITS

4 Impedance of Parallel *RLC* Circuits
5 Analysis of Parallel *RLC* Circuits
6 Parallel Resonance

PART 3: SERIES-PARALLEL CIRCUITS

7 Analysis of Series-Parallel *RLC* Circuits

PART 4: SPECIAL TOPICS

8 Bandwidth of Resonant Circuits
9 Applications
Application Activity

CHAPTER OBJECTIVES

PART 1: SERIES CIRCUITS

- Determine the impedance of a series *RLC* circuit
- Analyze series *RLC* circuits
- Analyze a circuit for series resonance

PART 2: PARALLEL CIRCUITS

- Determine the impedance of a parallel *RLC* circuit
- Analyze parallel *RLC* circuits
- Analyze a circuit for parallel resonance

PART 3: SERIES-PARALLEL CIRCUITS

- Analyze series-parallel *RLC* circuits

PART 4: SPECIAL TOPICS

- Determine the bandwidth of resonant circuits
- Discuss some applications of resonant circuits

KEY TERMS

- Series resonance
- Resonant frequency (f_r)
- Parallel resonance
- Tank circuit
- Half-power frequency
- Selectivity

APPLICATION ACTIVITY PREVIEW

In the application activity, the focus is on the resonant tuning circuit in the RF amplifier of an AM radio receiver. The tuning circuit is used to select any desired frequency within the AM band so that a desired station can be tuned in.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, analysis methods are extended to the coverage of circuits with combinations of resistive, inductive, and capacitive components. Series and parallel *RLC* circuits, plus series-parallel combinations, are studied.

Circuits with both inductance and capacitance can exhibit the property of resonance, which is important in many types of applications. Resonance is the basis for frequency selectivity in communication systems. For example, the ability of a radio or television receiver to select a certain frequency that is transmitted by a particular station and, at the same time, to eliminate frequencies from other stations is based on the principle of resonance. The conditions in *RLC* circuits that produce resonance and the characteristics of resonant circuits are covered in this chapter.

From Chapter 17 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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Part 1

SERIES CIRCUITS

1 IMPEDANCE OF SERIES *RLC* CIRCUITS

A series *RLC* circuit contains resistance, inductance, and capacitance. Since inductive reactance and capacitive reactance have opposite effects on the circuit phase angle, the total reactance is less than either individual reactance.

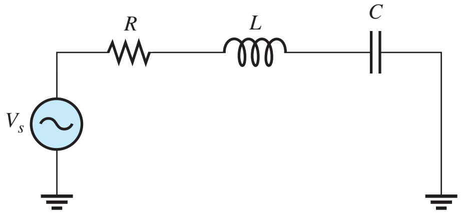
After completing this section, you should be able to

- **Determine the impedance of a series *RLC* circuit**
    - Calculate total reactance
    - Determine whether a circuit is predominately inductive or capacitive

A series *RLC* circuit is shown in Figure 1. It contains resistance, inductance, and capacitance.

FIGURE 1

Series *RLC* circuit.



As you know, inductive reactance ($\mathbf{X}_L$) causes the total current to lag the applied voltage. Capacitive reactance ($\mathbf{X}_C$) has the opposite effect: It causes the current to lead the voltage. Thus $\mathbf{X}_L$ and $\mathbf{X}_C$ tend to offset each other. When they are equal, they cancel, and the total reactance is zero. In any case, the magnitude of the total reactance in the series circuit is

Equation 1

$$X_{tot} = |X_L - X_C|$$

The term $|X_L - X_C|$ means the absolute value of the difference of the two reactances. That is, the sign of the result is considered positive no matter which reactance is greater. For example, $3 - 7 = -4$, but the absolute value is

$$|3 - 7| = 4$$

When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive.

The total impedance for the series *RLC* circuit is stated in rectangular form in Equation 2 and in polar form in Equation 3.

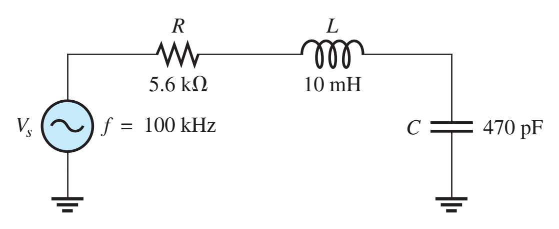
$$\mathbf{Z} = R + jX_L - jX_C \qquad \text{Equation 2}$$

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2}\angle \pm\tan^{-1}\left(\frac{X_{tot}}{R}\right) \qquad \text{Equation 3}$$

In Equation 3, $\sqrt{R^2 + (X_L - X_C)^2}$ is the magnitude and $\tan^{-1}(X_{tot}/R)$ is the phase angle between the total current and the applied voltage. If the circuit is predominately inductive, the phase angle is positive; and if predominately capacitive, the phase angle is negative.

EXAMPLE 1

For the series *RLC* circuit in Figure 2, determine the total impedance. Express it in both rectangular and polar forms.



▲ FIGURE 2

Solution First, find X_C and X_L.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ kHz})(470\text{ pF})} = 3.39\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(100\text{ kHz})(10\text{ mH}) = 6.28\text{ k}\Omega$$

In this case, X_L is greater than X_C, and thus the circuit is more inductive than capacitive. The magnitude of the total reactance is

$$X_{tot} = |X_L - X_C| = |6.28\text{ k}\Omega - 3.39\text{ k}\Omega| = 2.89\text{ k}\Omega \qquad \text{inductive}$$

The impedance in rectangular form is

$$\mathbf{Z} = R + (jX_L - jX_C) = 5.6\text{ k}\Omega + (j6.28\text{ k}\Omega - j3.39\text{ k}\Omega) = \mathbf{5.6\text{ k}\Omega + j2.89\text{ k}\Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_{tot}^2}\angle\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(5.6\text{ k}\Omega)^2 + (2.89\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2.89\text{ k}\Omega}{5.6\text{ k}\Omega}\right) = \mathbf{6.30\angle 27.3^\circ\text{ k}\Omega}$$

The positive angle shows that the circuit is inductive.

*Related Problem** Determine **Z** in polar form if *f* is increased to 200 kHz.

---

*Answers are at the end of the chapter.

As you have seen, when the inductive reactance is greater than the capacitive reactance, the circuit appears inductive; so the current lags the applied voltage. When the capacitive reactance is greater, the circuit appears capacitive, and the current leads the applied voltage.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. In a given series *RLC* circuit, X_C is 150 Ω and X_L is 80 Ω. What is the total reactance in ohms? Is it inductive or capacitive?
2. Determine the impedance in polar form for the circuit in Question 1 when $R = 47\ \Omega$. What is the magnitude of the impedance? What is the phase angle? Is the current leading or lagging the applied voltage?

2 ANALYSIS OF SERIES *RLC* CIRCUITS

Recall that capacitive reactance varies inversely with frequency and that inductive reactance varies directly with frequency. In this section, the combined effects of the reactances as a function of frequency are examined.

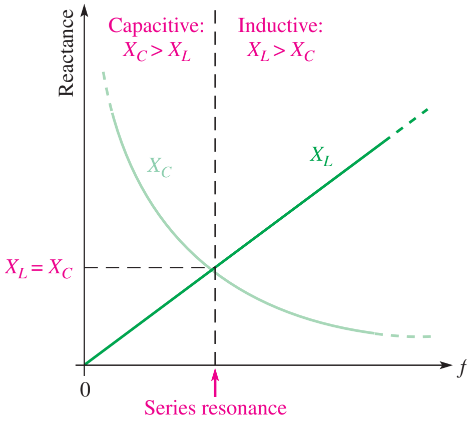
After completing this section, you should be able to

- **Analyze series *RLC* circuits**
  - Determine current in a series *RLC* circuit
  - Determine the voltages in a series *RLC* circuit
  - Determine the phase angle

Figure 3 shows that for a typical series *RLC* circuit the total reactance behaves as follows: Starting at a very low frequency, X_C is high, and X_L is low, and the circuit is predominantly capacitive. As the frequency is increased, X_C decreases and X_L increases until a value is reached where $X_C = X_L$ and the two reactances cancel, making the circuit purely resistive. This condition is **series resonance** and will be studied in Section 3. As the frequency is increased further, X_L becomes greater than X_C, and the circuit is predominantly inductive. Example 2 illustrates how the impedance and phase angle change as the source frequency is varied.

FIGURE 3

How X_C and X_L vary with frequency.



The graph of X_L is a straight line, and the graph of X_C is curved, as shown in Figure 3. The general equation for a straight line is $y = mx + b$, where m is the slope of the line and b is the y-axis intercept point. The formula $X_L = 2\pi fL$ fits this general straight-line formula, where $y = X_L$ (a variable), $m = 2\pi L$ (a constant), $x = f$ (a variable), and $b = 0$ as follows: $X_L = 2\pi Lf + 0$.

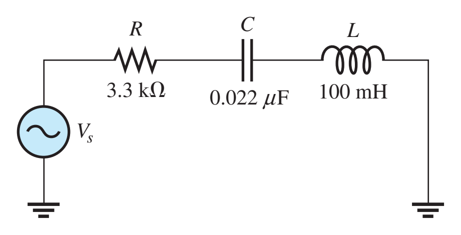
The X_C curve is called a *hyperbola,* and the general equation of a hyperbola is $xy = k$. The equation for capacitive reactance, $X_C = 1/2\pi fC$, can be rearranged as $X_C f = 1/2\pi C$ where $x = X_C$ (a variable), $y = f$ (a variable), and $k = 1/2\pi C$ (a constant).

EXAMPLE 2

For each of the following input frequencies, find the impedance in polar form for the circuit in Figure 4. Note the change in magnitude and phase angle with frequency.

(a) $f = 1\text{ kHz}$ **(b)** $f = 2\text{ kHz}$ **(c)** $f = 3.5\text{ kHz}$ **(d)** $f = 5\text{ kHz}$

▶ FIGURE 4



Solution

(a) At $f = 1\text{ kHz}$,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.022\ \mu\text{F})} = 7.23\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

The circuit is clearly capacitive, and the impedance is

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2}\angle -\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(3.3\text{ k}\Omega)^2 + (628\ \Omega - 7.23\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{6.60\text{ k}\Omega}{3.3\text{ k}\Omega}\right) = \mathbf{7.38\angle -63.4^\circ\ k\Omega}$$

The negative sign for the angle is used to indicate that the circuit is capacitive.

(b) At $f = 2\text{ kHz}$,

$$X_C = \frac{1}{2\pi(2\text{ kHz})(0.022\ \mu\text{F})} = 3.62\text{ k}\Omega$$

$$X_L = 2\pi(2\text{ kHz})(100\text{ mH}) = 1.26\text{ k}\Omega$$

The circuit is still capacitive, and the impedance is

$$\mathbf{Z} = \sqrt{(3.3\text{ k}\Omega)^2 + (1.26\text{ k}\Omega - 3.62\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{2.36\text{ k}\Omega}{3.3\text{ k}\Omega}\right)$$

$$= \mathbf{4.06\angle -35.6^\circ\ k\Omega}$$

(c) At $f = 3.5\text{ kHz}$,

$$X_C = \frac{1}{2\pi(3.5\text{ kHz})(0.022\ \mu\text{F})} = 2.07\text{ k}\Omega$$

$$X_L = 2\pi(3.5\text{ kHz})(100\text{ mH}) = 2.20\text{ k}\Omega$$

The circuit is very close to being purely resistive because X_C and X_L are nearly equal, but it is slightly inductive. The impedance is

$$\mathbf{Z} = \sqrt{(3.3\text{ k}\Omega)^2 + (2.20\text{ k}\Omega - 2.07\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{0.13\text{ k}\Omega}{3.3\text{ k}\Omega}\right)$$

$$= \mathbf{3.3\angle 2.26^\circ\ k\Omega}$$

(d) At $f = 5\text{ kHz}$,

$$X_C = \frac{1}{2\pi(5\text{ kHz})(0.022\ \mu\text{F})} = 1.45\text{ k}\Omega$$

$$X_L = 2\pi(5\text{ kHz})(100\text{ mH}) = 3.14\text{ k}\Omega$$

The circuit is now predominantly inductive. The impedance is

$$\mathbf{Z} = \sqrt{(3.3\text{ k}\Omega)^2 + (3.14\text{ k}\Omega - 1.45\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{1.69\text{ k}\Omega}{3.3\text{ k}\Omega}\right)$$

$$= \mathbf{3.71\angle 27.1^\circ\ k\Omega}$$

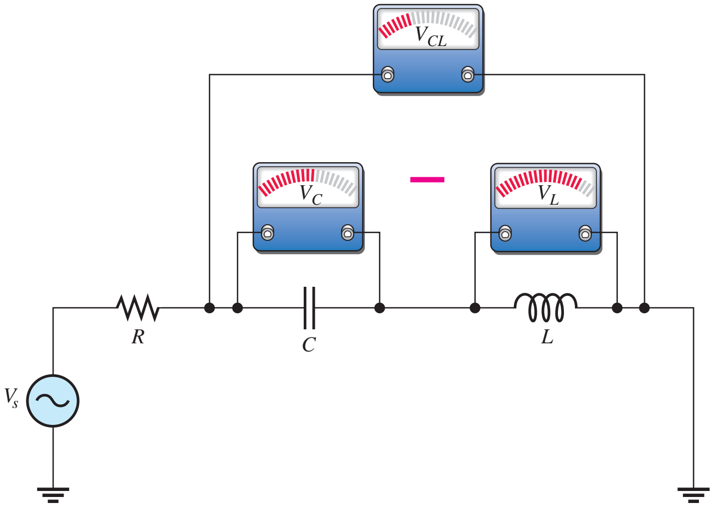
Notice how the circuit changed from capacitive to inductive as the frequency increased. The phase condition changed from the current leading to the current lagging as indicated by the sign of the angle. Note that the impedance magnitude decreased to a minimum equal to the resistance and then began increasing again. Also, notice that the negative phase angle decreased as the frequency increased and the angle became positive when the circuit became inductive and then increased with increasing frequency.

Related Problem Determine **Z** in polar form for $f = 7$ kHz and draw a graph of impedance vs. frequency using the values in this example.

In a series *RLC* circuit, the capacitor voltage and the inductor voltage are always 180° out of phase with each other. For this reason, V_C and V_L subtract from each other, and thus the voltage across *L* and *C* combined is always less than the larger individual voltage across either element, as illustrated in Figure 5 and in the waveform diagram of Figure 6.

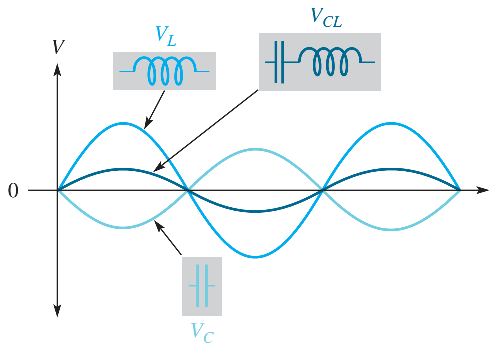
▶ FIGURE 5

The voltage across the series combination of *C* and *L* is always less than the larger individual voltage across either *C* or *L*.



▶ FIGURE 6

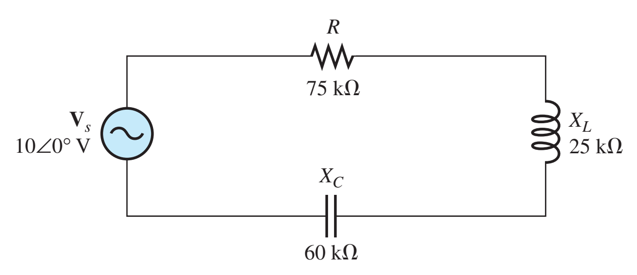
V_{CL} is the algebraic sum of V_L and V_C. Because of the phase relationship, V_L and V_C effectively subtract.



In the next example, Ohm's law is used to find the current and voltages in a series *RLC* circuit.

EXAMPLE 3

Find the current and the voltages across each component in Figure 7. Express each quantity in polar form, and draw a complete voltage phasor diagram.



▲ FIGURE 7

Solution First, find the total impedance.

$$\mathbf{Z} = R + jX_L - jX_C = 75\text{ k}\Omega + j25\text{ k}\Omega - j60\text{ k}\Omega = 75\text{ k}\Omega - j35\text{ k}\Omega$$

Convert to polar form for convenience in applying Ohm's law.

$$\mathbf{Z} = \sqrt{R^2 + X_{tot}^2}\angle -\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(75\text{ k}\Omega)^2 + (35\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{35\text{ k}\Omega}{75\text{ k}\Omega}\right) = 82.8\angle -25^\circ\text{ k}\Omega$$

where $X_{tot} = |X_L - X_C|$.

Apply Ohm's law to find the current.

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ\text{ V}}{82.8\angle -25^\circ\text{ k}\Omega} = \mathbf{121\angle 25.0^\circ\ \mu A}$$

Now, apply Ohm's law to find the voltages across *R*, *L*, and *C*.

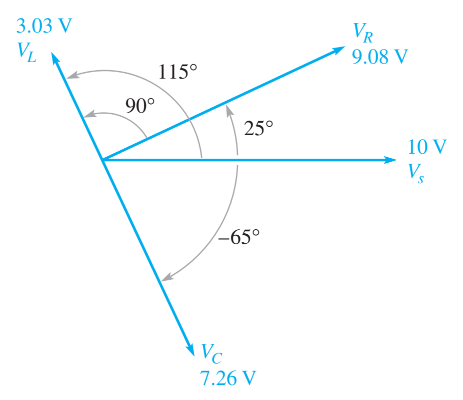
$$\mathbf{V}_R = \mathbf{IR} = (121\angle 25.0^\circ\ \mu\text{A})(75\angle 0^\circ\text{ k}\Omega) = \mathbf{9.08\angle 25.0^\circ\ V}$$

$$\mathbf{V}_L = \mathbf{IX}_L = (121\angle 25.0^\circ\ \mu\text{A})(25\angle 90^\circ\text{ k}\Omega) = \mathbf{3.03\angle 115^\circ\ V}$$

$$\mathbf{V}_C = \mathbf{IX}_C = (121\angle 25.0^\circ\ \mu\text{A})(60\angle -90^\circ\text{ k}\Omega) = \mathbf{7.26\angle -65.0^\circ\ V}$$

The phasor diagram is shown in Figure 8. The magnitudes represent rms values. Notice that V_L is leading V_R by 90°, and V_C is lagging V_R by 90°. Also, there is a 180° phase difference between V_L and V_C. If the current phasor were shown, it would be at

▶ FIGURE 8



the same angle as V_R. The current is leading V_s, the source voltage, by 25°, indicating a capacitive circuit ($X_C > X_L$). The phasor diagram is rotated 25° from its usual position because the reference is the source voltage, V_s, which is shown oriented along the x-axis.

Related Problem What will happen to the current as the frequency of the source voltage in Figure 7 is increased?

SECTION 2 CHECKUP

1. The following voltages occur in a certain series *RLC* circuit. Determine the source voltage: $V_R = 24\angle 30°$ V, $V_L = 15\angle 120°$ V, and $V_C = 45\angle -60°$ V.
2. When $R = 1.0\ \text{k}\Omega$, $X_C = 1.8\ \text{k}\Omega$, and $X_L = 1.2\ \text{k}\Omega$, does the current lead or lag the applied voltage?
3. Determine the total reactance in Question 2.

3 SERIES RESONANCE

In a series *RLC* circuit, series resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the **resonant frequency** and is designated f_r.

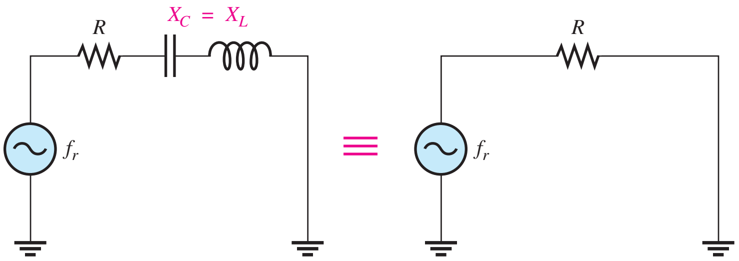
After completing this section, you should be able to

- **Analyze a circuit for series resonance**
    - Define *series resonance*
    - Determine the impedance at resonance
    - Explain why the reactances cancel at resonance
    - Determine the series resonant frequency
    - Calculate the current, voltages, and phase angle at resonance

Figure 9 illustrates the series resonant condition.

FIGURE 9

Series resonance. X_C and X_L cancel each other resulting in a purely resistive circuit.



Resonance is a condition in a series *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude; thus, they cancel each other and result in a purely resistive impedance. In a series *RLC* circuit, the total impedance was given in Equation 2 as

$$\mathbf{Z} = R + jX_L - jX_C$$

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

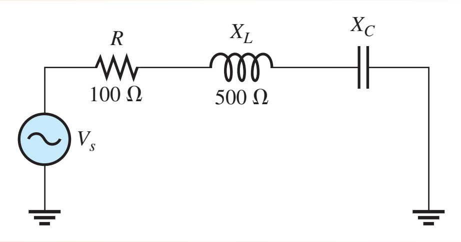
At resonance, $X_L = X_C$ and the j terms cancel; thus, the impedance is purely resistive. These resonant conditions are stated in the following equations:

$$X_L = X_C$$
$$Z_r = R$$

EXAMPLE 4

For the series *RLC* circuit in Figure 10, determine X_C and $\mathbf{Z}$ at resonance.

▶ FIGURE 10



Solution $X_L = X_C$ at the resonant frequency. Thus, $X_C = X_L = \mathbf{500\ \Omega}$. The impedance at resonance is

$$\mathbf{Z}_r = R + jX_L - jX_C = 100\ \Omega + j500\ \Omega - j500\ \Omega = \mathbf{100\angle 0°\ \Omega}$$

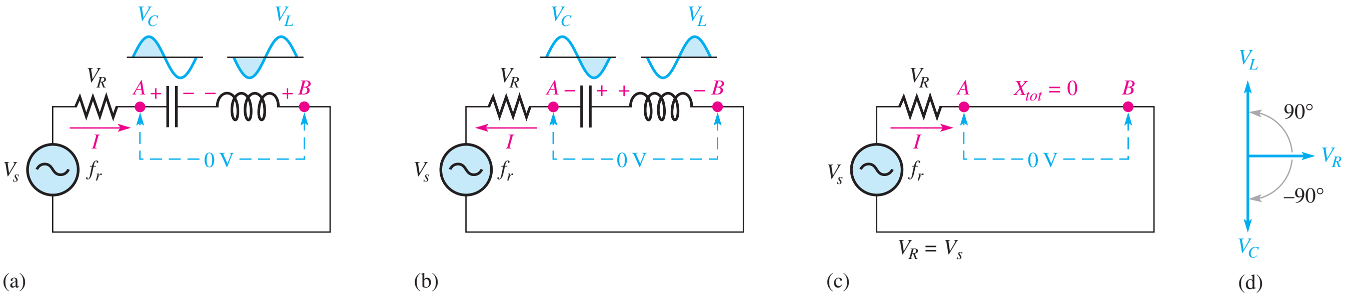
This shows that the impedance at resonance is equal to the resistance because the reactances are equal in magnitude and therefore cancel.

Related Problem Just below the resonant frequency, is the circuit more inductive or more capacitive?

X_L and X_C Cancel at Resonance

At the series resonant frequency (f_r), the voltages across C and L are equal in magnitude because the reactances are equal. The same current is through both since they are in series ($IX_C = IX_L$). Also, V_L and V_C are always 180° out of phase with each other.

During any given cycle, the polarities of the voltages across C and L are opposite, as shown in parts (a) and (b) of Figure 11. The equal and opposite voltages across C and L cancel, leaving zero volts from point A to point B as shown. Since there is no voltage drop from A to B but there is still current, the total reactance must be zero, as indicated in part (c). Also, the voltage phasor diagram in part (d) shows that V_C and V_L are equal in magnitude and 180° out of phase with each other.



▲ FIGURE 11

At the resonant frequency, f_r, the voltages across C and L are equal in magnitude. Since they are 180° out of phase with each other, they cancel, leaving 0 V across the LC combination (point A to point B). The section of the circuit from A to B effectively looks like a short at resonance.

Series Resonant Frequency

For a given series *RLC* circuit, resonance occurs at only one specific frequency. A formula for this resonant frequency is developed as follows:

$$X_L = X_C$$

Substitute the reactance formulas.

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

Then, multiplying both sides by $f_r/2\pi L$,

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

Take the square root of both sides. The formula for series resonant frequency is

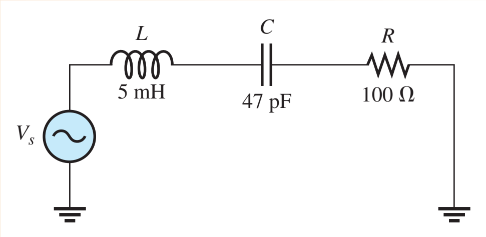
Equation 4

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

EXAMPLE 5

Find the series resonant frequency for the circuit in Figure 12.

▶ FIGURE 12



Solution The resonant frequency is

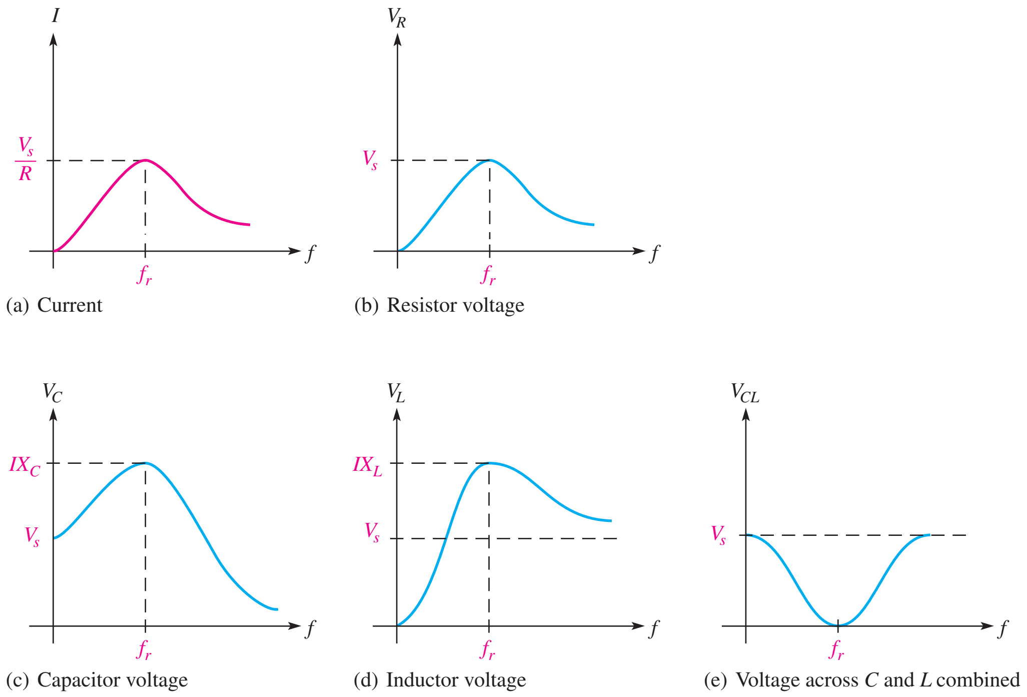
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5\text{ mH})(47\text{ pF})}} = \mathbf{328\ kHz}$$

Related Problem If $C = 0.01\ \mu\text{F}$ in Figure 12, what is the resonant frequency?

Use Multisim files E17-05A and E17-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

Current and Voltages in a Series *RLC* Circuit

At the series resonant frequency, the current is maximum ($I_{max} = V_s/R$). Above and below resonance, the current decreases because the impedance increases. A response curve showing the plot of current versus frequency is shown in Figure 13(a). The resistor voltage, V_R, follows the current and is maximum (equal to V_s) at resonance and zero at $f = 0$ and at $f = \infty$, as shown in Figure 13(b). The general shapes of the V_C and V_L curves are indicated in Figure 13(c) and (d). Notice that $V_C = V_s$ when $f = 0$, because the capacitor appears open. Also notice that V_L approaches V_s as f approaches infinity, because the inductor appears open. The voltage across the C and L combination decreases as the frequency increases below resonance, reaching a minimum of zero at the resonant frequency; then it increases above resonance, as shown in Figure 13(e).



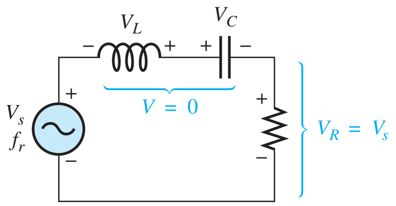
(a) Current (b) Resistor voltage

(c) Capacitor voltage (d) Inductor voltage (e) Voltage across *C* and *L* combined

▲ **FIGURE 13**

Generalized current and voltage magnitudes as a function of frequency in a series *RLC* circuit. V_C and V_L can be much larger than the source voltage. The shapes of the graphs depend on specific circuit values.

The voltages are maximum at resonance but drop off above and below f_r. The voltages across *L* and *C* at resonance are exactly equal in magnitude but 180° out of phase; so they cancel. Thus, the total voltage across both *L* and *C* is zero, and $V_R = V_s$ at resonance, as indicated in Figure 14. Individually, V_L and V_C can be much greater than the source voltage, as you will see later. Keep in mind that V_L and V_C are always opposite in polarity regardless of the frequency, but only at resonance are their magnitudes equal.



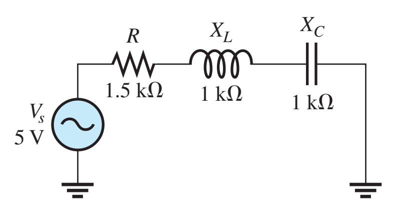
◀ **FIGURE 14**

Series *RLC* circuit at resonance.

EXAMPLE 6

Find *I*, V_R, V_L, and V_C at resonance for the circuit in Figure 15. The resonant values of X_L and X_C are shown.

▶ **FIGURE 15**



Solution At resonance, I is maximum and equal to V_s/R.

$$I = \frac{V_s}{R} = \frac{5\text{ V}}{1.5\text{ k}\Omega} = \mathbf{3.33\text{ mA}}$$

Apply Ohm's law to obtain the voltage magnitudes.

$$V_R = IR = (3.33\text{ mA})(2.2\text{ k}\Omega) = \mathbf{7.33\text{ V}}$$
$$V_L = IX_L = (3.33\text{ mA})(1\text{ k}\Omega) = \mathbf{3.33\text{ V}}$$
$$V_C = IX_C = (3.33\text{ mA})(1\text{ k}\Omega) = \mathbf{3.33\text{ V}}$$

Notice that all of the source voltage is dropped across the resistor. Also, of course, V_L and V_C are equal in magnitude but opposite in phase. This causes these voltages to cancel, making the total reactive voltage zero.

Related Problem What is the phase angle if the frequency is doubled?

Use Multisim files E17-06A and E17-06B to verify the calculated results in this example and to confirm your calculation for the related problem.

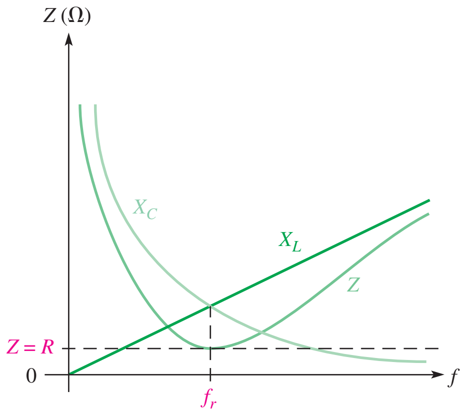
Series *RLC* Impedance

At frequencies below f_r, $X_C > X_L$; thus, the circuit is capacitive. At the resonant frequency, $X_C = X_L$, so the circuit is purely resistive. At frequencies above f_r, $X_L > X_C$; thus, the circuit is inductive.

The impedance magnitude is minimum at resonance ($Z = R$) and increases in value above and below the resonant point. The graph in Figure 16 illustrates how impedance changes with frequency. At zero frequency, both X_C and Z are infinitely large and X_L is zero because the capacitor looks like an open at 0 Hz and the inductor looks like a short. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L at frequencies below f_r, Z decreases along with X_C. At f_r, $X_C = X_L$ and $Z = R$. At frequencies above f_r, X_L becomes increasingly larger than X_C, causing Z to increase.

▶ **FIGURE 16**

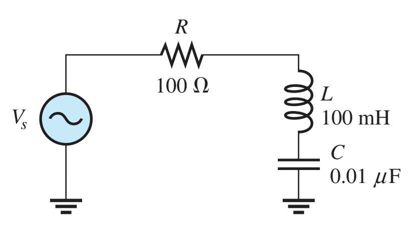
Series *RLC* impedance as a function of frequency.



EXAMPLE 7 For the circuit in Figure 17, determine the impedance magnitude at the following frequencies:

(a) f_r **(b)** 1000 Hz below f_r **(c)** 1000 Hz above f_r

▶ FIGURE 17



Solution **(a)** At f_r, the impedance is equal to R.

$$Z = R = \mathbf{100\ \Omega}$$

To determine the impedance above and below f_r, first calculate the resonant frequency.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.01\ \mu\text{F})}} = 5.03\text{ kHz}$$

(b) At 1000 Hz below f_r, the frequency and reactances are as follows:

$$f = f_r - 1\text{ kHz} = 5.03\text{ kHz} - 1\text{ kHz} = 4.03\text{ kHz}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(4.03\text{ kHz})(0.01\ \mu\text{F})} = 3.95\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(4.03\text{ kHz})(100\text{ mH}) = 2.53\text{ k}\Omega$$

Therefore, the impedance at $f_r - 1$ kHz is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100\ \Omega)^2 + (2.53\text{ k}\Omega - 3.95\text{ k}\Omega)^2} = \mathbf{1.42\ k\Omega}$$

(c) At 1000 Hz above f_r,

$$f = 5.03\text{ kHz} + 1\text{ kHz} = 6.03\text{ kHz}$$

$$X_C = \frac{1}{2\pi(6.03\text{ kHz})(0.01\ \mu\text{F})} = 2.64\text{ k}\Omega$$

$$X_L = 2\pi(6.03\text{ kHz})(100\text{ mH}) = 3.79\text{ k}\Omega$$

Therefore, the impedance at $f_r + 1$ kHz is

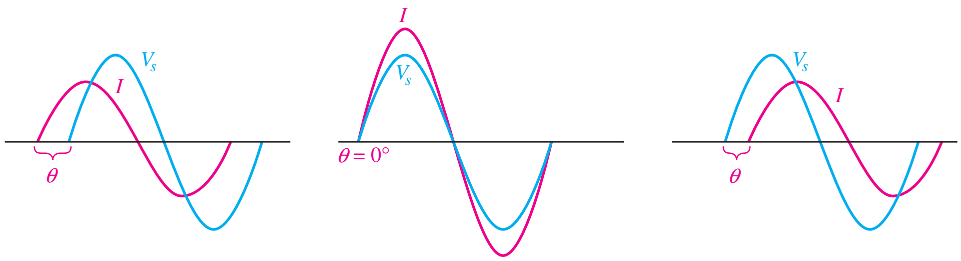
$$Z = \sqrt{(100\ \Omega)^2 + (3.79\text{ k}\Omega - 2.64\text{ k}\Omega)^2} = \mathbf{1.15\ k\Omega}$$

In part (b) Z is capacitive, and in part (c) Z is inductive.

Related Problem What happens to the impedance magnitude if f is decreased below 4.03 kHz? Above 6.03 kHz?

The Phase Angle of a Series *RLC* Circuit

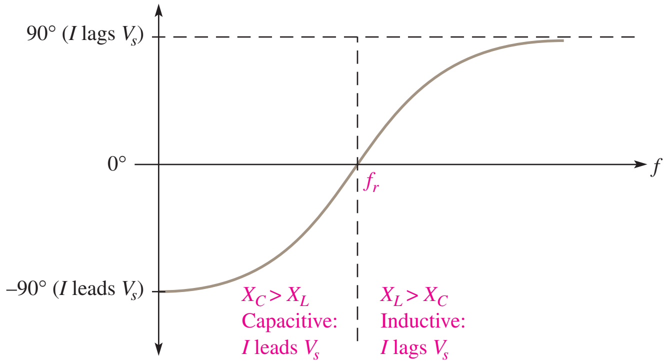
At frequencies below resonance, $X_C > X_L$, and the current leads the source voltage, as indicated in Figure 18(a). The phase angle decreases as the frequency approaches the resonant value and is 0° at resonance, as indicated in part (b). At frequencies above resonance, $X_L > X_C$, and the current lags the source voltage, as indicated in part (c). As the frequency goes higher, the phase angle approaches 90°. A plot of phase angle versus frequency is shown in part (d) of the figure.



(a) Below f_r, I leads V_s.

(b) At f_r, I is in phase with V_s.

(c) Above f_r, I lags V_s.



(d) Phase angle versus frequency.

FIGURE 18

The phase angle as a function of frequency in a series *RLC* circuit.

SECTION 3 CHECKUP

1. What is the condition for series resonance?
2. Why is the current maximum at the resonant frequency?
3. Calculate the resonant frequency for $C = 1000$ pF and $L = 1000\ \mu$H.
4. In Question 3, is the circuit inductive or capacitive at 50 kHz?

PARALLEL CIRCUITS

Part
2

4 IMPEDANCE OF PARALLEL *RLC* CIRCUITS

In this section, you will study the impedance and phase angle of a parallel *RLC* circuit. Also, conductance, susceptance, and admittance of a parallel *RLC* circuit are covered.

After completing this section, you should be able to

- **Determine the impedance of a parallel *RLC* circuit**
    - Calculate the conductance, susceptance, and admittance
    - Determine whether a circuit is predominately inductive or capacitive

Figure 19 shows a parallel *RLC* circuit. The total impedance can be calculated using the reciprocal of the sum-of-reciprocals method, just as was done for circuits with resistors in parallel.

$$\frac{1}{\mathbf{Z}} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}$$

or

$$\mathbf{Z} = \frac{1}{\dfrac{1}{R\angle 0^\circ} + \dfrac{1}{X_L\angle 90^\circ} + \dfrac{1}{X_C\angle -90^\circ}}$$

Equation 5

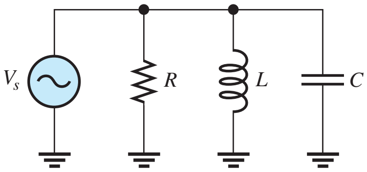


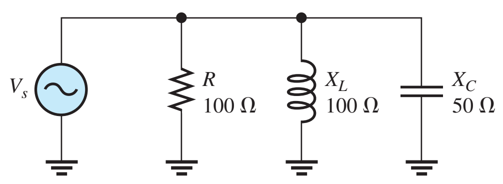
FIGURE 19

Parallel *RLC* circuit.

EXAMPLE 8

Find **Z** in polar form for the parallel *RLC* circuit in Figure 20.

FIGURE 20



Solution Use the sum-of-reciprocals formula.

$$\frac{1}{\mathbf{Z}} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ} = \frac{1}{100\angle 0^\circ\ \Omega} + \frac{1}{100\angle 90^\circ \Omega} + \frac{1}{50\angle -90^\circ\ \Omega}$$

Apply the rule for division of polar numbers.

$$\frac{1}{\mathbf{Z}} = 10\angle 0^\circ \text{ mS} + 10\angle -90^\circ \text{ mS} + 20\angle 90^\circ \text{ mS}$$

Recall that the sign of the denominator angle changes when dividing.

Next, convert each term to its rectangular equivalent and combine.

$$\frac{1}{\mathbf{Z}} = 10 \text{ mS} - j10 \text{ mS} + j20 \text{ mS} = 10 \text{ mS} + j10 \text{ mS}$$

Take the reciprocal to obtain **Z** and then convert to polar form.

$$\mathbf{Z} = \frac{1}{10 \text{ mS} + j10 \text{ mS}} = \frac{1}{\sqrt{(10 \text{ mS})^2 + (10 \text{ mS})^2}\angle \tan^{-1}\left(\frac{10 \text{ mS}}{10 \text{ mS}}\right)}$$

$$= \frac{1}{14.14\angle 45^\circ \text{ mS}} = \mathbf{70.7\angle -45^\circ\ \Omega}$$

The negative angle shows that the circuit is capacitive. This may surprise you because $X_L > X_C$. However, in a parallel circuit, the smaller quantity has the greater effect on the total current because its current is the greatest. Similar to the case of resistances in parallel, the smaller reactance draws more current and has the greater effect on the total Z.

In this circuit, the total current leads the total voltage by a phase angle of 45°.

Related Problem If the frequency in Figure 20 increases, does the impedance increase or decrease?

Conductance, Susceptance, and Admittance

The concepts of conductance (G), capacitive susceptance (B_C), inductive susceptance (B_L) and admittance (Y) should already be familiar to you. The phasor formulas are restated here.

Equation 6

$$\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

Equation 7

$$\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = jB_C$$

Equation 8

$$\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$$

Equation 9

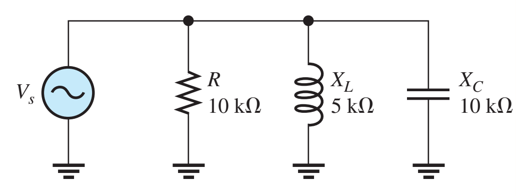
$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \pm\theta = G + jB_C - jB_L$$

As you know, the unit of each of these quantities is the siemens (S).

EXAMPLE 9

For the *RLC* circuit in Figure 21 determine the conductance, capacitive susceptance, inductive susceptance, and total admittance. Also, determine the impedance.

▶ FIGURE 21



Solution

$$\mathbf{G} = \frac{1}{R\angle 0^\circ} = \frac{1}{10\angle 0^\circ\text{ k}\Omega} = \mathbf{100\angle 0^\circ\ \mu S}$$

$$\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = \frac{1}{10\angle -90^\circ\text{ k}\Omega} = \mathbf{100\angle 90^\circ\ \mu S}$$

$$\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = \frac{1}{5\angle 90^\circ\text{ k}\Omega} = \mathbf{200\angle -90^\circ\ \mu S}$$

$$\begin{aligned}\mathbf{Y}_{tot} &= G + jB_C - jB_L = 100\ \mu\text{S} + j100\ \mu\text{S} - j200\ \mu\text{S}\\ &= 100\ \mu\text{S} - j100\ \mu\text{S} = \mathbf{141.4\angle -45^\circ\ \mu S}\end{aligned}$$

From $\mathbf{Y}_{tot}$, you can determine $\mathbf{Z}_{tot}$.

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}_{tot}} = \frac{1}{141.4\angle -45^\circ\ \mu\text{S}} = \mathbf{7.07\angle 45^\circ\ k\Omega}$$

Related Problem Is the circuit in Figure 21 predominately inductive or predominately capacitive?

SECTION 4 CHECKUP

1. In a certain parallel *RLC* circuit, the capacitive reactance is 60 Ω, and the inductive reactance is 100 Ω. Is the circuit predominantly capacitive or inductive?
2. Determine the admittance of a parallel circuit in which $R = 1.0\text{ k}\Omega$, $X_C = 500\ \Omega$, and $X_L = 1.2\text{ k}\Omega$.
3. In Question 2, what is the impedance?

5 ANALYSIS OF PARALLEL *RLC* CIRCUITS

As you have learned, the smaller reactance in a parallel circuit dominates because it results in the larger branch current.

After completing this section, you should be able to

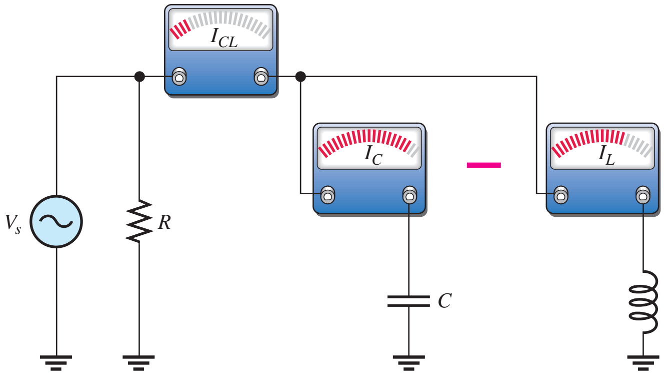
- **Analyze parallel *RLC* circuits**
  - Explain how the currents are related in terms of phase
  - Calculate impedance, currents, and voltages

Recall that capacitive reactance varies inversely with frequency and that inductive reactance varies directly with frequency. In a parallel *RLC* circuit at low frequencies, the inductive reactance is less than the capacitive reactance; therefore, the circuit is inductive. As the frequency is increased, X_L increases and X_C decreases until a value is reached where

$X_L = X_C$. This is the point of **parallel resonance**. As the frequency is increased further, X_C becomes smaller than X_L, and the circuit becomes capacitive.

Current Relationships

In a parallel *RLC* circuit, the current in the capacitive branch and the current in the inductive branch are *always* 180° out of phase with each other (neglecting any coil resistance). Because I_C and I_L add algebraically, the total current is actually the difference in their magnitudes. Thus, the total current into the parallel branches of L and C is always less than the largest individual branch current, as illustrated in Figure 22 and in the waveform diagram of Figure 23. Of course, the current in the resistive branch is always 90° out of phase with both reactive currents, as shown in the current phasor diagram of Figure 24. Notice that I_C is plotted on the positive y-axis and I_L is plotted on the negative y-axis.

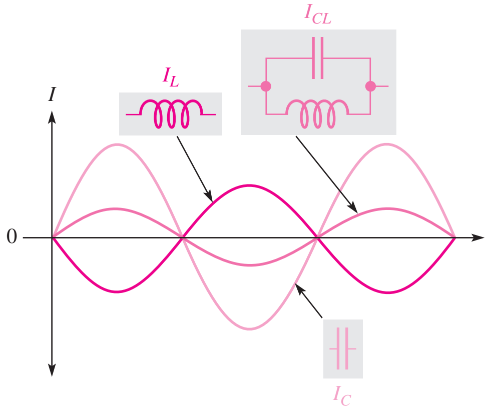


▲ FIGURE 22

The total current into the parallel combination of C and L is the difference of the two branch currents.

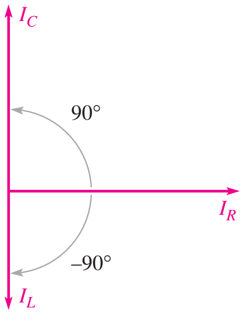
▶ FIGURE 23

I_C and I_L effectively subtract.



▶ FIGURE 24

Typical current phasor diagram for a parallel *RLC* circuit.



The total current can be expressed as

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2}\angle\tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$$

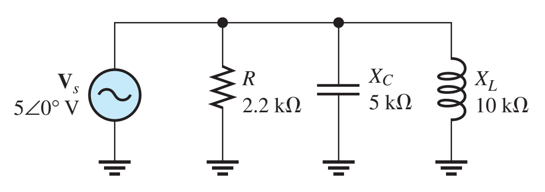
Equation 10

where I_{CL} is $I_C - I_L$, the total current into the L and C branches.

EXAMPLE 10

For the circuit in Figure 25 find each branch current and the total current. Draw a diagram of their relationship.

▶ **FIGURE 25**



Solution Use Ohm's law to find each branch current in phasor form.

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{5\angle 0^\circ\ \text{V}}{2.2\angle 0^\circ\ \text{k}\Omega} = \mathbf{2.27\angle 0^\circ\ mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}_s}{\mathbf{X}_C} = \frac{5\angle 0^\circ\ \text{V}}{5\angle -90^\circ\ \text{k}\Omega} = \mathbf{1\angle 90^\circ\ mA}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{X}_L} = \frac{5\angle 0^\circ\ \text{V}}{10\angle 90^\circ\ \text{k}\Omega} = \mathbf{0.5\angle -90^\circ\ mA}$$

The total current is the phasor sum of the branch currents. By Kirchhoff's law,

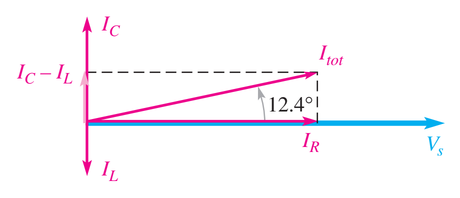
$$\begin{aligned}\mathbf{I}_{tot} &= \mathbf{I}_R + \mathbf{I}_C + \mathbf{I}_L \\ &= 2.27\angle 0^\circ\ \text{mA} + 1\angle 90^\circ\ \text{mA} + 0.5\angle -90^\circ\ \text{mA} \\ &= 2.27\ \text{mA} + j1\ \text{mA} - j0.5\ \text{mA} = 2.27\ \text{mA} + j0.5\ \text{mA}\end{aligned}$$

Converting to polar form yields

$$\begin{aligned}\mathbf{I}_{tot} &= \sqrt{I_R^2 + (I_C - I_L)^2}\angle\tan^{-1}\left(\frac{I_{CL}}{I_R}\right) \\ &= \sqrt{(2.27\ \text{mA})^2 + (0.5\ \text{mA})^2}\angle\tan^{-1}\left(\frac{0.5\ \text{mA}}{2.27\ \text{mA}}\right) = \mathbf{2.32\angle 12.4^\circ\ mA}\end{aligned}$$

The total current is 2.32 mA leading V_s by 12.4°. Figure 26 is the current phasor diagram for the circuit.

▶ **FIGURE 26**



Related Problem Will total current increase or decrease if the frequency in Figure 25 is increased?

SECTION 5 CHECKUP

1. In a three-branch parallel circuit, $R = 150\ \Omega$, $X_C = 100\ \Omega$, and $X_L = 50\ \Omega$. Determine the current in each branch when $V_s = 12$ V.
2. The impedance of a parallel *RLC* circuit is $2.8\angle -38.9°$ kΩ. Is the circuit capacitive or inductive?

6 PARALLEL RESONANCE

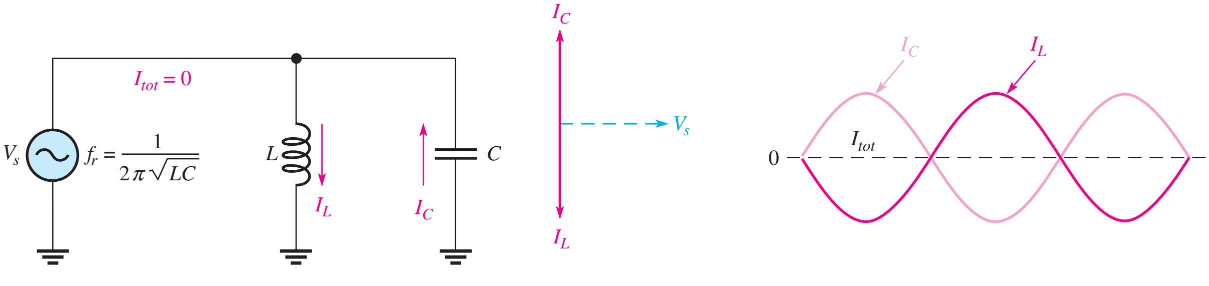
In this section, we will first look at the resonant condition in an ideal parallel *LC* circuit (no winding resistance). Then, we will examine the more realistic case where the resistance of the coil is taken into account.

After completing this section, you should be able to

- **Analyze a circuit for parallel resonance**
    - Describe parallel resonance in an ideal circuit
    - Describe parallel resonance in a nonideal circuit
    - Explain how impedance varies with frequency
    - Determine current and phase angle at resonance
    - Determine parallel resonant frequency

Condition for Ideal Parallel Resonance

Ideally, parallel resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the *resonant frequency,* just as in the series case. When $X_C = X_L$, the two branch currents, I_C and I_L, are equal in magnitude, and, of course, they are always 180° out of phase with each other. Thus, the two currents cancel and the total current is zero, as shown in Figure 27.



(a) Parallel circuit at resonance ($X_C = X_L$, $Z = \infty$) (b) Current phasors (c) Current waveforms

FIGURE 27

An ideal parallel *LC* circuit at resonance.

Since the total current is zero, the impedance of the parallel *LC* circuit is infinitely large (∞). These ideal resonant conditions are stated as follows:

$$X_L = X_C$$
$$Z_r = \infty$$

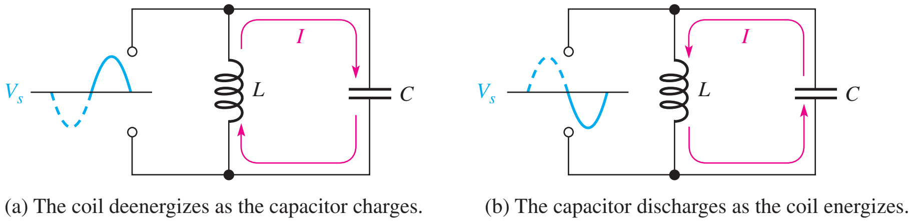
Parallel Resonant Frequency

For an ideal (no resistance) parallel resonant circuit, the frequency at which resonance occurs is determined by the same formula as in series resonant circuits; that is,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Tank Circuit

The parallel resonant *LC* circuit is often called a **tank circuit**. The term *tank circuit* refers to the fact that the parallel resonant circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor. The stored energy is transferred back and forth between the capacitor and the coil on alternate half-cycles as the current goes first one way and then the other when the inductor deenergizes and the capacitor charges, and vice versa. This concept is illustrated in Figure 28.



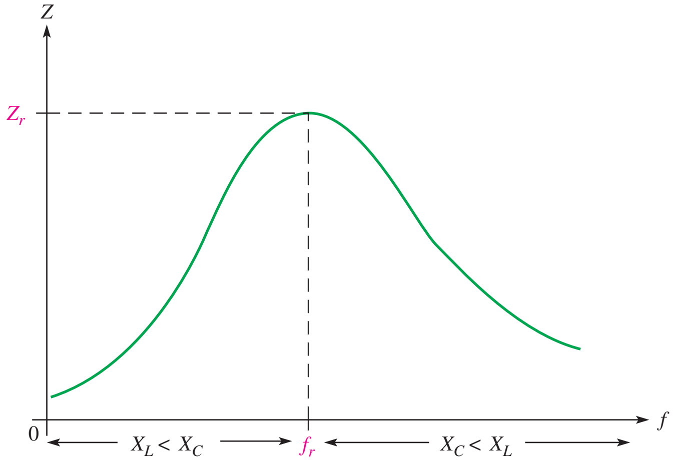
(a) The coil deenergizes as the capacitor charges.
(b) The capacitor discharges as the coil energizes.

▲ FIGURE 28

Energy storage in an ideal parallel resonant tank circuit.

Variation of the Impedance with Frequency

Ideally, the impedance of a parallel resonant circuit is infinite. In practice, the impedance is maximum at the resonant frequency and decreases at lower and higher frequencies, as indicated by the curve in Figure 29.



▲ FIGURE 29

Generalized impedance curve for a parallel resonant circuit. The circuit is inductive below f_r, resistive at f_r, and capacitive above f_r.

At very low frequencies, X_L is very small and X_C is very high, so the total impedance is essentially equal to that of the inductive branch. As the frequency goes up, the impedance also increases, and the inductive reactance dominates (because it is less than X_C) until the resonant frequency is reached. At this point, of course, $X_L \cong X_C$ (for $Q > 10$) and the impedance is at its maximum. As the frequency goes above resonance, the capacitive reactance dominates (because it is less than X_L) and the impedance decreases.

Current and Phase Angle at Resonance

In the ideal tank circuit, the total current from the source at resonance is zero because the impedance is infinite. In the nonideal case when the winding resistance is considered, there is some total current at the resonant frequency, and it is determined by the impedance at resonance.

Equation 11

$$I_{tot} = \frac{V_s}{Z_r}$$

The phase angle of the parallel resonant circuit is 0° because the impedance is purely resistive at the resonant frequency.

Effect of Winding Resistance on the Parallel Resonant Frequency

When the winding resistance is considered, the resonant condition can be expressed as

$$2\pi f_r L\left(\frac{Q^2 + 1}{Q^2}\right) = \frac{1}{2\pi f_r C}$$

where Q is the **quality factor** of the coil, X_L/R_W. Solving for f_r in terms of Q yields

Equation 12

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{Q^2 + 1}}$$

When $Q \geq 10$, the term with the Q factors is approximately 1.

$$\sqrt{\frac{Q^2}{Q^2 + 1}} = \sqrt{\frac{100}{101}} = 0.995 \cong 1$$

Therefore, the parallel resonant frequency is approximately the same as the series resonant frequency as long as Q is equal to or greater than 10.

$$f_r \cong \frac{1}{2\pi\sqrt{LC}} \qquad \text{for } Q \geq 10$$

A precise expression for f_r in terms of the circuit component values is

Equation 13

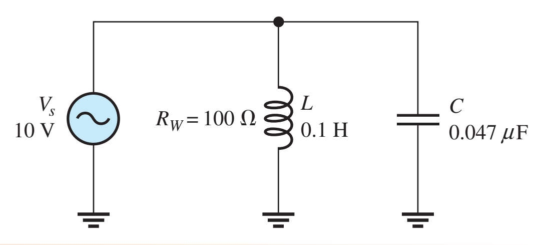
$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}}$$

This precise formula is seldom necessary and the simpler equation $f_r = 1/(2\pi\sqrt{LC})$ is sufficient for most practical situations.

EXAMPLE 11

Find the precise frequency and the value of Q at resonance for the circuit in Figure 30.

▶ **FIGURE 30**



Solution Use Equation 13 to find the resonant frequency.

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - [(100\ \Omega)^2(0.047\ \mu\text{F})/0.1\ \text{H}]}}{2\pi\sqrt{(0.047\ \mu\text{F})(0.1\ \text{H})}} = \mathbf{2.32\ kHz}$$

To calculate the quality factor, Q, first find X_L.

$$X_L = 2\pi f_r L = 2\pi(2.32\ \text{kHz})(0.1\ \text{H}) = 1.46\ \text{k}\Omega$$

$$Q = \frac{X_L}{R_W} = \frac{1.46\ \text{k}\Omega}{100\ \Omega} = \mathbf{14.6}$$

Note that since $Q > 10$, the approximate formula, $f_r \cong 1/(2\pi\sqrt{LC})$, can be used.

Related Problem For a smaller R_W, will f_r be less than or greater than 2.32 kHz?

Use Multisim files E17-11A and E17-11B to verify the calculated results in this example and to confirm your answer for the related problem.

SECTION 6 CHECKUP

1. Is the impedance minimum or maximum at parallel resonance?
2. Is the current minimum or maximum at parallel resonance?
3. For ideal parallel resonance, assume $X_L = 1500\ \Omega$. What is X_C?
4. A parallel tank circuit has the following values: $R_W = 4\ \Omega$, $L = 50$ mH, and $C = 10$ pF. Calculate f_r.
5. If $Q = 25$, $L = 50$ mH, and $C = 1000$ pF, what is f_r?
6. In Question 5, if $Q = 2.5$, what is f_r?

Part 3

SERIES-PARALLEL CIRCUITS

7 ANALYSIS OF SERIES-PARALLEL *RLC* CIRCUITS

In this section, series and parallel combinations of *R*, *L*, and *C* components are analyzed in specific examples. Also, conversion of a series-parallel circuit to an equivalent parallel circuit is covered and resonance in a nonideal parallel circuit is considered.

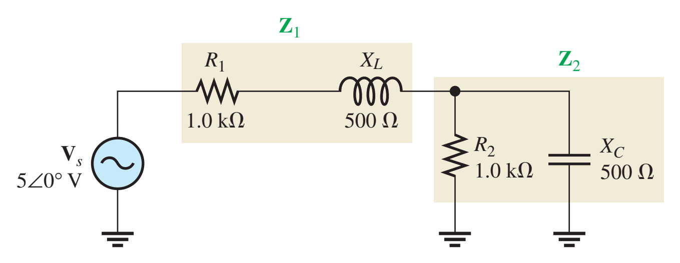
After completing this section, you should be able to

- **Analyze series-parallel *RLC* circuits**
  - Determine currents and voltages
  - Convert a series-parallel circuit to an equivalent parallel form
  - Analyze nonideal (with coil resistance) parallel circuits for parallel resonance
  - Examine the effect of a resistive load on a tank circuit

The following two examples illustrate an approach to the analysis of circuits with both series and parallel combinations of resistance, inductance, and capacitance.

EXAMPLE 12

In Figure 31, find the voltage across the capacitor in polar form. Is this circuit predominantly inductive or capacitive?



▲ FIGURE 31

Solution Use the voltage-divider formula in this analysis. The impedance of the series combination of R_1 and X_L is called $\mathbf{Z}_1$. In rectangular form,

$$\mathbf{Z}_1 = R_1 + jX_L = 1000\ \Omega + j500\ \Omega$$

Converting to polar form yields

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R_1}\right)$$

$$= \sqrt{(1000\ \Omega)^2 + (500\ \Omega)^2}\angle\tan^{-1}\left(\frac{500\ \Omega}{1000\ \Omega}\right) = 1118\angle 26.6^\circ\ \Omega$$

The impedance of the parallel combination of R_2 and X_C is called $\mathbf{Z}_2$. In polar form,

$$\mathbf{Z}_2 = \left(\frac{R_2X_C}{\sqrt{R_2^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R_2}{X_C}\right)$$

$$= \left[\frac{(1000\ \Omega)(500\ \Omega)}{\sqrt{(1000\ \Omega)^2 + (500\ \Omega)^2}}\right]\angle -\tan^{-1}\left(\frac{1000\ \Omega}{500\ \Omega}\right) = 447\angle -63.4^\circ\ \Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta + jZ_2\sin\theta$$

$$= (447\ \Omega)\cos(-63.4^\circ) + j447\sin(-63.4^\circ) = 200\ \Omega - j400\ \Omega$$

The total impedance $\mathbf{Z}_{tot}$ in rectangular form is

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = (1000\ \Omega + j500\ \Omega) + (200\ \Omega - j400\ \Omega) = 1200\ \Omega + j100\ \Omega$$

Converting to polar form yields

$$\mathbf{Z}_{tot} = \sqrt{(1200\ \Omega)^2 + (100\ \Omega)^2}\angle\tan^{-1}\left(\frac{100\ \Omega}{1200\ \Omega}\right) = 1204\angle 4.76^\circ\ \Omega$$

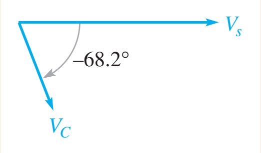
Now apply the voltage-divider formula to get $\mathbf{V}_C$.

$$\mathbf{V}_C = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_{tot}}\right)\mathbf{V}_s = \left(\frac{447\angle -63.4^\circ\ \Omega}{1204\angle 4.76^\circ\ \Omega}\right)5\angle 0^\circ\ \text{V} = \mathbf{1.86\angle -68.2^\circ\ V}$$

Therefore, V_C is 1.86 V and lags V_s by 68.2°.

The $+j$ term in $\mathbf{Z}_{tot}$, or the positive angle in its polar form, indicates that the circuit is more inductive than capacitive. However, it is just slightly more inductive because the angle is small. This result may surprise you, because $X_C = X_L = 500\ \Omega$. However, the capacitor is in parallel with a resistor, so the capacitor actually has less effect on the total impedance than does the inductor. Figure 32 shows the phasor relationship of V_C and V_s. Although $X_C = X_L$, this circuit is not at resonance because the j term of the total impedance is not zero due to the parallel combination of R_2 and X_C. You can see this by noting that the phase angle associated with $\mathbf{Z}_{tot}$ is 4.76° and not zero.

▶ FIGURE 32



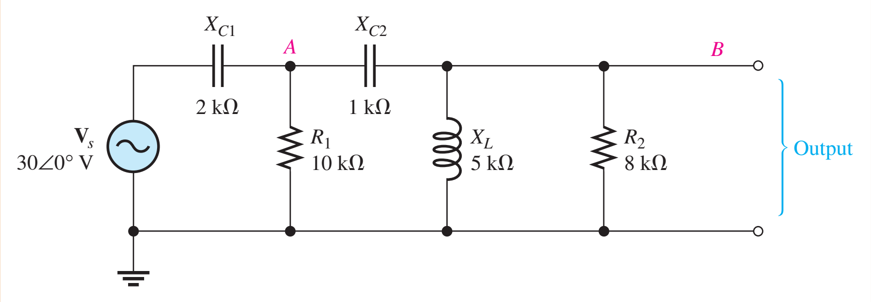
Related Problem Determine the voltage across the capacitor in polar form if R_1 is increased to 2.2 kΩ.

Open Multisim file E17-12 and verify the capacitor voltage.

EXAMPLE 13

For the reactive circuit in Figure 33, find the voltage at point *B* with respect to ground.

▶ **FIGURE 33**



Solution The voltage ($\mathbf{V}_B$) at point *B* is the voltage across the open output terminals. Use the voltage-divider approach. To do so, you must know the voltage ($\mathbf{V}_A$) at point *A* first; so you need to find the impedance from point *A* to ground as a starting point.

The parallel combination of X_L and R_2 is in series with X_{C2}. This combination is in parallel with R_1. Call this impedance from point *A* to ground, $\mathbf{Z}_A$. To find $\mathbf{Z}_A$, take the following steps. The impedance of the parallel combination of R_2 and X_L is called $\mathbf{Z}_1$.

$$\mathbf{Z}_1 = \left(\frac{R_2 X_L}{\sqrt{R_2^2 + X_L^2}}\right)\angle \tan^{-1}\left(\frac{R_2}{X_L}\right)$$

$$= \left(\frac{(8\text{ k}\Omega)(5\text{ k}\Omega)}{\sqrt{(8\text{ k}\Omega)^2 + (5\text{ k}\Omega)^2}}\right)\angle \tan^{-1}\left(\frac{8\text{ k}\Omega}{5\text{ k}\Omega}\right) = 4.24\angle 58.0°\text{ k}\Omega$$

Next, combine $\mathbf{Z}_1$ in series with $\mathbf{X}_{C2}$ to get an impedance $\mathbf{Z}_2$.

$$\begin{aligned}\mathbf{Z}_2 &= \mathbf{X}_{C2} + \mathbf{Z}_1\\ &= 1\angle -90°\text{ k}\Omega + 4.24\angle 58°\text{ k}\Omega = -j1\text{ k}\Omega + 2.25\text{ k}\Omega + j3.6\text{ k}\Omega\\ &= 2.25\text{ k}\Omega + j2.6\text{ k}\Omega\end{aligned}$$

Converting to polar form yields

$$\mathbf{Z}_2 = \sqrt{(2.25\text{ k}\Omega)^2 + (2.6\text{ k}\Omega)^2}\angle \tan^{-1}\left(\frac{2.6\text{ k}\Omega}{2.25\text{ k}\Omega}\right) = 3.44\angle 49.1°\text{ k}\Omega$$

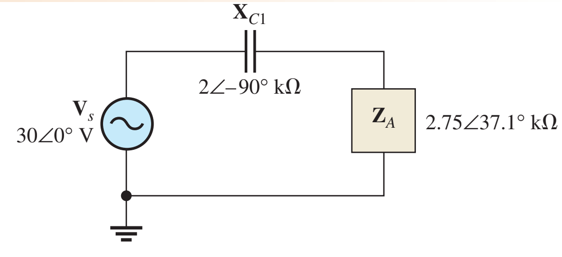
Finally, combine $\mathbf{Z}_2$ and $\mathbf{R}_1$ in parallel to get $\mathbf{Z}_A$.

$$\mathbf{Z}_A = \frac{\mathbf{R}_1\mathbf{Z}_2}{\mathbf{R}_1 + \mathbf{Z}_2} = \frac{(10\angle 0°\text{ k}\Omega)(3.44\angle 49.1°\text{ k}\Omega)}{10\text{ k}\Omega + 2.25\text{ k}\Omega + j2.6\text{ k}\Omega}$$

$$= \frac{34.4\angle 49.1°\text{ k}\Omega}{12.25\text{ k}\Omega + j2.6\text{ k}\Omega} = \frac{34.4\angle 49.1°\text{ k}\Omega}{12.5\angle 12.0°\text{ k}\Omega} = 2.75\angle 37.1°\text{ k}\Omega$$

The simplified circuit is shown in Figure 34.

▶ **FIGURE 34**



Next, use the voltage-divider principle to find the voltage ($\mathbf{V}_A$) at point A in Figure 33. The total impedance is

$$\begin{aligned}\mathbf{Z}_{tot} &= \mathbf{X}_{C1} + \mathbf{Z}_A \\ &= 2\angle-90^\circ\,\text{k}\Omega + 2.75\angle37.1^\circ\,\text{k}\Omega = -j2\,\text{k}\Omega + 2.19\,\text{k}\Omega + j1.66\,\text{k}\Omega \\ &= 2.19\,\text{k}\Omega - j0.340\,\text{k}\Omega\end{aligned}$$

Converting to polar form yields

$$\mathbf{Z}_{tot} = \sqrt{(2.19\,\text{k}\Omega)^2 + (0.340\,\text{k}\Omega)^2}\angle-\tan^{-1}\left(\frac{0.340\,\text{k}\Omega}{2.19\,\text{k}\Omega}\right) = 2.22\angle-8.82^\circ\,\text{k}\Omega$$

The voltage at point A is

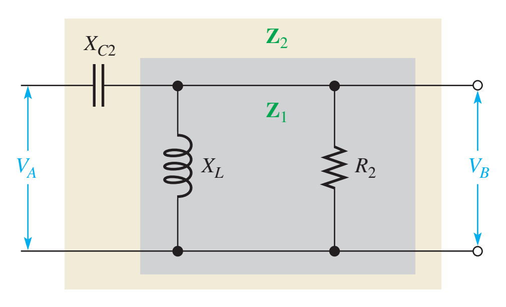
$$\mathbf{V}_A = \left(\frac{\mathbf{Z}_A}{\mathbf{Z}_{tot}}\right)\mathbf{V}_s = \left(\frac{2.75\angle37.1^\circ\,\text{k}\Omega}{2.22\angle-8.82^\circ\,\text{k}\Omega}\right)30\angle0^\circ\,\text{V} = 37.2\angle45.9^\circ\,\text{V}$$

Next, find the voltage ($\mathbf{V}_B$) at point B by dividing $\mathbf{V}_A$ down, as indicated in Figure 35. $\mathbf{V}_B$ is the open terminal output voltage.

$$\mathbf{V}_B = \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_2}\right)\mathbf{V}_A = \left(\frac{4.24\angle58^\circ\,\text{k}\Omega}{3.44\angle49.1^\circ\,\text{k}\Omega}\right)37.2\angle45.9^\circ\,\text{V} = \mathbf{45.9\angle54.8^\circ\,V}$$

Surprisingly, V_A is greater than V_s, and V_B is greater than V_A! This result is possible because of the out-of-phase relationship of the reactive voltages. Remember that X_C and X_L tend to cancel each other.

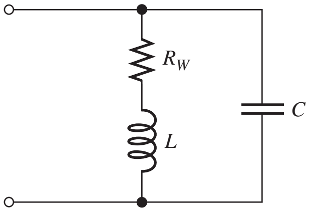
▶ FIGURE 35



Related Problem What is the voltage in polar form across C_1 in Figure 33?

Conversion of Series-Parallel to Parallel

The particular series-parallel configuration shown in Figure 36 is important because it represents a circuit having parallel L and C branches, with the winding resistance of the coil taken into account as a series resistance in the L branch.



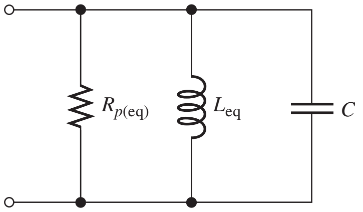
◀ FIGURE 36

A series-parallel *RLC* circuit ($Q = X_L/R_W$).

It is helpful to view the series-parallel circuit in Figure 36 in an equivalent parallel form, as indicated in Figure 37.

▶ FIGURE 37

Parallel equivalent form of the circuit in Figure 36.



The equivalent inductance, L_{eq}, and the equivalent parallel resistance, $R_{p(\text{eq})}$, are given by the following formulas:

Equation 14

$$L_{\text{eq}} = L\left(\frac{Q^2 + 1}{Q^2}\right)$$

Equation 15

$$R_{p(\text{eq})} = R_W(Q^2 + 1)$$

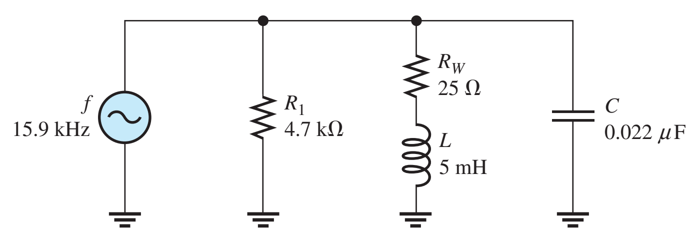
where Q is the quality factor of the coil, X_L/R_W. Derivations of these formulas are quite involved and thus are not given here. Notice in the equations that for a $Q \geq 10$, the value of L_{eq} is approximately the same as the original value of L. For example, if $L = 10$ mH and $Q = 10$, then

$$L_{\text{eq}} = 10\text{ mH}\left(\frac{10^2 + 1}{10^2}\right) = 10\text{ mH}(1.01) = 10.1\text{ mH}$$

The equivalency of the two circuits means that at a given frequency, when the same value of voltage is applied to both circuits, the same total current is in both circuits and the phase angles are the same. Basically, an equivalent circuit simply makes circuit analysis more convenient.

EXAMPLE 14

Convert the series-parallel circuit in Figure 38 to an equivalent parallel form at the given frequency.



▲ FIGURE 38

Solution Determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(15.9\text{ kHz})(5\text{ mH}) = 500\ \Omega$$

The Q of the coil is

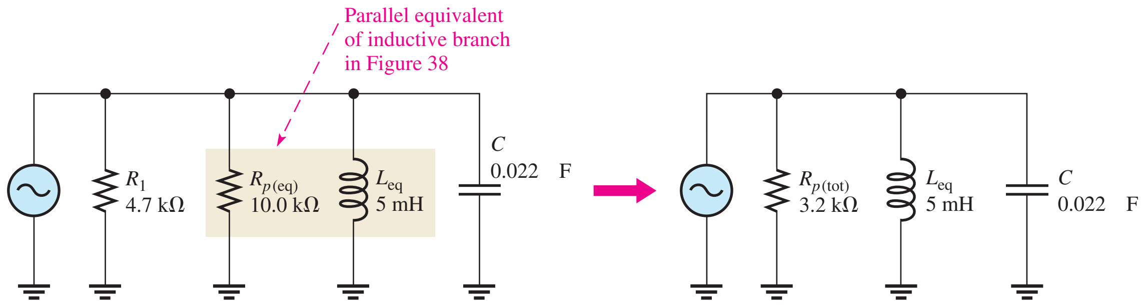
$$Q = \frac{X_L}{R_W} = \frac{500\ \Omega}{25\ \Omega} = 20$$

Since $Q > 10$, then $L_{\text{eq}} \cong L = 5$ mH.

The equivalent parallel resistance is

$$R_{p(\text{eq})} = R_W(Q^2 + 1) = (25\ \Omega)(20^2 + 1) = 10\ \text{k}\Omega$$

This equivalent resistance appears in parallel with R_1 as shown in Figure 39(a). When combined, they give a total parallel resistance ($R_{p(tot)}$) of 3.2 kΩ, as indicated in Figure 39(b).


Parallel equivalent of inductive branch in Figure 38
R_1 4.7 kΩ
$R_{p(\text{eq})}$ 10.0 kΩ
L_{eq} 5 mH
C 0.022 F
$R_{p(\text{tot})}$ 3.2 kΩ
L_{eq} 5 mH
C 0.022 F

(a) Parallel equivalent of the circuit in Figure 38

(b) $R_{p(tot)} = R_1 \parallel R_{p(\text{eq})} = 3.2\ \text{k}\Omega$

▲ **FIGURE 39**

Related Problem Find the equivalent parallel circuit if $R_W = 10\ \Omega$ in Figure 38.

Parallel Resonant Conditions in a Nonideal Circuit

The resonance of an ideal parallel *LC* circuit was examined in Section 6. Now, let's consider resonance in a tank circuit with the resistance of the coil taken into account. Figure 40 shows a nonideal tank circuit and its parallel *RLC* equivalent.

Recall that the quality factor, Q, of the circuit at resonance is simply the Q of the coil if no other resistance is in the circuit.

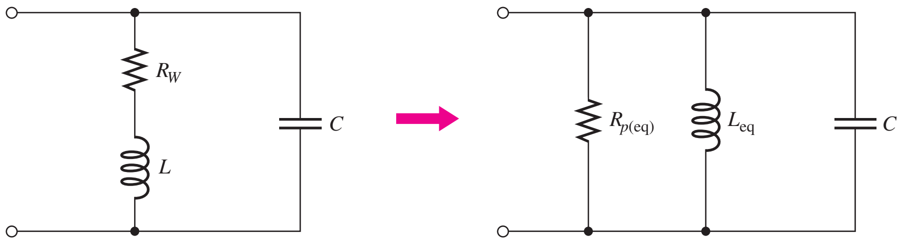
$$Q = \frac{X_L}{R_W}$$

The expressions for the equivalent inductance and the equivalent parallel resistance were given in Equations 14 and 15 as

$$L_{\text{eq}} = L\left(\frac{Q^2 + 1}{Q^2}\right)$$

$$R_{p(\text{eq})} = R_W(Q^2 + 1)$$

For $Q \geq 10$, $L_{\text{eq}} \cong L$.


R_W
L
C
$R_{p(\text{eq})}$
L_{eq}
C

(a) Nonideal tank circuit

(b) Parallel *RLC* equivalent

◀ **FIGURE 40**

A practical treatment of parallel resonant circuits must include the coil resistance.

At parallel resonance,

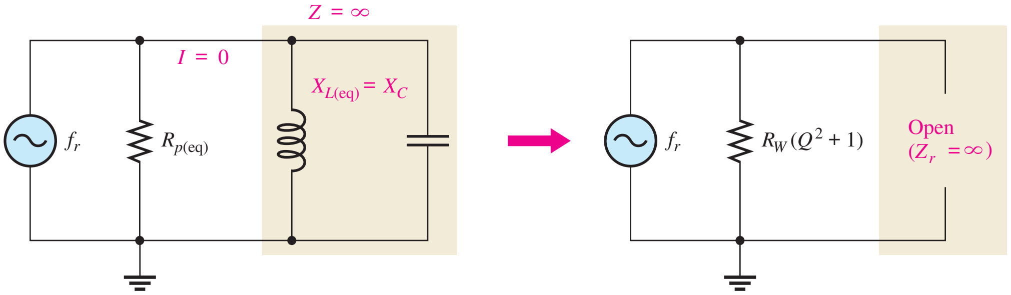
$$X_{L(\text{eq})} = X_C$$

In the parallel equivalent circuit, $R_{p(\text{eq})}$ is in parallel with an ideal coil and a capacitor, so the L and C branches act as an ideal tank circuit which has an infinite impedance at resonance as shown in Figure 41. Therefore, the total impedance of the nonideal tank circuit at resonance can be expressed as simply the equivalent parallel resistance.

Equation 16

$$Z_r = R_W(Q^2 + 1)$$

A derivation of Equation 16 is given in Appendix B.



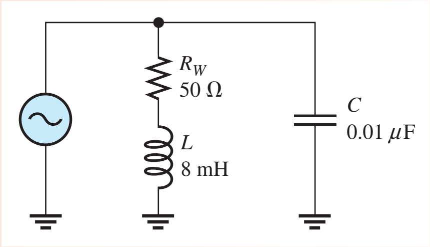
▲ FIGURE 41

At resonance, the parallel *LC* portion appears open and the source sees only $R_{p(eq)}$.

EXAMPLE 15

Determine the impedance of the circuit in Figure 42 at the resonant frequency ($f_r \cong$ 17,794 Hz).

▶ FIGURE 42



Solution Before you can calculate the impedance using Equation 16, you must find the quality factor. To get Q, first find the inductive reactance.

$$X_L = 2\pi f_r L = 2\pi(17{,}794\text{ Hz})(8\text{ mH}) = 894\ \Omega$$

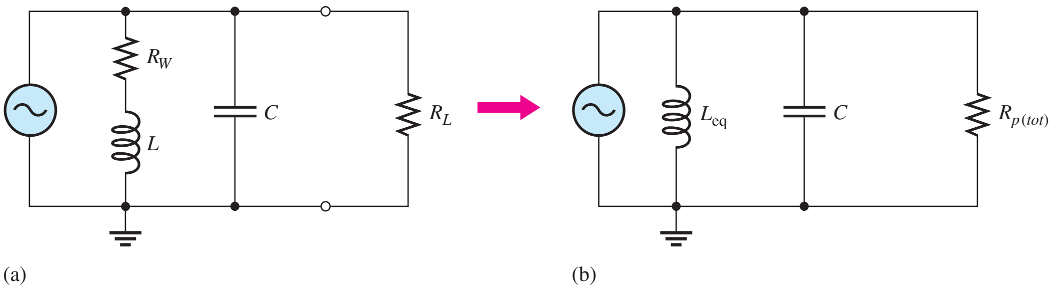
$$Q = \frac{X_L}{R_W} = \frac{894\ \Omega}{50\ \Omega} = 17.9$$

$$Z_r = R_W(Q^2 + 1) = 50\ \Omega(17.9^2 + 1) = \mathbf{16.1\ k\Omega}$$

Related Problem Determine Z_r for $R_W = 10\ \Omega$.

An External Load Resistance Affects a Tank Circuit

In most practical situations, an external load resistance appears in parallel with a nonideal tank circuit, as shown in Figure 43(a). Obviously, the external resistor (R_L) will dissipate more of the energy delivered by the source and thus will lower the overall Q of the circuit.



▲ **FIGURE 43**

Tank circuit with a parallel load resistor and its equivalent circuit.

The external resistor effectively appears in parallel with the equivalent parallel resistance of the coil, $R_{p(\text{eq})}$, and both are combined to determine a total parallel resistance, $R_{p(tot)}$, as indicated in Figure 43(b).

$$R_{p(tot)} = R_L \parallel R_{p(\text{eq})}$$

The overall Q, designated Q_O, for a parallel RLC circuit is expressed differently from the Q of a series circuit.

$$Q_O = \frac{R_{p(tot)}}{X_{L(\text{eq})}}$$

Equation 17

As you can see, the effect of loading the tank circuit is to reduce its overall Q (which is equal to the coil Q when unloaded).

SECTION 7 CHECKUP

1. A certain resonant circuit has a 100 μH inductor with a 2 Ω winding resistance in parallel with a 0.22 μF capacitor. If $Q = 8$, determine the parallel equivalent of this circuit.
2. Find the equivalent parallel inductance and resistance for a 20 mH coil with a winding resistance of 10 Ω at a frequency of 1 kHz.

Part 4

SPECIAL TOPICS

8 BANDWIDTH OF RESONANT CIRCUITS

The current in a series *RLC* is maximum at the resonant frequency because the reactances cancel. The current in a parallel *RLC* is minimum at the resonant frequency because the inductive and capacitive currents cancel. This circuit behavior relates to a characteristic called bandwidth.

After completing this section, you should be able to

- **Determine the bandwidth of resonant circuits**
  - Discuss the bandwidth of series and parallel resonant circuits
  - State the formula for bandwidth
  - Define *half-power frequency*
  - Define *selectivity*
  - Explain how the Q affects the bandwidth

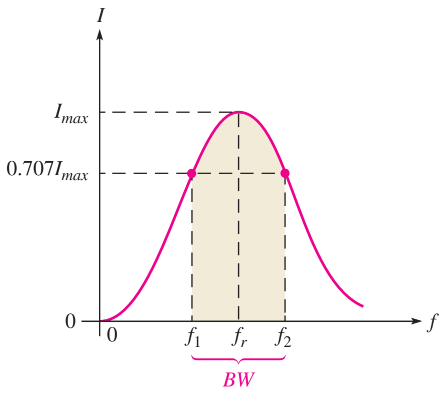
Series Resonant Circuits

The current in a series *RLC* circuit is maximum at the resonant frequency (also known as *center frequency*) and drops off on either side of this frequency. Bandwidth, sometimes abbreviated *BW*, is an important characteristic of a resonant circuit. The bandwidth is the range of frequencies for which the current is equal to or greater than 70.7% of its resonant value.

Figure 44 illustrates bandwidth on the response curve of a series *RLC* circuit. Notice that the frequency f_1 below f_r is the point at which the current is $0.707I_{max}$ and is commonly called the *lower critical frequency*. The frequency f_2 above f_r, where the current is again $0.707I_{max}$, is the *upper critical frequency*. Other names for f_1 and f_2 are *−3 dB frequencies, cutoff frequencies,* and *half-power frequencies*. The significance of the latter term is discussed later in the chapter.

► FIGURE 44

Bandwidth on series resonant response curve for *I*.



EXAMPLE 16 A certain series resonant circuit has a maximum current of 100 mA at the resonant frequency. What is the value of the current at the critical frequencies?

Solution Current at the critical frequencies is 70.7% of maximum.

$$I_{f1} = I_{f2} = 0.707I_{max} = 0.707(100\text{ mA}) = \mathbf{70.7\text{ mA}}$$

Related Problem A certain series resonant circuit has a current of 25 mA at the critical frequencies. What is the current at resonance?

Parallel Resonant Circuits

For a parallel resonant circuit, the impedance is maximum at the resonant frequency; so the total current is minimum. The bandwidth can be defined in relation to the impedance curve in the same manner that the current curve was used in the series circuit. Of course, f_r is the frequency at which Z is maximum; f_1 is the lower critical frequency at which $Z = 0.707Z_{max}$; and f_2 is the upper critical frequency at which again $Z = 0.707Z_{max}$. The bandwidth is the range of frequencies between f_1 and f_2, as shown in Figure 45.

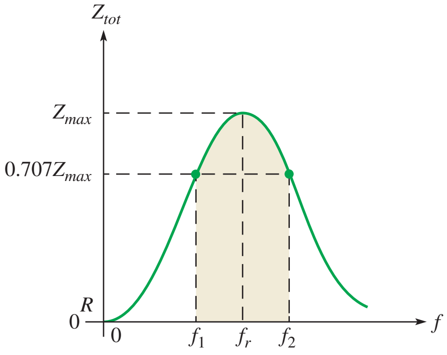


FIGURE 45

Bandwidth of the parallel resonant response curve for Z_{tot}.

Formula for Bandwidth

The bandwidth for either series or parallel resonant circuits is the range of frequencies between the critical frequencies for which the response curve (I or Z) is 0.707 of the maximum value. Thus, the bandwidth is actually the difference between f_2 and f_1.

$$BW = f_2 - f_1$$ **Equation 18**

Ideally, f_r is the center frequency and can be calculated as follows:

$$f_r = \frac{f_1 + f_2}{2}$$ **Equation 19**

EXAMPLE 17 A resonant circuit has a lower critical frequency of 8 kHz and an upper critical frequency of 12 kHz. Determine the bandwidth and center (resonant) frequency.

Solution

$$BW = f_2 - f_1 = 12\text{ kHz} - 8\text{ kHz} = \mathbf{4\text{ kHz}}$$

$$f_r = \frac{f_1 + f_2}{2} = \frac{12\text{ kHz} + 8\text{ kHz}}{2} = \mathbf{10\text{ kHz}}$$

Related Problem If the bandwidth of a resonant circuit is 2.5 kHz and its center frequency is 8 kHz, what are the lower and upper critical frequencies?

Half-Power Frequencies

As previously mentioned, the upper and lower critical frequencies are sometimes called the **half-power frequencies**. This term is derived from the fact that the power from the source at these frequencies is one-half the power delivered at the resonant frequency. The following shows that this is true for a series circuit. The same end result also applies to a parallel circuit. At resonance,

$$P_{max} = I_{max}^2R$$

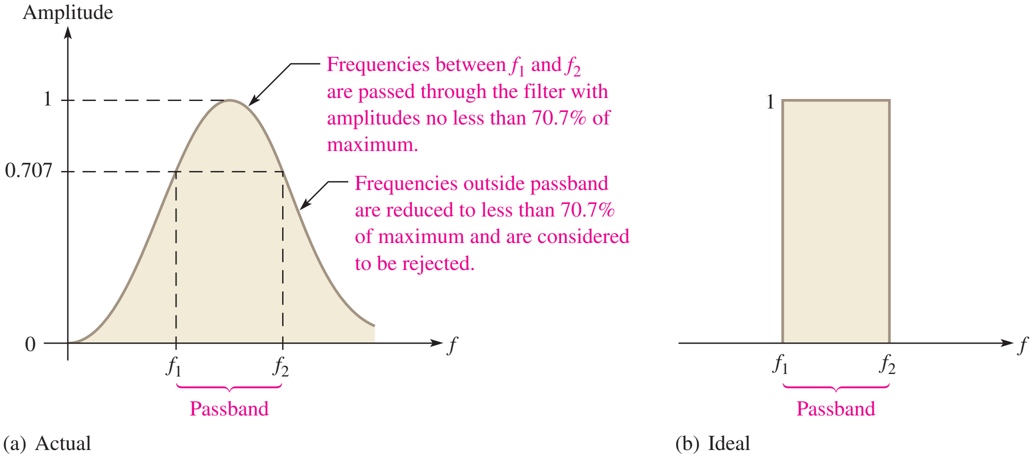
The power at f_1 or f_2 is

$$P_{f1} = I_{f1}^2R = (0.707I_{max})^2R = (0.707)^2I_{max}^2R = 0.5I_{max}^2R = 0.5P_{max}$$

Selectivity

The response curves in Figures 44 and 45 are also called *selectivity curves*. **Selectivity** defines how well a resonant circuit responds to a certain frequency and discriminates against all others. *The narrower the bandwidth, the greater the selectivity.*

Ideally, we assume that a resonant circuit accepts frequencies within its bandwidth and completely eliminates frequencies outside the bandwidth. Such is not actually the case, however, because signals with frequencies outside the bandwidth are not completely eliminated. Their magnitudes, however, are greatly reduced. The further the frequencies are from the critical frequencies, the greater the reduction, as illustrated in Figure 46(a). An ideal selectivity curve is shown in Figure 46(b).



(a) Actual (b) Ideal

▲ FIGURE 46

Generalized selectivity curve.

Another factor that influences selectivity is the steepness of the slopes of the response curve. The faster the curve drops off at the critical frequencies, the more selective the circuit is because it tends to respond only to the frequencies within the bandwidth. Figure 47 shows a general comparison of three response curves with varying degrees of selectivity.

Q Affects Bandwidth

A higher value of circuit Q results in a narrower bandwidth. A lower value of Q causes a wider bandwidth. A formula for the bandwidth of a resonant circuit in terms of Q is stated in the following equation:

Equation 20

$$BW = \frac{f_r}{Q}$$

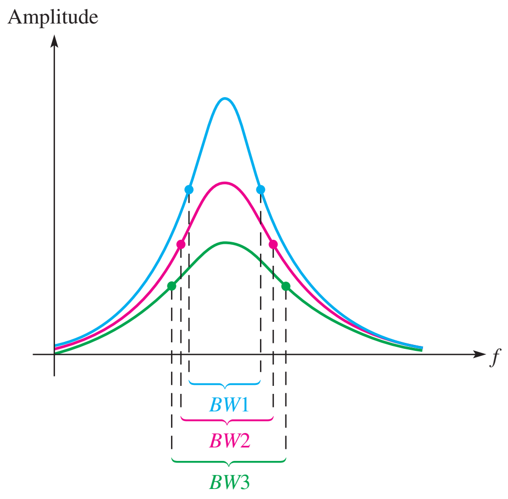


FIGURE 47

Comparative selectivity curves. The blue curve represents the greatest selectivity.

EXAMPLE 18

What is the bandwidth of each circuit in Figure 48?

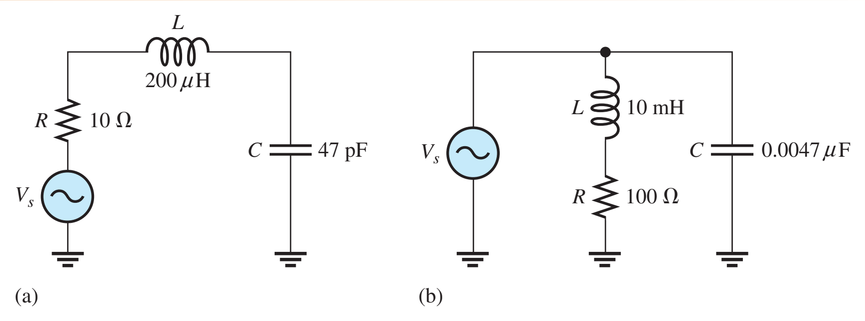


FIGURE 48

Solution For the circuit in Figure 48(a), determine the bandwidth as follows:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(200\ \mu\text{H})(47\ \text{pF})}} = 1.64\ \text{MHz}$$

$$X_L = 2\pi f_r L = 2\pi(1.64\ \text{MHz})(200\ \mu\text{H}) = 2.06\ \text{k}\Omega$$

$$Q = \frac{X_L}{R} = \frac{2.06\ \text{k}\Omega}{10\ \Omega} = 206$$

$$BW = \frac{f_r}{Q} = \frac{1.64\ \text{MHz}}{206} = \mathbf{7.96\ kHz}$$

For the circuit in Figure 48(b),

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} \cong \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10\ \text{mH})(0.0047\ \mu\text{F})}} = 23.2\ \text{kHz}$$

$$X_L = 2\pi f_r L = 2\pi(23.2\ \text{kHz})(10\ \text{mH}) = 1.46\ \text{k}\Omega$$

$$Q = \frac{X_L}{R} = \frac{1.46\ \text{k}\Omega}{100\ \Omega} = 14.6$$

$$BW = \frac{f_r}{Q} = \frac{23.2\ \text{kHz}}{14.6} = \mathbf{1.59\ kHz}$$

Related Problem Change C in Figure 48(a) to 1000 pF and determine the bandwidth.

Use Multisim files E17-18A, E17-18B, and E17-18C to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 8 CHECKUP

1. What is the bandwidth when $f_2 = 2.2$ MHz and $f_1 = 1.8$ MHz?
2. For a resonant circuit with the critical frequencies in Question 1, what is the center frequency?
3. The power-dissipated at resonance is 1.8 W. What is the power at the upper critical frequency?
4. Does a larger Q mean a narrower or a wider bandwidth?

9 APPLICATIONS

Resonant circuits are used in a wide variety of applications, particularly in communication systems. In this section, we will look briefly at a few common communication systems applications to illustrate the importance of resonant circuits in electronic communication.

After completing this section, you should be able to

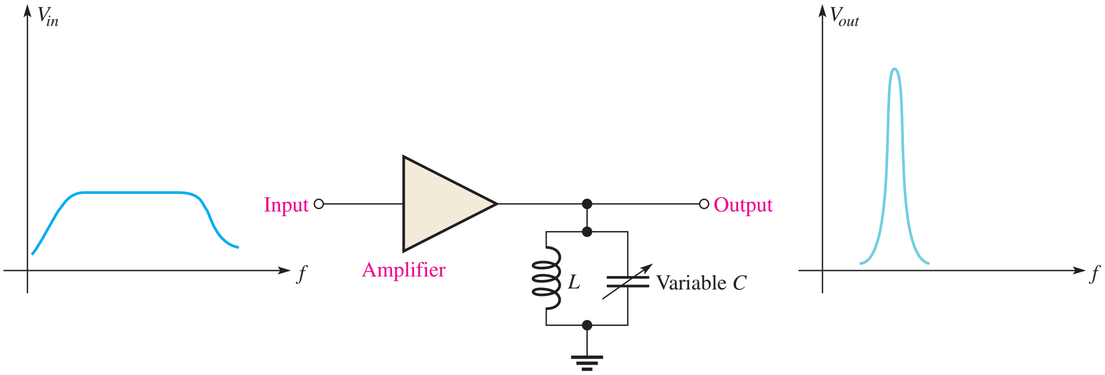
- **Discuss some applications of resonant circuits**
    - Describe a tuned amplifier application
    - Describe antenna coupling
    - Describe tuned amplifiers
    - Describe audio crossover networks
    - Describe a radio receiver

Tuned Amplifiers

A *tuned amplifier* is a circuit that amplifies signals within a specified band. Typically, a parallel resonant circuit is used in conjunction with an amplifier to achieve the selectivity. In terms of the general operation, input signals with frequencies that range over a wide band are accepted on the amplifier's input and are amplified. The resonant circuit allows only a relatively narrow band of those frequencies to be passed on. The variable capacitor allows tuning over the range of input frequencies so that a desired frequency can be selected, as indicated in Figure 49.

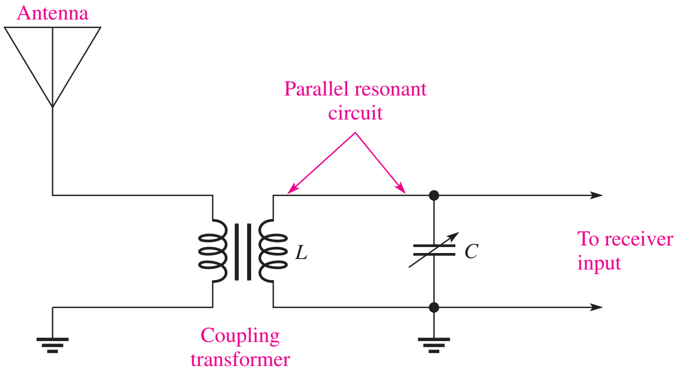
Antenna Input to a Receiver

Radio signals are sent out from a transmitter via electromagnetic waves that propagate through the atmosphere. When the electromagnetic waves cut across the receiving antenna, small voltages are induced. Out of all the wide range of electromagnetic frequencies, only


V_{in}
f
Input
Amplifier
L
Variable C
Output
V_{out}
f

▲ FIGURE 49

A basic tuned band-pass amplifier.


Antenna
Parallel resonant circuit
L
C
To receiver input
Coupling transformer

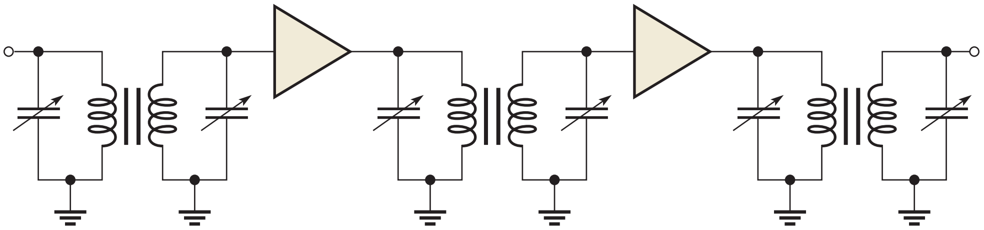
◀ FIGURE 50

Resonant coupling from an antenna.

one frequency or a limited band of frequencies must be extracted. Figure 50 shows a typical arrangement of an antenna coupled to the receiver input by a transformer. A variable capacitor is connected across the transformer secondary to form a parallel resonant circuit.

Double-Tuned Transformer Coupling in a Receiver

In some types of communication receivers, tuned amplifiers are transformer-coupled together to increase the amplification. Capacitors can be placed in parallel with the primary and secondary windings of the transformer, effectively creating two parallel resonant band-pass filters that are coupled together. This technique, illustrated in Figure 51, can result in



▲ FIGURE 51

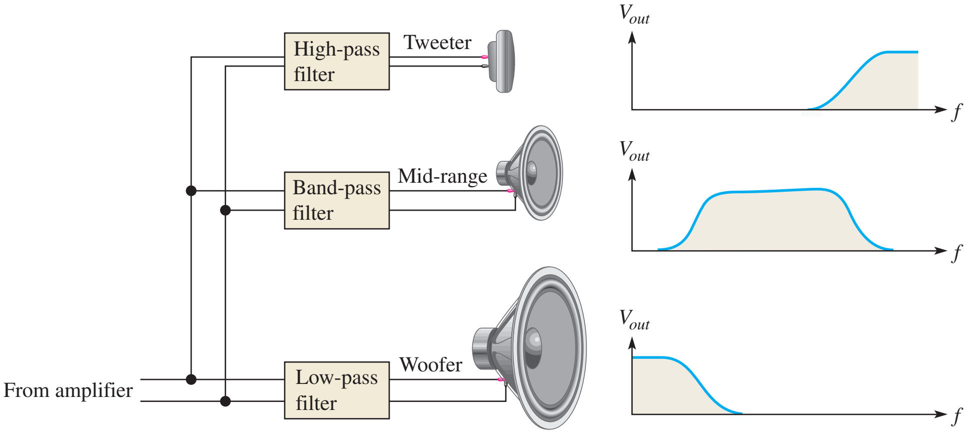
Double-tuned amplifiers.

a wider bandwidth and steeper slopes on the response curve, thus increasing the selectivity for a desired band of frequencies.

Audio Crossover Networks

Most quality stereo systems have speakers designed for specific parts of the audio frequency spectrum. Audio crossover networks are filter networks that separate the audio signal into different frequency bands for the speakers while maintaining an overall flat response. Crossover networks can be passive (meaning only inductors, capacitors, and resistors are used) or active (meaning transistor and operational amplifiers are used).

A three-way passive network includes a band-pass filter that is essentially a low-Q resonant filter designed to have a broad flat response. The design of passive crossover networks is complicated because of various factors that affect the overall response (such as variations of the speaker's impedance as a function of frequency), but the basic concept is simple. The network separates the audio spectrum into three parts and passes each part to the appropriate speaker, as indicated in Figure 52. The network has three filters: a high-pass filter that passes high frequencies to the tweeter (or high-frequency speaker), a band-pass filter that passes middle frequencies to the mid-range speaker, and a low-pass filter that passes low frequencies to the woofer (or low-frequency speaker).

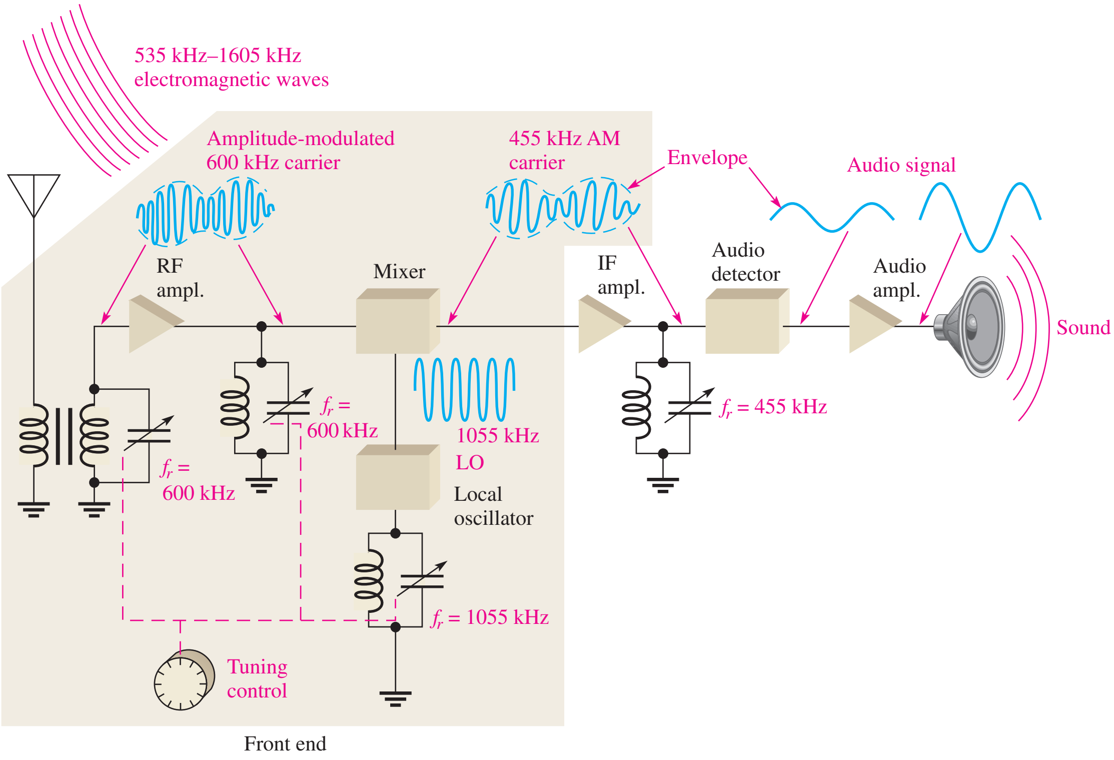


▲ **FIGURE 52**

A crossover network uses filters to separate the audio frequency bands.

Superheterodyne Receiver

Another example of resonant circuit (filter) applications is in the common AM (amplitude modulation) receiver. The AM broadcast band ranges from 535 kHz to 1605 kHz. Each AM station is assigned a 10 kHz bandwidth within that range. The tuned circuits are designed to pass only the signals from the desired radio station, rejecting all others. To reject stations outside the one that is tuned, the tuned circuits must be selective, passing on only the signals in the 10 kHz band and rejecting all others. Too much selectivity is not desirable either however. If the bandwidth is too narrow, some of the higher frequency modulated signals will be rejected, resulting in a loss of fidelity. Ideally, the resonant


535 kHz–1605 kHz
electromagnetic waves
Amplitude-modulated
600 kHz carrier
455 kHz AM
carrier
Envelope
Audio signal
RF
ampl.
Mixer
IF
ampl.
Audio
detector
Audio
ampl.
Sound
f_r =
600 kHz
f_r =
600 kHz
1055 kHz
LO
Local
oscillator
f_r = 455 kHz
f_r = 1055 kHz
Tuning
control
Front end

▲ **FIGURE 53**

A simplified diagram of a superheterodyne AM radio broadcast receiver showing an example of the application of tuned resonant circuits.

circuit must reject signals that are not in the desired passband. A simplified block diagram of a superheterodyne AM receiver is shown in Figure 53.

There are basically three parallel resonant circuits in the front end of the receiver. Each of these resonant circuits is gang-tuned by capacitors; that is, the capacitors are mechanically or electronically linked together so that they change together as the tuning knob is turned. The front end is tuned to receive a desired station, for example, one that transmits at 600 kHz. The input resonant circuit from the antenna and the RF (radio frequency) amplifier resonant circuit select only a frequency of 600 kHz out of all the frequencies crossing the antenna.

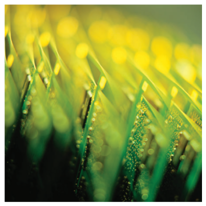
The actual audio (sound) signal is carried by the 600 kHz carrier frequency by modulating the amplitude of the carrier so that it follows the audio signal as indicated. The variation in the amplitude of the carrier corresponding to the audio signal is called the *envelope.* The 600 kHz is then applied to a circuit called the *mixer.*

The *local oscillator* (LO) is tuned to a frequency that is 455 kHz above the selected frequency (1055 kHz, in this case). By a process called *heterodyning* or *beating,* the AM signal and the local oscillator signal are mixed together, and the 600 kHz AM signal is converted by the mixer to a 455 kHz AM signal (1055 kHz $-$ 600 kHz $=$ 455 kHz).

The 455 kHz is the intermediate frequency (IF) for standard AM receivers. No matter which station within the broadcast band is selected, its frequency is always converted to the 455 kHz IF. The amplitude-modulated IF is amplified by the IF amplifier which is tuned to 455 kHz. The output of the IF amplifier is applied to an *audio detector* which removes the IF, leaving only the envelope which is the audio signal. The audio signal is then amplified and applied to the speaker.

SECTION 9 CHECKUP

1. Generally, why is a tuned filter necessary when a signal is coupled from an antenna to the input of a receiver?
2. What is a crossover network?
3. What is meant by *ganged tuning*?



Application Activity

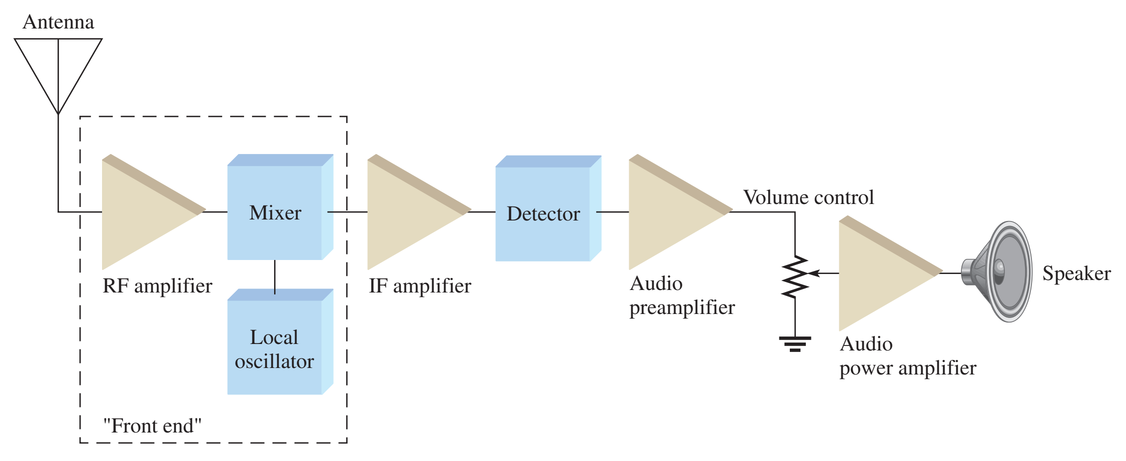
In this chapter, the receiver is used to illustrate one application of resonant circuits. We will focus on a part of the "front end" of the receiver system that contains resonant circuits. Generally, the front end includes the RF amplifier, the local oscillator, and the mixer. In this application activity, the RF amplifier is the focus. A knowledge of amplifier circuits is not necessary at this time.

A basic block diagram of an AM radio receiver is shown in Figure 54. In this particular system, the "front end" includes the circuitry used for tuning in a desired broadcasting station by frequency selection and then converting that selected frequency to a standard intermediate frequency (IF). AM radio stations transmit in the frequency range from 535 kHz to 1605 kHz. The purpose of the RF amplifier is to take the signals picked up by the antenna, reject all but the signal from the desired station, and amplify it to a higher level.

A schematic of the RF amplifier is shown in Figure 55. The parallel resonant tuning circuit consists of L, C_1 and C_2. This particular RF amplifier does not have a resonant circuit on the output. C_1 is a varactor, which is a semiconductor device that you will learn more about in a later course. All that you need to know at this point is that the varactor is basically a variable capacitor whose capacitance is varied by changing the dc voltage across it. In this circuit, the dc voltage comes from the wiper of the potentiometer used for tuning the receiver.

The voltage from the potentiometer can be varied from +1 V to +9 V. The particular varactor used in this circuit can be varied from 200 pF at 1 V to 5 pF at 9 V. The capacitor C_2 is a trimmer capacitor that is used for initially adjusting the resonant circuit. Once it is preset, it is left at that value. C_1 and C_2 are in parallel and their capacitances add to produce the total capacitance for the resonant circuit. C_3 has a minimal effect on the resonant circuit and can be ignored. The purpose of C_3 is to allow the dc voltage to be applied to the varactor while providing an ac ground.

In this application activity, you will focus on the RF amplifier circuit board in Figure 56. Although all of the amplifier components are on the board, the part that you are to focus on is the resonant circuit indicated by the highlighted area.



▲ FIGURE 54

Simplified block diagram of a basic radio receiver.

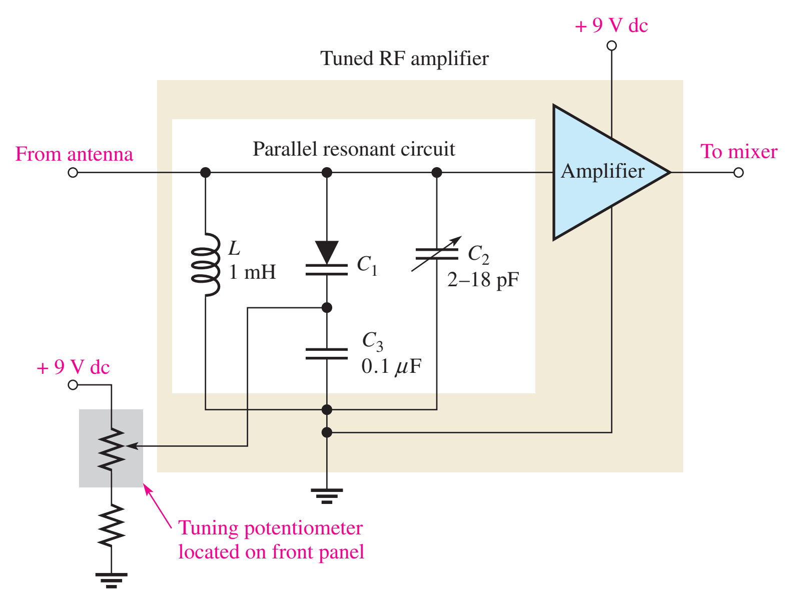

+ 9 V dc
Tuned RF amplifier
From antenna
Parallel resonant circuit
Amplifier
To mixer
L
1 mH
C_1
C_2
2–18 pF
C_3
0.1 μF
+ 9 V dc
Tuning potentiometer
located on front panel

FIGURE 55

Partial schematic of the RF amplifier showing the resonant tuning circuit.

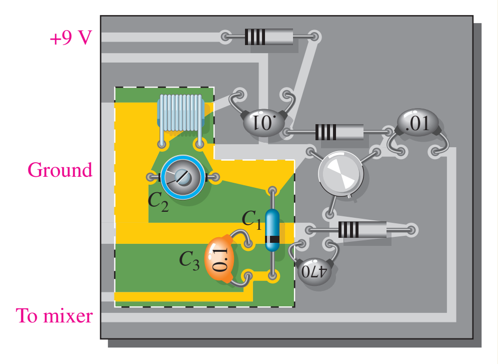

+9 V
Ground
To mixer
10
.01
C_2
C_1
C_3
0.1
470

FIGURE 56

RF amplifier circuit board.

Capacitance in the Resonant Circuit

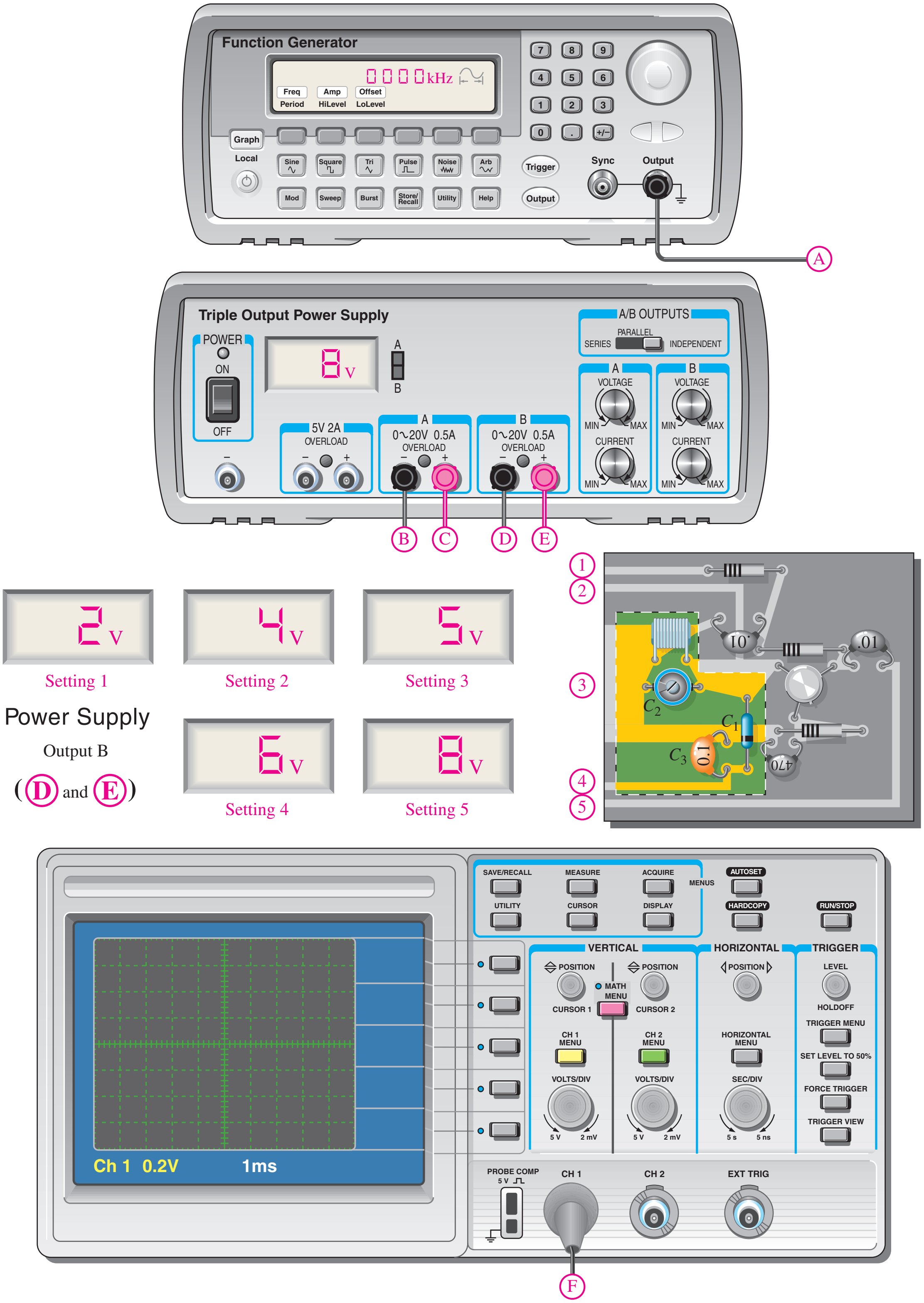
1. Calculate a capacitance setting for C_2 that will ensure a complete coverage of the AM frequency band as the varactor is varied over its capacitance range. C_3 can be ignored. The full range of resonant frequencies for the tuning circuit should more than cover the AM band, so that at the maximum varactor capacitance, the resonant frequency will be less than 535 kHz and at the minimum varactor capacitance, the resonant frequency will be greater than 1605 kHz.
2. Using the value of C_2 that you have calculated, determine the values of the varactor capacitance that will produce a resonant frequency of 535 kHz and 1605 kHz, respectively.

Testing the Resonant Circuit

3. Suggest a procedure for testing the resonant circuit using the instruments in the test bench setup of Figure 57. Develop a test setup by creating a point-to-point hook-up of the board and the instruments.
4. Using the graph in Figure 58 that shows the variation in varactor capacitance versus varactor voltage, determine the resonant frequency for each indicated setting from the *B* outputs of the dc power supply (rightmost output terminals). The *A* output of the power supply is used to provide 9 V to the amplifier. The *B* output of the power supply is used to simulate the potentiometer voltage.

Review

5. What is the AM frequency range?
6. State the purpose of the RF amplifier.
7. How is a particular frequency in the AM band selected?

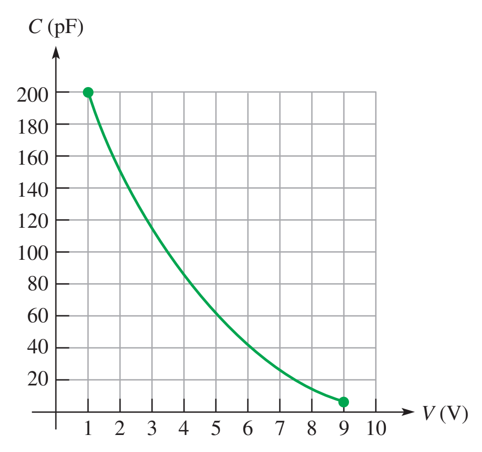

Function Generator
0000kHz
Freq
Amp
Offset
Period
HiLevel
LoLevel
Graph
Local
Sine
Square
Tri
Pulse
Noise
Arb
Mod
Sweep
Burst
Store/Recall
Utility
Help
Trigger
Output
Sync
Output
A
Triple Output Power Supply
POWER
ON
OFF
8V
A
B
5V 2A
OVERLOAD
A
0~20V 0.5A
OVERLOAD
B
0~20V 0.5A
OVERLOAD
A/B OUTPUTS
PARALLEL
SERIES
INDEPENDENT
A
VOLTAGE
MIN
MAX
CURRENT
B
VOLTAGE
CURRENT
B
C
D
E
1
2
3
4
5
10
.01
C_2
C_1
C_3
0.1
470
2V
Setting 1
4V
Setting 2
5V
Setting 3
Power Supply
Output B
(D and E)
6V
Setting 4
8V
Setting 5
SAVE/RECALL
MEASURE
ACQUIRE
MENUS
AUTOSET
UTILITY
CURSOR
DISPLAY
HARDCOPY
RUN/STOP
VERTICAL
HORIZONTAL
TRIGGER
POSITION
POSITION
POSITION
LEVEL
MATH MENU
CURSOR 1
CURSOR 2
HOLDOFF
TRIGGER MENU
CH 1 MENU
CH 2 MENU
HORIZONTAL MENU
SET LEVEL TO 50%
FORCE TRIGGER
VOLTS/DIV
VOLTS/DIV
SEC/DIV
TRIGGER VIEW
5 V
2 mV
5 V
2 mV
5 s
5 ns
Ch 1 0.2V 1ms
PROBE COMP
5 V
CH 1
CH 2
EXT TRIG
F

▲ **FIGURE 57**

Test bench setup.

▶ FIGURE 58

Varactor capacitance versus voltage.



SUMMARY

- X_L and X_C have opposing effects in an *RLC* circuit.
- In a series *RLC* circuit, the larger reactance determines the net reactance of the circuit.
- At series resonance, the inductive and capacitive reactances are equal.
- The impedance of a series *RLC* circuit is purely resistive at resonance.
- In a series *RLC* circuit, the current is maximum at resonance.
- The reactive voltages V_L and V_C cancel at resonance in a series *RLC* circuit because they are equal in magnitude and 180° out of phase.
- In a parallel *RLC* circuit, the smaller reactance determines the net reactance of the circuit.
- In a parallel resonant circuit, the impedance is maximum at the resonant frequency.
- A parallel resonant circuit is commonly called a *tank circuit.*
- The impedance of a parallel *RLC* circuit is purely resistive at resonance.
- The bandwidth of a series resonant circuit is the range of frequencies for which the current is $0.707I_{max}$ or greater.
- The bandwidth of a parallel resonant circuit is the range of frequencies for which the impedance is $0.707Z_{max}$ or greater.
- The critical frequencies are the frequencies above and below resonance where the circuit response is 70.7% of the maximum response.
- A higher Q produces a narrower bandwidth.

KEY TERMS

Half-power frequency The frequency at which the output power of a resonant circuit is 50% of the maximum (the output voltage is 70.7% of maximum); another name for *critical* or *cutoff frequency.*

Parallel resonance A condition in a parallel *RLC* circuit in which the reactances ideally are equal and the impedance is maximum.

Resonant frequency The frequency at which resonance occurs; also known as the *center frequency.*

Selectivity A measure of how effectively a resonant circuit passes certain desired frequencies and rejects all others. Generally, the narrower the bandwidth, the greater the selectivity.

Series resonance A condition in a series *RLC* circuit in which the reactances ideally cancel and the impedance is minimum.

Tank circuit A parallel resonant circuit.

FORMULAS

Series *RLC* Circuits

1 $X_{tot} = |X_L - X_C|$

2 $\mathbf{Z} = R + jX_L - jX_C$

3 $\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2} \angle \pm \tan^{-1}\left(\frac{X_{tot}}{R}\right)$

Series Resonance

4 $f_r = \frac{1}{2\pi\sqrt{LC}}$

Parallel *RLC* Circuits

5 $\mathbf{Z} = \cfrac{1}{\cfrac{1}{R\angle 0^\circ} + \cfrac{1}{X_L\angle 90^\circ} + \cfrac{1}{X_C\angle -90^\circ}}$

6 $\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$

7 $\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = jB_C$

8 $\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$

9 $\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta = G + jB_C - jB_L$

10 $\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2} \angle \tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$

Parallel Resonance

11 $I_{tot} = \frac{V_s}{Z_r}$

12 $f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{Q^2 + 1}}$

13 $f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}}$

14 $L_{\text{eq}} = L\left(\frac{Q^2 + 1}{Q^2}\right)$

15 $R_{p(\text{eq})} = R_W(Q^2 + 1)$

16 $Z_r = R_W(Q^2 + 1)$

17 $Q_O = \frac{R_{p(tot)}}{X_{L(\text{eq})}}$

18 $BW = f_2 - f_1$

19 $f_r = \frac{f_1 + f_2}{2}$

20 $BW = \frac{f_r}{Q}$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The total reactance of a series *RLC* circuit is the difference between the capacitive reactance and the inductive reactance.
2. The total impedance of a series *RLC* circuit is the algebraic sum of R, X_C, and X_L.

3. Series resonance occurs when $X_L = X_C$.
4. Below the resonant frequency, a series *RLC* circuit is predominately capacitive.
5. At the series resonant frequency, the *RLC* circuit is resistive and the current is minimum.
6. In an ideal parallel *RLC* circuit, the total current is zero at resonance.
7. At parallel resonance, the total impedance of an *RLC* circuit is maximum.
8. Below the parallel resonant frequency, the circuit is predominately capacitive.
9. The bandwidth of a parallel resonant circuit is the difference between the upper and lower critical frequencies.
10. Selectivity of a resonant circuit is better when the bandwidth is narrow.

SELF–TEST

Answers are at the end of the chapter.

1. The total reactance of a series *RLC* circuit at resonance is
   (a) zero **(b)** equal to the resistance **(c)** infinity **(d)** capacitive
2. The phase angle between the source voltage and current of a series *RLC* circuit at resonance is
   (a) $-90°$ **(b)** $+90°$ **(c)** $0°$ **(d)** dependent on the reactance
3. The impedance at the resonant frequency of a series *RLC* circuit with $L = 15$ mH, $C = 0.015\ \mu$F, and $R_W = 80\ \Omega$ is
   (a) 15 kΩ **(b)** 80 Ω **(c)** 30 Ω **(d)** 0 Ω
4. In a series *RLC* circuit that is operating below the resonant frequency, the current
   (a) is in phase with the applied voltage **(b)** lags the applied voltage
   (c) leads the applied voltage
5. If the value of *C* in a series *RLC* circuit is increased, the resonant frequency
   (a) is not affected **(b)** increases **(c)** remains the same **(d)** decreases
6. In a certain series resonant circuit, $V_C = 150$ V, $V_L = 150$ V, and $V_R = 50$ V. The value of the source voltage is
   (a) 150 V **(b)** 300 V **(c)** 50 V **(d)** 350 V
7. A certain series resonant circuit has a bandwidth of 1 kHz. If the existing coil is replaced with one having a lower value of *Q*, the bandwidth will
   (a) increase **(b)** decrease **(c)** remain the same **(d)** be more selective
8. At frequencies below resonance in a parallel *RLC* circuit, the current
   (a) leads the source voltage **(b)** lags the source voltage
   (c) is in phase with the source voltage
9. The total current into the *L* and *C* branches of a parallel circuit at resonance is ideally
   (a) maximum **(b)** low **(c)** high **(d)** zero
10. To tune a parallel resonant circuit to a lower frequency, the capacitance should be
    (a) increased **(b)** decreased **(c)** left alone **(d)** replaced with inductance
11. The resonant frequency of a parallel circuit is approximately the same as a series circuit when
    (a) the *Q* is very low **(b)** the *Q* is very high
    (c) there is no resistance **(d)** either answer (b) or (c)
12. If the resistance in parallel with a parallel resonant circuit is reduced, the bandwidth
    (a) disappears **(b)** decreases **(c)** becomes sharper **(d)** increases

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 60.

1. If R_1 opens, the total current
   (a) increases **(b)** decreases **(c)** stays the same

2. If C_1 opens, the voltage across C_2

(a) increases **(b)** decreases **(c)** stays the same

3. If L_2 opens, the voltage across it

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 63.

4. If L opens, the voltage across R

(a) increases **(b)** decreases **(c)** stays the same

5. If f is adjusted to its resonant value, the current through R

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 64.

6. If L is increased to 100 mH, the resonant frequency

(a) increases **(b)** decreases **(c)** stays the same

7. If C is increased to 100 pF, the resonant frequency

(a) increases **(b)** decreases **(c)** stays the same

8. If L becomes open, the voltage across C

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 66.

9. If R_2 becomes open, the voltage across L

(a) increases **(b)** decreases **(c)** stays the same

10. If C becomes shorted, the voltage across R_1

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 69.

11. If L_1 opens, the voltage from point a to point b

(a) increases **(b)** decreases **(c)** stays the same

12. If the frequency of the source is increased, the voltage from a to b

(a) increases **(b)** decreases **(c)** stays the same

13. If the frequency of the source voltage is increased, the current through R_1

(a) increases **(b)** decreases **(c)** stays the same

14. If the frequency of the source voltage is decreased, the voltage across C

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

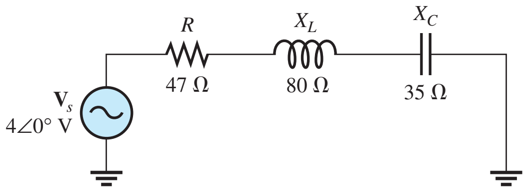
More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

PART 1: SERIES CIRCUITS

SECTION 1 Impedance of Series *RLC* Circuits

1. A certain series *RLC* circuit has the following values: $R = 10\ \Omega$, $C = 0.047\ \mu\text{F}$, and $L = 5$ mH. Determine the impedance in polar form. What is the net reactance? The source frequency is 5 kHz.
2. Find the impedance in Figure 59, and express it in polar form.

▶ FIGURE 59

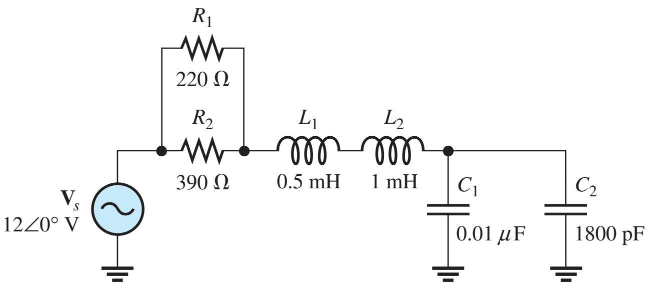


3. If the frequency of the source voltage in Figure 59 is doubled from the value that produces the indicated reactances, how does the magnitude of the impedance change?
4. For the circuit of Figure 59, determine the net reactance that will make the impedance magnitude equal to 100 Ω.

SECTION 2 Analysis of Series *RLC* Circuits

5. For the circuit in Figure 59, find $\mathbf{I}_{tot}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in polar form.
6. Draw the voltage phasor diagram for the circuit in Figure 59.
7. Analyze the circuit in Figure 60 for the following ($f = 25$ kHz):

   (a) $\mathbf{I}_{tot}$ **(b)** P_{true} **(c)** P_r **(d)** P_a

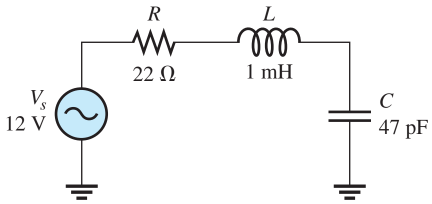


▲ FIGURE 60

SECTION 3 Series Resonance

8. For the circuit in Figure 59, is the resonant frequency higher or lower than the setting indicated by the reactance values?
9. For the circuit in Figure 61, what is the voltage across R at resonance?
10. Find X_L, X_C, Z, and I at the resonant frequency in Figure 61.

▶ FIGURE 61

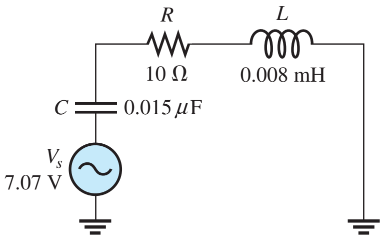


11. A certain series resonant circuit has a maximum current of 50 mA and a V_L of 100 V. The applied voltage is 10 V. What is Z? What are X_L and X_C?
12. For the *RLC* circuit in Figure 62, determine the resonant frequency.
13. What is the value of the current at the half-power points in Figure 62?
14. Determine the phase angle between the applied voltage and the current at the critical frequencies in Figure 62. What is the phase angle at resonance?

*15. Design a circuit in which the following series resonant frequencies are switch-selectable:

   (a) 500 kHz **(b)** 1000 kHz **(c)** 1500 kHz **(d)** 2000 kHz

▶ FIGURE 62



PART 2: PARALLEL CIRCUITS

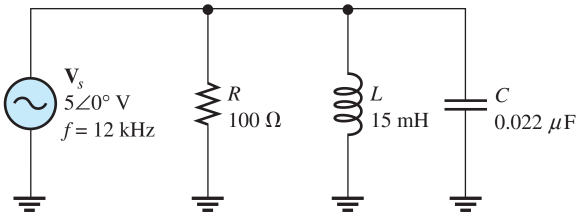
SECTION 4 Impedance of Parallel *RLC* Circuits

16. Express the impedance of the circuit in Figure 63 in polar form.

17. Is the circuit in Figure 63 capacitive or inductive? Explain.

18. At what frequency does the circuit in Figure 63 change its reactive characteristic (from inductive to capacitive or vice versa)?

▶ FIGURE 63


$\mathbf{V}_s$
$5\angle 0°$ V
$f = 12$ kHz
R
100 Ω
L
15 mH
C
0.022 μF

SECTION 5 Analysis of Parallel *RLC* Circuits

19. For the circuit in Figure 63, find all the currents and voltages in polar form.

20. Find the total impedance of the circuit in Figure 63 at 50 kHz.

21. Change the frequency to 100 kHz in Figure 63 and repeat Problem 19.

SECTION 6 Parallel Resonance

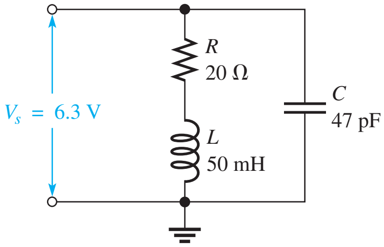
22. What is the impedance of an ideal parallel resonant circuit (no resistance in either branch)?

23. Find Z at resonance and f_r for the tank circuit in Figure 64.

24. How much current is drawn from the source in Figure 64 at resonance? What are the inductive current and the capacitive current at the resonant frequency?

25. Find P_{true}, P_r, and P_a in the circuit of Figure 64 at resonance.

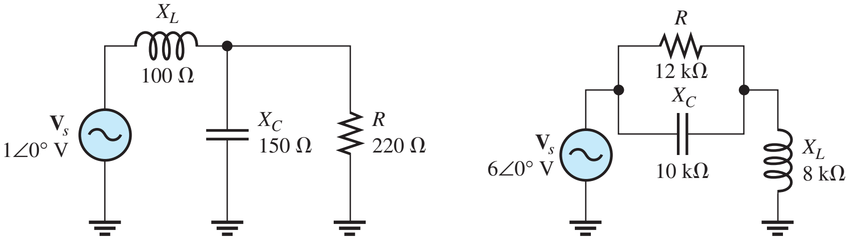
▶ FIGURE 64


$V_s = 6.3$ V
R
20 Ω
L
50 mH
C
47 pF

PART 3: SERIES-PARALLEL CIRCUITS

SECTION 7 Analysis of Series-Parallel *RLC* Circuits

26. Find the total impedance for each circuit in Figure 65.


X_L
100 Ω
$\mathbf{V}_s$
$1\angle 0°$ V
X_C
150 Ω
R
220 Ω
R
12 kΩ
X_C
10 kΩ
$\mathbf{V}_s$
$6\angle 0°$ V
X_L
8 kΩ

(a) (b)

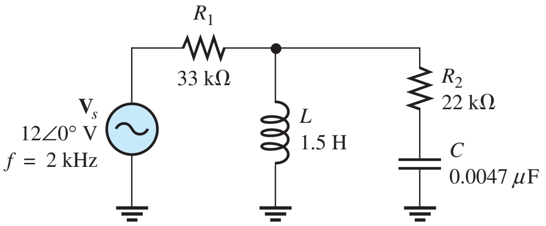
▲ FIGURE 65

27. For each circuit in Figure 65, determine the phase angle between the source voltage and the total current.

28. Determine the voltage across each element in Figure 66, and express each in polar form.

29. Convert the circuit in Figure 66 to an equivalent series form.

▶ FIGURE 66

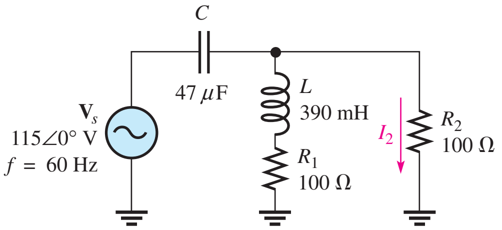


R_1
33 kΩ
$\mathbf{V}_s$
12∠0° V
f = 2 kHz
L
1.5 H
R_2
22 kΩ
C
0.0047 μF

30. What is the current through R_2 in Figure 67?

31. In Figure 67, what is the phase angle between I_2 and the source voltage?

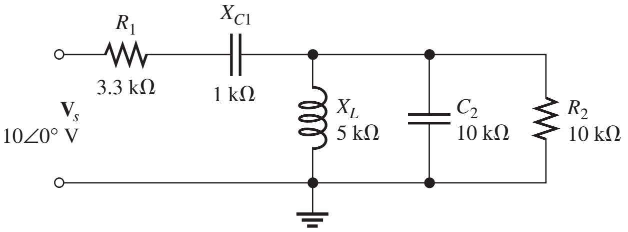
▶ FIGURE 67



C
47 μF
$\mathbf{V}_s$
115∠0° V
f = 60 Hz
L
390 mH
R_1
100 Ω
I_2
R_2
100 Ω

***32.** Determine the total resistance and the total reactance in Figure 68.

***33.** Find the current through each component in Figure 68. Find the voltage across each component.



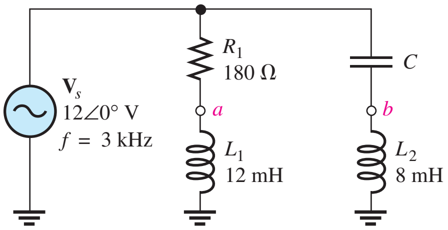
R_1
3.3 kΩ
X_{C1}
1 kΩ
$\mathbf{V}_s$
10∠0° V
X_L
5 kΩ
C_2
10 kΩ
R_2
10 kΩ

▲ FIGURE 68

34. Determine if there is a value of C that will make $V_{ab} = 0$ V in Figure 69. If not, explain.

***35.** If the value of C is 0.22 μF, what is the current through a 100 Ω resistor connected from a to b in Figure 69?

▶ FIGURE 69

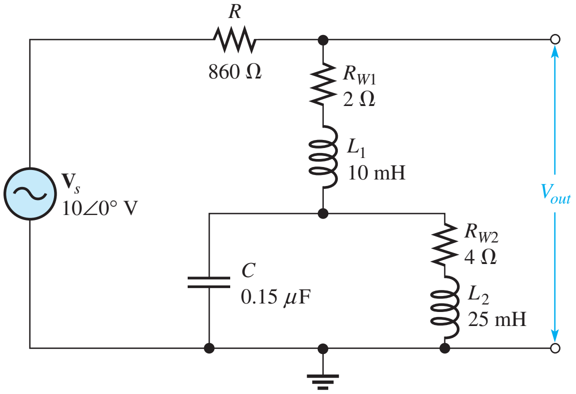


$\mathbf{V}_s$
12∠0° V
f = 3 kHz
R_1
180 Ω
a
L_1
12 mH
C
b
L_2
8 mH

***36.** How many resonant frequencies are there in the circuit of Figure 70? Why?

***37.** Determine the resonant frequencies and the output voltage at each frequency in Figure 70.

▶ FIGURE 70



***38.** Design a parallel-resonant network using a single coil and switch-selectable capacitors to produce the following resonant frequencies: 8 MHz, 9 MHz, 10 MHz, and 11 MHz. Assume a 10 μH coil with a winding resistance of 5 Ω.

PART 4: SPECIAL TOPICS

SECTION 8 Bandwidth of Resonant Circuits

39. At resonance, $X_L = 2\text{ k}\Omega$ and $R_W = 25\ \Omega$ in a parallel *RLC* circuit. The resonant frequency is 5 kHz. Determine the bandwidth.

40. If the lower critical frequency is 2400 Hz and the upper critical frequency is 2800 Hz, what is the bandwidth? What is the resonant frequency?

41. In a certain *RLC* circuit, the power at resonance is 2.75 W. What is the power at the lower critical frequency?

***42.** What values of L and C should be used in a tank circuit to obtain a resonant frequency of 8 kHz? The bandwidth must be 800 Hz. The winding resistance of the coil is 10 Ω.

43. A parallel resonant circuit has a Q of 50 and a BW of 400 Hz. If Q is doubled, what is the bandwidth for the same f_r?

Multisim Troubleshooting and Analysis

These problems require Multisim.

44. Open file P17-44 and determine if there is a fault. If so, find the fault.

45. Open file P17-45 and determine if there is a fault. If so, find the fault.

46. Open file P17-46 and determine if there is a fault. If so, find the fault.

47. Open file P17-47 and determine if there is a fault. If so, find the fault.

48. Open file P17-48 and determine if there is a fault. If so, find the fault.

49. Open file P17-49 and determine if there is a fault. If so, find the fault.

50. Open file P17-50 and determine the resonant frequency of the circuit.

51. Open file P17-51 and determine the resonant frequency of the circuit.

ANSWERS

SECTION CHECKUPS

SECTION 1 Impedance of Series *RLC* Circuits

1. $X_{tot} = 70\ \Omega$; capacitive

2. $\mathbf{Z} = 84.3\angle -56.1^\circ\ \Omega$; $Z = 84.3\ \Omega$; $\theta = -56.1^\circ$; current is leading V_s.

SECTION 2 Analysis of Series *RLC* Circuits

1. $\mathbf{V}_s = 38.4\angle -21.3^\circ$ V
2. Current leads the voltage.
3. $X_{tot} = 600\ \Omega$

SECTION 3 Series Resonance

1. For series resonance, $X_L = X_C$.
2. The current is maximum because the impedance is minimum.
3. $f_r = 159$ kHz
4. The circuit is capacitive.

SECTION 4 Impedance of Parallel *RLC* Circuits

1. The circuit is capacitive.
2. $\mathbf{Y} = 1.54\angle 49.4^\circ$ mS
3. $\mathbf{Z} = 651\angle -49.4^\circ\ \Omega$

SECTION 5 Analysis of Parallel *RLC* Circuits

1. $I_R = 80$ mA, $I_C = 120$ mA, $I_L = 240$ mA
2. The circuit is capacitive.

SECTION 6 Parallel Resonance

1. Impedance is maximum at parallel resonance.
2. The current is minimum.
3. $X_C = 1500\ \Omega$
4. $f_r = 225$ kHz
5. $f_r = 22.5$ kHz
6. $f_r = 20.9$ kHz

SECTION 7 Analysis of Series-Parallel *RLC* Circuits

1. $R_{p(\text{eq})} = 130\ \Omega$, $L_{\text{eq}} = 101.6\ \mu\text{H}$, $C = 0.22\ \mu\text{F}$
2. $L_{(\text{eq})} = 20.1$ mH, $R_{p(\text{eq})} = 1.59$ kΩ

SECTION 8 Bandwidth of Resonant Circuits

1. $BW = f_2 - f_1 = 400$ kHz
2. $f_r = 2$ MHz
3. $P_{f2} = 0.9$ W
4. Larger Q means narrower BW.

SECTION 9 Applications

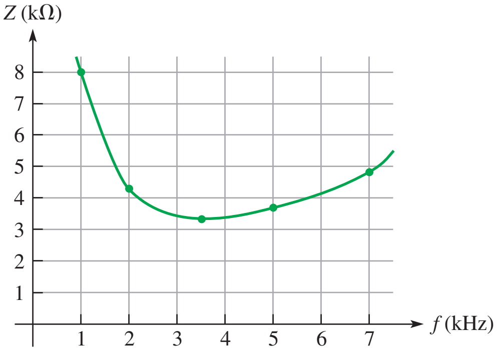
1. A tuned filter is used to select a narrow band of frequencies.
2. A crossover network is a filter network that separates the audio signal into different frequency bands for the speakers while maintaining an overall flat response.
3. Ganged tuning is done with several capacitors (or inductors) whose values can be varied simultaneously with a common control.

RELATED PROBLEMS FOR EXAMPLES

1 $\mathbf{Z} = 12.7\angle 82.3^\circ\ \text{k}\Omega$

2 $\mathbf{Z} = 4.72\angle 45.6^\circ\ \text{k}\Omega$. See Figure 71.

▶ FIGURE 71



3 Current will increase with frequency to a certain point and then it will decrease.

4 The circuit is more capacitive.

5 $f_r = 22.5$ kHz

6 45°

7 Z increases; Z increases.

8 Z decreases.

9 Inductive

10 I_{tot} increases.

11 Greater

12 $\mathbf{V}_C = 0.93\angle -65.8^\circ$ V

13 $\mathbf{V}_{C1} = 27.1\angle -81.1^\circ$ V

14 $R_{p(\text{eq})} = 25\ \text{k}\Omega, L_{\text{eq}} = 5$ mH; $C = 0.022\ \mu$F

15 $Z_r = 79.9\ \text{k}\Omega$

16 $I = 35.4$ mA

17 $f_1 = 6.75$ kHz; $f_2 = 9.25$ kHz

18 $BW = 7.96$ kHz

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F **6.** T **7.** T **8.** F **9.** T **10.** T

SELF-TEST

1. (a) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (c) **7.** (a) **8.** (b)
9. (d) **10.** (a) **11.** (b) **12.** (d)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (b) **7.** (b) **8.** (c)
9. (a) **10.** (a) **11.** (a) **12.** (a) **13.** (b) **14.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. $520\angle -88.9^\circ\ \Omega$; $520\ \Omega$ capacitive

3. Impedance increases to $150\ \Omega$

5. $\mathbf{I}_{tot} = 61.4\angle -43.8^\circ$ mA
$\mathbf{V}_R = 2.89\angle -43.8^\circ$ V
$\mathbf{V}_L = 4.91\angle 46.2^\circ$ V
$\mathbf{V}_C = 2.15\angle -134^\circ$ V

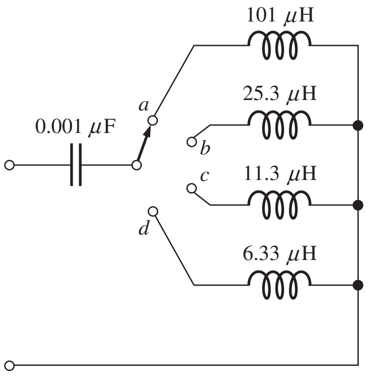
7. **(a)** $35.8\angle 65.1^\circ$ mA
(b) 181 mW
(c) 390 mVAR
(d) 430 mVA

9. 12 V

11. $Z = 200\ \Omega$, $X_C = X_L = 2\ \text{k}\Omega$

13. 500 mA

15. See Figure 72.



▲ **FIGURE 72**

17. The phase angle of -4.43° indicates a slightly capacitive circuit.

19. $\mathbf{I}_R = 50\angle 0^\circ$ mA
$\mathbf{I}_L = 4.42\angle -90^\circ$ mA
$\mathbf{I}_C = 8.29\angle 90^\circ$ mA
$\mathbf{I}_{tot} = 50.2\angle 4.43^\circ$ mA
$\mathbf{V}_R = \mathbf{V}_L = \mathbf{V}_C = 5\angle 0^\circ$ V

21. $\mathbf{I}_R = 50\angle 0^\circ$ mA, $\mathbf{I}_L = 531\angle -90^\circ\ \mu$A,
$\mathbf{I}_C = 69.1\angle 90^\circ\ \mu$A, $\mathbf{I}_{tot} = 84.9\angle 53.9^\circ$ mA

23. $53.5\ \text{M}\Omega$, 104 kHz

25. $P_r = 0$ VAR, $P_a = 7.45\ \mu$VA, $P_{\text{true}} = 538$ mW

27. **(a)** -1.97° (V_s lags I_{tot}) **(b)** 23.0° (V_s leads I_{tot})

29. $49.1\ \text{k}\Omega$ resistor in series with 1.38 H inductor

31. 45.2° (I_2 leads V_s)

33. $\mathbf{I}_{R1} = \mathbf{I}_{C1} = 1.09\angle -25.7^\circ$ mA
$\mathbf{I}_{R2} = 767\angle 19.3^\circ\ \mu$A
$\mathbf{I}_{C2} = 767\angle 109.3^\circ\ \mu$A
$\mathbf{I}_L = 1.53\angle -70.7^\circ$ mA
$\mathbf{V}_{R2} = \mathbf{V}_{C2} = \mathbf{V}_L = 7.67\angle 19.3^\circ$ V
$\mathbf{V}_{R1} = 3.60\angle -25.7^\circ$ V
$\mathbf{V}_{C1} = 1.09\angle -116^\circ$ V

35. $52.2\angle 126^\circ$ mA

37. $f_{r(\text{series})} = 4.11$ kHz
$\mathbf{V}_{out} = 4.83\angle -61.0^\circ$ V
$f_{r(\text{parallel})} = 2.6$ kHz
$\mathbf{V}_{out} \cong 10\angle 0^\circ$ V

39. 62.5 Hz

41. 1.38 W

43. 200 Hz

45. C_1 leaky

47. C_1 leaky

49. No fault

51. $f_c \approx 338.698$ kHz

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Passive Filters

CHAPTER OUTLINE

1 Low-Pass Filters
2 High-Pass Filters
3 Band-Pass Filters
4 Band-Stop Filters
Application Activity

CHAPTER OBJECTIVES

- Analyze the operation of *RC* and *RL* low-pass filters
- Analyze the operation of *RC* and *RL* high-pass filters
- Analyze the operation of band-pass filters
- Analyze the operation of band-stop filters

KEY TERMS

- Low-pass filter
- Passband
- Critical frequency (f_c)
- Roll-off
- Attenuation
- Decade
- Bode plot
- High-pass filter
- Band-pass filter
- Center frequency (f_0)
- Band-stop filter

APPLICATION ACTIVITY PREVIEW

In the application activity, you will plot the frequency responses of filters based on oscilloscope measurements and identify the types of filters.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

Passive filters are discussed in this chapter. Passive filters use various combinations of resistors, capacitors, and inductors. You should have already seen how basic *RC*, *RL*, and *RLC* circuits can be used as filters. Now, you will learn that passive filters can be placed in four general categories according to their response characteristics: low-pass, high-pass, band-pass, and band-stop. Within each category, there are several common types that will be examined.

From Chapter 18 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 LOW-PASS FILTERS

A **low-pass filter** allows signals with lower frequencies to pass from input to output while rejecting higher frequencies.

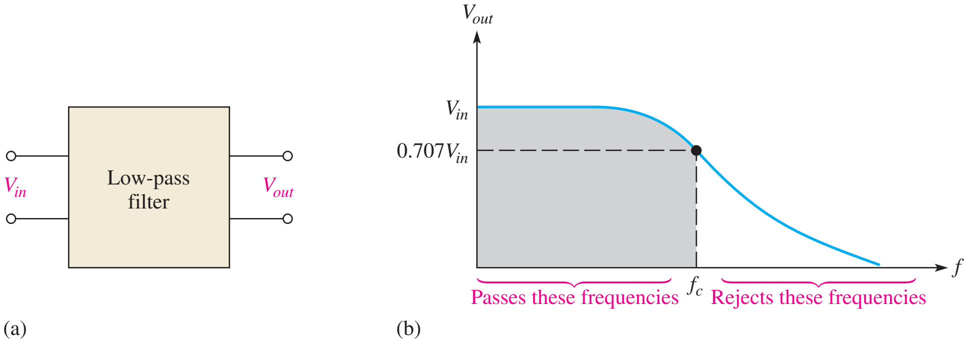
After completing this section, you should be able to

- **Analyze the operation of *RC* and *RL* low-pass filters**
    - Express the voltage and power ratios of a filter in decibels
    - Determine the critical frequency of a low-pass filter
    - Explain the difference between actual and ideal low-pass response curves
    - Define *roll-off*
    - Generate a Bode plot for a low-pass filter
    - Discuss phase shift in a low-pass filter

Figure 1 shows a block diagram and a general response curve for a low-pass filter. The range of frequencies passed by a filter within specified limits is called the **passband** of the filter. The point considered to be the upper end of the passband is at the critical frequency, f_c, as illustrated in Figure 1(b). The **critical frequency (f_c)** is the frequency at which the filter's output voltage is 70.7% of the maximum. The filter's critical frequency is also called the *cutoff frequency, break frequency,* or *−3 dB frequency* because the output voltage is down 3 dB from its maximum at this frequency. The term *dB* (*decibel*) is a commonly used unit in filter measurements.

FIGURE 1

Low-pass filter block diagram and general response curve.



Decibels

The basis for the decibel unit stems from the logarithmic response of the human ear to the intensity of sound. The **decibel** is a logarithmic measurement of the ratio of one power to another or one voltage to another, which can be used to express the input-to-output relationship of a filter. The following equation expresses a power ratio in decibels:

Equation 1

$$\text{dB} = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

From the properties of logarithms, the following decibel formula for a voltage ratio is derived.

$$\text{dB} = 20\log\left(\frac{V_{out}}{V_{in}}\right)$$ **Equation 2**

EXAMPLE 1 At a certain frequency, the output voltage of a filter is 5 V and the input is 10 V. Express the voltage ratio in decibels.

Solution

$$20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log\left(\frac{5\text{ V}}{10\text{ V}}\right) = 20\log(0.5) = \mathbf{-6.02\ dB}$$

*Related Problem** Express the ratio $V_{out}/V_{in} = 0.85$ in decibels.

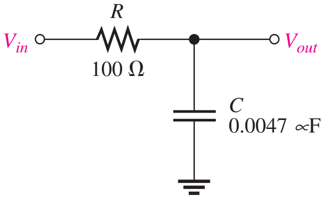
---

*Answers are at the end of the chapter.

RC Low-Pass Filter

A basic *RC* low-pass filter is shown in Figure 2. Notice that the output voltage is taken across the capacitor.

When the input is dc (0 Hz), the output voltage equals the input voltage because X_C is infinitely large. As the input frequency is increased, X_C decreases and, as a result, V_{out} gradually decreases until a frequency is reached where $X_C = R$. This is the critical frequency, f_c, of the filter.



▲ **FIGURE 2**

$$X_C = \frac{1}{2\pi f_c C} = R$$

Solving for f_c,

$$f_c = \frac{1}{2\pi RC}$$ **Equation 3**

At any frequency, by application of the voltage-divider formula, the output voltage magnitude is

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

Since $X_C = R$ at f_c, the output voltage at the critical frequency can be expressed as

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + R^2}}\right)V_{in} = \left(\frac{R}{\sqrt{2R^2}}\right)V_{in} = \left(\frac{R}{R\sqrt{2}}\right)V_{in} = \left(\frac{1}{\sqrt{2}}\right)V_{in} = 0.707V_{in}$$

These calculations show that the output is 70.7% of the input when $X_C = R$. The frequency at which this occurs is, by definition, the critical frequency.

The ratio of output voltage to input voltage at the critical frequency can be expressed in decibels as follows:

$$V_{out} = 0.707V_{in}$$

$$\frac{V_{out}}{V_{in}} = 0.707$$

$$20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log(0.707) = -3\text{ dB}$$

EXAMPLE 2 Determine the critical frequency and V_{out} at f_c for an input of 10 V for the *RC* low-pass filter in Figure 2.

Solution

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(100\ \Omega)(0.0047\ \mu\text{F})} = \mathbf{339\ kHz}$$

The output voltage is 3 dB below V_{in} at this frequency (V_{out} has a maximum value of V_{in}).

$$V_{out} = 0.707V_{in} = \mathbf{7.07\ V}$$

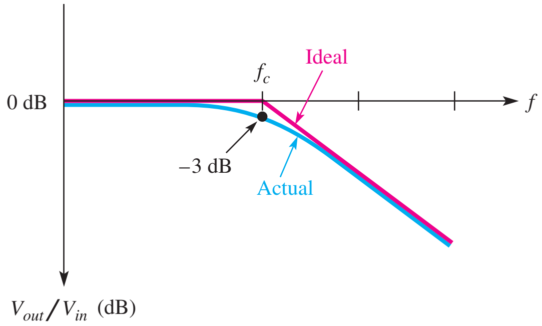
Related Problem A certain *RC* low-pass filter has $R = 1.0\ \text{k}\Omega$ and $C = 0.022\ \mu\text{F}$. Determine its critical frequency.

Roll-Off of the Response Curve

The blue line in Figure 3 shows an actual response curve for a low-pass filter. The maximum output is defined to be 0 dB as a reference. Zero decibels corresponds to $V_{out} = V_{in}$ because $20 \log(V_{out}/V_{in}) = 20 \log 1 = 0$ dB. The output drops from 0 dB to -3 dB at the critical frequency and then continues to decrease at a fixed rate. This pattern of decrease is called the **roll-off** of the frequency response. The red line shows an ideal output response that is considered to be "flat" out to the critical frequency. The output then decreases at the fixed rate.

FIGURE 3

Actual and ideal response curves for a low-pass filter.



As you have seen, the output voltage of a low-pass filter decreases by 3 dB when the frequency is increased to the critical value f_c. As the frequency continues to increase above f_c, the output voltage continues to decrease. In fact, for each tenfold increase in frequency above f_c, there is a 20 dB reduction in the output, as shown in the following steps.

Let's take a frequency that is ten times the critical frequency ($f = 10f_c$). Since $R = X_C$ at f_c, then $R = 10X_C$ at $10f_c$ because of the inverse relationship of X_C and f.

The **attenuation** is the reduction in voltage expressed as the ratio V_{out}/V_{in} and is developed as follows:

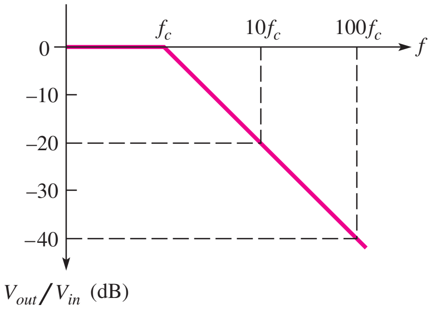
$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{X_C}{\sqrt{(10X_C)^2 + X_C^2}}$$

$$= \frac{X_C}{\sqrt{100X_C^2 + X_C^2}} = \frac{X_C}{\sqrt{X_C^2(100 + 1)}} = \frac{X_C}{X_C\sqrt{101}} = \frac{1}{\sqrt{101}} \cong \frac{1}{10} = 0.1$$

The dB attenuation is

$$20 \log\left(\frac{V_{out}}{V_{in}}\right) = 20 \log(0.1) = -20\ \text{dB}$$

A tenfold change in frequency is called a **decade**. So, for an *RC* circuit, the output voltage is reduced by 20 dB for each decade increase in frequency. A similar result can be derived for a high-pass circuit. The roll-off is a constant −20 dB/decade for a basic *RC* or *RL* filter. Figure 4 shows the ideal frequency response plot on a semilog scale, where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is called a **Bode plot**.

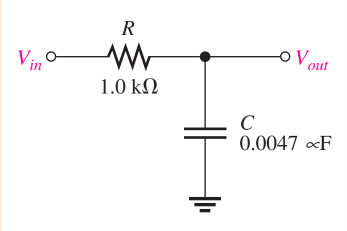


▲ **FIGURE 4**

Frequency roll-off for an *RC* low-pass filter (Bode plot).

EXAMPLE 3

Make a Bode plot for the filter in Figure 5 for three decades of frequency. Use semilog graph paper.



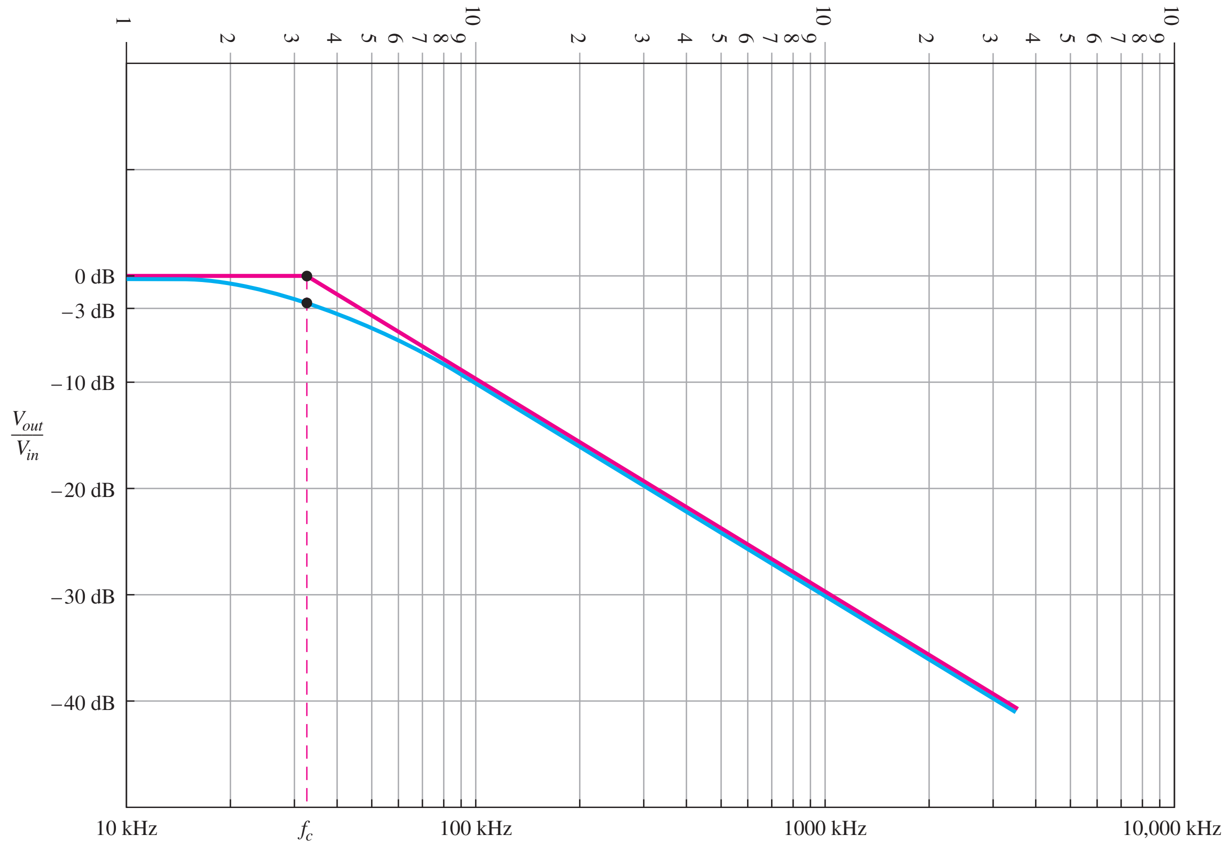
▲ **FIGURE 5**

Solution The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.0\ \text{k}\Omega)(0.0047\ \mu\text{F})} = 33.9\ \text{kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 6. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

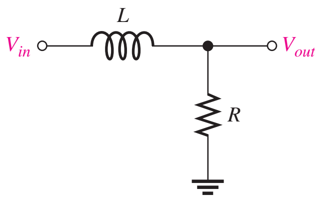
The output is flat below f_c (33.9 kHz). As the frequency is increased above f_c, the output drops at a −20 dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at −3 dB rather than 0 dB at the critical frequency.



▲ FIGURE 6

Bode plot for Figure 5. The red line represents the ideal response curve and the blue line represents the actual response.

Related Problem What happens to the critical frequency and roll-off rate if C is reduced to 0.001 μF in Figure 5?



▲ FIGURE 7

RL low-pass filter.

RL Low-Pass Filter

A basic *RL* low-pass filter is shown in Figure 7. Notice that the output voltage is taken across the resistor.

When the input is dc (0 Hz), the output voltage ideally equals the input voltage because X_L is a short (if R_W is neglected). As the input frequency is increased, X_L increases and, as a result, V_{out} gradually decreases until the critical frequency is reached. At this point, $X_L = R$ and the frequency is

$$2\pi f_c L = R$$

$$f_c = \frac{R}{2\pi L}$$

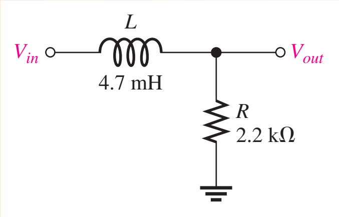
Equation 4

$$f_c = \frac{1}{2\pi(L/R)}$$

Just as in the *RC* low-pass filter, $V_{out} = 0.707V_{in}$ and, thus, the output voltage is −3 dB below the input voltage at the critical frequency.

EXAMPLE 4 Make a Bode plot for the filter in Figure 8 for three decades of frequency. Use semilog graph paper.

FIGURE 8



Solution The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi(L/R)} = \frac{1}{2\pi(4.7\text{ mH}/2.2\text{ k}\Omega)} = 74.5\text{ kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 9. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

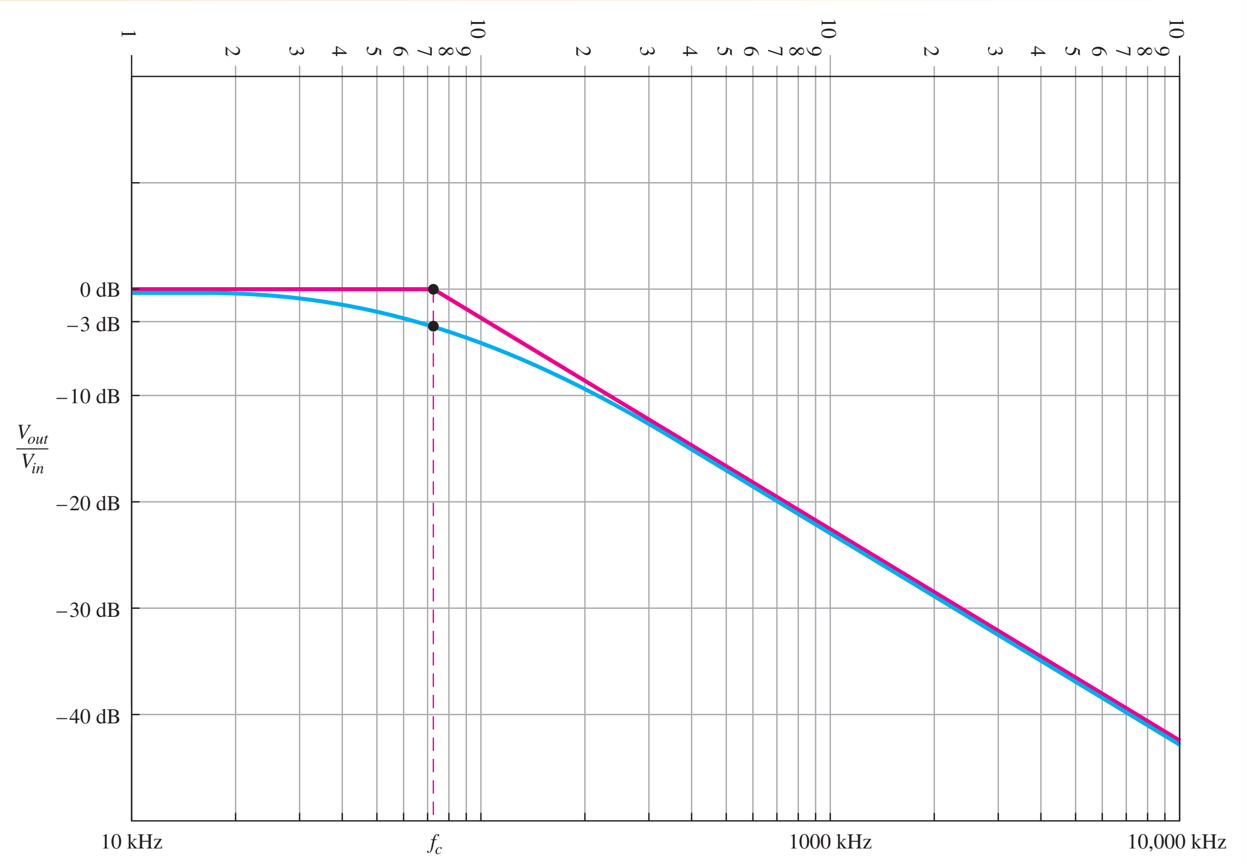


FIGURE 9

Bode plot for Figure 8. The red line is the ideal response curve and the blue line is the actual response.

The output is flat below f_c (74.5 kHz). As the frequency is increased above f_c, the output drops at a -20 dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at -3 dB rather than 0 dB at the critical frequency.

Related Problem What happens to the critical frequency and roll-off rate if L is reduced to 1 mH in Figure 8?

Use Multisim files E18-04A and E18-04B to verify the calculated results in this example and to confirm your calculations for the related problem.

Phase Shift in a Low-Pass Filter

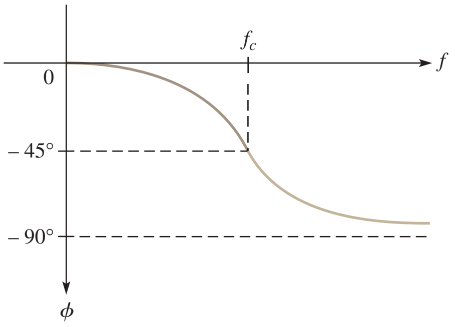
The *RC* low-pass filter acts as a lag circuit. Recall that the phase shift from input to output is expressed as

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

At the critical frequency, $X_C = R$ and, therefore, $\phi = -45°$. As the input frequency is reduced, ϕ decreases and approaches 0° when the frequency approaches zero. Figure 10, illustrates this phase characteristic.

FIGURE 10

Phase characteristic of a low-pass filter.



The *RL* low-pass filter also acts as a lag circuit. Recall that the phase shift is expressed as

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

As in the *RC* filter, the phase shift from input to output is $-45°$ at the critical frequency and decreases for frequencies below f_c.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. In a certain low-pass filter, $f_c = 2.5$ kHz. What is its passband?
2. In a certain low-pass filter, $R = 100\ \Omega$ and $X_C = 2\ \Omega$ at a frequency, f_1. Determine $\mathbf{V}_{out}$ at f_1 when $\mathbf{V}_{in} = 5\angle 0°$ V rms.
3. $V_{out} = 400$ mV, and $V_{in} = 1.2$ V. Express the ratio V_{out}/V_{in} in dB.

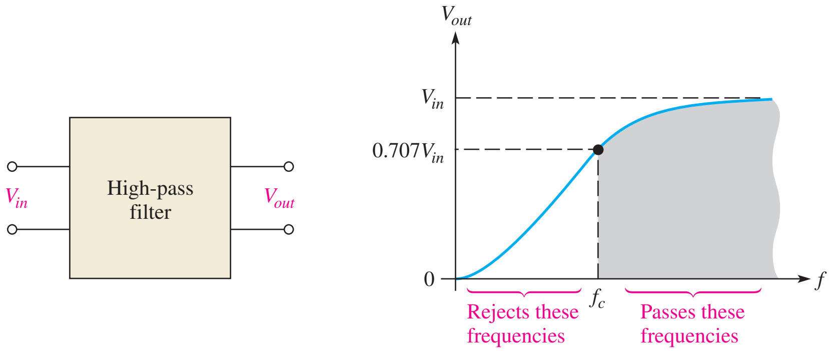
2 HIGH-PASS FILTERS

A **high-pass filter** allows signals with higher frequencies to pass from input to output while rejecting lower frequencies.

After completing this section, you should be able to

- **Analyze the operation of *RC* and *RL* high-pass filters**
    - Determine the critical frequency of a high-pass filter
    - Explain the difference between actual and ideal response curves
    - Generate a Bode plot for a high-pass filter
    - Discuss phase shift in a high-pass filter

Figure 11 shows a block diagram and a general response curve for a high-pass filter. The frequency considered to be the lower end of the passband is called the *critical frequency.* Just as in the low-pass filter, it is the frequency at which the output is 70.7% of the maximum, as indicated in the figure.

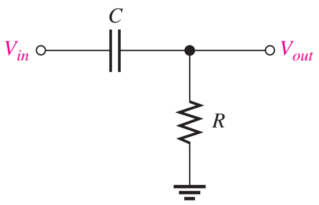


▲ FIGURE 11

High-pass filter block diagram and response curve.

RC High-Pass Filter

A basic *RC* high-pass filter is shown in Figure 12. Notice that the output voltage is taken across the resistor.



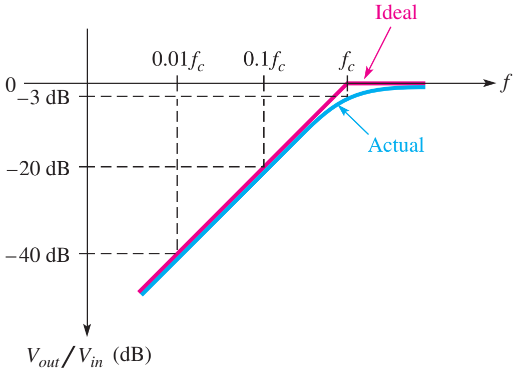
◀ FIGURE 12

RC high-pass filter.

When the input frequency is at its critical value, $X_C = R$ and the output voltage is $0.707V_{in}$, just as in the case of the low-pass filter. As the input frequency increases above the critical frequency, X_C decreases and, as a result, the output voltage increases and approaches a value equal to V_{in}. The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi RC}$$

Below f_c, the output voltage decreases (rolls off) at a rate of -20 dB/decade. Figure 13 shows an actual and an ideal response curve for a high-pass filter.



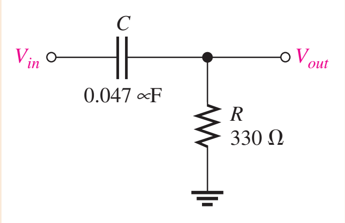
▲ **FIGURE 13**

Actual and ideal response curves for a high-pass filter.

EXAMPLE 5

Make a Bode plot for the filter in Figure 14 for three decades of frequency. Use semilog graph paper.

▶ **FIGURE 14**



Solution The critical frequency for this high-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(330\ \Omega)(0.047\ \mu\text{F})} = 10.3\ \text{kHz} \cong 10\ \text{kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 15. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

The output is flat above f_c (approximately 10 kHz). As the frequency is reduced below f_c, the output drops at a -20 dB/decade rate. Thus, for the ideal curve, every time the frequency is reduced by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at -3 dB rather than 0 dB at the critical frequency.

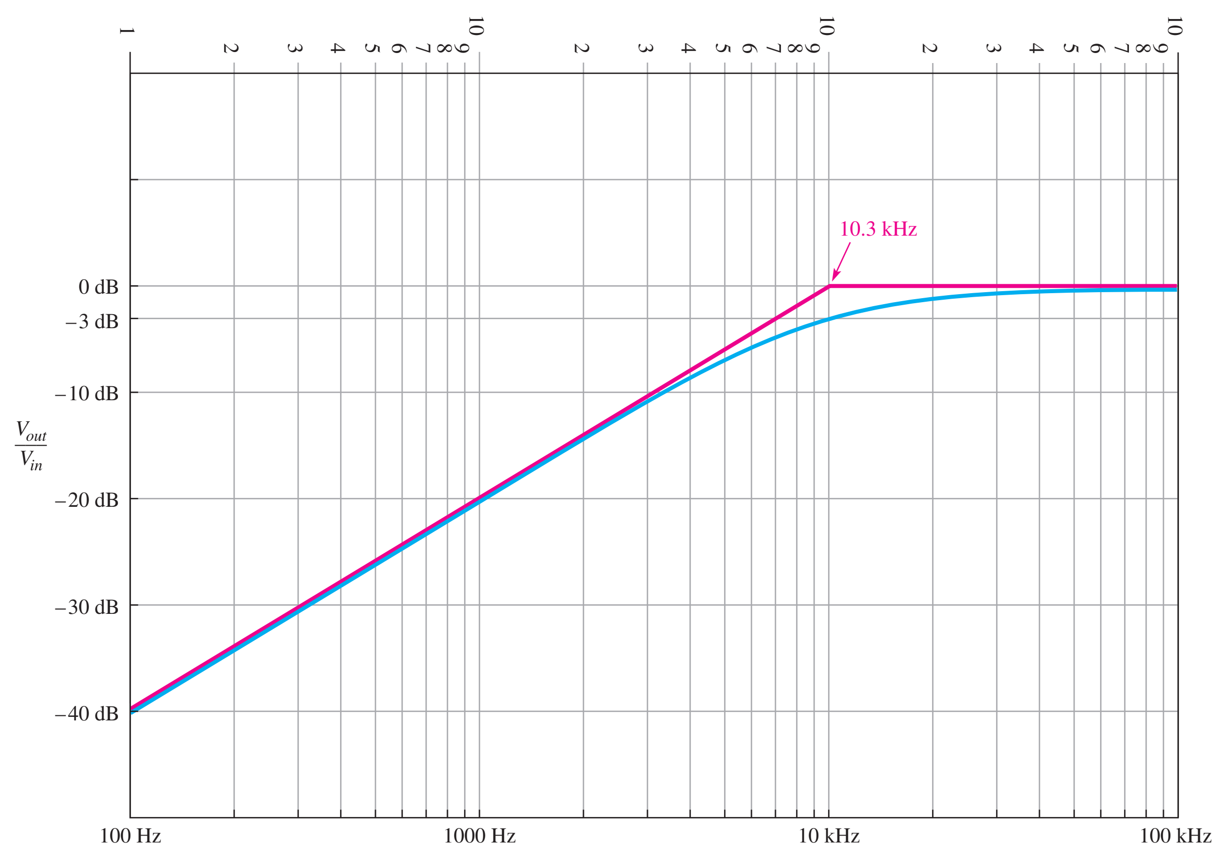


FIGURE 15

Bode plot for Figure 14. The red line is the ideal response curve and the blue line is the actual response curve.

Related Problem If the frequency for the high-pass filter is decreased to 10 Hz, what is the output to input ratio in decibels?

Use Multisim file E18-05 to verify the calculated results in this example and to confirm your calculation for the related problem.

RL High-Pass Filter

A basic *RL* high-pass filter is shown in Figure 16. Notice that the output is taken across the inductor.

When the input frequency is at its critical value, $X_L = R$, and the output voltage is $0.707V_{in}$. As the frequency increases above f_c, X_L increases and, as a result, the output

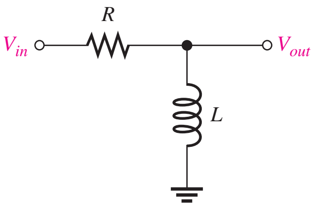


FIGURE 16

RL high-pass filter.

voltage increases until it equals V_{in}. The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi(L/R)}$$

Phase Shift in a High-Pass Filter

Both the *RC* and the *RL* high-pass filters act as lead circuits. Recall that the phase shift from input to output for the *RC* lead circuit is

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

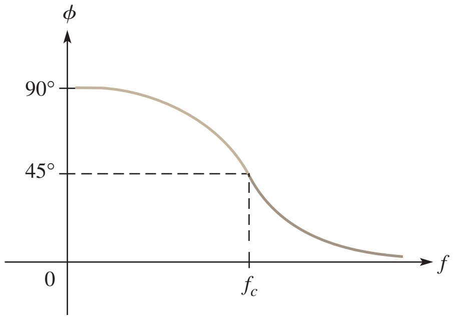
and the phase shift for the *RL* lead circuit is

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

At the critical frequency, $X_L = R$ and, therefore, $\phi = 45°$. As the frequency is increased, ϕ decreases toward 0°, as shown in Figure 17.

FIGURE 17

Phase characteristic of a high-pass filter.

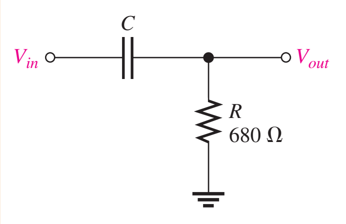


EXAMPLE 6

(a) In Figure 18, find the value of C so that X_C is approximately ten times less than R at an input frequency of 10 kHz.

(b) If a 5 V sine wave with a dc level of 10 V is applied, what are the output voltage magnitude and the phase shift?

FIGURE 18



Solution **(a)** Determine the value of C as follows:

$$X_C = 0.1R = 0.1(680\ \Omega) = 68\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(10\text{ kHz})(68\ \Omega)} = \mathbf{0.234\ \mu F}$$

Using the nearest standard value of $C = 0.22\ \mu\text{F}$,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(10\text{ kHz})(0.22\ \mu\text{F})} = 72\ \Omega$$

(b) Determine the magnitude of the sinusoidal output using the voltage-divider formula.

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in} = \left(\frac{680\ \Omega}{\sqrt{(680\ \Omega)^2 + (72\ \Omega)^2}}\right)5\text{ V} = \mathbf{4.97\ V}$$

The phase shift is

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{72\ \Omega}{680\ \Omega}\right) = \mathbf{6.04^\circ}$$

At $f = 10$ kHz, which is a decade above the critical frequency, the sinusoidal output is almost equal to the input in magnitude, and the phase shift is very small. The 10 V dc level has been filtered out and does not appear at the output.

Related Problem Repeat parts (a) and (b) of the example if R is changed to 220 Ω.

Use Multisim files E18-06A and E18-06B to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 2 CHECKUP

1. The input voltage of a high-pass filter is 1 V. What is V_{out} at the critical frequency?
2. In a certain high-pass filter, $V_{in} = 10\angle 0^\circ$ V, $R = 1.0$ kΩ, and $X_L = 15$ kΩ. Determine V_{out}.

3 BAND-PASS FILTERS

A **band-pass filter** allows a certain band of frequencies to pass and attenuates or rejects all frequencies below and above the passband.

After completing this section, you should be able to

- **Analyze the operation of band-pass filters**
  - Define *bandwidth*
  - Show how a band-pass filter is implemented with low-pass and high-pass filters
  - Explain the series-resonant band-pass filter
  - Explain the parallel-resonant band-pass filter
  - Calculate the bandwidth and output voltage of a band-pass filter

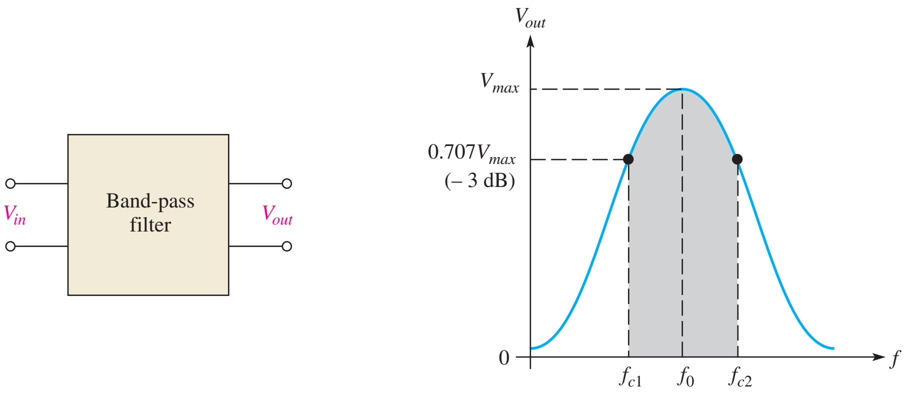
The bandwidth of a band-pass filter is the range of frequencies for which the current, and therefore the output voltage, is equal to or greater than 70.7% of its value at the resonant frequency.

As you know, bandwidth is often abbreviated *BW* and can be calculated as

$$BW = f_{c2} - f_{c1}$$

where f_{c1} is the lower cutoff frequency and f_{c2} is the upper cutoff frequency.

Figure 19 shows a typical band-pass response curve.



▲ **FIGURE 19**

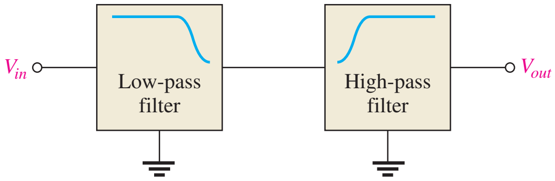
Typical band-pass response curve.

Low-Pass/High-Pass Filter

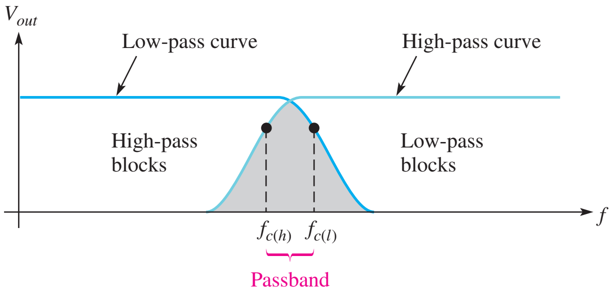
A combination of a low-pass and a high-pass filter can be used to form a band-pass filter, as illustrated in Figure 20. The loading effect of the second filter on the first must be taken into account.

▶ **FIGURE 20**

A low-pass and a high-pass filter are used to form a band-pass filter.



If the critical frequency of the low-pass filter, $f_{c(l)}$, is higher than the critical frequency of the high-pass filter, $f_{c(h)}$, the responses overlap. Thus, all frequencies except those between $f_{c(h)}$ and $f_{c(l)}$ are eliminated, as shown in Figure 21.



▲ **FIGURE 21**

Overlapping response curves of a low-pass/high-pass filter.

EXAMPLE 7

A high-pass filter with $f_c = 2$ kHz and a low-pass filter with $f_c = 2.5$ kHz are used to construct a band-pass filter. Assuming no loading effect, what is the bandwidth of the passband?

Solution

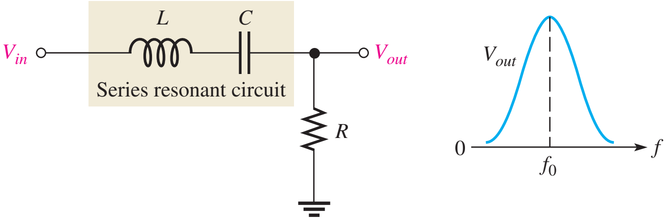
$$BW = f_{c(l)} - f_{c(h)} = 2.5\text{ kHz} - 2\text{ kHz} = \mathbf{500\ Hz}$$

Related Problem

If $f_{c(l)} = 9$ kHz and the bandwidth is 1.5 kHz, what is $f_{c(h)}$?

Series Resonant Band-Pass Filter

A type of series resonant band-pass filter is shown in Figure 22. A series resonant circuit has minimum impedance and maximum current at the resonant frequency, f_r. Thus, most of the input voltage is dropped across the resistor at the resonant frequency. Therefore, the output across R has a band-pass characteristic with a maximum output at the frequency of resonance. The resonant frequency is called the **center frequency, f_0**. The bandwidth is determined by the quality factor, Q, of the circuit and the resonant frequency. Recall that $Q = X_L/R$, where R is the resistance in the circuit.



▲ FIGURE 22

Series resonant band-pass filter.

A higher value of Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth. A formula for the bandwidth of a resonant circuit in terms of Q is stated in the following equation:

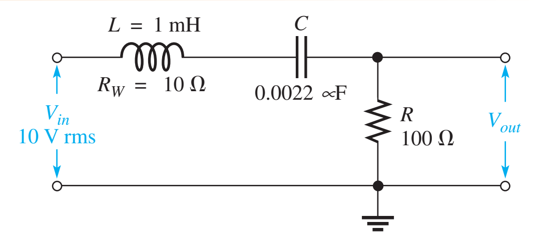
$$BW = \frac{f_0}{Q}$$

Equation 5

EXAMPLE 8

Determine the output voltage magnitude at the center frequency (f_0) and the bandwidth for the filter in Figure 23.

▶ FIGURE 23



Solution At f_0, X_C and X_L cancel and the impedance of the resonant circuit is equal to the winding resistance, R_W. The total circuit resistance is $R_L + R_W$. By the voltage-divider formula,

$$V_{out} = \left(\frac{R_L}{R_L + R_W}\right)V_{in} = \left(\frac{100\ \Omega}{110\ \Omega}\right)10\ \text{V} = \mathbf{9.09\ V}$$

The center frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\ \text{mH})(0.0022\ \mu\text{F})}} = 107\ \text{kHz}$$

At f_0, the inductive reactance is

$$X_L = 2\pi fL = 2\pi(107\text{ kHz})(1\text{ mH}) = 672\ \Omega$$

and the total resistance is

$$R_{tot} = R_L + R_W = 100\ \Omega + 10\ \Omega = 110\ \Omega$$

Therefore, the circuit Q is

$$Q = \frac{X_L}{R_{tot}} = \frac{672\ \Omega}{110\ \Omega} = 6.11$$

The bandwidth is

$$BW = \frac{f_0}{Q} = \frac{107\text{ kHz}}{6.11} = \mathbf{17.5\ kHz}$$

Related Problem If a 1 mH coil with a winding resistance of 18 Ω replaces the existing coil in Figure 23, how is the bandwidth affected?

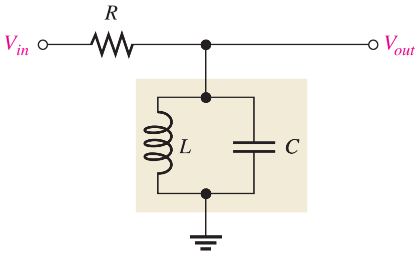
Use Multisim files E18-08A and E18-08B to verify the calculated results in this example and to confirm your answer for the related problem.

Parallel Resonant Band-Pass Filter

A type of band-pass filter using a parallel resonant circuit is shown in Figure 24. Recall that a parallel resonant circuit has maximum impedance at resonance. The circuit in Figure 24 acts as a voltage divider. At resonance, the impedance of the tank circuit is much greater than the resistance. Thus, most of the input voltage is across the tank circuit, producing a maximum output voltage at the resonant (center) frequency.

▶ FIGURE 24

Parallel resonant band-pass filter.

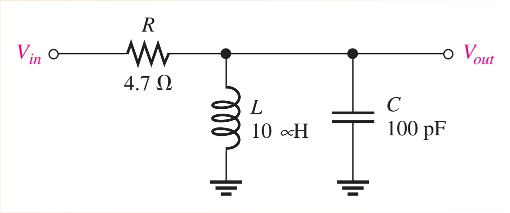


For frequencies above or below resonance, the tank circuit impedance drops off, and more of the input voltage is across R. As a result, the output voltage across the tank circuit drops off, creating a band-pass characteristic.

EXAMPLE 9

What is the center frequency of the filter in Figure 25? Assume $R_W = 0\ \Omega$.

▶ FIGURE 25



Solution The center frequency of the filter is its resonant frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10\ \mu\text{H})(100\ \text{pF})}} = \mathbf{5.03\ MHz}$$

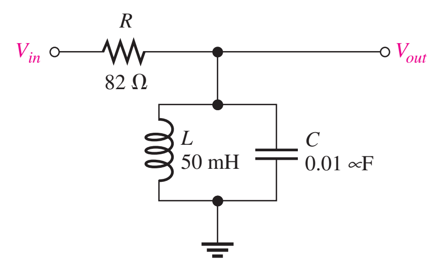
Related Problem Determine f_0 in Figure 25 if C is changed to 1000 pF.

Use Multisim files E18-09A and E18-09B to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 10

Determine the center frequency and bandwidth for the band-pass filter in Figure 26 if the inductor has a winding resistance of 15 Ω.

▶ FIGURE 26



Solution Recall that the resonant (center) frequency of a nonideal tank circuit is

$$f_0 = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (15\ \Omega)^2(0.01\ \mu\text{F})/50\ \text{mH}}}{2\pi\sqrt{(50\ \text{mH})(0.01\mu\text{F})}} = \mathbf{7.12\ kHz}$$

The Q of the coil at resonance is

$$Q = \frac{X_L}{R_W} = \frac{2\pi f_0 L}{R_W} = \frac{2\pi(7.12\ \text{kHz})(50\ \text{mH})}{15\ \Omega} = 149$$

The bandwidth of the filter is

$$BW = \frac{f_0}{Q} = \frac{7.12\ \text{kHz}}{149} = \mathbf{47.8\ Hz}$$

Note that since $Q > 10$, the simpler formula, $f_0 = 1/(2\pi\sqrt{LC})$, could have been used to calculate f_0.

Related Problem Knowing the value of Q, recalculate f_0 using the simpler formula.

SECTION 3 CHECKUP

1. For a band-pass filter, $f_{c(h)} = 29.8$ kHz and $f_{c(l)} = 30.2$ kHz. What is the bandwidth?
2. A parallel resonant band-pass filter has the following values: $R_W = 15\ \Omega$, $L = 50\ \mu\text{H}$, and $C = 470$ pF. Determine the approximate center frequency.

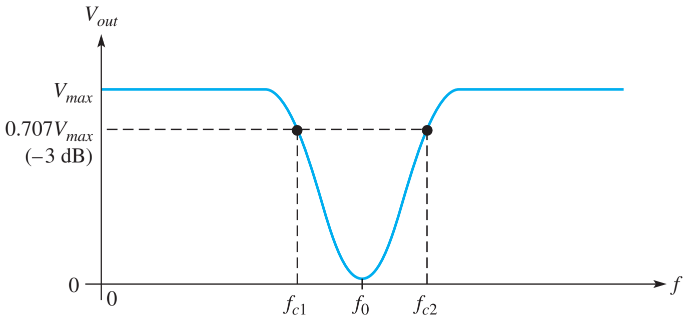
4 BAND-STOP FILTERS

A band-stop filter is essentially the opposite of a band-pass filter in terms of the responses. A **band-stop filter** allows all frequencies to pass except those lying within a certain stopband.

After completing this section, you should be able to

- **Analyze the operation of band-stop filters**
  - Show how a band-stop filter is implemented with low-pass and high-pass filters
  - Explain the series-resonant band-stop filter
  - Explain the parallel-resonant band-stop filter
  - Calculate the bandwidth and output voltage of a band-stop filter

Figure 27 shows a general band-stop response curve.

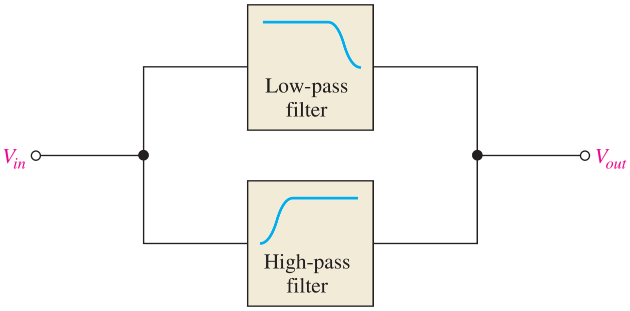


▲ FIGURE 27

General band-stop response curve.

Low-Pass/High-Pass Filter

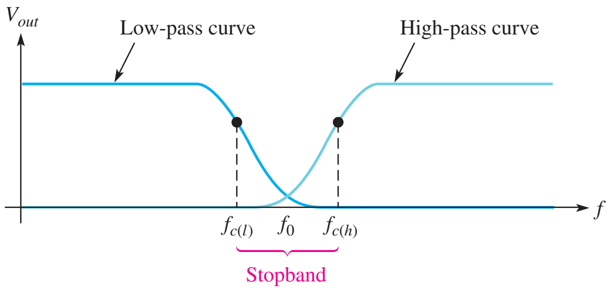
A band-stop filter can be formed from a low-pass and a high-pass filter, as shown in Figure 28.



▲ FIGURE 28

A low-pass and a high-pass filter are used to form a band-stop filter.

If the low-pass critical frequency, $f_{c(l)}$, is set lower than the high-pass critical frequency, $f_{c(h)}$, a band-stop characteristic is formed as illustrated in Figure 29.

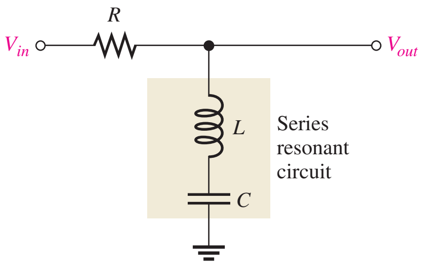


▲ **FIGURE 29**

Band-stop response curve.

Series Resonant Band-Stop Filter

A series resonant circuit used in a band-stop configuration is shown in Figure 30. Basically, it works as follows: At the resonant frequency, the impedance is minimum, and therefore the output voltage is minimum. Most of the input voltage is dropped across R. At frequencies above and below resonance, the impedance increases, causing more voltage across the output.



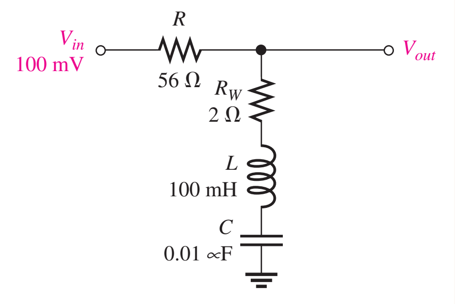
◀ **FIGURE 30**

Series resonant band-stop filter.

EXAMPLE 11

Find the output voltage magnitude at f_0 and the bandwidth in Figure 31.

▶ **FIGURE 31**



Solution Since $X_L = X_C$ at resonance, they cancel, leaving R_W. The output voltage is determined using the voltage-divider formula.

$$V_{out} = \left(\frac{R_W}{R + R_W}\right)V_{in} = \left(\frac{2\ \Omega}{58\ \Omega}\right)100\text{ mV} = \mathbf{3.45\ mV}$$

To determine the bandwidth, first calculate the center frequency and Q of the circuit.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.01\ \mu\text{F})}} = 5.03\text{ kHz}$$

$$Q = \frac{X_L}{R_{(tot)}} = \frac{2\pi f L}{R + R_W} = \frac{2\pi(5.03\text{ kHz})(100\text{ mH})}{58\ \Omega} = \frac{3.16\text{ k}\Omega}{58\ \Omega} = 54.5$$

$$BW = \frac{f_0}{Q} = \frac{5.03\text{ kHz}}{54.5} = \mathbf{92.3\ Hz}$$

Related Problem Assume $R_W = 10\ \Omega$ in Figure 31. Determine V_{out} and the bandwidth.

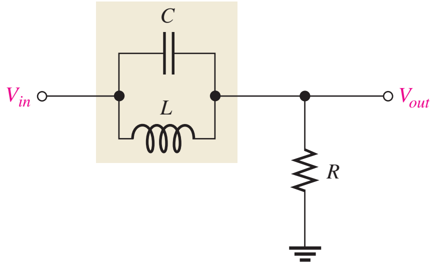
Use Multisim files E18-11A and E18-11B to verify the calculated results in this example and to confirm your calculation for the related problem.

Parallel Resonant Band-Stop Filter

A parallel resonant circuit used in a band-stop configuration is shown in Figure 32. At the resonant frequency, the tank impedance is maximum, and so most of the input voltage appears across it. Very little voltage is across R at resonance. As the tank impedance decreases above and below resonance, the output voltage increases.

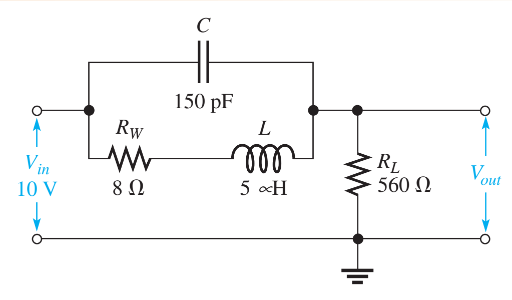
▶ FIGURE 32

Parallel resonant band-stop filter.



EXAMPLE 12

Find the center frequency of the filter in Figure 33. Draw the output response curve showing the minimum and maximum voltages.



▲ FIGURE 33

Solution The center frequency is

$$f_0 = \frac{\sqrt{1 - R_W^2 C/L}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (8\ \Omega)^2(150\ \text{pF})/5\ \mu\text{H}}}{2\pi\sqrt{(5\ \mu\text{H})(150\ \text{pF})}} = \mathbf{5.79\ MHz}$$

At the center (resonant) frequency,

$$X_L = 2\pi f_0 L = 2\pi(5.79\ \text{MHz})(5\ \mu\text{H}) = 182\ \Omega$$

$$Q = \frac{X_L}{R_W} = \frac{182\ \Omega}{8\ \Omega} = 22.8$$

$$Z_r = R_W(Q^2 + 1) = 8\ \Omega(22.8^2 + 1) = 4.17\ \text{k}\Omega \quad \text{(purely resistive)}$$

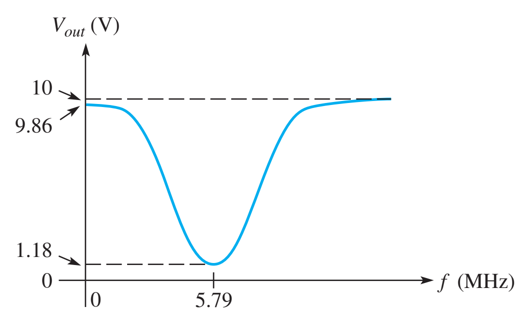
Next, use the voltage-divider formula to find the minimum output voltage magnitude.

$$V_{out(min)} = \left(\frac{R_L}{R_L + Z_r}\right)V_{in} = \left(\frac{560\ \Omega}{4.73\ \text{k}\Omega}\right)10\ \text{V} = 1.18\ \text{V}$$

At zero frequency, the impedance of the tank circuit is R_W because $X_C = \infty$ and $X_L = 0\ \Omega$. Therefore, the maximum output voltage below resonance is

$$V_{out(max)} = \left(\frac{R_L}{R_L + R_W}\right)V_{in} = \left(\frac{560\ \Omega}{568\ \Omega}\right)10\ \text{V} = 9.86\ \text{V}$$

As the frequency increases much higher than f_0, X_C approaches 0 Ω, and V_{out} approaches V_{in} (10 V). Figure 34 shows the response curve.



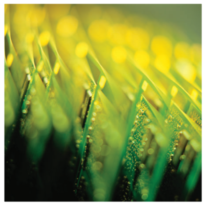
▲ FIGURE 34

Related Problem What is the minimum output voltage if $R_L = 1.0\ \text{k}\Omega$ in Figure 33?

Use Multisim files E18-12A and E18-12B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 4 CHECKUP

1. How does a band-stop filter differ from a band-pass filter?
2. Name three basic ways to construct a band-stop filter.

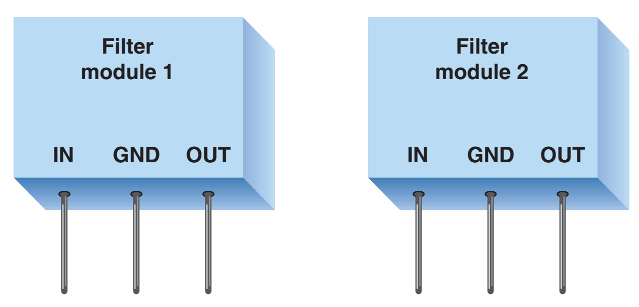


Application Activity

In this application activity, you will plot the frequency responses of two types of filters based on a series of oscilloscope measurements and identify the type of filter in each case. The filters are contained in sealed modules as shown in Figure 35. You are concerned only with determining the filter response characteristics and not the types of internal components.

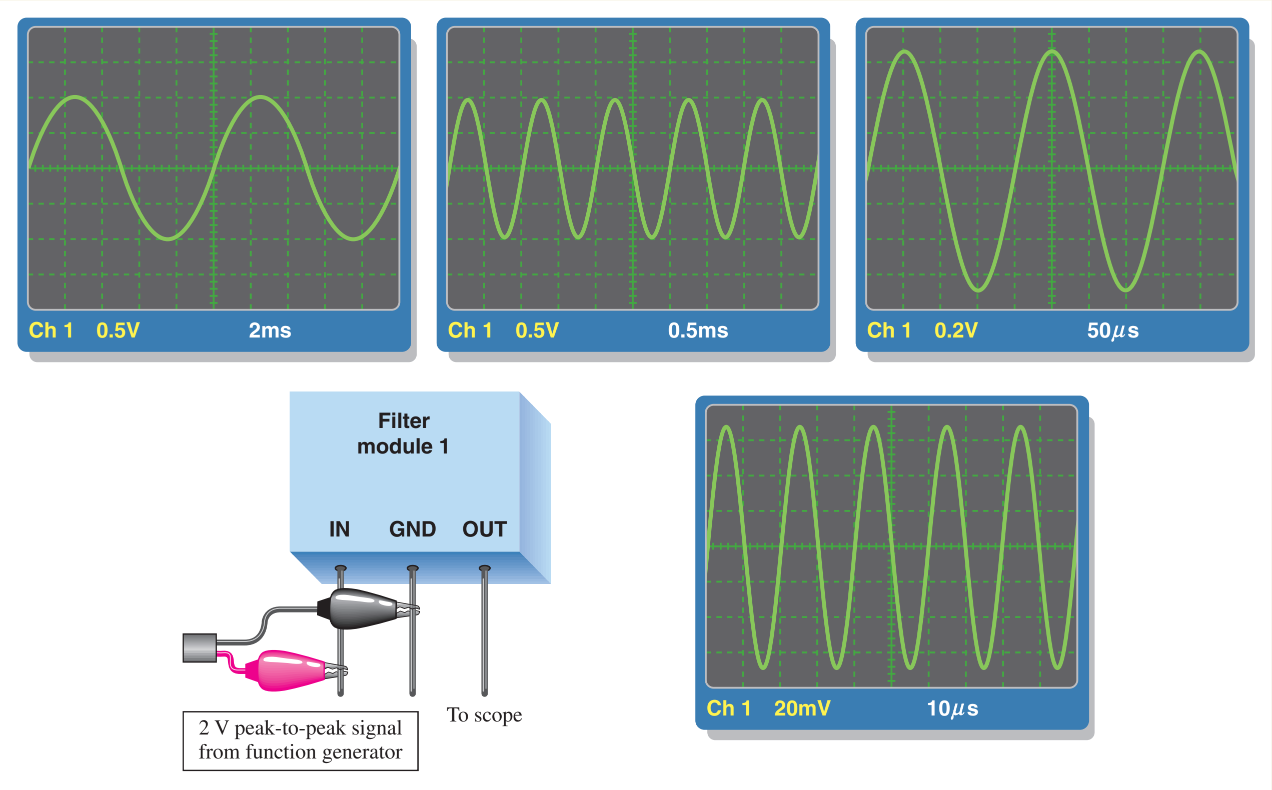
Filter Measurement and Analysis

1. Refer to Figure 36. Based on the series of four oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.
2. Refer to Figure 37. Based on the series of six oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.

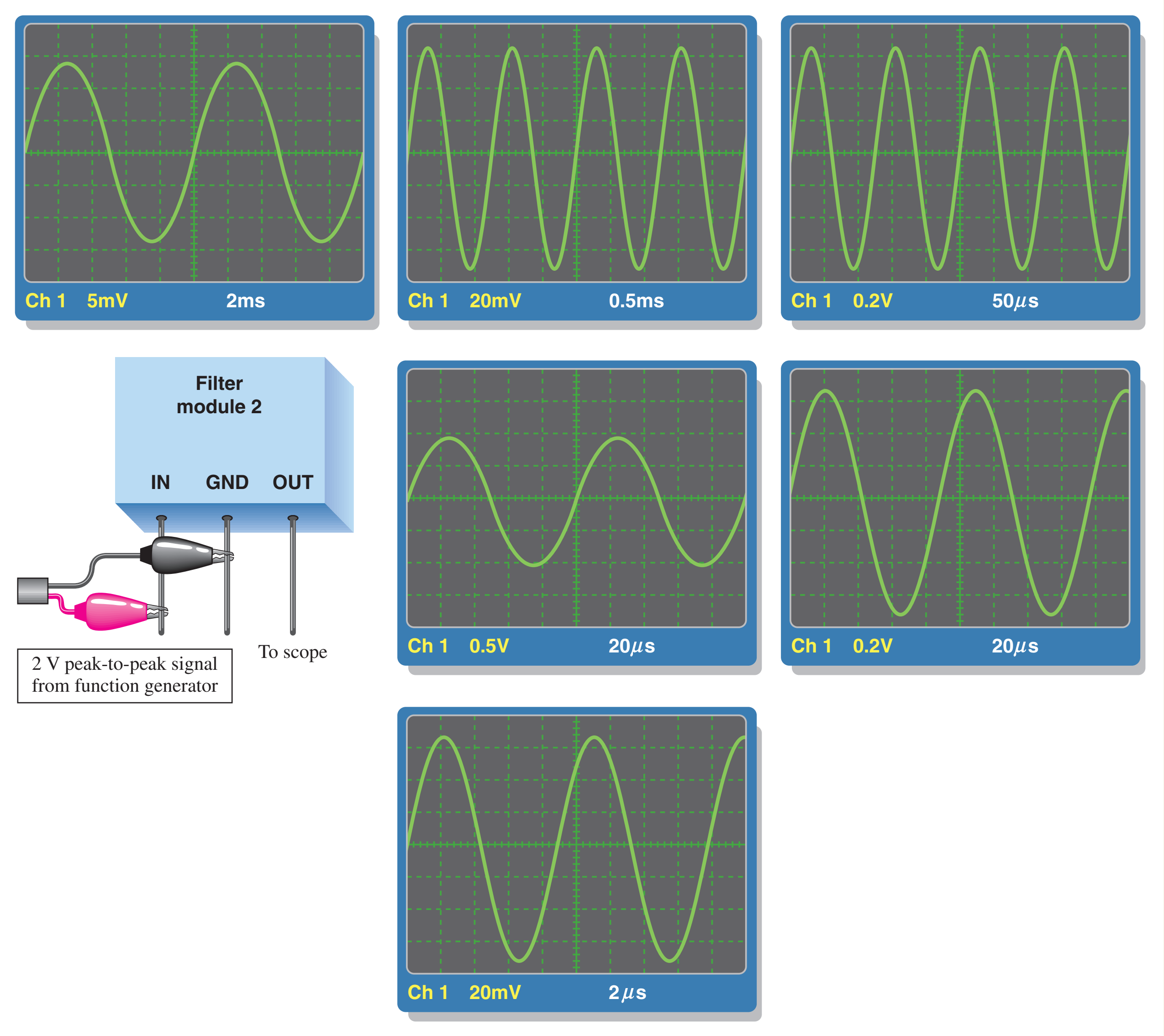

Filter module 1
IN
GND
OUT
Filter module 2
IN
GND
OUT

▶ FIGURE 35

Filter modules.


Ch 1 0.5V 2ms
Ch 1 0.5V 0.5ms
Ch 1 0.2V 50μs
Filter module 1
IN
GND
OUT
2 V peak-to-peak signal from function generator
To scope
Ch 1 20mV 10μs

▲ FIGURE 36


Ch 1 5mV 2ms
Ch 1 20mV 0.5ms
Ch 1 0.2V 50μs
Filter module 2
IN GND OUT
2 V peak-to-peak signal from function generator
To scope
Ch 1 0.5V 20μs
Ch 1 0.2V 20μs
Ch 1 20mV 2μs

▲ FIGURE 37

Review

3. Explain how the waveforms in Figure 36 indicate the type of filter.
4. Explain how the waveforms in Figure 37 indicate the type of filter.

SUMMARY

- Four categories of passive filters according to their response characteristics are low-pass, high-pass, band-pass, and band-stop.
- In an *RC* low-pass filter, the output voltage is taken across the capacitor and the output lags the input.
- In an *RL* low-pass filter, the output voltage is taken across the resistor and the output lags the input.
- In an *RC* high-pass filter, the output is taken across the resistor and the output leads the input.
- In an *RL* high-pass filter, the output is taken across the inductor and the output leads the input.

- The roll-off rate of a basic *RC* or *RL* filter is −20 dB per decade.
- A band-pass filter passes frequencies between the lower and upper critical frequencies and rejects all others.
- A band-stop filter rejects frequencies between its lower and upper critical frequencies and passes all others.
- The bandwidth of a resonant filter is determined by the quality factor (Q) of the circuit and the resonant frequency.
- Critical frequencies are also called −3 dB frequencies.
- The output voltage is 70.7% of its maximum at the critical frequencies.

KEY TERMS

Attenuation A reduction of the output signal compared to the input signal, resulting in a ratio with a value of less than 1 for the output voltage to the input voltage of a circuit.

Band-pass filter A filter that passes a range of frequencies lying between two critical frequencies and rejects frequencies above and below that range.

Band-stop filter A filter that rejects a range of frequencies lying between two critical frequencies and passes frequencies above and below that range.

Bode plot The graph of a filter's frequency response showing the change in the output voltage to input voltage ratio expressed in dB as a function of frequency for a constant input voltage.

Center frequency (f_0) The resonant frequency of a band-pass or band-stop filter.

Critical frequency (f_c) The frequency at which a filter's output voltage is 70.7% of the maximum.

Decade A tenfold change in frequency or other parameter.

High-pass filter A type of filter that passes all frequencies above a critical frequency and rejects all frequencies below that critical frequency.

Low-pass filter A type of filter that passes all frequencies below a critical frequency and rejects all frequencies above that critical frequency.

Passband The range of frequencies passed by a filter.

Roll-off The rate of decrease of a filter's frequency response.

FORMULAS

1	$\mathrm{dB} = 10\log\left(\frac{P_{out}}{P_{in}}\right)$	Power ratio in decibels
2	$\mathrm{dB} = 20\log\left(\frac{V_{out}}{V_{in}}\right)$	Voltage ratio in decibels
3	$f_c = \frac{1}{2\pi RC}$	Critical frequency
4	$f_c = \frac{1}{2\pi(L/R)}$	Critical frequency
5	$BW = \frac{f_0}{Q}$	Bandwidth

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The bandwidth of a low-pass filter ideally extends from 0 Hz up to a specified critical frequency.
2. The decibel is a unit for expressing the ratio of one frequency to another.
3. In an *RC* low-pass filter, the output is taken across the capacitor.
4. Roll-off is the rate at which the output of a low-pass filter decreases above the critical frequency.
5. Attenuation is the ratio of input voltage to output voltage.
6. In an *RL* high-pass filter, the output is taken across the inductor.

7. A high-pass filter acts as a lag circuit.

8. If the output voltage of a band-pass filter is 1 V at the resonant frequency, it is 0.707 V at the critical frequencies.

9. The frequency at which resonance occurs in a band-pass filter is sometimes called the center frequency.

10. The output of a band-stop filter is maximum at the resonant frequency.

SELF-TEST

Answers are at the end of the chapter.

1. The maximum output voltage of a certain low-pass filter is 10 V. The output voltage at the critical frequency is

(a) 10 V **(b)** 0 V **(c)** 7.07 V **(d)** 1.414 V

2. A sinusoidal voltage with a peak-to-peak value of 15 V is applied to an *RC* low-pass filter. If the reactance at the input frequency is zero, the output voltage is

(a) 15 V peak-to-peak **(b)** zero
(c) 10.6 V peak-to-peak **(d)** 7.5 V peak-to-peak

3. The same signal in Question 2 is applied to an *RC* high-pass filter. If the reactance is zero at the input frequency, the output voltage is

(a) 15 V peak-to-peak **(b)** zero
(c) 10.6 V peak-to-peak **(d)** 7.5 V peak-to-peak

4. At the critical frequency, the output of a filter is down from its maximum by

(a) 0 dB **(b)** -3 dB **(c)** -20 dB **(d)** -6 dB

5. If the output of a low-pass *RC* filter is 12 dB below its maximum at $f = 1$ kHz, then at $f = 10$ kHz, the output is below its maximum by

(a) 3 dB **(b)** 10 dB **(c)** 20 dB **(d)** 32 dB

6. In a passive filter, the ratio V_{out}/V_{in} is called

(a) roll-off **(b)** gain **(c)** attenuation **(d)** critical reduction

7. For each decade increase in frequency above the critical frequency, the output of a low-pass filter decreases by

(a) 20 dB **(b)** 3 dB **(c)** 10 dB **(d)** 0 dB

8. At the critical frequency, the phase shift through a high-pass filter is

(a) 90° **(b)** 0° **(c)** 45° **(d)** dependent on the reactance

9. In a series resonant band-pass filter, a higher value of *Q* results in

(a) a higher resonant frequency **(b)** a smaller bandwidth
(c) a higher impedance **(d)** a larger bandwidth

10. At series resonance,

(a) $X_C = X_L$ **(b)** $X_C > X_L$ **(c)** $X_C < X_L$

11. In a certain parallel resonant band-pass filter, the resonant frequency is 10 kHz. If the bandwidth is 2 kHz, the lower critical frequency is

(a) 5 kHz **(b)** 12 kHz **(c)** 9 kHz **(d)** not determinable

12. In a band-pass filter, the output voltage at the resonant frequency is

(a) minimum **(b)** maximum
(c) 70.7% of maximum **(d)** 70.7% of minimum

13. In a band-stop filter, the output voltage at the critical frequencies is

(a) minimum **(b)** maximum
(c) 70.7% of maximum **(d)** 70.7% of minimum

14. At a sufficiently high value of *Q*, the resonant frequency for a parallel resonant filter is ideally

(a) much greater than the resonant frequency of a series resonant filter
(b) much less than the resonant frequency of a series resonant filter
(c) equal to the resonant frequency of a series resonant filter

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 38(a).

1. If the frequency of the input voltage is increased, V_{out}

(a) increases **(b)** decreases **(c)** stays the same

2. If C is increased, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 38(d).

3. If the frequency of the input voltage is increased, V_{out}

(a) increases **(b)** decreases **(c)** stays the same

4. If L is increased, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 40.

5. If the switch is thrown from position 1 to position 2, the critical frequency

(a) increases **(b)** decreases **(c)** stays the same

6. If the switch is thrown from position 2 to position 3, the critical frequency

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 41(a).

7. If the frequency of the input voltage is increased, V_{out}

(a) increases **(b)** decreases **(c)** stays the same

8. If R is increased to 180 Ω, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 42.

9. If the switch is thrown from position 1 to position 2, the critical frequency

(a) increases **(b)** decreases **(c)** stays the same

10. If the switch is in position 3 and R_5 opens, V_{out}

(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 48.

11. If L_2 opens, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

12. If C becomes shorted, the output voltage

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

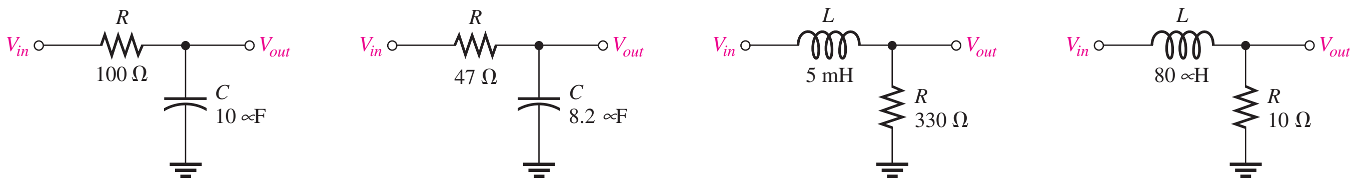
SECTION 1 Low-Pass Filters

1. In a certain low-pass filter, $X_C = 500\ \Omega$ and $R = 2.2\ \text{k}\Omega$. What is the output voltage ($\mathbf{V}_{out}$) when the input is 10 V rms?

2. A certain low-pass filter has a critical frequency of 3 kHz. Determine which of the following frequencies are passed and which are rejected:

(a) 100 Hz **(b)** 1 kHz **(c)** 2 kHz **(d)** 3 kHz **(e)** 5 kHz

3. Determine the output voltage ($\mathbf{V}_{out}$) of each filter in Figure 38 at the specified frequency when $V_{in} = 10$ V.



R
V_in
V_out
100 Ω
C
10 ∝F
R
V_in
V_out
47 Ω
C
8.2 ∝F
L
V_in
V_out
5 mH
R
330 Ω
L
V_in
V_out
80 ∝H
R
10 Ω

(a) $f = 60$ Hz (b) $f = 400$ Hz (c) $f = 1$ kHz (d) $f = 2$ kHz

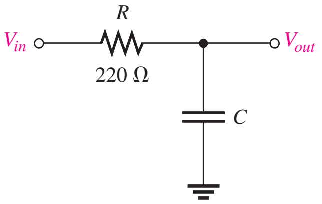
▲ **FIGURE 38**

4. What is f_c for each filter in Figure 38? Determine the output voltage at f_c in each case when $V_{in} = 5$ V.

5. For the filter in Figure 39, calculate the value of C required for each of the following critical frequencies:

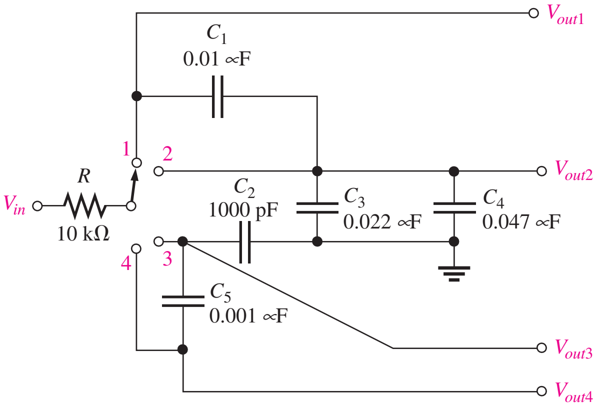
(a) 60 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 5 kHz

► **FIGURE 39**



R
V_in
V_out
220 Ω
C

***6.** Determine the critical frequency for each switch position on the switched filter network of Figure 40.



V_out1
C_1
0.01 ∝F
1
2
V_out2
R
V_in
10 kΩ
C_2
1000 pF
C_3
0.022 ∝F
C_4
0.047 ∝F
4
3
C_5
0.001 ∝F
V_out3
V_out4

▲ **FIGURE 40**

7. Draw a Bode plot for each part of Problem 5.

8. For each following case, express the voltage ratio in dB:

(a) $V_{in} = 1$ V, $V_{out} = 1$ V **(b)** $V_{in} = 5$ V, $V_{out} = 3$ V

(c) $V_{in} = 10$ V, $V_{out} = 7.07$ V **(d)** $V_{in} = 25$ V, $V_{out} = 5$ V

9. The input voltage to a low-pass *RC* filter is 8 V rms. Find the output voltage at the following dB levels:

(a) -1 dB **(b)** -3 dB **(c)** -6 dB **(d)** -20 dB

10. For a basic *RC* low-pass filter, find the output voltage in dB relative to a 0 dB input for the following frequencies ($f_c = 1$ kHz):

(a) 10 kHz **(b)** 100 kHz **(c)** 1 MHz

11. Repeat Problem 10 for $f_c = 100$ kHz.

SECTION 2 High-Pass Filters

12. Repeat Problem 11 for a high-pass filter.

13. In a high-pass filter, $X_C = 500\ \Omega$ and $R = 2.2\ \text{k}\Omega$. What is the output voltage ($\mathbf{V}_{out}$) when $V_{in} = 10$ V rms?

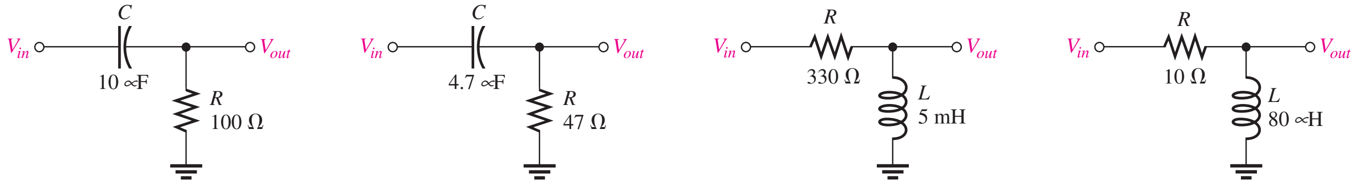
14. A high-pass filter has a critical frequency of 50 Hz. Determine which of the following frequencies are passed and which are rejected:

(a) 1 Hz **(b)** 20 Hz **(c)** 50 Hz **(d)** 60 Hz **(e)** 30 kHz

15. Determine the output voltage of each filter in Figure 41 at the specified frequency when $V_{in} = 10$ V.

16. What is f_c for each filter in Figure 41? Determine the output voltage at f_c in each case ($V_{in} = 10$ V).

17. Draw the Bode plot for each filter in Figure 41.

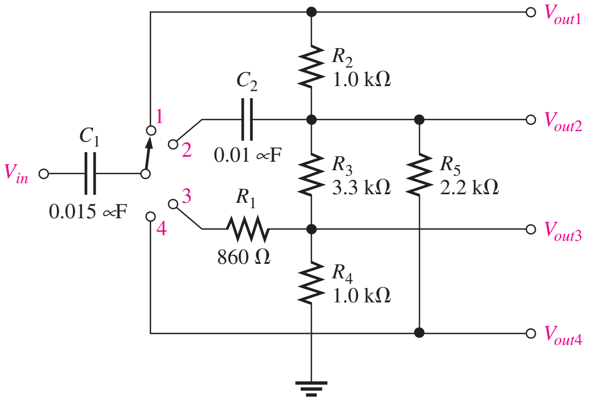


C
Vin
Vout
10 ∝F
R
100 Ω
C
Vin
Vout
4.7 ∝F
R
47 Ω
R
Vin
Vout
330 Ω
L
5 mH
R
Vin
Vout
10 Ω
L
80 ∝H

(a) $f = 60$ Hz (b) $f = 400$ Hz (c) $f = 1$ kHz (d) $f = 2$ kHz

▲ FIGURE 41

***18.** Determine f_c for each switch position in Figure 42.

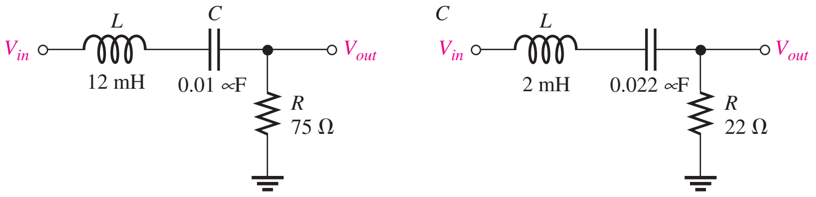


Vout1
R2
1.0 kΩ
C2
1
2
0.01 ∝F
Vout2
C1
Vin
0.015 ∝F
3
4
R1
860 Ω
R3
3.3 kΩ
R5
2.2 kΩ
Vout3
R4
1.0 kΩ
Vout4

▲ FIGURE 42

SECTION 3 Band-Pass Filters

19. Determine the center frequency for each filter in Figure 43.



L
C
Vin
Vout
12 mH
0.01 ∝F
R
75 Ω
C
L
Vin
Vout
2 mH
0.022 ∝F
R
22 Ω

(a) (b)

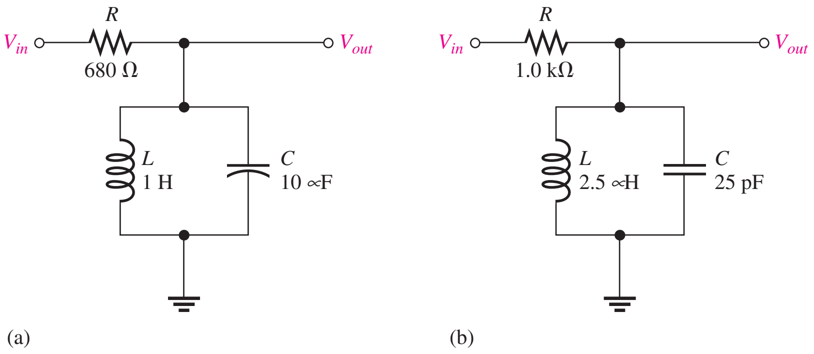
▲ FIGURE 43

20. Assuming that the coils in Figure 43 have a winding resistance of 10 Ω, find the bandwidth for each filter.

21. What are the upper and lower critical frequencies for each filter in Figure 43? Assume the response is symmetrical about f_0.

22. For each filter in Figure 44, find the center frequency of the passband. Neglect R_W.

23. If the coils in Figure 44 have a winding resistance of 4 Ω, what is the output voltage at resonance when $V_{in} = 120\text{ V}$?



V_{in}
R
680 Ω
V_{out}
L
1 H
C
10 ∝F
V_{in}
R
1.0 kΩ
V_{out}
L
2.5 ∝H
C
25 pF

(a)

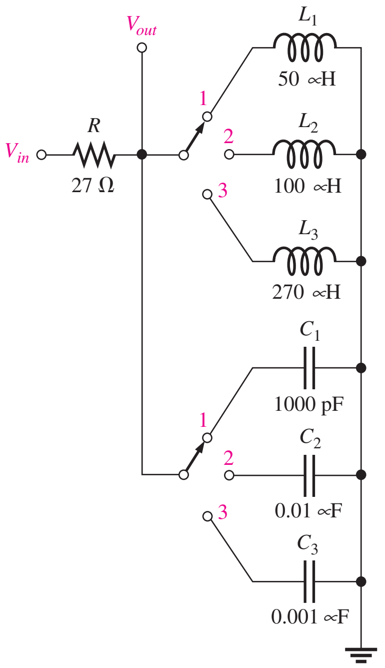
(b)

▲ **FIGURE 44**

***24.** Determine the separation of center frequencies for all switch positions in Figure 45. Do any of the responses overlap? Assume $R_W = 0\ \Omega$ for each coil.

***25.** Design a band-pass filter using a parallel resonant circuit to meet all of the following specifications: $BW = 500\text{ Hz}$; $Q = 40$; and $I_{C(max)} = 20\text{ mA}$, $V_{C(max)} = 2.5\text{ V}$.

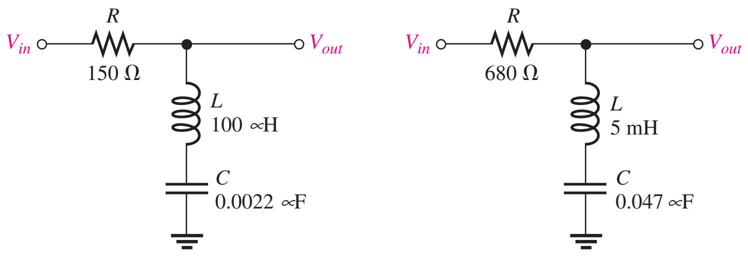
▶ **FIGURE 45**



V_{out}
L_1
50 ∝H
1
2
L_2
100 ∝H
3
L_3
270 ∝H
V_{in}
R
27 Ω
C_1
1000 pF
1
2
C_2
0.01 ∝F
3
C_3
0.001 ∝F

SECTION 4 Band-Stop Filters

26. Determine the center frequency for each filter in Figure 46.

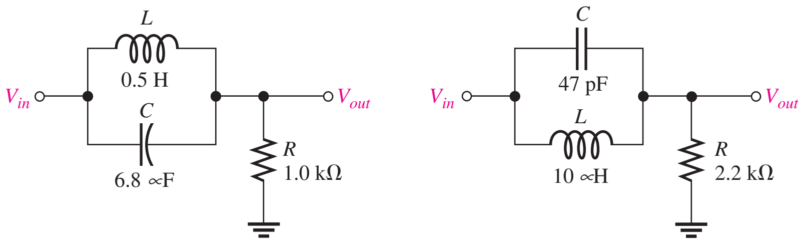

R
V_{in}
V_{out}
150 Ω
L
100 ∝H
C
0.0022 ∝F
R
V_{in}
V_{out}
680 Ω
L
5 mH
C
0.047 ∝F

(a) (b)

▲ **FIGURE 46**

27. For each filter in Figure 47, find the center frequency of the stopband.

28. If the coils in Figure 47 have a winding resistance of 8 Ω, what is the output voltage at resonance when $V_{in} = 50$ V?

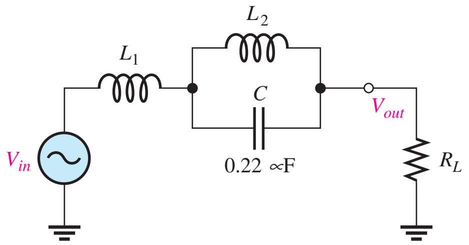

L
0.5 H
C
V_{in}
V_{out}
6.8 ∝F
R
1.0 kΩ
C
47 pF
L
V_{in}
V_{out}
10 ∝H
R
2.2 kΩ

(a) (b)

▲ **FIGURE 47**

***29.** Determine the values of L_1 and L_2 in Figure 48 to pass a signal with a frequency of 1200 kHz and stop (reject) a signal with a frequency of 456 kHz.

▶ **FIGURE 48**


L_2
L_1
C
V_{out}
V_{in}
0.22 ∝F
R_L

Multisim Troubleshooting and Analysis

These problems require Multisim.

30. Open file P18-30 and determine if there is a fault. If so, find the fault.

31. Open file P18-31 and determine if there is a fault. If so, find the fault.

32. Open file P18-32 and determine if there is a fault. If so, find the fault.

33. Open file P18-33 and determine if there is a fault. If so, find the fault.

34. Open file P18-34 and determine if there is a fault. If so, find the fault.

35. Open file P18-35 and determine if there is a fault. If so, find the fault.

36. Open file P18-36 and determine the center frequency of the circuit.

37. Open file P18-37 and determine the bandwidth of the circuit.

ANSWERS

SECTION CHECKUPS

SECTION 1 Low-Pass Filters

1. The passband is 0 Hz to 2.5 kHz.

2. $\mathbf{V}_{out} = 100\angle{-88.9^\circ}$ mV rms

3. $20\log(V_{out}/V_{in}) = -9.54$ dB

SECTION 2 High-Pass Filters

1. $V_{out} = 0.707$ V

2. $\mathbf{V}_{out} = 9.98\angle 3.81^\circ$ V

SECTION 3 Band-Pass Filters

1. $BW = 30.2\text{ kHz} - 29.8\text{ kHz} = 400\text{ Hz}$

2. $f_0 \cong 1.04$ MHz

SECTION 4 Band-Stop Filters

1. A band-stop filter rejects, rather than passes, a certain band of frequencies.

2. High-pass/low-pass combination, series resonant circuit, and parallel resonant circuit

RELATED PROBLEMS FOR EXAMPLES

1 -1.41 dB

2 7.23 kHz

3 f_c increases to 159 kHz. Roll-off rate remains -20 dB/decade.

4 f_c increases to 350 kHz. Roll-off rate remains -20 dB/decade.

5 -60 dB

6 $C = 0.723\ \mu$F; $V_{out} = 4.98$ V; $\phi = 5.7^\circ$

7 10.5 kHz

8 BW increases to 18.8 kHz.

9 1.59 MHz

10 7.12 kHz (no significant difference)

11 $V_{out} = 15.2$ mV; $BW = 105$ Hz

12 1.94 V

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F
6. T **7.** F **8.** T **9.** T **10.** F

SELF-TEST

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (d) **6.** (c) **7.** (a)
8. (c) **9.** (b) **10.** (a) **11.** (c) **12.** (b) **13.** (c) **14.** (c)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (b) **3.** (b) **4.** (b) **5.** (b) **6.** (a)
7. (a) **8.** (a) **9.** (a) **10.** (c) **11.** (b) **12.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. $2.22\angle{-77.2^\circ}$ V rms

3. **(a)** $9.36\angle{-20.7^\circ}$ V **(b)** $7.18\angle{-44.1^\circ}$ V
(c) $9.96\angle{-5.44^\circ}$ V **(d)** $9.95\angle{-5.74^\circ}$ V

5. **(a)** 12.1 μF **(b)** 1.45 μF
(c) 0.723 μF **(d)** 0.144 μF

7. See Figure 49.

9. **(a)** 7.13 V **(b)** 5.67 V **(c)** 4.01 V **(d)** 0.800 V

11. **(a)** 0 dB **(b)** -3 dB (0 dB ideal) **(c)** -20 dB

13. $9.75\angle{12.8^\circ}$ V

15. **(a)** $3.53\angle{69.3^\circ}$ V **(b)** $4.85\angle{61.0^\circ}$ V
(c) $947\angle{84.6^\circ}$ mV **(d)** $995\angle{84.3^\circ}$ mV

17. See Figure 50.

19. **(a)** 14.5 kHz **(b)** 24.0 kHz

21. **(a)** 15.06 kHz, 13.94 kHz **(b)** 25.3 kHz, 22.7 kHz

23. **(a)** 117 V **(b)** 115 V

25. $C = 0.064\ \mu\text{F}$, $L = 989\ \mu\text{H}$, $f_r = 20$ kHz

27. **(a)** 86.3 Hz **(b)** 7.34 MHz

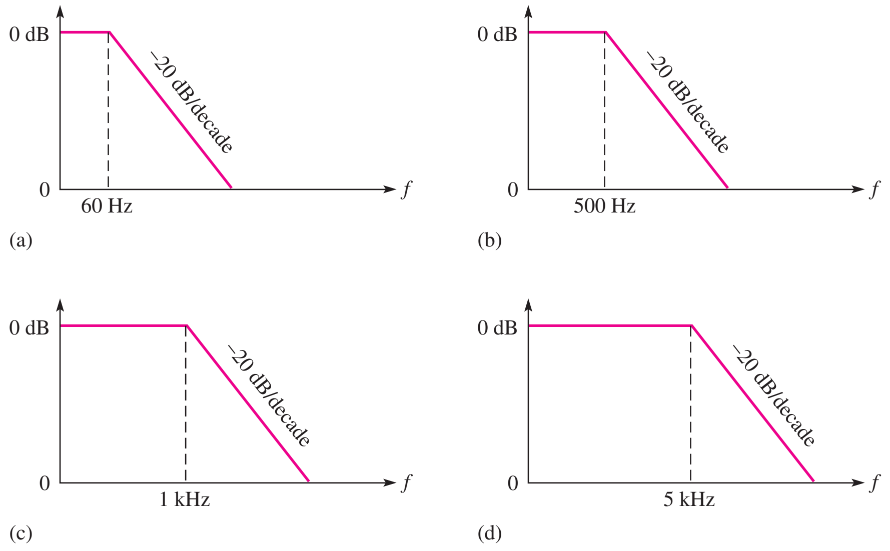
29. $L_1 = 0.08\ \mu\text{H}$,
$L_2 = 0.554\ \mu\text{H}$

31. C_2 leaky

33. C_1 shorted

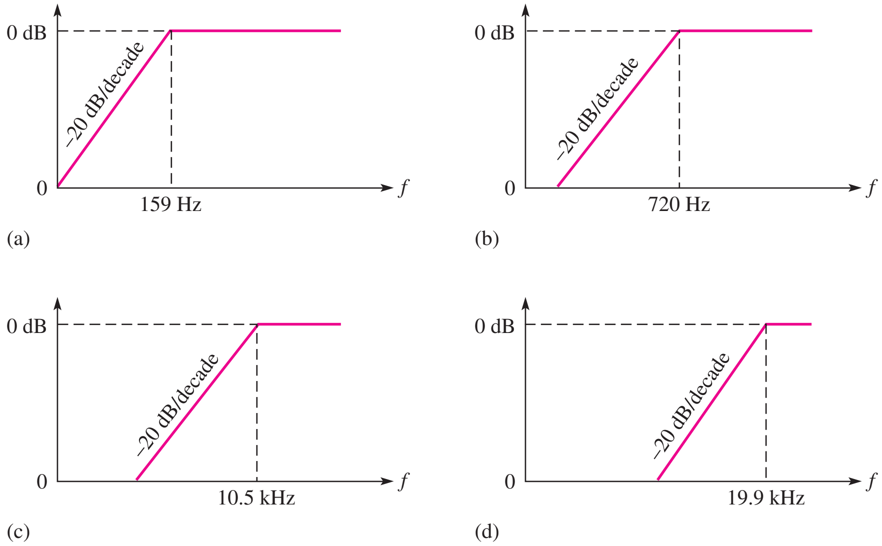
35. No fault

37. $BW \approx 88.93$ MHz



▲ FIGURE 49

FIGURE 50


0 dB
0
–20 dB/decade
159 Hz
f
(a)
0 dB
0
–20 dB/decade
720 Hz
f
(b)
0 dB
0
–20 dB/decade
10.5 kHz
f
(c)
0 dB
0
–20 dB/decade
19.9 kHz
f
(d)

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CIRCUIT THEOREMS IN AC ANALYSIS

From Chapter 19 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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CIRCUIT THEOREMS IN AC ANALYSIS

CHAPTER OUTLINE

1 The Superposition Theorem
2 Thevenin's Theorem
3 Norton's Theorem
4 Maximum Power Transfer Theorem
Application Activity

CHAPTER OBJECTIVES

- Apply the superposition theorem to ac circuit analysis
- Apply Thevenin's theorem to simplify reactive ac circuits for analysis
- Apply Norton's theorem to simplify reactive ac circuits
- Apply the maximum power transfer theorem

KEY TERMS

- Superposition theorem
- Thevenin's theorem
- Equivalent circuit
- Norton's theorem
- Complex conjugate

APPLICATION ACTIVITY PREVIEW

In the application activity, you will evaluate a band-pass filter module to determine its internal component values. You will apply Thevenin's theorem to determine an optimum load impedance for maximum power transfer.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

The theorems in this chapter make analysis easier for certain types of circuits. These methods do not replace Ohm's law and Kirchhoff's laws, but they are normally used in conjunction with those laws in certain situations.

The superposition theorem helps you to deal with circuits that have multiple sources. Thevenin's and Norton's theorems provide methods for reducing a circuit to a simple equivalent form for easier analysis. The maximum power transfer theorem is used in applications where it is important for a given circuit to provide maximum power to a load.

1 THE SUPERPOSITION THEOREM

In this section, the superposition theorem is applied to circuits with ac sources and reactive components.

After completing this section, you should be able to

- **Apply the superposition theorem to ac circuit analysis**
  - State the superposition theorem
  - List the steps in applying the theorem

The **superposition theorem** can be stated as follows:

The current in any given branch of a multiple-source circuit can be found by determining the currents in that particular branch produced by each source acting alone, with all other sources replaced by their internal impedances. The total current in the given branch is the phasor sum of the individual currents in that branch.

The procedure for the application of the superposition theorem is as follows:

Step 1: Leave one of the voltage (current) sources in the circuit, and replace all others with their internal impedance. For ideal voltage sources, the internal impedance is zero. For ideal current sources, the internal impedance is infinite. We will call this procedure *zeroing* the source.

Step 2: Find the current in the branch of interest produced by the one remaining source.

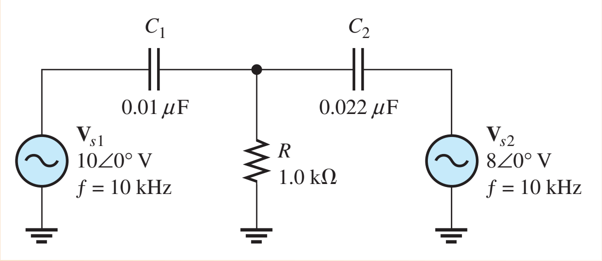
Step 3: Repeat Steps 1 and 2 for each source in turn. When complete, you will have a number of current values equal to the number of sources in the circuit.

Step 4: Add the individual current values as phasor quantities.

Example 1 illustrates this procedure for a circuit containing two ideal voltage sources, V_{s1} and V_{s2}.

EXAMPLE 1

Find the current in R of Figure 1 using the superposition theorem. Assume the internal source impedances are zero.

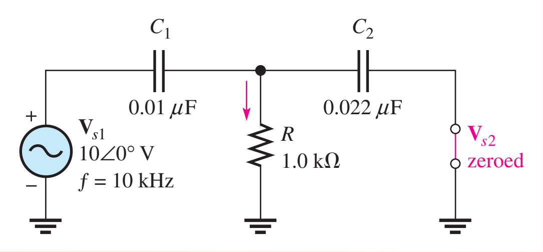


▲ FIGURE 1

Solution **Step 1:** Replace V_{s2} with its internal impedance (zero in this case), and find the current in R due to V_{s1}, as indicated in Figure 2.

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(10\text{ kHz})(0.022\ \mu\text{F})} = 723\ \Omega$$



▲ FIGURE 2

Looking from V_{s1}, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 1.59\angle -90^\circ\text{ k}\Omega + \frac{(1.0\angle 0^\circ\text{ k}\Omega)(723\angle -90^\circ\ \Omega)}{1.0\text{ k}\Omega - j723\ \Omega}$$

$$= 1.59\angle -90^\circ\text{ k}\Omega + 588\angle -54.1^\circ\ \Omega$$

$$= -j1.59\text{ k}\Omega + 345\ \Omega - j476\ \Omega = 345\ \Omega - j2.07\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = 2.10\angle -80.5^\circ\text{ k}\Omega$$

The total current from V_{s1} is

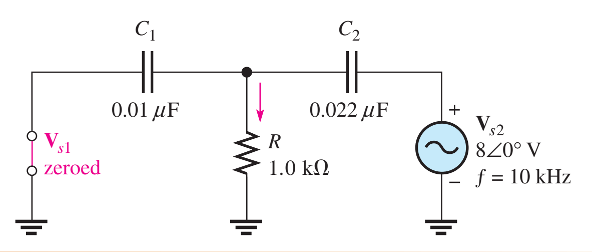
$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{10\angle 0^\circ\text{ V}}{2.10\angle -80.5^\circ\text{ k}\Omega} = 4.76\angle 80.5^\circ\text{ mA}$$

Use the current-divider formula. The current through R due to V_{s1} is

$$\mathbf{I}_{R1} = \left(\frac{X_{C2}\angle -90^\circ}{R - jX_{C2}}\right)\mathbf{I}_{s1} = \left(\frac{723\angle -90^\circ\ \Omega}{1.0\text{ k}\Omega - j723\ \Omega}\right)4.76\angle 80.5^\circ\text{ mA}$$

$$= (0.588\angle -54.9^\circ\ \Omega)(4.76\angle 80.5^\circ\text{ mA}) = 2.80\angle 25.6^\circ\text{ mA}$$

Step 2: Find the current in R due to source V_{s2} by replacing V_{s1} with its internal impedance (zero), as shown in Figure 3.



▲ FIGURE 3

Looking from V_{s2}, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 723\angle -90^\circ\ \Omega + \frac{(1.0\angle 0^\circ\ \text{k}\Omega)(1.59\angle -90^\circ\ \text{k}\Omega)}{1.0\ \text{k}\Omega - j1.59\ \text{k}\Omega}$$

$$= 723\angle -90^\circ\ \Omega + 847\angle -32.2^\circ\ \Omega$$

$$= -j723\ \Omega + 717\ \Omega - j451\ \Omega = 717\ \Omega - j1174\ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 1376\angle -58.6^\circ\ \Omega$$

The total current from V_{s2} is

$$\mathbf{I}_{s2} = \frac{\mathbf{V}_{s2}}{\mathbf{Z}} = \frac{8\angle 0^\circ\ \text{V}}{1376\angle -58.6^\circ\ \Omega} = 5.81\angle 58.6^\circ\ \text{mA}$$

Use the current-divider formula. The current through R due to V_{s2} is

$$\mathbf{I}_{R2} = \left(\frac{X_{C1}\angle -90^\circ}{R - jX_{C1}}\right)\mathbf{I}_{s2}$$

$$= \left(\frac{1.59\angle -90^\circ\ \text{k}\Omega}{1.0\ \text{k}\Omega - j1.59\ \text{k}\Omega}\right)5.81\angle 58.6^\circ\ \text{mA} = 4.91\angle 26.4^\circ\ \text{mA}$$

Step 3: Convert the two individual resistor currents to rectangular form and add to get the total current through R.

$$\mathbf{I}_{R1} = 2.80\angle 25.6^\circ\ \text{mA} = 2.53\ \text{mA} + j1.21\ \text{mA}$$

$$\mathbf{I}_{R2} = 4.91\angle 26.4^\circ\ \text{mA} = 4.40\ \text{mA} + j2.18\ \text{mA}$$

$$\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2} = 6.93\ \text{mA} + j3.39\ \text{mA} = \mathbf{7.71\angle 26.1^\circ\ mA}$$

*Related Problem** Determine $\mathbf{I}_R$ if $\mathbf{V}_{s2} = 8\angle 180^\circ$ V in Figure 1.

Use Multisim files E19-01A and E19-01B to verify the calculated results in this example and to confirm your calculation for the related problem.

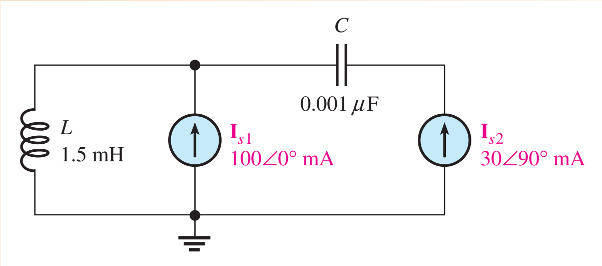
---

*Answers are at the end of the chapter.

Example 2 illustrates the application of the superposition theorem for a circuit with two current sources, I_{s1} and I_{s2}.

EXAMPLE 2

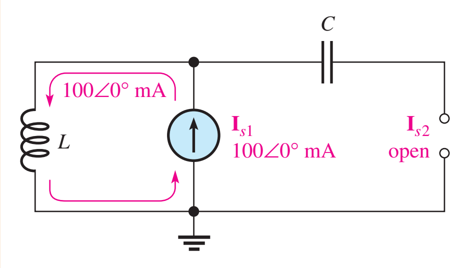
Find the inductor current in Figure 4. Assume the current sources are ideal.



▲ FIGURE 4

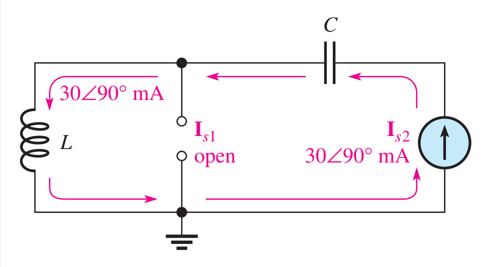
Solution **Step 1:** Find the current through the inductor due to current source I_{s1} by replacing source I_{s2} with an open, as shown in Figure 5. As you can see, the entire 100 mA from the current source I_{s1} is through the inductor.

▶ **FIGURE 5**



Step 2: Find the current through the inductor due to current source I_{s2} by replacing source I_{s1} with an open, as indicated in Figure 6. Notice that all of the 30 mA from source I_{s2} is through the inductor.

▶ **FIGURE 6**



Step 3: To get the total inductor current, superimpose the two individual currents and add as phasor quantities.

$$\begin{aligned}\mathbf{I}_L &= \mathbf{I}_{L1} + \mathbf{I}_{L2}\\ &= 100\angle 0^\circ\text{ mA} + 30\angle 90^\circ\text{ mA} = 100\text{ mA} + j30\text{ mA}\\ &= \mathbf{104\angle 16.7^\circ\ mA}\end{aligned}$$

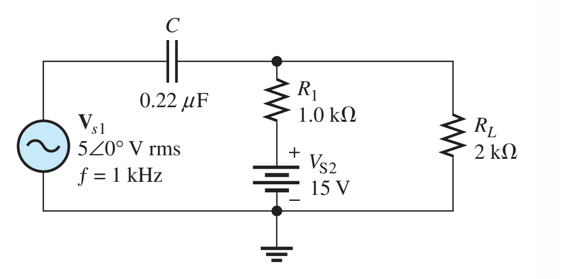
Related Problem Find the current through the capacitor in Figure 4.

Example 3 illustrates the analysis of a circuit with an ac voltage source and a dc voltage source. This situation is common in many amplifier applications.

EXAMPLE 3

Find the total current in the load resistor, R_L, in Figure 7. Assume the sources are ideal.

▶ **FIGURE 7**



Solution **Step 1:** Find the current through R_L due to the ac source V_{s1} by zeroing (replacing with its internal impedance) the dc source V_{S2}, as shown in Figure 8. Looking from V_{s1}, the impedance is

$$\mathbf{Z} = \mathbf{X}_C + \frac{\mathbf{R}_1\mathbf{R}_L}{\mathbf{R}_1 + \mathbf{R}_L}$$

$$X_C = \frac{1}{2\pi(1.0\text{ kHz})(0.22\ \mu\text{F})} = 723\ \Omega$$

$$\mathbf{Z} = 723\angle -90^\circ\ \Omega + \frac{(1.0\angle 0^\circ\text{ k}\Omega)(2\angle 0^\circ\text{ k}\Omega)}{3\angle 0^\circ\text{ k}\Omega}$$

$$= -j723\ \Omega + 667\ \Omega = 984\angle -47.3^\circ\ \Omega$$

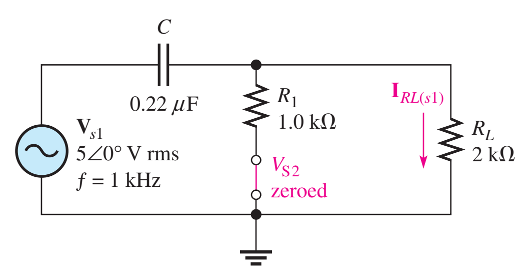
The total current from the ac source is

$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{5\angle 0^\circ\text{ V}}{984\angle -47.3^\circ\ \Omega} = 5.08\angle 47.3^\circ\text{ mA}$$

Use the current-divider approach. The current in R_L due to V_{s1} is

$$\mathbf{I}_{RL(s1)} = \left(\frac{R_1}{R_1 + R_L}\right)\mathbf{I}_{s1} = \left(\frac{1.0\text{ k}\Omega}{3\text{ k}\Omega}\right)5.08\angle 47.3^\circ\text{ mA} = 1.69\angle 47.3^\circ\text{ mA}$$

▶ FIGURE 8


C
0.22 μF
$\mathbf{V}_{s1}$
5∠0° V rms
f = 1 kHz
R_1
1.0 kΩ
V_{S2}
zeroed
$\mathbf{I}_{RL(s1)}$
R_L
2 kΩ

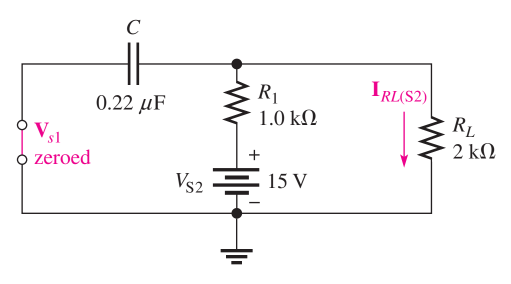
Step 2: Find the current in R_L due to the dc source V_{S2} by zeroing V_{s1} (replacing with its internal impedance), as shown in Figure 9. The impedance magnitude as seen by V_{S2} is

$$Z = R_1 + R_L = 3\text{ k}\Omega$$

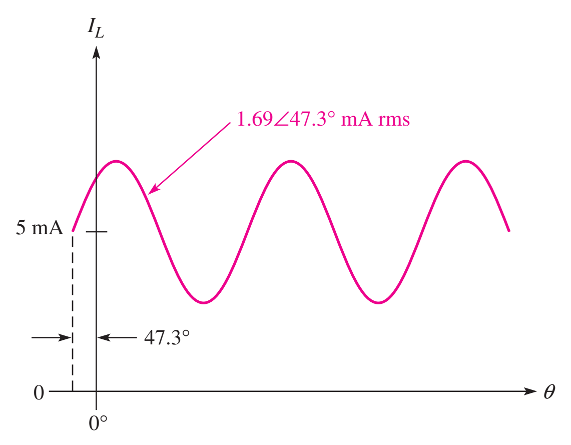
The current produced by V_{S2} is

$$I_{RL(S2)} = \frac{V_{S2}}{Z} = \frac{15\text{ V}}{3\text{ k}\Omega} = 5\text{ mA dc}$$

▶ FIGURE 9


C
0.22 μF
$\mathbf{V}_{s1}$
zeroed
R_1
1.0 kΩ
+
V_{S2}
15 V
−
$\mathbf{I}_{RL(S2)}$
R_L
2 kΩ

Step 3: By superposition, the total current in R_L is $1.69\angle 47.3^\circ$ mA riding on a dc level of 5 mA, as indicated in Figure 10.



▲ FIGURE 10

Related Problem Determine the current through R_L if V_{S2} is changed to 9 V.

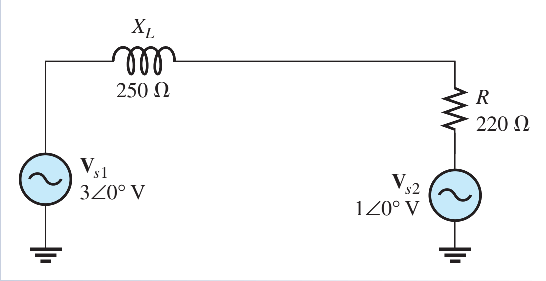
Use Multisim files E19-03A and E19-03B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. If two equal currents are in opposing directions at any instant of time in a given branch of a circuit, what is the net current at that instant?
2. Why is the superposition theorem useful in the analysis of multiple-source circuits?
3. Using the superposition theorem, find the magnitude of the current through R in Figure 11.

▶ FIGURE 11



2 THEVENIN'S THEOREM

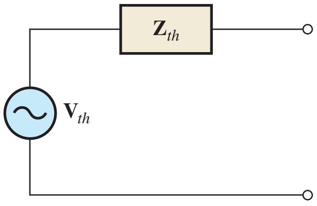
Thevenin's theorem, as applied to ac circuits, provides a method for reducing any circuit to an equivalent form that consists of an equivalent ac voltage source in series with an equivalent impedance.

After completing this section, you should be able to

- **Apply Thevenin's theorem to simplify reactive ac circuits for analysis**
  - Describe the form of a Thevenin equivalent circuit
  - Obtain the Thevenin equivalent ac voltage source
  - Obtain the Thevenin equivalent impedance
  - List the steps in applying Thevenin's theorem to an ac circuit

Equivalency

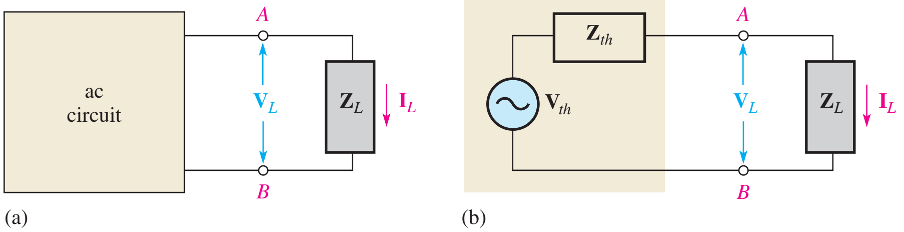
The form of Thevenin's **equivalent circuit** is shown in Figure 12. Regardless of how complex the original circuit is, it can always be reduced to this equivalent form. The equivalent voltage source is designated $\mathbf{V}_{th}$; the equivalent impedance is designated $\mathbf{Z}_{th}$ (lowercase italic subscript denotes ac quantity). Notice that the impedance is represented by a block in the circuit diagram. This is because the equivalent impedance can be of several forms: purely resistive, purely capacitive, purely inductive, or a combination of a resistance and a reactance.



▲ FIGURE 12

Thevenin's equivalent circuit.

Figure 13(a) shows a block diagram that represents an ac circuit of any given complexity. This circuit has two output terminals, *A* and *B*. A load impedance, $\mathbf{Z}_L$, is connected to the terminals. The circuit produces a certain voltage, $\mathbf{V}_L$, and a certain current, $\mathbf{I}_L$, as illustrated.



▲ FIGURE 13

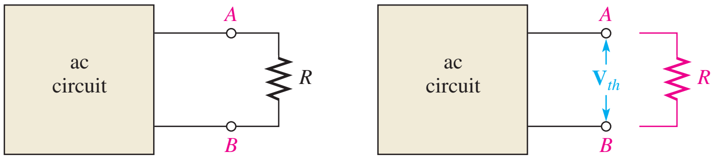
An ac circuit of any complexity can be reduced to a Thevenin equivalent for analysis purposes.

By Thevenin's theorem, the circuit in the block can be reduced to an equivalent form, as indicated in the beige area of Figure 13(b). The term *equivalent* means that when the same value of load is connected to both the original circuit and Thevenin's equivalent circuit, the load voltages and currents are equal for both. Therefore, as far as the load is concerned, there is no difference between the original circuit and Thevenin's equivalent circuit. The load "sees" the same current and voltage regardless of whether it is connected to the original circuit or to the Thevenin equivalent. For ac circuits, the equivalent circuit is for one particular frequency. When the frequency is changed, the equivalent circuit must be recalculated.

Thevenin's Equivalent Voltage ($\mathbf{V}_{th}$)

As you have seen, the equivalent voltage, $\mathbf{V}_{th}$, is one part of the complete Thevenin equivalent circuit.

> **Thevenin's equivalent voltage is defined as the open circuit voltage between two specified terminals in a circuit.**



(a) Circuit (b) With R removed

▲ **FIGURE 14**

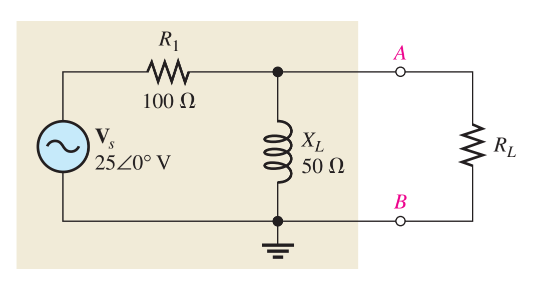
How $\mathbf{V}_{th}$ is determined.

To illustrate, let's assume that an ac circuit of some type has a resistor connected between two specified terminals, *A* and *B*, as shown in Figure 14(a). We wish to find the Thevenin equivalent circuit for the circuit as "seen" by *R*. $\mathbf{V}_{th}$ is the voltage across terminals *A* and *B*, with *R* removed, as shown in part (b) of the figure. The circuit is viewed from the open terminals *A* and *B*, and *R* is considered external to the circuit for which the Thevenin equivalent is to be found.

The following three examples show how to find $\mathbf{V}_{th}$.

EXAMPLE 4

Refer to Figure 15. Determine $\mathbf{V}_{th}$ for the circuit within the beige box as viewed from terminals *A* and *B*.



▲ **FIGURE 15**

Solution Remove R_L and determine the voltage from *A* to *B* ($\mathbf{V}_{th}$). In this case, the voltage from *A* to *B* is the same as the voltage across X_L. This is determined using the voltage-divider method.

$$\mathbf{V}_L = \left(\frac{X_L\angle 90^\circ}{R_1 + jX_L}\right)\mathbf{V}_s = \left(\frac{50\angle 90^\circ\ \Omega}{100\ \Omega + j50\ \Omega}\right)\mathbf{V}_s$$

$$= \left(\frac{50\angle 90^\circ\ \Omega}{112\angle 26.6^\circ\ \Omega}\right)25\angle 0^\circ\text{ V} = 11.2\angle 63.4^\circ\text{ V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{V}_L = \mathbf{11.2\angle 63.4^\circ\ V}$$

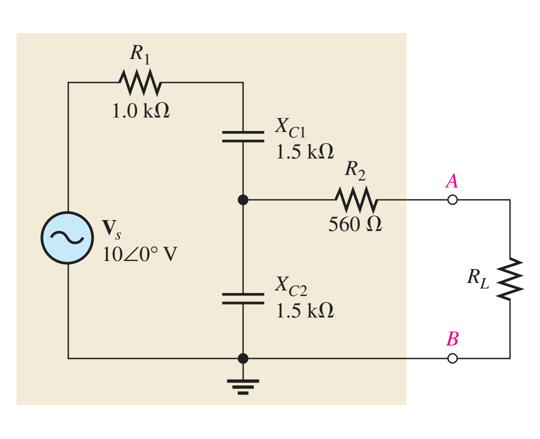
Related Problem Determine $\mathbf{V}_{th}$ if R_1 is changed to 47 Ω in Figure 15.

Use Multisim files E19-04A and E19-04B to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 5

Refer to Figure 16. Determine the Thevenin voltage for the circuit within the beige box as viewed from terminals *A* and *B*.

▶ **FIGURE 16**



Solution Thevenin's voltage for the circuit between terminals *A* and *B* is the voltage that appears across *A* and *B* with R_L removed from the circuit.

There is no voltage drop across R_2 because the open between terminals *A* and *B* prevents current through it. Thus, $\mathbf{V}_{AB}$ is the same as $\mathbf{V}_{C2}$ and can be found by the voltage-divider formula.

$$\mathbf{V}_{AB} = \mathbf{V}_{C2} = \left(\frac{X_{C2}\angle -90^\circ}{R_1 - jX_{C1} - jX_{C2}}\right)\mathbf{V}_s = \left(\frac{1.5\angle -90^\circ\ \text{k}\Omega}{1.0\ \text{k}\Omega - j3\ \text{k}\Omega}\right)10\angle 0^\circ\ \text{V}$$

$$= \left(\frac{1.5\angle -90^\circ\ \text{k}\Omega}{3.16\angle -71.6^\circ\ \text{k}\Omega}\right)10\angle 0^\circ\ \text{V} = 4.75\angle -18.4^\circ\ \text{V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{4.75\angle -18.4^\circ\ V}$$

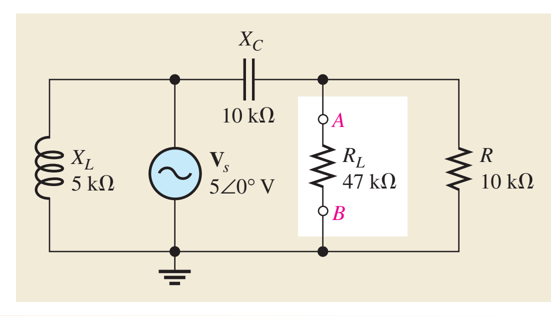
Related Problem Determine $\mathbf{V}_{th}$ if R_1 is changed to 2.2 kΩ in Figure 16.

Use Multisim files E19-05A and E19-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 6

Refer to Figure 17. Find $\mathbf{V}_{th}$ for the circuit within the beige box as viewed from terminals *A* and *B*.

▶ **FIGURE 17**



Solution First remove R_L and determine the voltage across the resulting open terminals, which is $\mathbf{V}_{th}$. Find $\mathbf{V}_{th}$ by applying the voltage-divider formula to X_C and R.

$$\mathbf{V}_{th} = \mathbf{V}_R = \left(\frac{R\angle 0^\circ}{R - jX_C}\right)\mathbf{V}_s = \left(\frac{10\angle 0^\circ\text{ k}\Omega}{10\text{ k}\Omega - j10\text{ k}\Omega}\right)5\angle 0^\circ\text{ V}$$

$$= \left(\frac{10\angle 0^\circ\text{ k}\Omega}{14.1\angle -45^\circ\text{ k}\Omega}\right)5\angle 0^\circ\text{ V} = \mathbf{3.55\angle 45^\circ\ V}$$

Notice X_L has no effect on the result, since the 5 V source appears across X_C and R in combination.

Related Problem Find $\mathbf{V}_{th}$ if R is 22 kΩ and R_L is 39 kΩ in Figure 17.

Use Multisim files E19-06A and E19-06B to verify the calculated results in this example and to confirm your calculation for the related problem.

Thevenin's Equivalent Impedance ($\mathbf{Z}_{th}$)

The previous examples illustrated how to find $\mathbf{V}_{th}$. Now, let's determine the Thevenin equivalent impedance, $\mathbf{Z}_{th}$, the second part of a Thevenin equivalent circuit. As defined by Thevenin's theorem,

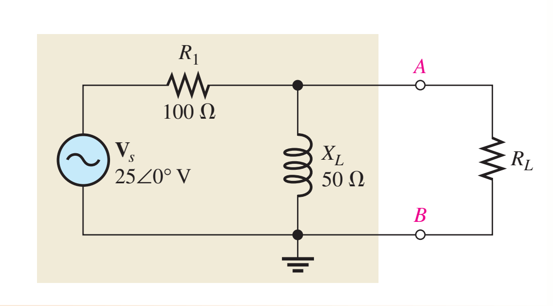
> **Thevenin's equivalent impedance is the total impedance appearing between two specified terminals in a given circuit with all sources replaced by their internal impedances.**

To find $\mathbf{Z}_{th}$ between any two terminals in a circuit, replace all the voltage sources by a short (any internal impedance remains in series). Replace all the current sources by an open (any internal impedance remains in parallel). Then determine the total impedance between the two terminals. The following three examples illustrate how to find $\mathbf{Z}_{th}$.

EXAMPLE 7

Find $\mathbf{Z}_{th}$ for the part of the circuit in Figure 18 that is within the beige box as viewed from terminals A and B. This is the same circuit used in Example 4.

▶ FIGURE 18

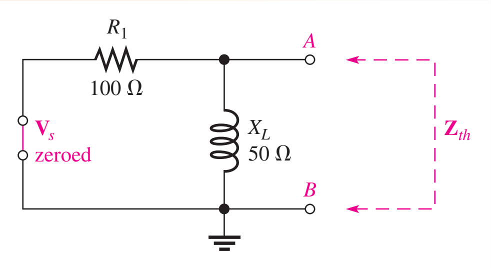


Solution First, replace $\mathbf{V}_s$ with its internal impedance (zero in this case), as shown in Figure 19. Looking in between terminals A and B, R_1 and X_L are in parallel. Thus,

$$\mathbf{Z}_{th} = \frac{(R_1\angle 0^\circ)(X_L\angle 90^\circ)}{R_1 + jX_L} = \frac{(100\angle 0^\circ\ \Omega)(50\angle 90^\circ\ \Omega)}{100\ \Omega + j50\ \Omega}$$

$$= \frac{(100\angle 0^\circ\ \Omega)(50\angle 90^\circ\ \Omega)}{112\angle 26.6^\circ\ \Omega} = \mathbf{44.6\angle 63.4^\circ\ \Omega}$$

▶ **FIGURE 19**

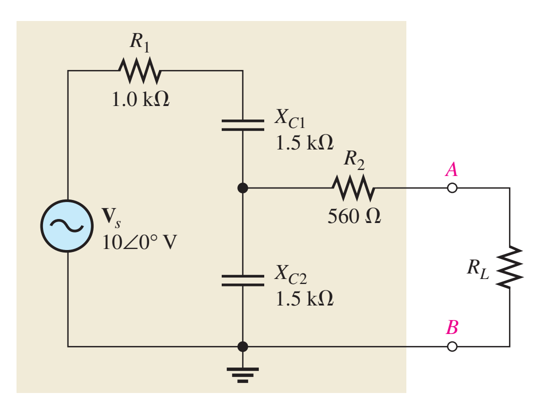


R_1
100 Ω
A
$\mathbf{V}_s$
zeroed
X_L
50 Ω
B
$\mathbf{Z}_{th}$

Related Problem Change R_1 to 47 Ω and determine $\mathbf{Z}_{th}$.

EXAMPLE 8

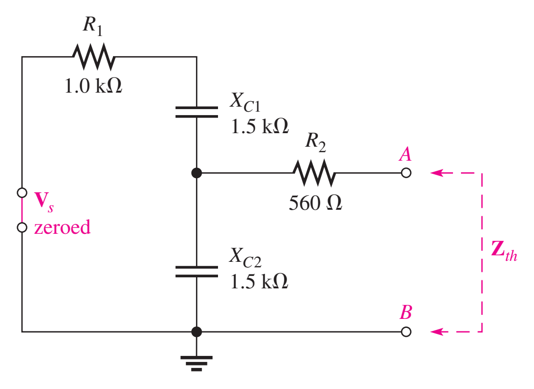
Refer to Figure 20. Determine $\mathbf{Z}_{th}$ for the circuit within the beige box as viewed from terminals *A* and *B*. This is the same circuit used in Example 5.



R_1
1.0 kΩ
X_{C1}
1.5 kΩ
R_2
560 Ω
A
$\mathbf{V}_s$
10∠0° V
R_L
X_{C2}
1.5 kΩ
B

▲ **FIGURE 20**

Solution First, replace the voltage source with its internal impedance (zero in this case), as shown in Figure 21.



R_1
1.0 kΩ
X_{C1}
1.5 kΩ
R_2
A
560 Ω
$\mathbf{V}_s$
zeroed
$\mathbf{Z}_{th}$
X_{C2}
1.5 kΩ
B

▲ **FIGURE 21**

Looking from terminals A and B, C_2 appears in parallel with the series combination of R_1 and C_1. This entire combination is in series with R_2. The calculation for $\mathbf{Z}_{th}$ is as follows:

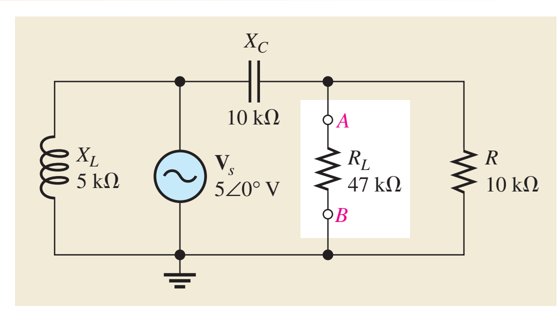
$$\begin{aligned}\mathbf{Z}_{th} &= R_2\angle 0^\circ + \frac{(X_{C2}\angle -90^\circ)(R_1 - jX_{C1})}{R_1 - jX_{C1} - jX_{C2}}\\ &= 560\angle 0^\circ\ \Omega + \frac{(1.5\angle -90^\circ\ \text{k}\Omega)(1.0\ \text{k}\Omega - j1.5\ \text{k}\Omega)}{1.0\ \text{k}\Omega - j3\ \text{k}\Omega}\\ &= 560\angle 0^\circ\ \Omega + \frac{(1.5\angle -90^\circ\ \text{k}\Omega)(1.8\angle -56.3^\circ\ \text{k}\Omega)}{3.16\angle -71.6^\circ\ \text{k}\Omega}\\ &= 560\angle 0^\circ\ \Omega + 854\angle -74.7^\circ\ \Omega = 560\ \Omega + 225\ \Omega - j824\ \Omega\\ &= 785\ \Omega - j824\ \Omega = \mathbf{1138\angle -46.4^\circ\ \Omega}\end{aligned}$$

Related Problem Determine $\mathbf{Z}_{th}$ if R_1 is changed to 2.2 kΩ in Figure 20.

EXAMPLE 9

Refer to Figure 22. Determine $\mathbf{Z}_{th}$ for the portion of the circuit within the beige box as viewed from terminals A and B. This is the same circuit as in Example 6.

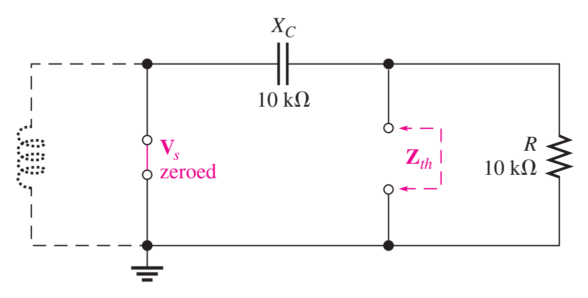
FIGURE 22



Solution With the voltage source replaced by its internal impedence (zero in this case), X_L is effectively out of the circuit. R and C appear in parallel when viewed from the open terminals, as indicated in Figure 23. $\mathbf{Z}_{th}$ is calculated as follows:

$$\begin{aligned}\mathbf{Z}_{th} &= \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C} = \frac{(10\angle 0^\circ\ \text{k}\Omega)(10\angle -90^\circ\ \text{k}\Omega)}{10\ \Omega - j10\ \text{k}\Omega}\\ &= \frac{(10\angle 0^\circ\ \text{k}\Omega)(10\angle -90^\circ\ \text{k}\Omega)}{14.1\angle -45^\circ\ \text{k}\Omega} = \mathbf{7.07\angle -45^\circ\ k\Omega}\end{aligned}$$

FIGURE 23



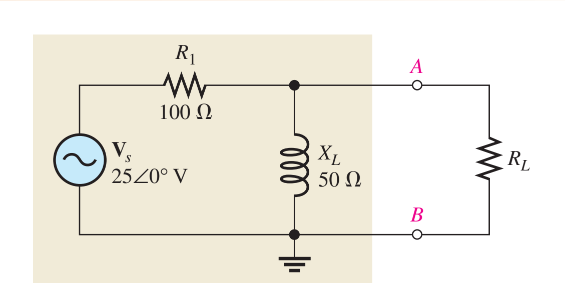
Related Problem Find $\mathbf{Z}_{th}$ if R is 22 kΩ and R_L is 39 kΩ in Figure 22.

Thevenin's Equivalent Circuit

The previous six examples have shown how to find the two equivalent components of a Thevenin circuit, $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$. Keep in mind that you can find $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$ for any circuit. Once you have determined these equivalent values, you must connect them in series to form the Thevenin equivalent circuit. The following three examples use the previous examples to illustrate this final step.

EXAMPLE 10

Refer to Figure 24. Draw the Thevenin equivalent for the circuit within the beige box as viewed from terminals *A* and *B*. This is the circuit used in Examples 4 and 7.

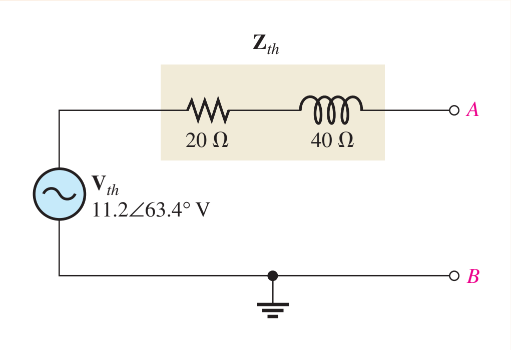


▲ FIGURE 24

Solution From Examples 4 and 7, respectively, $\mathbf{V}_{th} = 11.2\angle 63.4^\circ$ V and $\mathbf{Z}_{th} = 44.6\angle 63.4^\circ\ \Omega$. In rectangular form, the impedance is

$$\mathbf{Z}_{th} = 20\ \Omega + j40\ \Omega$$

This form indicates that the impedance is a 20 Ω resistor in series with a 40 Ω inductive reactance. The Thevenin equivalent circuit is shown in Figure 25.

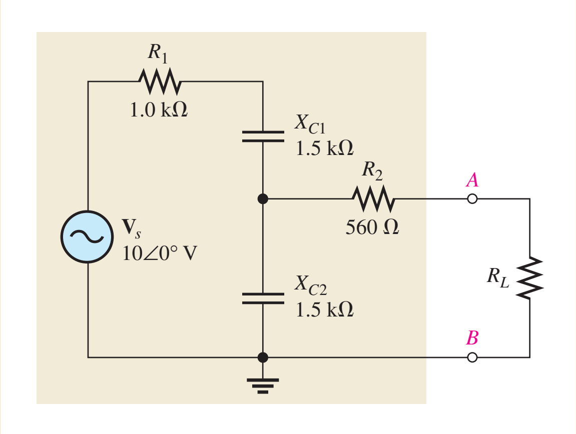


▲ FIGURE 25

Related Problem Draw the Thevenin equivalent circuit for Figure 24 with $R_1 = 47\ \Omega$.

EXAMPLE 11

Refer to Figure 26. Draw the Thevenin equivalent for the circuit within the beige box as viewed from terminals *A* and *B*. This is the circuit used in Examples 5 and 8.



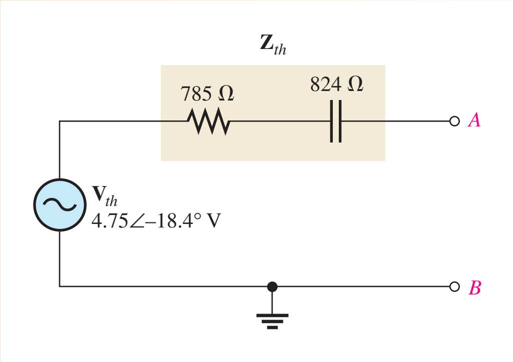
R_1
1.0 kΩ
X_{C1}
1.5 kΩ
R_2
560 Ω
A
$\mathbf{V}_s$
10∠0° V
R_L
X_{C2}
1.5 kΩ
B

▲ FIGURE 26

Solution From Examples 5 and 8, respectively, $\mathbf{V}_{th} = 4.75\angle -18.4^\circ$ V and $\mathbf{Z}_{th} = 11.38\angle -46.4^\circ\ \Omega$. In rectangular form, the impedance is

$$\mathbf{Z}_{th} = 785\ \Omega - j824\ \Omega$$

The Thevenin equivalent circuit is shown in Figure 27.



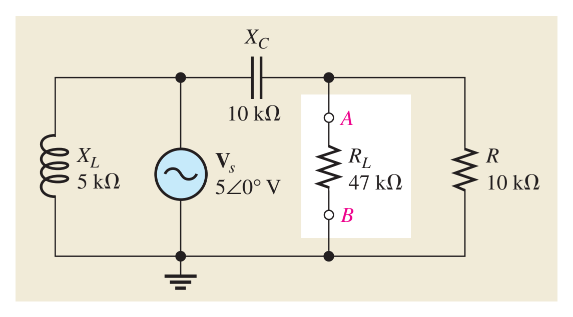
$\mathbf{Z}_{th}$
785 Ω
824 Ω
A
$\mathbf{V}_{th}$
4.75∠–18.4° V
B

▲ FIGURE 27

Related Problem Draw the Thevenin equivalent for the circuit in Figure 26 with $R_1 = 2.2\ \text{k}\Omega$.

EXAMPLE 12

Refer to Figure 28. Determine the Thevenin equivalent for the circuit within the beige box as viewed from terminals *A* and *B*. This is the circuit used in Examples 6 and 9.



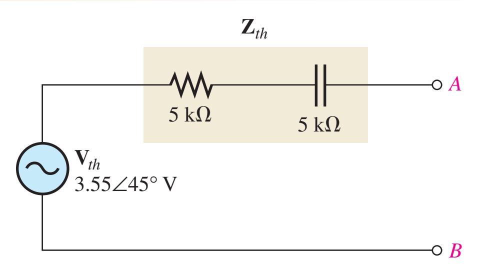
▲ FIGURE 28

Solution From Examples 6 and 9, respectively, $\mathbf{V}_{th} = 3.55\angle 45°\text{ V}$, and $\mathbf{Z}_{th} = 7.07\angle -45°\text{ k}\Omega$. The impedance in rectangular form is

$$\mathbf{Z}_{th} = 5\text{ k}\Omega - j5\text{ k}\Omega$$

Thus, the Thevenin equivalent circuit is as shown in Figure 29.

▶ FIGURE 29



Related Problem Change R to 22 kΩ and R_L to 39 kΩ in Figure 28 and draw the Thevenin equivalent circuit.

Summary of Thevenin's Theorem

Remember that the Thevenin equivalent circuit is always a voltage source in series with an impedance regardless of the original circuit that it replaces. The significance of Thevenin's theorem is that the equivalent circuit can replace the original circuit as far as any external load is concerned. Any load connected between the terminals of a Thevenin equivalent circuit experiences the same current and voltage as if it were connected to the terminals of the original circuit.

A summary of steps for applying Thevenin's theorem follows.

Step 1: Open the two terminals between which you want to find the Thevenin circuit. This is done by removing the component from which the circuit is to be viewed.

Step 2: Determine the voltage across the two open terminals.

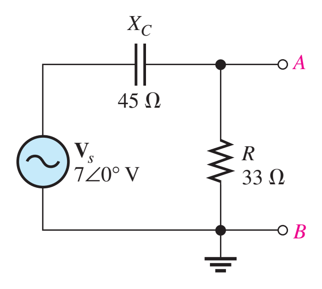
Step 3: Determine the impedance viewed from the two open terminals with ideal voltage sources replaced with shorts and ideal current sources replaced with opens (zeroed).

Step 4: Connect $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$ in series to produce the complete Thevenin equivalent circuit.

SECTION 2 CHECKUP

1. What are the two basic components of a Thevenin equivalent ac circuit?
2. For a certain circuit, $\mathbf{Z}_{th} = 25\ \Omega - j50\ \Omega$, and $\mathbf{V}_{th} = 5\angle 0°\ \text{V}$. Draw the Thevenin equivalent circuit.
3. For the circuit in Figure 30, find the Thevenin equivalent looking from terminals *A* and *B*.

▶ FIGURE 30

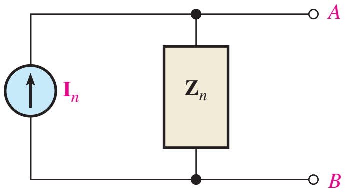


3 NORTON'S THEOREM

Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler, more manageable form for analysis. The basic difference is that **Norton's theorem** gives an equivalent current source (rather than a voltage source) in parallel (rather than in series) with an equivalent impedance.

After completing this section, you should be able to

- **Apply Norton's theorem to simplify reactive ac circuits**
  - Describe the form of a Norton equivalent circuit
  - Obtain the Norton equivalent ac current source
  - Obtain the Norton equivalent impedance



▲ FIGURE 31

Norton equivalent circuit.

The form of Norton's equivalent circuit is shown in Figure 31. Regardless of how complex the original circuit is, it can be reduced to this equivalent form. The equivalent current source is designated $\mathbf{I}_n$, and the equivalent impedance is $\mathbf{Z}_n$ (lowercase italic subscript denotes ac quantity).

Norton's theorem shows you how to find $\mathbf{I}_n$ and $\mathbf{Z}_n$. Once they are known, simply connect them in parallel to get the complete Norton equivalent circuit.

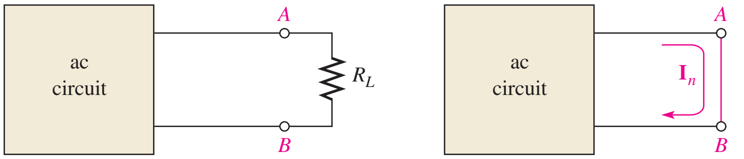
Norton's Equivalent Current Source ($\mathbf{I}_n$)

$\mathbf{I}_n$ is one part of the Norton equivalent circuit; $\mathbf{Z}_n$ is the other part.

> **Norton's equivalent current is defined as the short-circuit current between two specified terminals in a given circuit.**

Any load connected between these two terminals effectively "sees" a current source $\mathbf{I}_n$ in parallel with $\mathbf{Z}_n$.

To illustrate, let's suppose that the circuit shown in Figure 32 has a load resistor connected to terminals *A* and *B*, as indicated in part (a), and we wish to find the Norton equivalent


ac
circuit
A
B
R_L
ac
circuit
A
B
$\mathbf{I}_n$

(a) Circuit with load resistor

(b) Load is replaced by a short, resulting in a short circuit current, $\mathbf{I}_n$.

▲ **FIGURE 32**

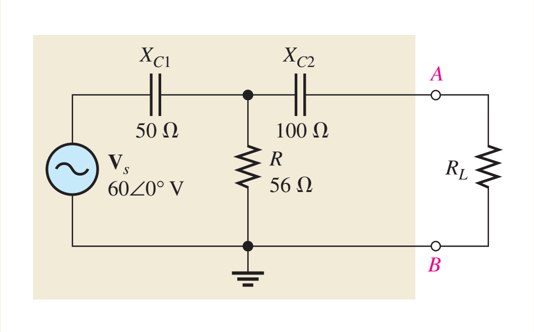
How $\mathbf{I}_n$ is determined.

for the circuit as viewed from terminals *A* and *B*. To find $\mathbf{I}_n$, calculate the current between terminals *A* and *B* with those terminals shorted, as shown in part (b). Example 13 shows how to find $\mathbf{I}_n$.

EXAMPLE 13

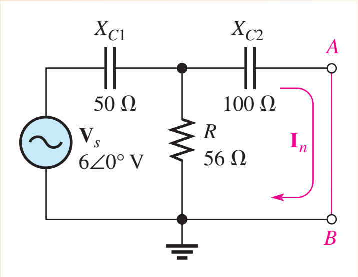
In Figure 33, determine $\mathbf{I}_n$ for the circuit as “seen” by the load resistor. The beige area identifies the portion of the circuit to be nortonized.

▶ **FIGURE 33**


X_{C1}
X_{C2}
A
50 Ω
100 Ω
$\mathbf{V}_s$
60∠0° V
R
56 Ω
R_L
B

Solution Short the terminals *A* and *B*, as shown in Figure 34.

▶ **FIGURE 34**


X_{C1}
X_{C2}
A
50 Ω
100 Ω
$\mathbf{V}_s$
6∠0° V
R
56 Ω
$\mathbf{I}_n$
B

$\mathbf{I}_n$ is the current through the short and is calculated as follows. First, the total impedance viewed from the source is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 50\angle{-90^\circ}\,\Omega + \frac{(56\angle 0^\circ\,\Omega)(100\angle{-90^\circ}\,\Omega)}{56\,\Omega - j100\,\Omega}$$
$$= 50\angle{-90^\circ}\,\Omega + 48.9\angle{-29.3^\circ}\,\Omega$$
$$= -j50\,\Omega + 42.6\,\Omega - j23.9\,\Omega = 42.6\,\Omega - j73.9\,\Omega$$

Converting to polar form yields

$$\mathbf{Z} = 85.3\angle{-60.0^\circ}\,\Omega$$

Next, the total current from the source is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{6\angle 0^\circ\text{ V}}{85.3\angle -60.0^\circ\ \Omega} = 70.3\angle 60.0^\circ\text{ mA}$$

Finally, apply the current-divider formula to get $\mathbf{I}_n$ (the current through the short between terminals A and B).

$$\mathbf{I}_n = \left(\frac{\mathbf{R}}{\mathbf{R} + \mathbf{X}_{C2}}\right)\mathbf{I}_s = \left(\frac{56\angle 0^\circ\ \Omega}{56\ \Omega - j100\ \Omega}\right)70.3\angle 60.0^\circ\text{ mA} = \mathbf{34.4\angle 121^\circ\ mA}$$

This is the value for the equivalent Norton current source.

Related Problem Determine $\mathbf{I}_n$ if $\mathbf{V}_s$ is changed to $2.5\angle 0^\circ$ V and R is changed to 33 Ω in Figure 33.

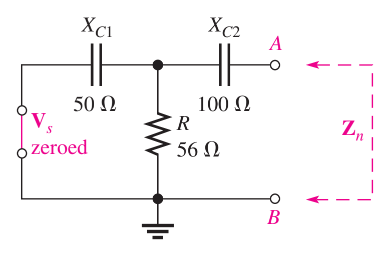
Norton's Equivalent Impedance ($\mathbf{Z}_n$)

$\mathbf{Z}_n$ is defined the same as $\mathbf{Z}_{th}$: It is the total impedance appearing between two specified terminals of a given circuit viewed from the open terminals with all sources replaced by their internal impedances.

EXAMPLE 14 Find $\mathbf{Z}_n$ for the circuit in Figure 33 (Example 13) viewed from the open across terminals A and B.

Solution First, replace $\mathbf{V}_s$ with its internal impedance (zero), as indicated in Figure 35.

▶ FIGURE 35



Looking in between terminals A and B, C_2 is in series with the parallel combination of R and C_1. Thus,

$$\mathbf{Z}_n = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 100\angle -90^\circ\ \Omega + \frac{(56\angle 0^\circ\ \Omega)(50\angle -90^\circ\ \Omega)}{56\ \Omega - j50\ \Omega}$$

$$= 100\angle -90^\circ\ \Omega + 37.3\angle -48.2^\circ\ \Omega$$

$$= -j100\ \Omega + 24.8\ \Omega - j27.8\ \Omega = \mathbf{24.8\ \Omega - \mathit{j}128\ \Omega}$$

The Norton equivalent impedance is a 24.8 Ω resistance in series with a 128 Ω capacitive reactance.

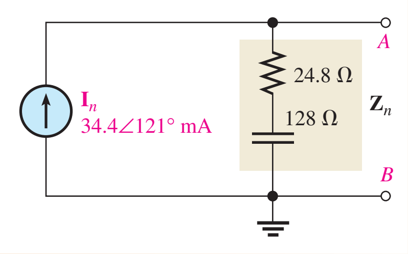
Related Problem Find $\mathbf{Z}_n$ in Figure 33 if $R = 33\ \Omega$.

Examples 13 and 14 showed how to find the two equivalent components of a Norton equivalent circuit. Keep in mind that you can find these values for any given ac circuit. Once you know $\mathbf{I}_n$ and $\mathbf{Z}_n$, connect them in parallel to form the Norton equivalent circuit, as Example 15 illustrates.

EXAMPLE 15 Show the complete Norton equivalent circuit for the circuit in Figure 33 (Example 13).

Solution From Examples 13 and 14, respectively, $\mathbf{I}_n = 34.4\angle 121°$ mA and $\mathbf{Z}_n = 24.8\ \Omega - j128\ \Omega$. The Norton equivalent circuit is shown in Figure 36.

FIGURE 36



Related Problem Show the Norton equivalent for the circuit in Figure 33 if $\mathbf{V}_s = 2.5\angle 0°$ V and $R = 33\ \Omega$.

Summary of Norton's Theorem

Any load connected between the terminals of a Norton equivalent circuit will have the same current through it and the same voltage across it as it would when connected to the terminals of the original circuit. A summary of steps for theoretically applying Norton's theorem is as follows:

Step 1: Replace the load connected to the two terminals between which the Norton circuit is to be determined with a short.

Step 2: Determine the current through the short. This is $\mathbf{I}_n$.

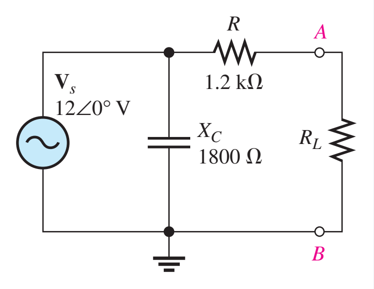
Step 3: Open the terminals and determine the impedance between the two open terminals with all sources replaced with their internal impedances. This is $\mathbf{Z}_n$.

Step 4: Connect $\mathbf{I}_n$ and $\mathbf{Z}_n$ in parallel.

SECTION 3 CHECKUP

1. For a given circuit, $I_n = 5\angle 0°$ mA, and $Z_n = 150\Omega + j100\Omega$. Draw the Norton equivalent circuit.
2. Find the Norton circuit as seen by R_L in Figure 37.

FIGURE 37



4 MAXIMUM POWER TRANSFER THEOREM

Maximum power is transferred to a load connected to a circuit when the load impedance is the complex conjugate of the circuit's output impedance.

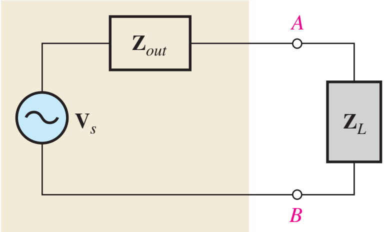
After completing this section, you should be able to

- **Apply the maximum power transfer theorem**
    - Explain the maximum power transfer theorem
    - Determine the value of load impedance for which maximum power is transferred from a given circuit

The **complex conjugate** of $R - jX_C$ is $R + jX_L$ and vice versa, where the resistances are equal in magnitude and the reactances are equal in magnitude but opposite in sign. The output impedance is effectively Thevenin's equivalent impedance viewed from the output terminals. When $\mathbf{Z}_L$ is the complex conjugate of $\mathbf{Z}_{out}$, maximum power is transferred from the circuit to the load with a power factor of 1. An equivalent circuit with its output impedance and load is shown in Figure 38.

▶ FIGURE 38

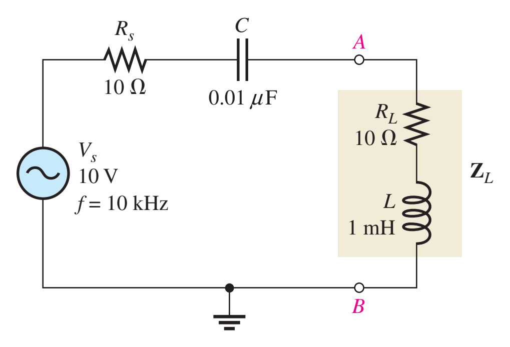
Equivalent circuit with load.



Example 16 shows that maximum power occurs when the impedances are conjugately matched.

EXAMPLE 16

The circuit to the left of terminals A and B in Figure 39 provides power to the load $\mathbf{Z}_L$. This can be viewed as simulating a power amplifier delivering power to a complex load. It is the Thevenin equivalent of a more complex circuit. Calculate and plot a



▲ FIGURE 39

graph of the power delivered to the load for each of the following frequencies: 10 kHz, 30 kHz, 50 kHz, 80 kHz, and 100 kHz.

Solution For $f = 10\text{ kHz}$,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$
$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(1\text{ mH}) = 62.8\ \Omega$$

The magnitude of the total impedance is

$$Z_{tot} = \sqrt{(R_s + R_L)^2 + (X_L - X_C)^2} = \sqrt{(20\ \Omega)^2 + (1.53\text{ k}\Omega)^2} = 1.53\text{ k}\Omega$$

The current is

$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{1.53\text{ k}\Omega} = 6.54\text{ mA}$$

The load power is

$$P_L = I^2R_L = (6.54\text{ mA})^2(10\ \Omega) = \mathbf{428\ \mu W}$$

For $f = 30\text{ kHz}$,

$$X_C = \frac{1}{2\pi(30\text{ kHz})(0.01\ \mu\text{F})} = 531\ \Omega$$
$$X_L = 2\pi(30\text{ kHz})(1\text{ mH}) = 189\ \Omega$$
$$Z_{tot} = \sqrt{(20\ \Omega)^2 + (342\ \Omega)^2} = 343\ \Omega$$
$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{343\ \Omega} = 29.2\text{ mA}$$
$$P_L = I^2R_L = (29.2\text{ mA})^2(10\ \Omega) = \mathbf{8.53\ mW}$$

For $f = 50\text{ kHz}$,

$$X_C = \frac{1}{2\pi(50\text{ kHz})(0.01\ \mu\text{F})} = 318\ \Omega$$
$$X_L = 2\pi(50\text{ kHz})(1\text{ mH}) = 314\ \Omega$$

Note that X_C and X_L are very close to being equal which makes the impedances approximately complex conjugates. The exact frequency at which $X_L = X_C$ is 50.3 kHz.

$$Z_{tot} = \sqrt{(20\ \Omega)^2 + (4\ \Omega)^2} = 20.4\ \Omega$$
$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{20.4\ \Omega} = 490\text{ mA}$$
$$P_L = I^2R_L = (490\text{ mA})^2(10\ \Omega) = \mathbf{2.40\ W}$$

For $f = 80\text{ kHz}$,

$$X_C = \frac{1}{2\pi(80\text{ kHz})(0.01\ \mu\text{F})} = 199\ \Omega$$
$$X_L = 2\pi(80\text{ kHz})(1\text{ mH}) = 503\ \Omega$$
$$Z_{tot} = \sqrt{(20\ \Omega)^2 + (304\ \Omega)^2} = 305\ \Omega$$
$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{305\ \Omega} = 32.8\text{ mA}$$
$$P_L = I^2R_L = (32.8\text{ mA})^2(10\ \Omega) = \mathbf{10.8\ mW}$$

For $f = 100\text{ kHz}$,

$$X_C = \frac{1}{2\pi(100\text{ kHz})(0.01\ \mu\text{F})} = 159\ \Omega$$

$$X_L = 2\pi(100\text{ kHz})(1\text{ mH}) = 628\ \Omega$$

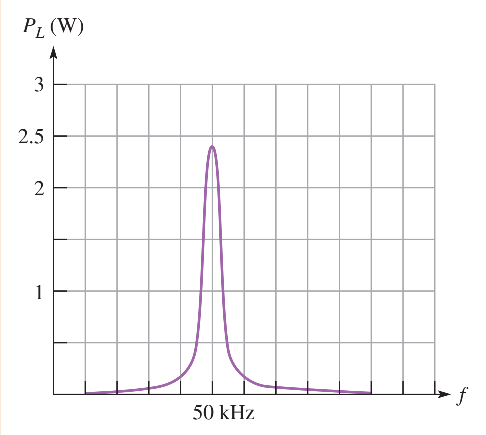
$$Z_{tot} = \sqrt{(20\ \Omega)^2 + (469\ \Omega)^2} = 469\ \Omega$$

$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{469\ \Omega} = 21.3\text{ mA}$$

$$P_L = I^2R_L = (21.3\text{ mA})^2(10\ \Omega) = \mathbf{4.54\ mW}$$

As you can see from the results, the power to the load peaks at the frequency (50 kHz) for which the load impedance is the complex conjugate of the output impedance (when the reactances are equal in magnitude). A graph of the load power versus frequency is shown in Figure 40. Since the maximum power is so much larger than the other values, an accurate plot is difficult to achieve without intermediate values.

FIGURE 40



Related Problem If $R = 47\ \Omega$ and $C = 0.022\ \mu\text{F}$ in a series RC circuit, what is the complex conjugate of the impedance at 100 kHz?

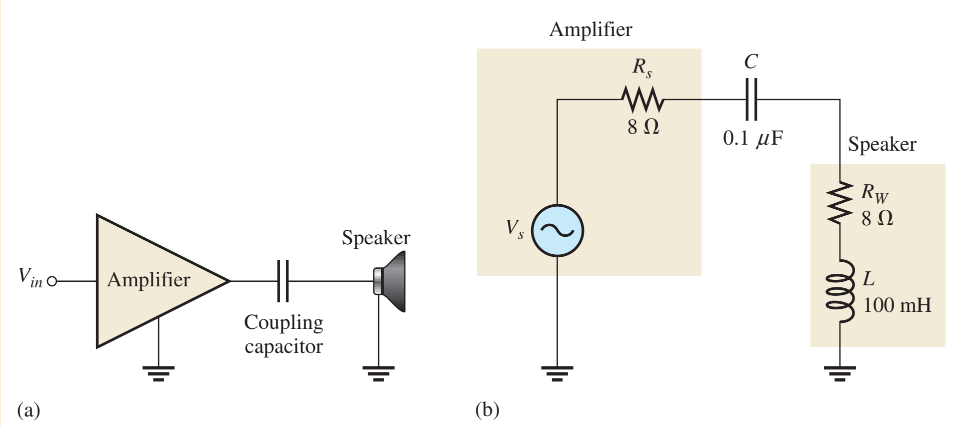
Use Multisim files E19-16A and E19-16B to verify the calculated results in this example.

Example 17 illustrates that the frequency at which maximum load power occurs is the value that makes the source and load impedances complex conjugates.

EXAMPLE 17

(a) Determine the frequency at which maximum power is transferred from the amplifier to the speaker in Figure 41(a). The amplifier and coupling capacitor are the source, and the speaker is the load, as shown in the equivalent circuit of Figure 41(b).

(b) How many watts of power are delivered to the speaker at this frequency if $V_s =$ 3.8 V rms?



▲ FIGURE 41

Solution **(a)** When the power to the speaker is maximum, the source impedance ($R_s - jX_C$) and the load impedance ($R_W + jX_L$) are complex conjugates, so

$$X_C = X_L$$

$$\frac{1}{2\pi fC} = 2\pi fL$$

Solving for f,

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.1\ \mu\text{F})}} \cong \mathbf{1.59\ kHz}$$

(b) Calculate the power to the speaker as follows:

$$Z_{tot} = R_s + R_W = 8\ \Omega + 8\ \Omega = 16\ \Omega$$

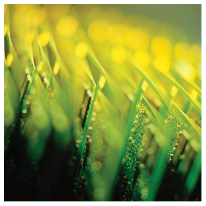
$$I = \frac{V_s}{Z_{tot}} = \frac{3.8\text{ V}}{16\ \Omega} = 238\text{ mA}$$

$$P_{max} = I^2 R_W = (238\text{ mA})^2(8\ \Omega) = \mathbf{453\ mW}$$

Related Problem Determine the frequency at which maximum power is transferred from the amplifier to the speaker in Figure 41 if the coupling capacitor is 1 μF.

SECTION 4 CHECKUP

1. If the output impedance of a certain driving circuit is 50 Ω − j10Ω, what value of load impedance will result in the maximum power to the load?
2. For the circuit in Question 1, how much power is delivered to the load when the load impedance is the complex conjugate of the output impedance and when the load current is 2 A?



Application Activity

In this application activity, you have a sealed band-pass filter module that has been removed from a system and two schematics. Both schematics indicate that the band-pass filter is implemented with a low-pass/high-pass combination. It is uncertain which schematic corresponds to the filter module, but one of them does. By certain measurements, you will determine which schematic represents the filter so that the filter circuit can be reproduced. Also, you will determine the proper load for maximum power transfer. The filter circuit contained in a sealed module and two schematics, one of which corresponds to the filter circuit, are shown in Figure 42.

Filter Measurement and Analysis

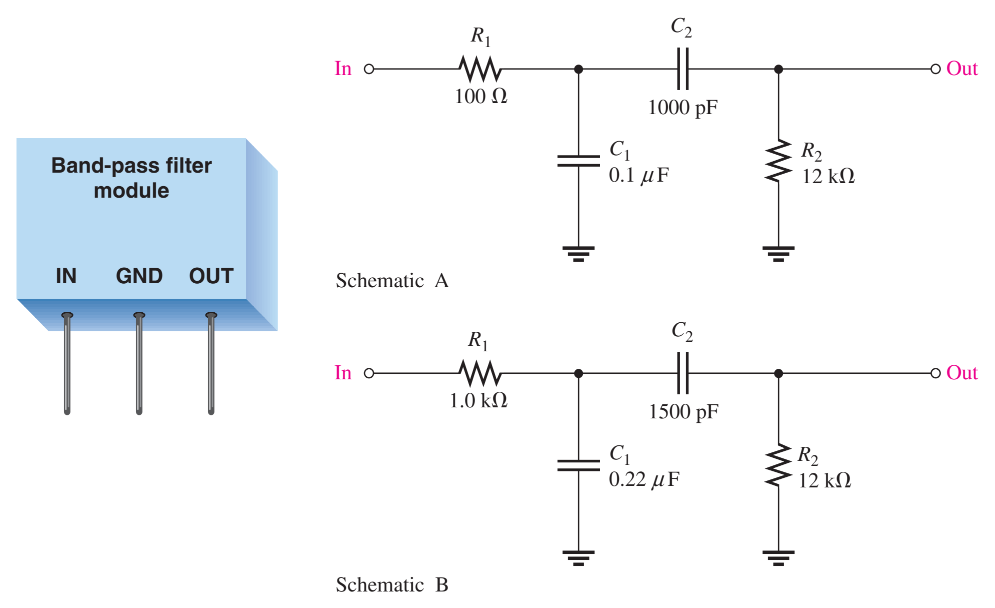
1. Based on the oscilloscope measurement of the filter output shown in Figure 43, determine which schematic in Figure 42 represents the component values of the filter circuit in the module. A 10 V peak-to-peak voltage is applied to the input.
2. Based on the oscilloscope measurement in Figure 43, determine if the filter is operating at its approximate center frequency.
3. Using Thevenin's theorem, determine the load impedance that will provide for maximum power transfer at the center frequency when connected to the output of the filter. Assume the source impedance is zero.

Review

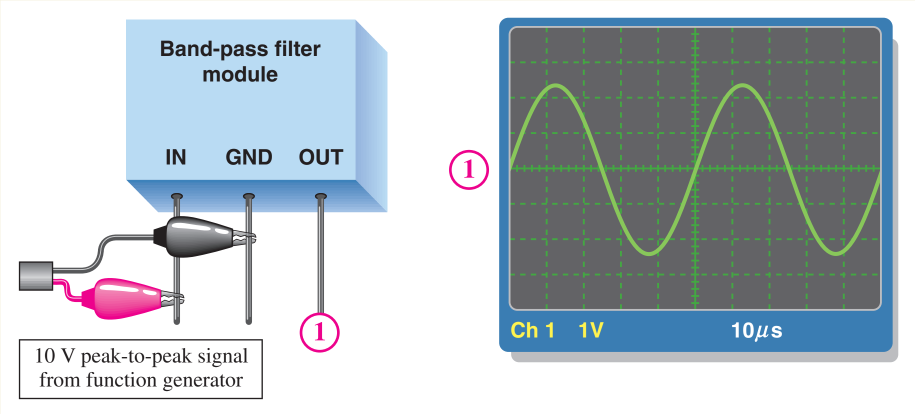
4. Determine the peak-to-peak output voltage at the frequency shown in Figure 43 of the circuit in Figure 42 that was determined not to be in the module.
5. Find the center frequency of the circuit in Figure 42 that was determined not to be in the module.

▶ FIGURE 42

Filter module and schematics.


Band-pass filter module
IN
GND
OUT
In
R_1
100 Ω
C_2
1000 pF
Out
C_1
0.1 μF
R_2
12 kΩ
Schematic A
In
R_1
1.0 kΩ
C_2
1500 pF
Out
C_1
0.22 μF
R_2
12 kΩ
Schematic B

▶ FIGURE 43


Band-pass filter module
IN
GND
OUT
①
10 V peak-to-peak signal from function generator
①
Ch 1 1V
10μs

SUMMARY

- The superposition theorem is useful for the analysis of both ac and dc multiple-source circuits.
- Thevenin's theorem provides a method for the reduction of any ac circuit to an equivalent form consisting of an equivalent voltage source in series with an equivalent impedance.
- The term *equivalency,* as used in Thevenin's and Norton's theorems, means that when a given load impedance is connected to the equivalent circuit, it will have the same voltage across it and the same current through it as when it is connected to the original circuit.
- Norton's theorem provides a method for the reduction of any ac circuit to an equivalent form consisting of an equivalent current source in parallel with an equivalent impedance.
- Maximum power is transferred to a load when the load impedance is the complex conjugate of the impedance of the driving circuit.

KEY TERMS

Complex conjugate A complex number having the same real part and an oppositely signed imaginary part; an impedance containing the same resistance and a reactance opposite in phase but equal in magnitude to that of a given impedance.

Equivalent circuit A circuit that produces the same voltage and current to a given load as the original circuit that it replaces.

Norton's theorem A method for simplifying a two-terminal circuit to an equivalent circuit with only a current source in parallel with an impedance.

Superposition theorem A method for the analysis of circuits with more than one source.

Thevenin's theorem A method for simplifying a two-terminal circuit to an equivalent circuit with only a voltage source in series with an impedance.

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The superposition theorem can be applied to ac circuits with multiple sources.
2. If the current through a given component from one source is 100 mA and the current through the same component from a second source is 60 mA in the opposite direction, the total current through the component is 160 mA.
3. The purpose of Thevenin's theorem is to simplify a circuit to make analysis easier.
4. The Thevenin equivalent for an ac circuit consists of an equivalent impedance and an equivalent voltage source.
5. If the output voltage across the open terminals of a certain ac circuit is 5 V, the Thevenin equivalent voltage must be less than 5 V.
6. To determine the Thevenin equivalent impedance of a circuit with an ac voltage source, the source must be replaced by an open.
7. Norton's theorem can be applied to a circuit with an ac current source.
8. The Norton equivalent impedance is found by replacing the ac current source with an open.
9. The complex conjugate of $10\ \Omega - j12\ \Omega$ is $12\ \Omega + j10\ \Omega$.
10. Maximum power is transferred from a circuit to the load when the load is the complex conjugate of the output impedance.

SELF-TEST

Answers are at the end of the chapter.

1. In applying the superposition theorem,
   (a) all sources are considered simultaneously
   (b) all voltage sources are considered simultaneously
   (c) the sources are considered one at a time with all others replaced by a short
   (d) the sources are considered one at a time with all others replaced by their internal impedances

2. A Thevenin ac equivalent circuit always consists of an equivalent ac voltage source
(a) and an equivalent capacitance **(b)** and an equivalent inductive reactance
(c) and an equivalent impedance **(d)** in series with an equivalent capacitive reactance

3. One circuit is equivalent to another when
(a) the same load has the same voltage and current when connected to either circuit
(b) different loads have the same voltage and current when connected to either circuit
(c) the circuits have equal voltage sources and equal series impedances
(d) the circuits produce the same output voltage

4. The Thevenin equivalent voltage is
(a) the open circuit voltage **(b)** the short circuit voltage
(c) the voltage across an equivalent load **(d)** none of the above

5. The Thevenin equivalent impedance is the impedance looking from
(a) the source with the output shorted
(b) the source with the output open
(c) any two specified open terminals with all sources replaced by their internal impedances
(d) any two specified open terminals with all sources replaced by a short

6. A Norton ac equivalent circuit always consists of
(a) an equivalent ac current source in series with an equivalent impedance
(b) an equivalent ac current source in parallel with an equivalent reactance
(c) an equivalent ac current source in parallel with an equivalent impedance
(d) an equivalent ac voltage source in parallel with an equivalent impedance

7. The Norton equivalent current is
(a) the total current from the source **(b)** the short circuit current
(c) the current to an equivalent load **(d)** none of the above

8. The complex conjugate of $50\ \Omega + j100\ \Omega$ is
(a) $50\ \Omega - j50\ \Omega$ **(b)** $100\ \Omega + j50\ \Omega$ **(c)** $100\ \Omega - j50\ \Omega$ **(d)** $50\ \Omega - j100\ \Omega$

9. In order to get maximum power transfer from a capacitive source, the load must
(a) have a capacitance equal to the source capacitance
(b) have an impedance equal in magnitude to the source impedance
(c) be inductive
(d) have an impedance that is the complex conjugate of the source impedance
(e) answers (a) and (d)

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 47.

1. If the dc voltage source is shorted out, the voltage at point *A* with respect to ground
(a) increases **(b)** decreases **(c)** stays the same

2. If C_2 opens, the voltage across R_5
(a) increases **(b)** decreases **(c)** stays the same

3. If C_2 opens, the dc voltage across R_5
(a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 49(c).

4. If V_2 is reduced to 0 V, the voltage across R_L
(a) increases **(b)** decreases **(c)** stays the same

5. If the frequency of the voltage sources is increased, the current through R_L

(**a**) increases (**b**) decreases (**c**) stays the same

Refer to Figure 50.

6. If the frequency of the source voltage is increased, the voltage across R_1

(**a**) increases (**b**) decreases (**c**) stays the same

7. If R_L opens, the voltage across it

(**a**) increases (**b**) decreases (**c**) stays the same

Refer to Figure 51.

8. If the source frequency is increased, the voltage across R_3

(**a**) increases (**b**) decreases (**c**) stays the same

9. If the capacitor value is reduced, the current from the source

(**a**) increases (**b**) decreases (**c**) stays the same

Refer to Figure 54.

10. If R_2 opens, the current from the current source

(**a**) increases (**b**) decreases (**c**) stays the same

11. If the frequency of the voltage source increase, X_{C2}

(**a**) increases (**b**) decreases (**c**) stays the same

12. If the load is removed, the voltage across R_3

(**a**) increases (**b**) decreases (**c**) stays the same

13. If the load is removed, the voltage across R_2

(**a**) increases (**b**) decreases (**c**) stays the same

PROBLEMS

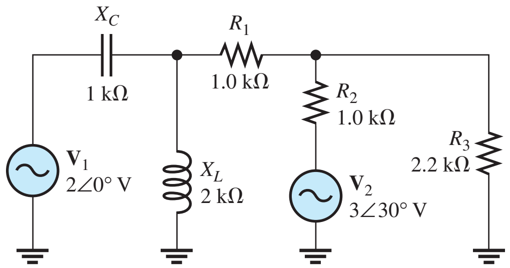
More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The Superposition Theorem

1. Using the superposition method, calculate the current through R_3 in Figure 44.

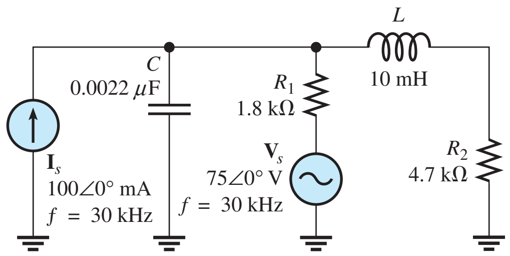
2. Use the superposition theorem to find the current in and the voltage across the R_2 branch of Figure 44.

FIGURE 44

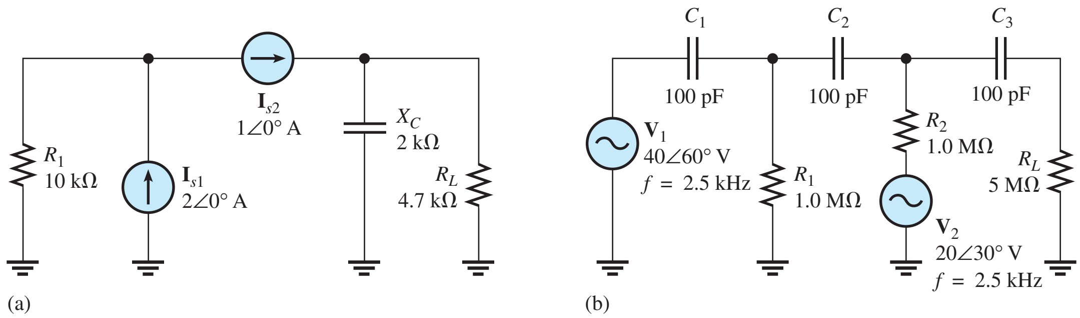


3. Using the superposition theorem, solve for the current through R_1 in Figure 45.

FIGURE 45



4. Using the superposition theorem, find the current through R_L in each circuit of Figure 46.

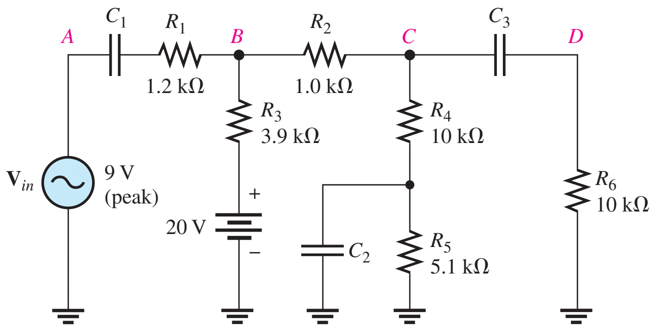

R_1
10 kΩ
$\mathbf{I}_{s1}$
2∠0° A
$\mathbf{I}_{s2}$
1∠0° A
X_C
2 kΩ
R_L
4.7 kΩ
(a)
C_1
C_2
C_3
100 pF
100 pF
100 pF
$\mathbf{V}_1$
40∠60° V
f = 2.5 kHz
R_1
1.0 MΩ
R_2
1.0 MΩ
$\mathbf{V}_2$
20∠30° V
f = 2.5 kHz
R_L
5 MΩ
(b)

▲ FIGURE 46

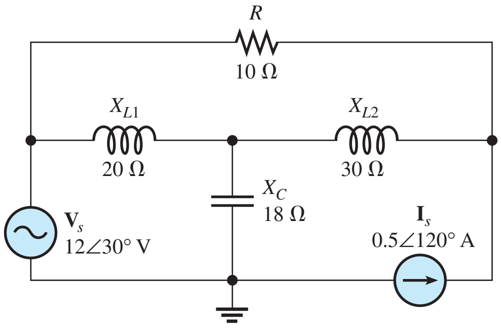
*5. Determine the voltage at each point (A, B, C, D) in Figure 47. Assume $X_C = 0$ for all capacitors. Draw the voltage waveforms at each of the points.

*6. Use the superposition theorem to find the capacitor current in Figure 48.

*7. Use the superposition theorem to find the resistor current in Figure 48.


A
C_1
R_1
1.2 kΩ
B
R_2
1.0 kΩ
C
C_3
D
R_3
3.9 kΩ
R_4
10 kΩ
$\mathbf{V}_{in}$
9 V
(peak)
+
20 V
−
C_2
R_5
5.1 kΩ
R_6
10 kΩ

▲ FIGURE 47

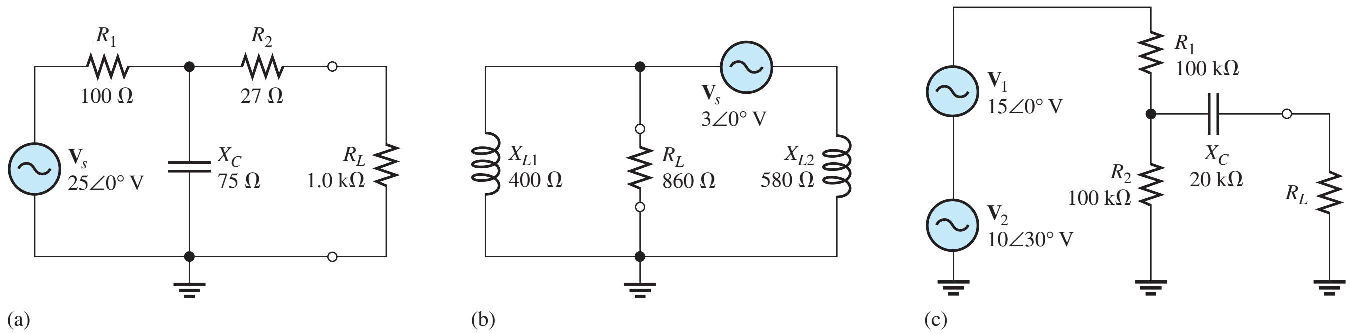

R
10 Ω
X_{L1}
20 Ω
X_{L2}
30 Ω
X_C
18 Ω
$\mathbf{V}_s$
12∠30° V
$\mathbf{I}_s$
0.5∠120° A

▲ FIGURE 48

SECTION 2 Thevenin's Theorem

*8. Using Thevenin's theorem, determine the ac equivalent circuit for the circuit in Figure 47 as viewed from R_6. Assume all $X_C = 0$.

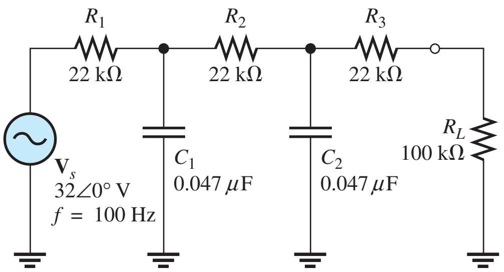
9. For each circuit in Figure 49, determine the Thevenin equivalent circuit for the portion of the circuit viewed by R_L.


R_1
100 Ω
R_2
27 Ω
$\mathbf{V}_s$
25∠0° V
X_C
75 Ω
R_L
1.0 kΩ
(a)
X_{L1}
400 Ω
R_L
860 Ω
$\mathbf{V}_s$
3∠0° V
X_{L2}
580 Ω
(b)
$\mathbf{V}_1$
15∠0° V
R_1
100 kΩ
X_C
20 kΩ
R_2
100 kΩ
R_L
$\mathbf{V}_2$
10∠30° V
(c)

▲ FIGURE 49

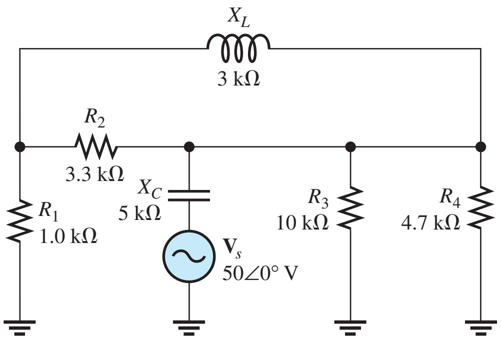
10. Using Thevenin's theorem, determine the current through the load R_L in Figure 50.

***11.** Using Thevenin's theorem, find the voltage across R_4 in Figure 51.



R1
22 kΩ
R2
22 kΩ
R3
22 kΩ
Vs
32∠0° V
f = 100 Hz
C1
0.047 μF
C2
0.047 μF
RL
100 kΩ

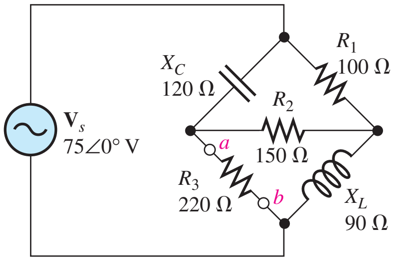
▲ FIGURE 50



XL
3 kΩ
R2
3.3 kΩ
XC
5 kΩ
R1
1.0 kΩ
Vs
50∠0° V
R3
10 kΩ
R4
4.7 kΩ

▲ FIGURE 51

***12.** Simplify the circuit external to R_3 in Figure 52 to its Thevenin equivalent.



XC
120 Ω
R1
100 Ω
R2
150 Ω
Vs
75∠0° V
a
R3
220 Ω
b
XL
90 Ω

▲ FIGURE 52

SECTION 3 Norton's Theorem

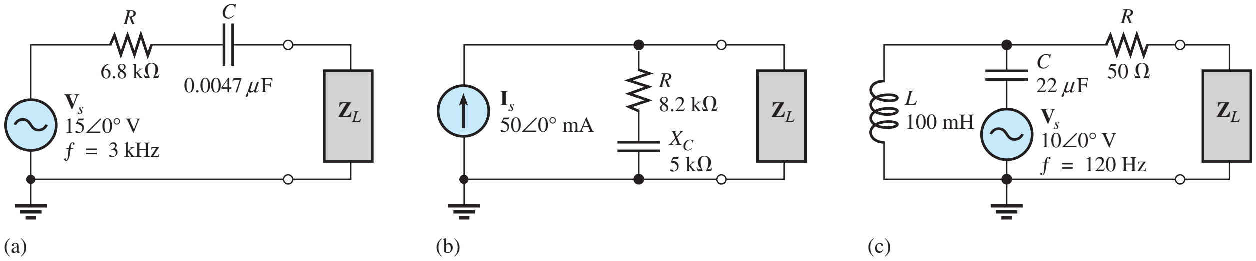
13. For each circuit in Figure 49, determine the Norton equivalent as seen by R_L.

14. Using Norton's theorem, find the current through the load resistor R_L in Figure 50.

***15.** Using Norton's theorem, find the voltage across R_4 in Figure 51.

SECTION 4 Maximum Power Transfer Theorem

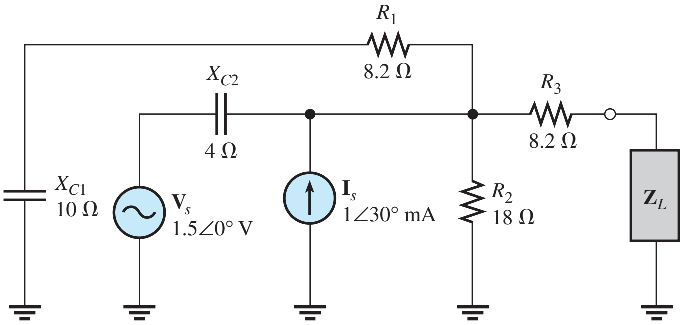
16. For each circuit in Figure 53, maximum power is to be transferred to the load R_L. Determine the appropriate value for the load impedance in each case.



R
6.8 kΩ
C
0.0047 μF
Vs
15∠0° V
f = 3 kHz
ZL
(a)
Is
50∠0° mA
R
8.2 kΩ
XC
5 kΩ
ZL
(b)
L
100 mH
C
22 μF
R
50 Ω
Vs
10∠0° V
f = 120 Hz
ZL
(c)

▲ FIGURE 53

***17.** Determine $\mathbf{Z}_L$ for maximum power in Figure 54.

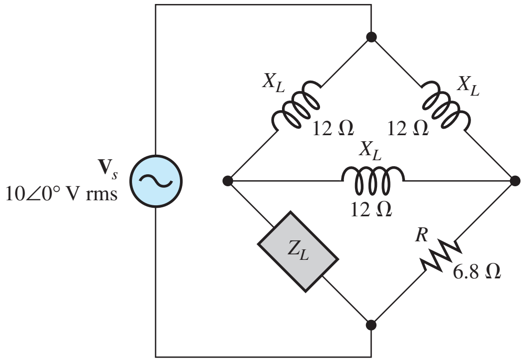


▲ FIGURE 54

***18.** Find the load impedance required for maximum power transfer to Z_L in Figure 55. Determine the maximum true power.

***19.** A load is to be connected in the place of R_2 in Figure 52 to achieve maximum power transfer. Determine the type of load, and express it in rectangular form.

▶ FIGURE 55



Multisim Troubleshooting and Analysis

These problems require Multisim.

20. Open file P19-20 and determine if there is a fault. If so, find the fault.

21. Open file P19-21 and determine if there is a fault. If so, find the fault.

22. Open file P19-22 and determine if there is a fault. If so, find the fault.

23. Open file P19-23 and determine if there is a fault. If so, find the fault.

24. Open file P19-24 and determine the Thevenin equivalent circuit by measurement looking from Point *A*.

25. Open file P19-25 and determine the Norton equivalent circuit by measurement looking from Point *A*.

ANSWERS

SECTION CHECKUPS

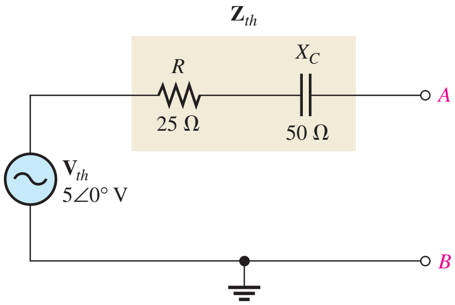
SECTION 1 The Superposition Theorem

1. The net current is zero.
2. The circuit can be analyzed one source at a time using superposition.
3. $I_R = 12$ mA

SECTION 2 Thevenin's Theorem

1. The components of a Thevenin equivalent ac circuit are equivalent voltage source and equivalent series impedance.
2. See Figure 56.
3. $\mathbf{Z}_{th} = 21.5\ \Omega - j15.7\ \Omega$; $\mathbf{V}_{th} = 4.14\angle 53.8°\ \text{V}$

▶ FIGURE 56

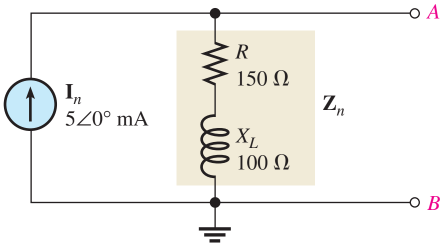


$\mathbf{Z}_{th}$
R
25 Ω
X_C
50 Ω
A
$\mathbf{V}_{th}$
$5\angle 0°$ V
B

SECTION 3 Norton's Theorem

1. See Figure 57.
2. $\mathbf{Z}_n = R\angle 0° = 1.2\angle 0°\ \text{k}\Omega$; $\mathbf{I}_n = 10\angle 0°\ \text{mA}$

▶ FIGURE 57



A
$\mathbf{I}_n$
$5\angle 0°$ mA
R
150 Ω
X_L
100 Ω
$\mathbf{Z}_n$
B

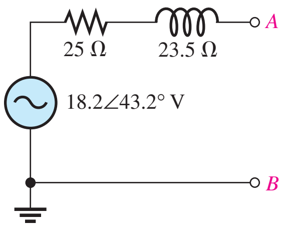
SECTION 4 Maximum Power Transfer Theorem

1. $\mathbf{Z}_L = 50\ \Omega + j10\ \Omega$
2. $P_L = 200\ \text{W}$

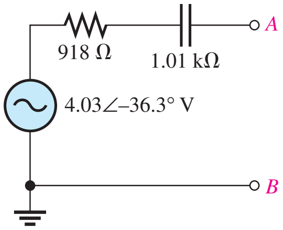
RELATED PROBLEMS FOR EXAMPLES

1 $2.11\angle -153°$ mA
2 $30\angle 90°$ mA
3 $1.69\angle 47.3°$ mA riding on a dc level of 3 mA
4 $18.2\angle 43.2°$ V
5 $4.03\angle -36.3°$ V
6 $4.55\angle 24.4°$ V
7 $34.3\angle 43.2°\ \Omega$
8 $1.37\angle -47.8°\ \text{k}\Omega$
9 $9.10\angle -65.6°\ \text{k}\Omega$

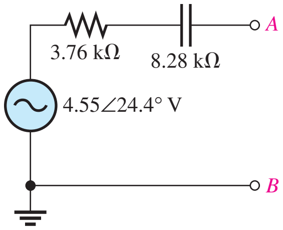
10 See Figure 58.
11 See Figure 59.
12 See Figure 60.


A
25 Ω
23.5 Ω
18.2∠43.2° V
B

▲ **FIGURE 58**


A
918 Ω
1.01 kΩ
4.03∠–36.3° V
B

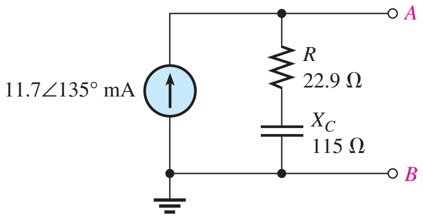
▲ **FIGURE 59**


A
3.76 kΩ
8.28 kΩ
4.55∠24.4° V
B

▲ **FIGURE 60**

13 $11.7\angle 135°$ mA
14 $117\angle -78.7°$ Ω
15 See Figure 61.

► **FIGURE 61**


A
R
22.9 Ω
X_C
115 Ω
11.7∠135° mA
B

16 47 Ω + j72.3 Ω
17 503 Hz

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F
6. T **7.** T **8.** F **9.** F **10.** T

SELF-TEST

1. (d) **2.** (c) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (b) **8.** (d) **9.** (d)

CIRCUIT DYNAMICS QUIZ

1. (c) **2.** (a) **3.** (c) **4.** (b) **5.** (a) **6.** (a) **7.** (a)
8. (a) **9.** (b) **10.** (c) **11.** (b) **12.** (b) **13.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. $1.22\angle 28.6^\circ$ mA

3. $81.0\angle -11.9^\circ$ mA

5. $V_{A(\text{dc})} = 0\text{ V}$, $V_{B(\text{dc})} = 16.1\text{ V}$, $V_{C(\text{dc})} = 15.1\text{ V}$,
$V_{D(\text{dc})} = 0\text{ V}$, $V_{A(\text{peak})} = 9\text{ V}$, $V_{B(\text{peak})} = 5.96\text{ V}$,
$V_{C(\text{peak})} = V_{D(\text{peak})} = 4.96\text{ V}$

7. $766\angle -71.7^\circ$ mA

9. **(a)** $\mathbf{V}_{th} = 15\angle -53.1^\circ\text{ V}$
$\mathbf{Z}_{th} = 63\ \Omega - j48\ \Omega = 79.2\angle -37.3^\circ\ \Omega$
(b) $\mathbf{V}_{th} = 1.22\angle 0^\circ\text{ V}$
$\mathbf{Z}_{th} = j237\ \Omega = 237\angle 90^\circ\ \Omega$
(c) $\mathbf{V}_{th} = 12.1\angle 11.9^\circ\text{ V}$
$\mathbf{Z}_{th} = 50\text{ k}\Omega - j20\text{ k}\Omega = 53.9\angle -21.8^\circ\text{ k}\Omega$

11. $16.9\angle 88.2^\circ$ V

13. **(a)** $\mathbf{I}_n = 189\angle -15.8^\circ\text{ mA}$
$\mathbf{Z}_n = 63\ \Omega - j48\ \Omega$
(b) $\mathbf{I}_n = 5.15\angle -90^\circ\text{ mA}$
$\mathbf{Z}_n = j237\ \Omega$
(c) $\mathbf{I}_n = 224\angle 33.7^\circ\ \mu\text{A}$
$\mathbf{Z}_n = 50\text{ k}\Omega - j20\text{ k}\Omega$

15. $16.8\angle 88.5^\circ$ V

17. $9.18\ \Omega + j2.90\ \Omega$

19. $95.2\ \Omega + j42.7\ \Omega$

21. C_2 leaky

23. No fault

25. $\mathbf{I}_n = 30.142\angle -113.1^\circ\ \mu\text{A}$
$\mathbf{Z}_n = 30.3\angle 64.28^\circ\text{ k}\Omega$

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TIME RESPONSE OF REACTIVE CIRCUITS

CHAPTER OUTLINE

1 The *RC* Integrator
2 Response of an *RC* Integrator to a Single Pulse
3 Response of *RC* Integrators to Repetitive Pulses
4 Response of an *RC* Differentiator to a Single Pulse
5 Response of *RC* Differentiators to Repetitive Pulses
6 Response of *RL* Integrators to Pulse Inputs
7 Response of *RL* Differentiators to Pulse Inputs
8 Relationship of Time Response to Frequency Response
9 Troubleshooting
Application Activity

CHAPTER OBJECTIVES

- Explain the operation of an *RC* integrator
- Analyze an *RC* integrator with a single input pulse
- Analyze an *RC* integrator with repetitive input pulses
- Analyze an *RC* differentiator with a single input pulse
- Analyze an *RC* differentiator with repetitive input pulses
- Analyze the operation of an *RL* integrator
- Analyze the operation of an *RL* differentiator
- Explain the relationship of time response to frequency response
- Troubleshoot *RC* integrators and *RC* differentiators

KEY TERMS

- Integrator
- Transient time
- Steady-state
- Differentiator
- DC component

APPLICATION ACTIVITY PREVIEW

In the application activity, you will have to specify the wiring in a time-delay circuit. You will also determine component values to meet certain specifications and then determine instrument settings to properly test the circuit.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, the time response of *RC* and *RL* circuits with pulse inputs is examined. Understanding exponential changes of voltages and currents in capacitors and inductors is crucial to the study of time response.

With pulse inputs, the time responses of circuits are important. In the areas of pulse and digital circuits, technicians are often concerned with how a circuit responds over an interval of time to rapid changes in voltages or current. The relationship of the circuit time constant to the input pulse characteristics, such as pulse width and period, determines the wave shapes of voltages in the circuit.

Integrator and *differentiator,* terms used throughout this chapter, refer to mathematical functions that are approximated by these circuits under certain conditions. Mathematical integration is a summation process, and mathematical differentiation is a process for establishing an instantaneous rate of change of a quantity.

From Chapter 20 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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1 THE *RC* INTEGRATOR

In terms of time response, a series *RC* circuit in which the output voltage is taken across the capacitor is known as an **integrator**. Recall that in terms of frequency response, it is a low-pass filter. The term *integrator* is derived from the mathematical process of integration, which this type of circuit approximates under certain conditions.

After completing this section, you should be able to

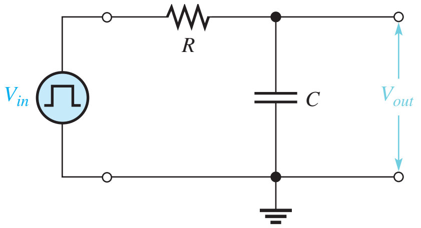
- **Explain the operation of an *RC* integrator**
    - Describe how a capacitor charges and discharges
    - Explain how a capacitor reacts to an instantaneous change in voltage or current
    - Describe the basic output voltage waveform

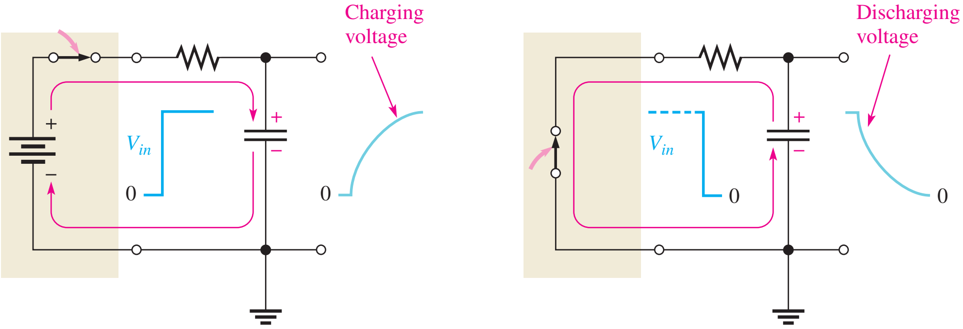
Charging and Discharging of a Capacitor

When a pulse generator is connected to the input of an *RC* integrator, as shown in Figure 1, the capacitor will charge and discharge in response to the pulses. When the input goes from its low level to its high level, the capacitor charges toward the high level of the pulse through the resistor. This charging action is analogous to connecting a battery through a closed switch to the *RC* circuit, as illustrated in Figure 2(a). When the pulse goes from its high level back to its low level, the capacitor discharges back through the source. Compared to the resistance of the resistor, the resistance of the source is assumed to be negligible. This discharging action is analogous to replacing the source with a closed switch, as illustrated in Figure 2(b).

▶ FIGURE 1

An *RC* integrator with a pulse generator connected.





(a) When the input pulse goes HIGH, the source effectively acts as a battery in series with a closed switch, thereby charging the capacitor.

(b) When the input pulse goes back LOW, the source effectively acts as a closed switch, providing a discharge path for the capacitor.

▲ FIGURE 2

The equivalent action when a pulse source charges and discharges the capacitor.

A capacitor will charge and discharge following an exponential curve. Its rate of charging and discharging, of course, depends on the *RC* **time constant**, a fixed time interval determined by *R* and *C* ($\tau = RC$).

For an ideal pulse, both edges are considered to be instantaneous. Two basic rules of capacitor behavior help in understanding the response of *RC* circuits to pulse inputs.

1. The capacitor appears as a short to an instantaneous change in current and as an open to dc.
2. The voltage across the capacitor cannot change instantaneously—it can change only exponentially.

Capacitor Voltage

In an *RC* integrator, the output is the capacitor voltage. The capacitor charges during the time that the pulse is high. If the pulse is at its high level long enough, the capacitor will fully charge to the voltage amplitude of the pulse, as illustrated in Figure 3. The capacitor discharges during the time that the pulse is low. If the low time between pulses is long enough, the capacitor will fully discharge to zero, as shown in the figure. Then when the next pulse occurs, it will charge again.

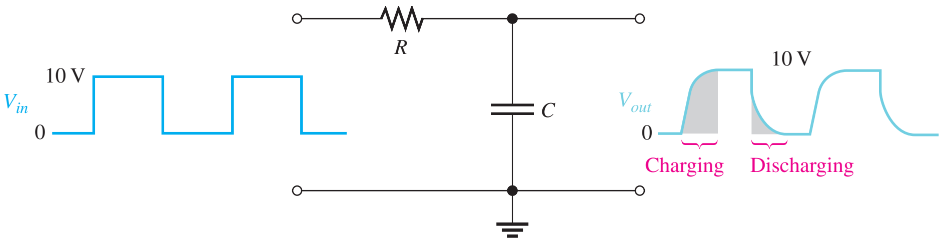


FIGURE 3

Illustration of a capacitor fully charging and discharging in response to a pulse input.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. Define the term *integrator* in relation to an *RC* circuit.
2. What causes the capacitor in an *RC* circuit to charge and discharge?

2 RESPONSE OF AN *RC* INTEGRATOR TO A SINGLE PULSE

From the previous section, you have a general idea of how an *RC* integrator responds to an input pulse. In this section, the response to a single pulse is examined in detail.

After completing this section, you should be able to

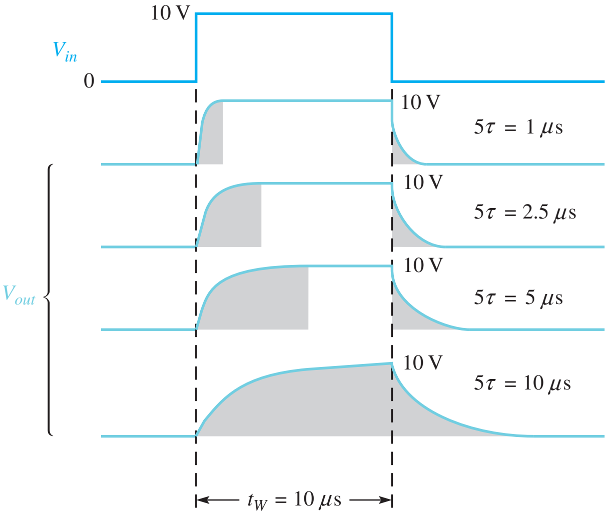
- **Analyze an *RC* integrator with a single input pulse**
  - Discuss the importance of the circuit time constant
  - Define *transient time*
  - Determine the response when the pulse width is equal to or greater than five time constants
  - Determine the response when the pulse width is less than five time constants

Two conditions of pulse response must be considered:

1. When the input pulse width (t_W) is equal to or greater than five time constants ($t_W \geq 5\tau$)
2. When the input pulse width is less than five time constants ($t_W < 5\tau$)

Recall that five time constants is accepted as the time for a capacitor to fully charge or fully discharge; this time is often called the **transient time**. A capacitor will fully charge if the pulse width is equal to or greater than five time constants (5τ). This condition is expressed as $t_W \geq 5\tau$. At the end of the pulse, the capacitor fully discharges back through the source.

Figure 4 illustrates the output waveforms for various *RC* transient times and a fixed input pulse width. Notice that as the transient time becomes shorter, compared to the pulse width, the shape of the output pulse approaches that of the input. In each case, the output reaches the full amplitude of the input.



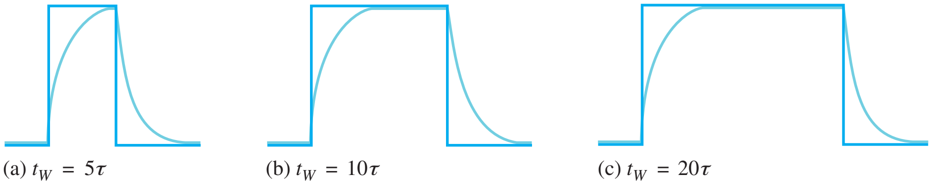
▲ **FIGURE 4**

Variation of an *RC* integrator's output pulse shape with time constant. The shaded areas indicate when the capacitor is charging or discharging.

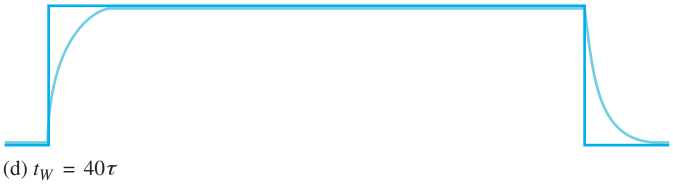
Figure 5 shows how a fixed time constant and a variable input pulse width affect the integrator output. Notice that as the pulse width is increased, the shape of the output pulse approaches that of the input. Again, this means that the transient time is short compared to the pulse width.

▶ **FIGURE 5**

Variation of an *RC* integrator's output pulse shape with input pulse width (the time constant is fixed). Dark blue is input and light blue is output.



(a) $t_W = 5\tau$ (b) $t_W = 10\tau$ (c) $t_W = 20\tau$



(d) $t_W = 40\tau$

Now let's examine the case in which the width of the input pulse is less than five time constants of the *RC* integrator. This condition is expressed as $t_W < 5\tau$. As you know, the capacitor charges for the duration of the pulse. However, because the pulse width is less than the time it takes the capacitor to fully charge (5τ), the output voltage will *not* reach the full input voltage before the end of the pulse. The capacitor only partially charges, as illustrated in Figure 6 for several values of *RC* time constants. Notice that for longer time constants, the output reaches a lower voltage because the capacitor cannot charge as much. Of course, in the examples with a single pulse input, the capacitor fully discharges after the pulse ends.

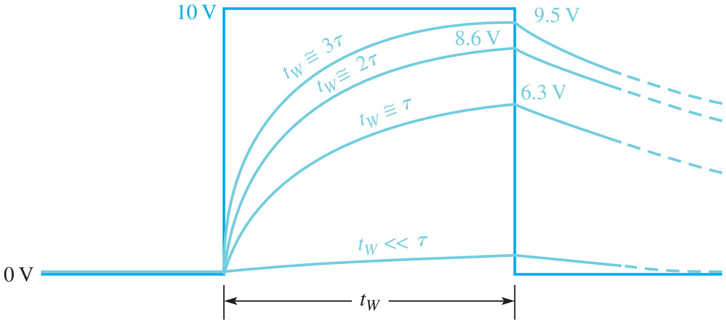
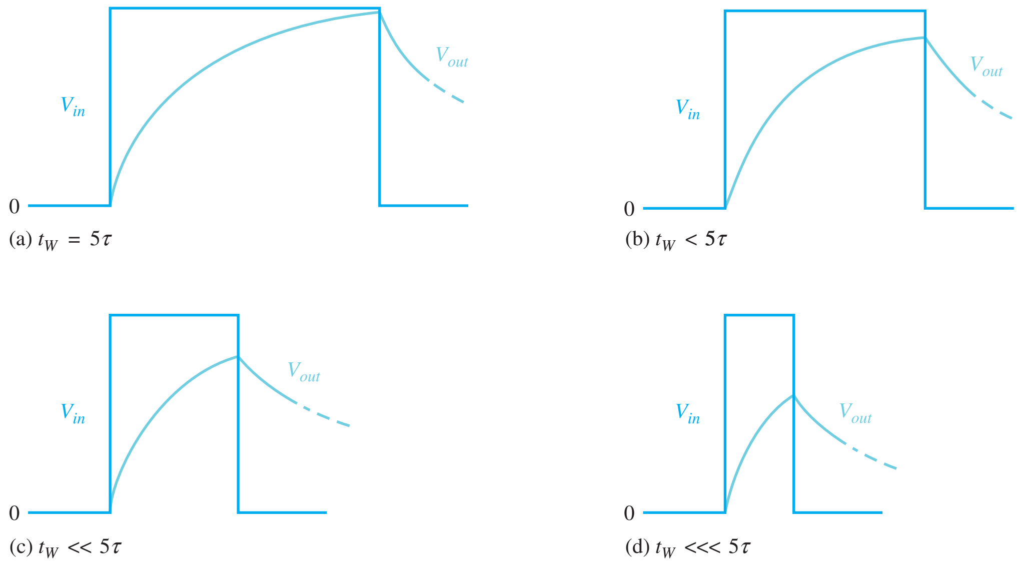


FIGURE 6

Capacitor voltage for various time constants that are longer than the input pulse width. Dark blue is input and light blue is output.

When the time constant is much greater than the input pulse width, the capacitor charges very little, and, as a result, the output voltage becomes almost negligible, as indicated in Figure 6.

Figure 7 illustrates the effect of reducing the input pulse width for a fixed time constant value. As the input pulse width is reduced, the output voltage decreases because the capacitor has less time to charge. However, it takes the capacitor approximately the same length of time (5τ) to discharge back to zero for each condition after the pulse is removed.



(a) $t_W = 5\tau$

(b) $t_W < 5\tau$

(c) $t_W << 5\tau$

(d) $t_W <<< 5\tau$

FIGURE 7

The capacitor charges less and less as the input pulse width is reduced. The time constant is fixed.

EXAMPLE 1

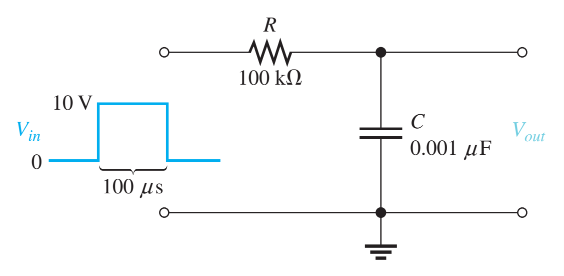
A single 10 V pulse with a width of 100 μs is applied to the *RC* integrator in Figure 8.

(a) To what voltage will the capacitor charge?

(b) How long will it take the capacitor to discharge if the internal resistance of the pulse source is 50 Ω?

(c) Draw the output voltage waveform.

▶ **FIGURE 8**



Solution

(a) The circuit time constant is

$$\tau = RC = (100\text{ k}\Omega)(0.001\ \mu\text{F}) = 100\ \mu\text{s}$$

Notice that the pulse width is exactly equal to the time constant. Thus, the capacitor will charge approximately 63% of the full input amplitude in one time constant, so the output will reach a maximum voltage of

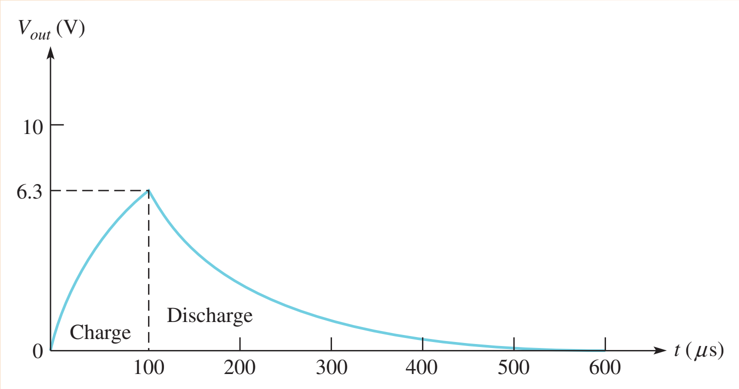
$$V_{out} = (0.63)10\text{ V} = \mathbf{6.3\ V}$$

(b) The capacitor discharges back through the source when the pulse ends. You can neglect the 50 Ω source resistance in series with 100 kΩ. The total approximate discharge time, therefore, is

$$5\tau = 5(100\ \mu\text{s}) = \mathbf{500\ \mu s}$$

(c) The output charging and discharging curve is shown in Figure 9.

▶ **FIGURE 9**



*Related Problem**

If the input pulse width in Figure 8 is increased to 200 μs, to what voltage will the capacitor charge?

Use Multisim files E20-01A and E20-01B to verify the calculated results in this example and the related problem.

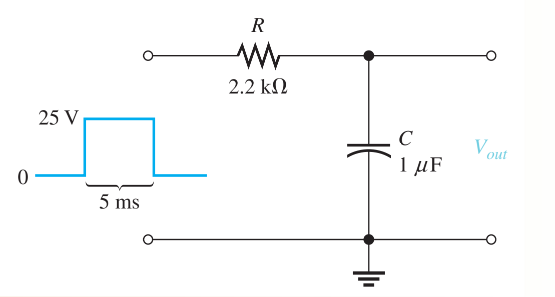
---

*Answers are at the end of the chapter.

EXAMPLE 2

Determine how much the capacitor in Figure 10 will charge when the single pulse is applied to the input.

▶ FIGURE 10



Solution Calculate the time constant.

$$\tau = RC = (2.2\text{ k}\Omega)(1\ \mu\text{F}) = 2.2\text{ ms}$$

Because the pulse width is 5 ms, the capacitor charges for 2.27 time constants (5 ms/2.2 ms = 2.27). Use an exponential formula to find the voltage to which the capacitor will charge. With $V_F = 25$ V and $t = 5$ ms, the calculation is as follows:

$$\begin{aligned} v &= V_F(1 - e^{-t/RC}) \\ &= (25\text{ V})(1 - e^{-5\text{ms}/2.2\text{ms}}) = (25\text{ V})(1 - e^{-2.27}) \\ &= (25\text{ V})(1 - 0.103) = (25\text{ V})(0.897) = \mathbf{22.4\ V} \end{aligned}$$

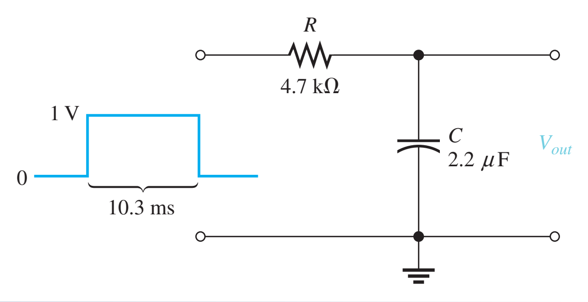
These calculations show that the capacitor charges to 22.4 V during the 5 ms duration of the input pulse. It will discharge back to zero when the pulse goes back to zero.

Related Problem Determine how much C will charge if the pulse width is increased to 10 ms.

SECTION 2 CHECKUP

1. When an input pulse is applied to an *RC* integrator, what condition must exist in order for the output voltage to reach full amplitude?
2. For the circuit in Figure 11, which has a single input pulse, find the maximum output voltage and determine how long the capacitor will discharge.
3. For Figure 11, draw the approximate shape of the output voltage with respect to the input pulse.
4. If an integrator time constant equals the input pulse width, will the capacitor fully charge?
5. Describe the condition under which the output voltage has the approximate shape of a rectangular input pulse.

▶ FIGURE 11



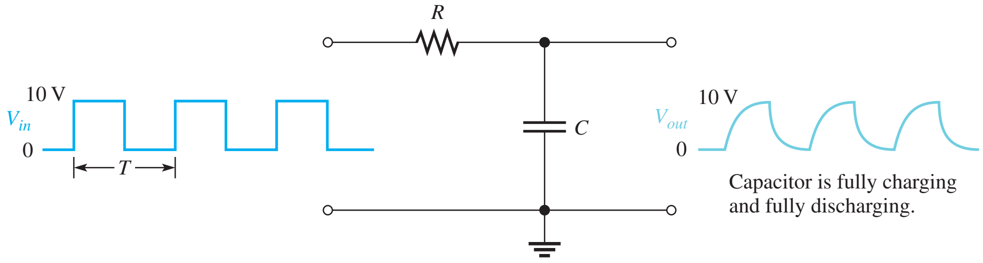
3 RESPONSE OF *RC* INTEGRATORS TO REPETITIVE PULSES

In electronic systems, you will encounter waveforms with repetitive pulses much more often than single pulses. However, an understanding of the integrator's response to single pulses is necessary in order to understand how these circuits respond to repeated pulses.

After completing this section, you should be able to

- **Analyze an *RC* integrator with repetitive input pulses**
  - Determine the response when the capacitor does not fully charge or discharge
  - Define *steady state*
  - Describe the effect of an increase in time constant on circuit response

If a **periodic** pulse waveform is applied to an *RC* integrator, as shown in Figure 12, the output waveshape depends on the relationship of the circuit time constant and the frequency (period) of the input pulses. The capacitor, of course, charges and discharges in response to a pulse input. The amount of charge and discharge of the capacitor depends both on the circuit time constant and on the input frequency, as mentioned.



▲ FIGURE 12

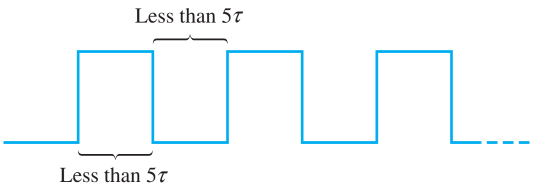
RC integrator with a repetitive pulse waveform input ($T = 10\tau$).

If the pulse width and the time between pulses are each equal to or greater than five time constants, the capacitor will fully charge and fully discharge during each period (*T*) of the input waveform. This case is shown in Figure 12.

When the pulse width and the time between pulses are shorter than five time constants, as illustrated in Figure 13 for a square wave, the capacitor will *not* completely charge or discharge. We will now examine the effects of this situation on the output voltage of the *RC* integrator.

▶ FIGURE 13

Input waveform that does not allow full charge or discharge of the capacitor in an *RC* integrator.



This icon indicates selected websites for further information on topics in this section. See the Companion Website provided with this text.

For illustration, let's use an *RC* integrator with a charging and discharging time constant equal to the pulse width of a 10 V square wave input, as shown in Figure 14. This choice will simplify the analysis and will demonstrate the basic action of the integrator under these conditions. At this point, we do not care what the exact time constant value is because we know that an *RC* circuit charges approximately 63% during one time constant interval.

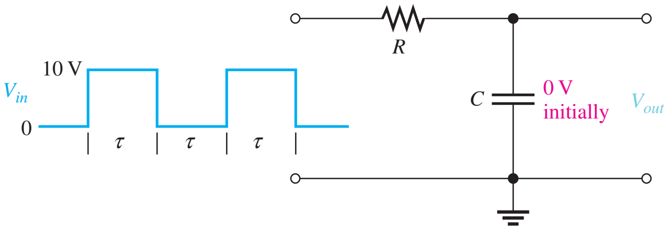


FIGURE 14

RC integrator with a square wave input having a period equal to two time constants ($T = 2\tau$).

Let's assume that the capacitor in Figure 14 begins initially uncharged and examine the output voltage on a pulse-by-pulse basis. Figure 15 shows the charging and discharging shapes of five pulses.

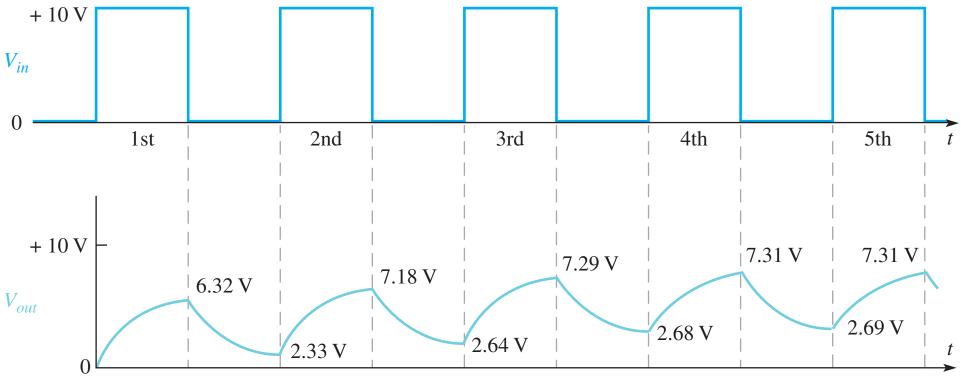


FIGURE 15

Input and output for the initially uncharged integrator in Figure 14.

First pulse During the first pulse, the capacitor charges. The output voltage reaches 6.32 V (63.2% of 10 V), as shown in Figure 15.

Between first and second pulses The capacitor discharges, and the voltage decreases to 36.8% of the voltage at the beginning of this interval: 0.368(6.32 V) = 2.33 V.

Second pulse The capacitor voltage begins at 2.33 V and increases 63.2% of the way to 10 V. This calculation is as follows: The total charging range is 10 V − 2.33 V = 7.67 V. The capacitor voltage will increase an additional 63.2% of 7.67 V, which is 4.85 V. Thus, at the end of the second pulse, the output voltage is 2.33 V + 4.85 V = 7.18 V, as shown in Figure 15. Notice that the average is building up.

Between second and third pulses The capacitor discharges during this time, and therefore the voltage decreases to 36.8% of the initial voltage by the end of the second pulse: 0.368(7.18 V) = 2.64 V.

Third pulse At the start of the third pulse, the capacitor voltage begins at 2.64 V. The capacitor charges 63.2% of the way from 2.64 V to 10 V: 0.632(10 V − 2.64 V) = 4.65 V. Therefore, the voltage at the end of the third pulse is 2.64 V + 4.65 V = 7.29 V.

Between third and fourth pulses The voltage during this interval decreases due to capacitor discharge. It will decrease to 36.8% of its value by the end of the third pulse. The final voltage in this interval is 0.368(7.29 V) = 2.68 V.

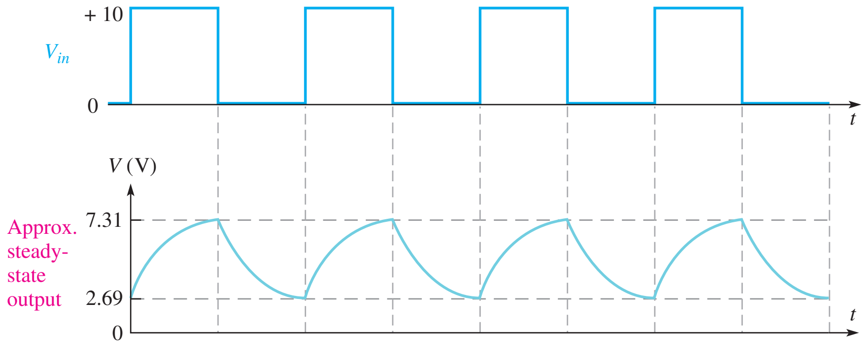
Fourth pulse At the start of the fourth pulse, the capacitor voltage is 2.68 V. The voltage increases by 0.632(10 V − 2.68 V) = 4.63 V. Therefore, at the end of the fourth pulse, the capacitor voltage is 2.68 V + 4.63 V = 7.31 V. Notice that the values are leveling off as the pulses continue.

Between fourth and fifth pulses Between these pulses, the capacitor voltage drops to 0.368(7.31 V) = 2.69 V.

Fifth pulse During the fifth pulse, the capacitor charges 0.632(10 V − 2.69 V) = 4.62 V. Since it started at 2.69 V, the voltage at the end of the pulse is 2.69 V + 4.62 V = 7.31 V.

Steady-State Time Response

In the preceding discussion, the output voltage gradually built up and then began leveling off. It takes approximately 5τ for the output voltage to build up to a constant average value. This interval is the transient time of the circuit. Once the output voltage reaches the average value of the input voltage, a **steady-state** condition is reached that continues as long as the periodic input continues. This condition is illustrated in Figure 16 based on the values obtained in the preceding discussion.



▲ FIGURE 16

Output reaches steady state after 5τ.

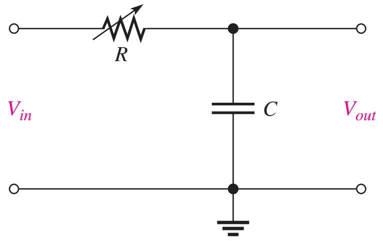
The transient time for our example circuit is the time from the beginning of the first pulse to the end of the third pulse. The reason for this interval is that the capacitor voltage at the end of the third pulse is 7.29 V, which is about 99% of the final voltage.

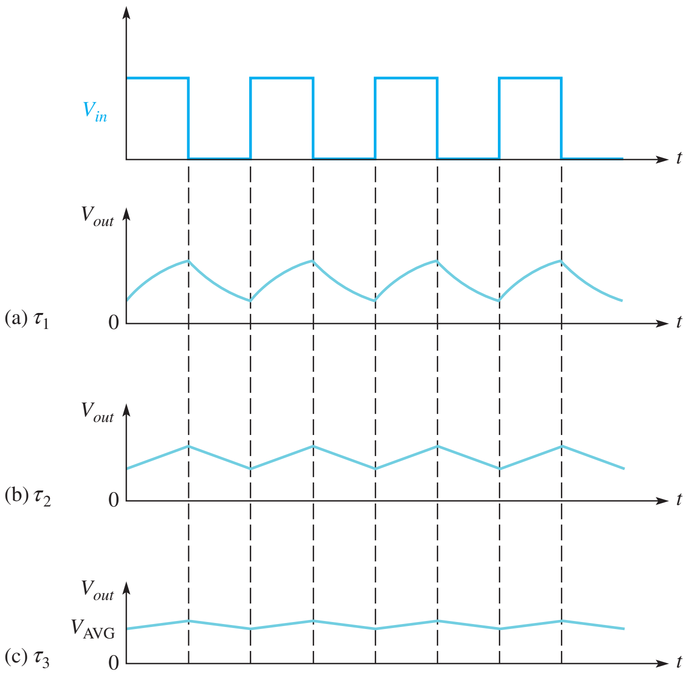
The Effect of an Increase in Time Constant

What happens to the output voltage if the *RC* time constant of the integrator is increased with a variable resistor, as indicated in Figure 17? As the time constant is increased, the capacitor charges less during a pulse and discharges less between pulses. The result is a

▶ FIGURE 17

Integrator with a variable *RC* time constant.





▲ FIGURE 18

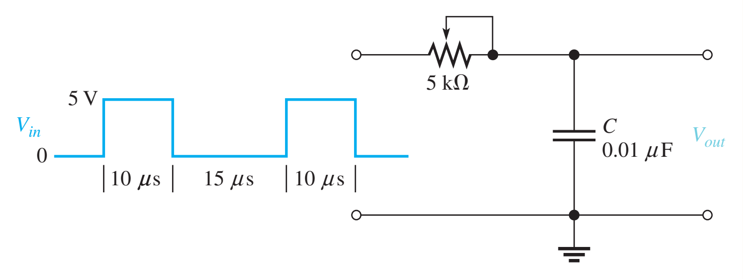
Effect of longer time constants on the output of an *RC* integrator ($\tau_3 > \tau_2 > \tau_1$).

smaller fluctuation in the output voltage for increasing values of time constant, as shown in Figure 18.

As the time constant becomes extremely long compared to the pulse width, the output voltage approaches a constant dc voltage, as shown in Figure 18(c). This value is the average value of the input. For a square wave, it is one-half the amplitude.

EXAMPLE 3

Determine the output voltage waveform for the first two pulses applied to the *RC* integrator in Figure 19. Assume that the capacitor is initially uncharged and the rheostat is set to 5 kΩ.



▲ FIGURE 19

Solution First, calculate the circuit time constant.

$$\tau = RC = (5\text{ k}\Omega)(0.01\ \mu\text{F}) = 50\ \mu\text{s}$$

Obviously, the time constant is much longer than the input pulse width or the interval between pulses (notice that the input is not a square wave). In this case, the exponential formulas must be applied, and the analysis is relatively difficult. Follow the solution carefully.

1. *Calculation for first pulse:* Use the equation for an increasing exponential because C is charging. Note that V_F is 5 V, and t equals the pulse width of 10 μs. Therefore,

$$v_C = V_F(1 - e^{-t/RC}) = (5\text{ V})(1 - e^{-10\mu\text{s}/50\mu\text{s}})$$
$$= (5\text{ V})(1 - 0.819) = \mathbf{906\text{ mV}}$$

This result is plotted in Figure 20(a).

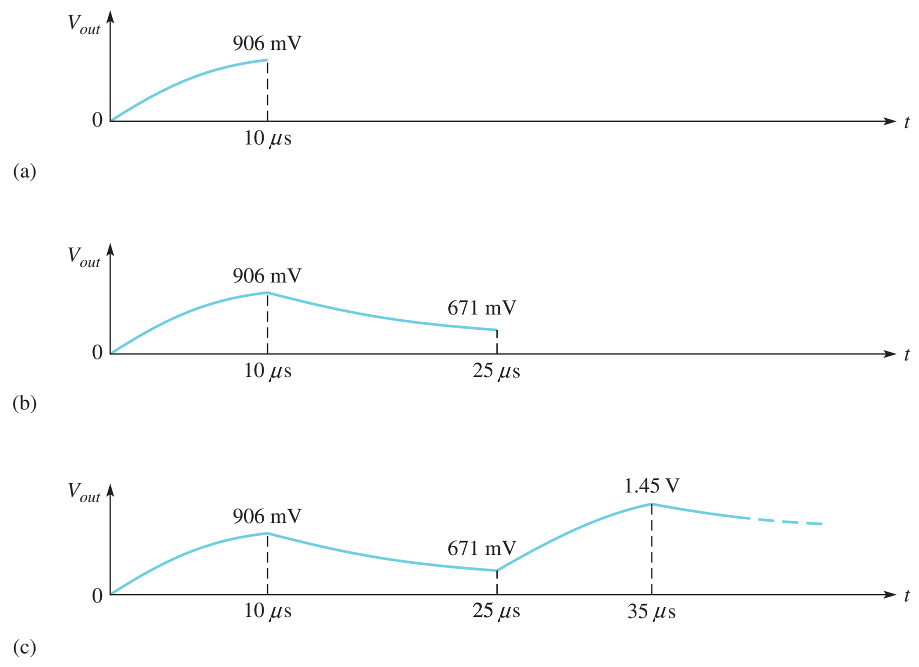
2. *Calculation for interval between first and second pulse:* Use the equation for a decreasing exponential because C is discharging. Note that V_i is 906 mV because C begins to discharge from this value at the end of the first pulse. The discharge time is 15 μs. Therefore,

$$v_C = V_i e^{-t/RC} = (906\text{ mV})e^{-15\mu\text{s}/50\mu\text{s}}$$
$$= (906\text{ mV})(0.741) = \mathbf{671\text{ mV}}$$

This result is shown in Figure 20(b).

3. *Calculation for second pulse:* At the beginning of the second pulse, the output voltage is 671 mV. During the second pulse, the capacitor will again charge. In this case, it does not begin at zero volts. It already has 671 mV from the previous charge and discharge. To handle this situation, you must use the general exponential formula.

$$v = V_F + (V_i - V_F)e^{-t/\tau}$$



▲ FIGURE 20

Using this equation, you can calculate the voltage across the capacitor at the end of the second pulse as follows:

$$
\begin{aligned}
v_C &= V_F + (V_i - V_F)e^{-t/RC} \\
&= 5\text{ V} + (671\text{ mV} - 5\text{ V})e^{-10\mu\text{s}/50\mu\text{s}} \\
&= 5\text{ V} + (-4.33\text{ V})(0.819) = 5\text{ V} - 3.55\text{ V} = \mathbf{1.45\text{ V}}
\end{aligned}
$$

This result is shown in Figure 20(c).

Notice that the output waveform builds up on successive input pulses. After approximately 5τ, it will reach its steady state and will fluctuate between a constant maximum and a constant minimum, with an average equal to the average value of the input. You can see this pattern by carrying the analysis in this example further.

Related Problem Determine V_{out} at the beginning of the third pulse.

Use Multisim file E20-03 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 3 CHECKUP

1. What conditions allow an *RC* integrator capacitor to fully charge and discharge when a periodic pulse waveform is applied to the input?
2. What will the output waveform look like if the circuit time constant is extremely short compared to the pulse width of a square wave input?
3. What is the time called that is required for the output voltage to build up to a constant average value when 5τ is greater than the pulse width of an input square wave?
4. Define *steady-state response*.
5. Describe the output of an *RC* integrator when the input is a square wave with a period much less than 1τ.

4 RESPONSE OF AN *RC* DIFFERENTIATOR TO A SINGLE PULSE

In terms of time response, a series *RC* circuit in which the output voltage is taken across the resistor is known as a **differentiator**. Recall that in terms of frequency response, it is a high-pass filter. The term *differentiator* is derived from the mathematical process of differentiation, which this type of circuit approximates under certain conditions.

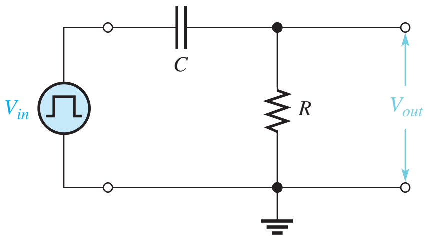
After completing this section, you should be able to

- **Analyze an *RC* differentiator with a single input pulse**
  - Describe the response at the rising edge of the input pulse
  - Determine the response during and at the end of a pulse for various pulse width–time constant relationships

Figure 21 shows an *RC* differentiator with a pulse input. The same action occurs in a differentiator as in an integrator, except the output voltage is taken across the resistor rather than across the capacitor. The capacitor charges exponentially at a rate depending on the *RC* time constant. The shape of the differentiator's resistor voltage is determined by the charging and discharging action of the capacitor.

▶ **FIGURE 21**

An *RC* differentiator with a pulse generator connected.

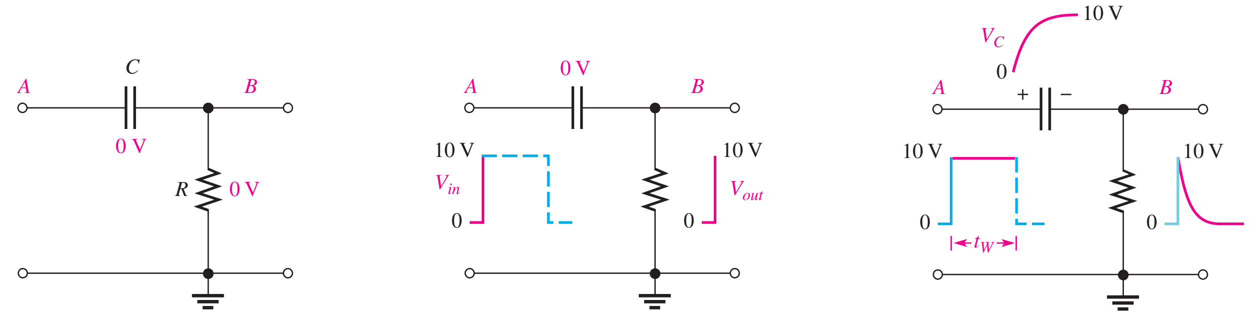


Pulse Response

To understand how the output voltage is shaped by a differentiator, you must consider the following:

1. The response to the rising pulse edge
2. The response between the rising and falling edges
3. The response to the falling pulse edge

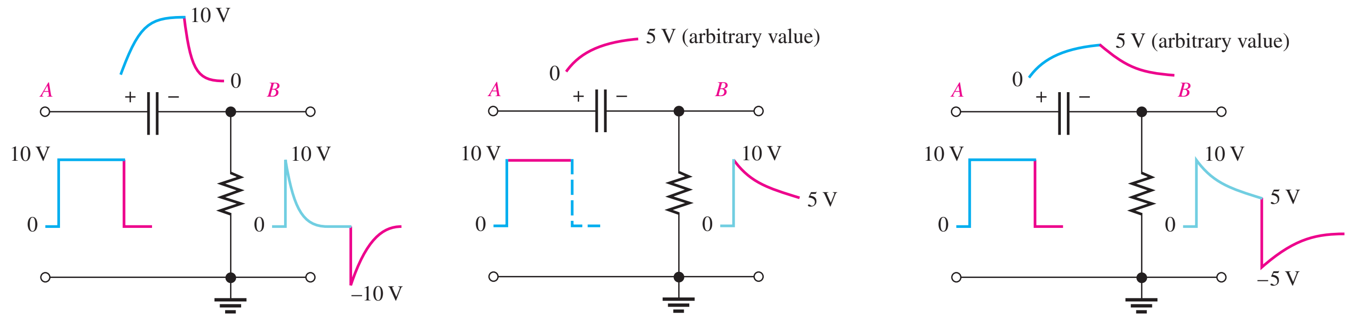
Let's assume that the capacitor is initially uncharged prior to the rising pulse edge. Prior to the pulse, the input is zero volts. Thus, there are zero volts across the capacitor and also zero volts across the resistor, as indicated in Figure 22(a).


C
A
B
0 V
R
0 V
0 V
10 V
V_{in}
0
10 V
V_{out}
0
V_C
10 V
0
+
−
10 V
0
t_W
10 V
0

(a) Before pulse is applied

(b) At rising edge of input pulse

(c) During level part of pulse when $t_W \geq 5\tau$


10 V
0
A
+
−
B
10 V
0
10 V
0
−10 V
5 V (arbitrary value)
0
10 V
0
10 V
0
5 V
5 V (arbitrary value)
0
10 V
0
10 V
0
5 V
−5 V

(d) At falling edge of pulse when $t_W \geq 5\tau$

(e) During level part of pulse when $t_W < 5\tau$

(f) At falling edge of pulse when $t_W < 5\tau$

▲ **FIGURE 22**

Examples of the response of an *RC* differentiator to a single input pulse under two conditions: $t_W \geq 5\tau$ and $t_W < 5\tau$.

Response to the Rising Edge of the Input Pulse Let's also assume that a 10 V pulse is applied to the input. When the rising edge occurs, point *A* goes to +10 V. Recall that the voltage across a capacitor cannot change instantaneously, and thus the capacitor appears instantaneously as a short. Therefore, if point *A* goes instantly to $+10$ V, then point *B must* also go instantly to $+10$ V, keeping the capacitor voltage zero for the instant of the rising edge. The capacitor voltage is the voltage from point *A* to point *B*.

The voltage at point *B* with respect to ground is the voltage across the resistor (and the output voltage). Thus, the output voltage suddenly goes to $+10$ V in response to the rising pulse edge, as indicated in Figure 22(b).

Response During Pulse When $t_W \geq 5\tau$ While the pulse is at its high level between the rising edge and the falling edge, the capacitor is charging. When the pulse width is equal to or greater than five time constants ($t_W \geq 5\tau$), the capacitor has time to fully charge.

As the voltage across the capacitor builds up exponentially, the voltage across the resistor decreases exponentially until it reaches zero volts at the time the capacitor reaches full charge ($+10$ V in this case). This decrease in the resistor voltage occurs because the sum of the capacitor voltage and the resistor voltage at any instant must be equal to the applied voltage, in compliance with Kirchhoff's voltage law ($v_C + v_R = v_{in}$). This part of the response is illustrated in Figure 22(c).

Response to Falling Edge When $t_W \geq 5\tau$ Let's examine the case in which the capacitor is fully charged at the end of the pulse ($t_W \geq 5\tau$). Refer to Figure 22(d). On the falling edge, the input pulse suddenly goes from $+10$ V back to zero. An instant before the falling edge, the capacitor is charged to 10 V, so point *A* is $+10$ V and point *B* is 0 V. The voltage across a capacitor cannot change instantaneously, so when point *A* makes a transition from $+10$ V to zero on the falling edge, point *B must* also make a 10 V transition from zero to -10 V. This keeps the voltage across the capacitor at 10 V for the instant of the falling edge.

The capacitor now begins to discharge exponentially. As a result, the resistor voltage goes from -10 V to zero in an exponential curve, as indicated in red in Figure 22(d).

Response During Pulse When $t_W < 5\tau$ When the pulse width is less than five time constants ($t_W < 5\tau$), the capacitor does not have time to fully charge. Its partial charge depends on the relation of the time constant and the pulse width.

Because the capacitor does not reach the full $+10$ V, the resistor voltage will not reach zero volts by the end of the pulse. For example, if the capacitor charges to +5 V during the pulse interval, the resistor voltage will decrease to +5 V, as illustrated in Figure 22(e).

Response to Falling Edge When $t_W < 5\tau$ Now, let's examine the case in which the capacitor is only partially charged at the end of the pulse ($t_W < 5\tau$). For example, if the capacitor charges to $+5$ V, the resistor voltage at the instant before the falling edge is also $+5$ V because the capacitor voltage plus the resistor voltage must add up to $+10$ V, as illustrated in Figure 22(e).

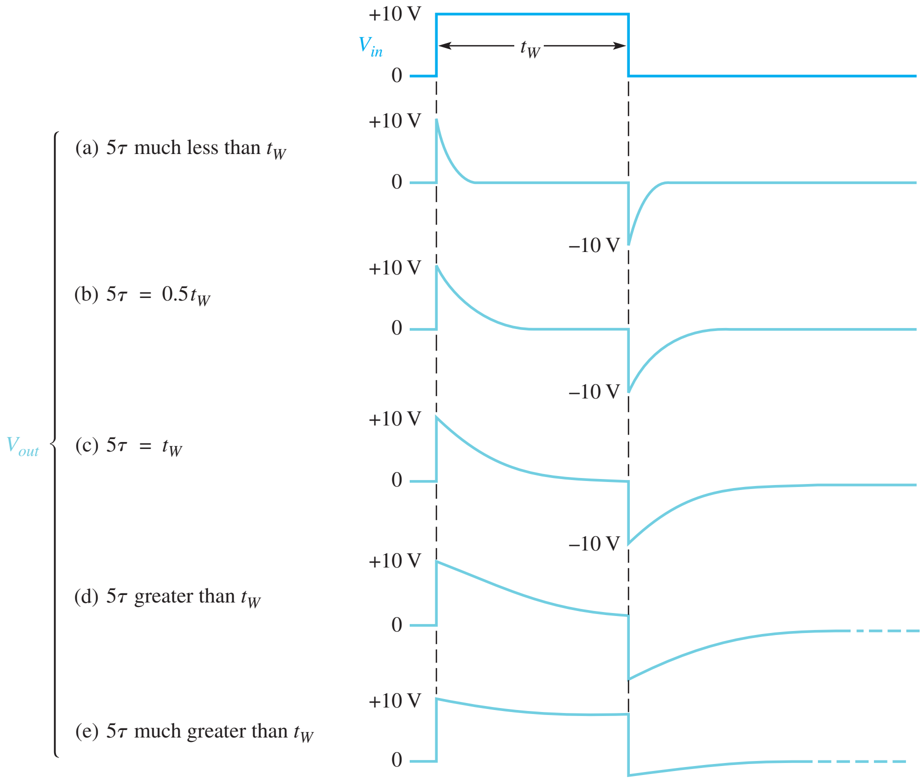
When the falling edge occurs, point *A* goes from $+10$ V to zero. As a result, point *B* goes from $+5$ V to -5 V, as illustrated in Figure 22(f). This decrease occurs, of course, because the capacitor voltage cannot change at the instant of the falling edge. Immediately after the falling edge, the capacitor begins to discharge to zero. As a result, the resistor voltage goes from -5 V to zero, as shown.

Summary of *RC* Differentiator Response to a Single Pulse

A good way to summarize this section is to look at the general output waveforms of a differentiator as the time constant is varied from one extreme, when 5τ is much less than the pulse width, to the other extreme, when 5τ is much greater than the pulse width. These situations are illustrated in Figure 23. In part (a) of the figure, the output consists of narrow positive and negative "spikes." In part (e), the output approaches the shape of the input. Various conditions between these extremes are illustrated in parts (b), (c), and (d).

TECH TIP

You may have observed a pulse that looks similar to Figure 23(e) when you ac couple a pulse to an oscilloscope. In this case the capacitor in the oscilloscope coupling circuit can act as an unwanted differentiating circuit, causing the pulse to droop. To avoid this, you can dc couple the scope and check the probe compensation.

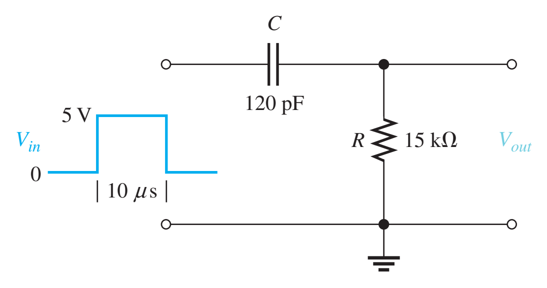


▲ **FIGURE 23**

Effects of a change in time constant on the shape of the output voltage of an *RC* differentiator.

EXAMPLE 4

Draw the output voltage for the *RC* differentiator in Figure 24.



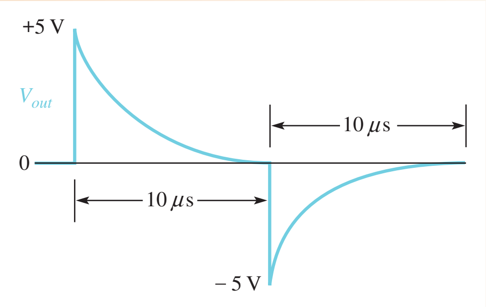
▲ **FIGURE 24**

Solution First, calculate the time constant.

$$\tau = RC = (15\text{ k}\Omega)(120\text{ pF}) = 1.8\ \mu\text{s}$$

In this case, $t_W > 5\tau$, so the capacitor reaches full charge before the end of the pulse.

On the rising edge, the resistor voltage jumps to +5 V and then decreases exponentially to zero by the end of the pulse. On the falling edge, the resistor voltage jumps to −5 V and then goes back to zero exponentially. The resistor voltage is, of course, the output, and its shape is shown in Figure 25.



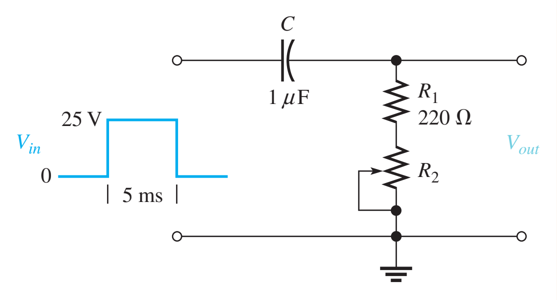
▲ FIGURE 25

Related Problem Draw the output voltage if C is changed to 12 pF in Figure 24.

Use Multisim files E20-04A and E20-04B to verify the calculated results in this example and to confirm your answer for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle (long period).

EXAMPLE 5

Determine the output voltage waveform for the RC differentiator in Figure 26 with the rheostat set so that the total resistance of R_1 and R_2 is 2 kΩ.



▲ FIGURE 26

Solution First, calculate the time constant.

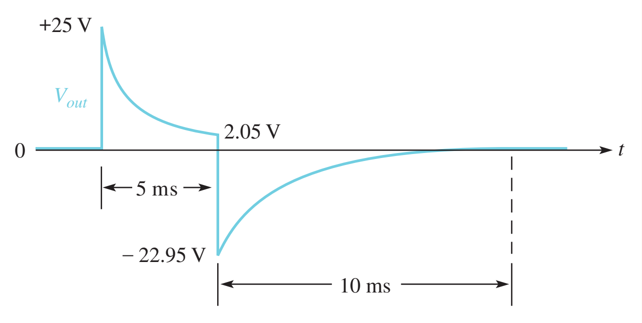
$$\tau = R_{tot}C = (2\ \text{k}\Omega)(1\ \mu\text{F}) = 2\ \text{ms}$$

On the rising edge, the resistor voltage immediately jumps to +25 V. Because the pulse width is 5 ms, the capacitor charges for 2.5 time constants and therefore does not reach full charge. Thus, you must use the formula for a decreasing exponential in order to calculate to what voltage the output decreases by the end of the pulse.

$$v_{out} = V_i e^{-t/RC} = 25e^{-5\text{ms}/2\text{ms}} = 25(0.082) = 2.05\ \text{V}$$

where $V_i = 25$ V and $t = 5$ ms. This calculation gives the resistor voltage (v_{out}) at the end of the 5 ms pulse width interval.

On the falling edge, the resistor voltage immediately jumps from +2.05 V down to −22.95 V (a 25 V transition). The resulting waveform of the output voltage is shown in Figure 27.



▲ FIGURE 27

Related Problem Determine the voltage at the end of the pulse in Figure 26 if the rheostat is set so that the total resistance is 1.5 kΩ.

Use Multisim files E20-05A and E20-05B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 4 CHECKUP

1. Draw the output of a differentiator for a 10 V input pulse when $5\tau = 0.5t_W$.
2. Under what condition does the output pulse shape most closely resemble the input pulse for a differentiator?
3. What does the differentiator output look like when 5τ is much less than the pulse width of the input?
4. If the resistor voltage in a differentiating circuit is down to $+5\ V$ at the end of a 15 V input pulse, to what negative value will the resistor voltage go in response to the falling edge of the input?

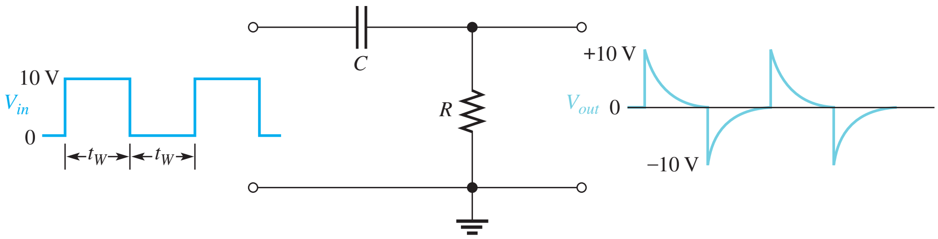
5 RESPONSE OF *RC* DIFFERENTIATORS TO REPETITIVE PULSES

The *RC* differentiator response to a single pulse, covered in the last section, is extended in this section to repetitive pulses.

After completing this section, you should be able to

- **Analyze an *RC* differentiator with repetitive input pulses**
    - Determine the response when the pulse width is less than five time constants

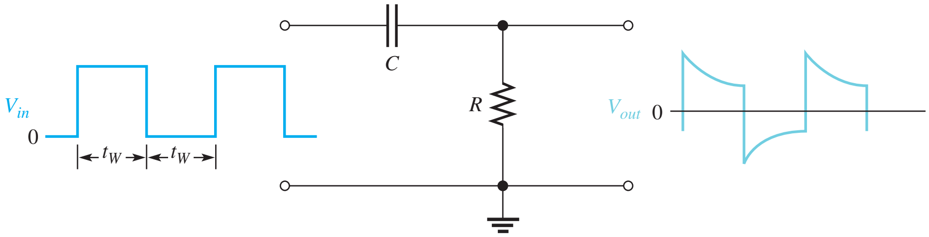
If a periodic pulse waveform is applied to an *RC* differentiating circuit, two conditions again are possible: $t_W \geq 5\tau$ or $t_W < 5\tau$. Figure 28 shows the output when $t_W = 5\tau$. As the time constant is reduced, both the positive and the negative portions of the output become narrower. Notice that the average value of the output is zero. An average value of zero means that the waveform has equal positive and negative portions. The average value of a waveform is its **dc component**. Because a capacitor blocks dc, the dc component of the input is prevented from passing through to the output, resulting in an average value of zero.



▲ **FIGURE 28**

Example of differentiator response when $t_W = 5\tau$.

Figure 29 shows the steady-state output when $t_W < 5\tau$. As the time constant is increased, the positively and negatively sloping portions become flatter. For a very long time constant, the output approaches the shape of the input, but with an average value of zero.



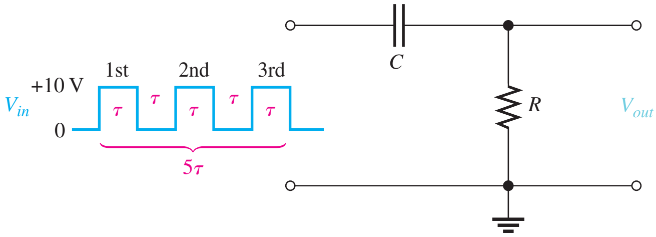
▲ **FIGURE 29**

Example of differentiator response when $t_W < 5\tau$.

Analysis of a Repetitive Waveform

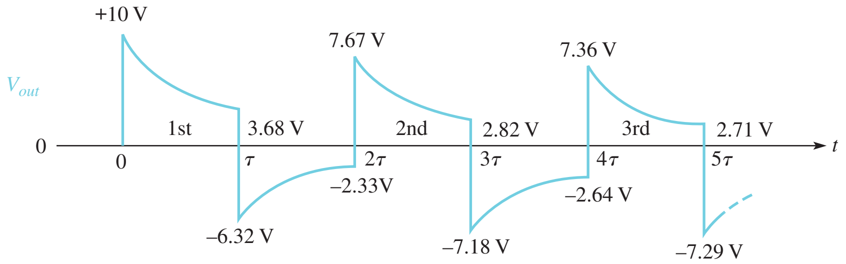
Like the integrator, the differentiator output takes time (5τ) to reach steady state. To illustrate the response, let's take an example in which the time constant equals the input pulse width. At this point, we do not care what the circuit time constant is because we know that the resistor voltage will decrease to approximately 37% of its maximum value during one pulse (1τ).

Let's assume that the capacitor in Figure 30 begins initially uncharged and then examine the output voltage on a pulse-by-pulse basis. The results of the analysis that follows are shown in Figure 31.



◀ **FIGURE 30**

RC differentiator with $\tau = t_W$.



◀ **FIGURE 31**

Differentiator output waveform during transient time for the circuit in Figure 30.

First pulse On the rising edge, the output instantaneously jumps to +10 V. Then the capacitor partially charges to 63.2% of 10 V, which is 6.32 V. Thus, the output voltage must decrease to 3.68 V, as shown in Figure 31. On the falling edge, the output instantaneously makes a negative-going 10 V transition to −6.32 V (−10 V + 3.68 V = −6.32 V).

Between first and second pulses The capacitor discharges to 36.8% of 6.32 V, which is 2.33 V. Thus, the resistor voltage, which starts at −6.32 V, must increase to −2.33 V. Why? Because at the instant prior to the next pulse, the input voltage is zero. Therefore, the sum of v_C and v_R must be zero (2.33 V − 2.33 V = 0). Remember that $v_C + v_R = v_{in}$ at all times, in accordance with Kirchhoff's voltage law.

Second pulse On the rising edge, the output makes an instantaneous, positive-going, 10 V transition from −2.33 V to 7.67 V. Then by the end of the pulse the capacitor charges 0.632 × (10 V − 2.33 V) = 4.85 V. Thus, the capacitor voltage increases from 2.33 V to 2.33 V + 4.85 V = 7.18 V. The output voltage drops to 0.368 × 7.67 V = 2.82 V.

On the falling edge, the output instantaneously makes a negative-going transition from 2.82 V to −7.18 V, as shown in Figure 31.

Between second and third pulses The capacitor discharges to 36.8% of 7.18 V, which is 2.64 V. Thus, the output voltage starts at −7.18 V and increases to −2.64 V because the capacitor voltage and the resistor voltage must add up to zero at the instant prior to the third pulse (the input is zero).

Third pulse On the rising edge, the output makes an instantaneous 10 V transition from −2.64 V to +7.36 V. Then the capacitor charges 0.632 × (10 V − 2.64 V) = 4.65 V to 2.64 V + 4.65 V = 7.29 V. As a result, the output voltage drops to 0.368 × 7.36 V = 2.71 V. On the falling edge, the output instantly goes from +2.71 V down to −7.29 V.

After the third pulse, five time constants have elapsed, and the output voltage is close to its steady state. Thus, it will continue to vary from a positive maximum of about +7.3 V to a negative maximum of about −7.3 V, with an average value of zero.

SECTION 5 CHECKUP

1. What conditions allow an *RC* differentiator to fully charge and discharge when a periodic pulse waveform is applied to the input?
2. What will the output waveform look like if the circuit time constant is extremely short compared to the pulse width of a square wave input?
3. What does the average value of the differentiator output voltage equal during steady state?

6 RESPONSE OF *RL* INTEGRATORS TO PULSE INPUTS

A series *RL* circuit in which the output voltage is taken across the resistor is known as an integrator in terms of time response. Although only the response to a single pulse is discussed, it can be extended to repetitive pulses, as described for the *RC* integrator.

After completing this section, you should be able to

- **Analyze the operation of an *RL* integrator**
  - Determine the response to a single input pulse

Figure 32 shows an *RL* integrator. The output waveform is taken across the resistor and, under equivalent conditions, is the same shape as that for the *RC* integrator. Recall that in the *RC* case, the output was across the capacitor.

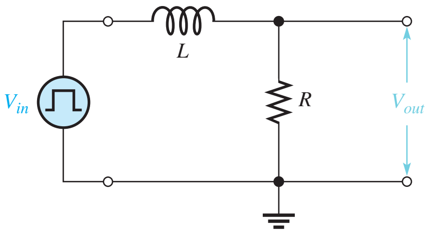


FIGURE 32

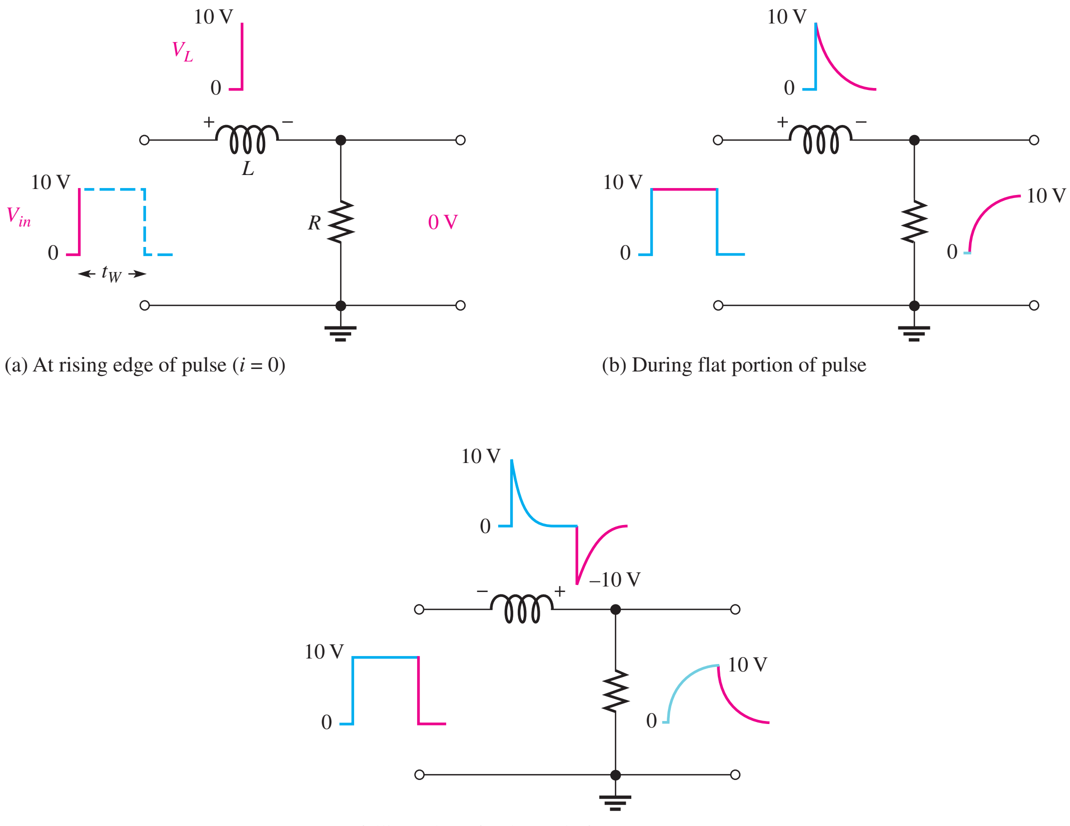
An *RL* integrator with a pulse generator connected.

As you know, each edge of an ideal pulse is considered to be instantaneous. Two basic rules for inductor behavior will aid in analyzing *RL* circuit responses to pulse inputs:

1. The inductor appears as an open to an instantaneous change in current and as a short (ideally) to dc.
2. The current in an inductor cannot change instantaneously—it can change only exponentially.

Response of the *RL* Integrator to a Single Pulse

When a pulse generator is connected to the input of the integrator and the voltage pulse goes from its low level to its high level, the inductor prevents a sudden change in current. As a result, the inductor acts as an open, and all of the input voltage is across it at the instant of the rising pulse edge. This situation is indicated in Figure 33(a).



(a) At rising edge of pulse ($i = 0$)

(b) During flat portion of pulse

(c) At falling edge of pulse and after

FIGURE 33

Illustration of the pulse response of an *RL* integrator ($t_W > 5\tau$).

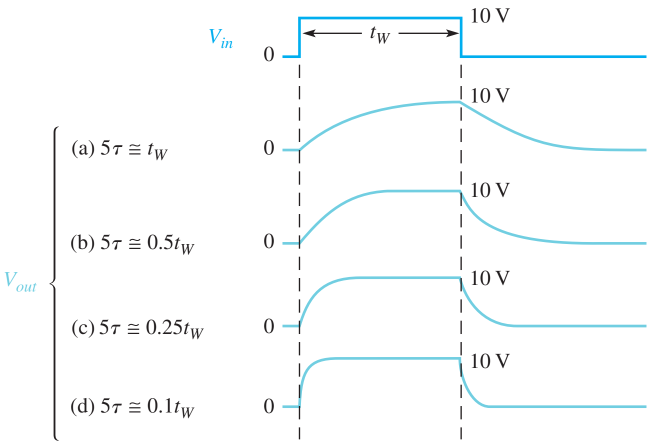
After the rising edge, the current builds up, and the output voltage follows the current as it increases exponentially, as shown in Figure 33(b). The current can reach a maximum of V_p/R if the transient time is shorter than the pulse width ($V_p = 10$ V in this example).

When the pulse goes from its high level to its low level, an induced voltage with reversed polarity is created across the coil in an effort to keep the current equal to V_p/R. The output voltage begins to decrease exponentially, as shown in Figure 33(c).

The exact shape of the output depends on the L/R time constant as summarized in Figure 34 for various relationships between the time constant and the pulse width. You should note that the response of this RL circuit in terms of the shape of the output is identical to that of the RC integrator. The relationship of the L/R time constant to the input pulse width has the same effect as the RC time constant that was shown in Figure 4. For example, when $t_W < 5\tau$, the output voltage will not reach its maximum possible value.

▶ **FIGURE 34**

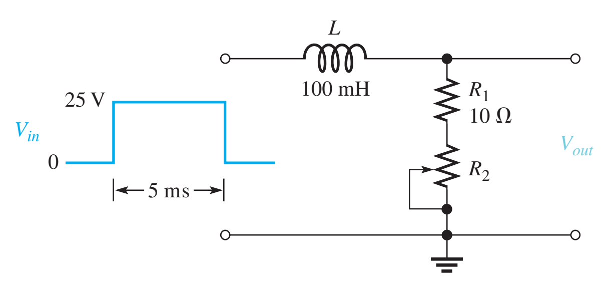
Illustration of the variation in *RL* integrator output pulse shape with time constant.



EXAMPLE 6

Determine the maximum output voltage for the RL integrator in Figure 35 when a single pulse is applied as shown. The rheostat is set so that the total resistance is 50 Ω.

▶ **FIGURE 35**



Solution Calculate the time constant.

$$\tau = \frac{L}{R} = \frac{100\text{ mH}}{50\ \Omega} = 2\text{ ms}$$

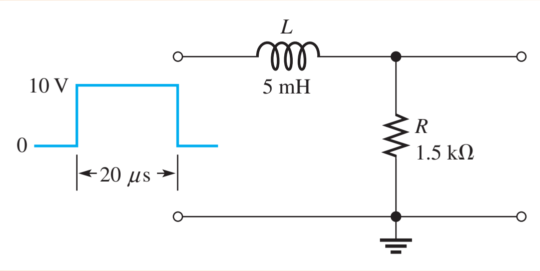
Because the pulse width is 5 ms, the inductor charges for 2.5τ. Use the exponential formula derived from $v = V_F + (V_i - V_F)e^{-Rt/L}$ with $V_i = 0$ and $\tau = L/R$ to calculate the voltage.

$$v_{out(max)} = V_F(1 - e^{-t/\tau}) = 25(1 - e^{-5\text{ms}/2\text{ms}})$$
$$= 25(1 - e^{-2.5}) = 25(1 - 0.082) = 25(0.918) = \mathbf{22.9\ V}$$

Related Problem To what resistance must the rheostat, R_2, be set for the output voltage to reach 25 V by the end of the pulse in Figure 35?

EXAMPLE 7

A pulse is applied to the *RL* integrator in Figure 36. Determine the complete waveshapes and the values for I, V_R, and V_L.



▲ FIGURE 36

Solution The circuit time constant is

$$\tau = \frac{L}{R} = \frac{5\text{ mH}}{1.5\text{ k}\Omega} = 3.33\ \mu\text{s}$$

Since $5\tau = 16.7\ \mu\text{s}$ is less than t_W, the current will reach its maximum value and remain there until the end of the pulse.

At the rising edge of the pulse,

$$i = 0\text{ A}$$
$$v_R = 0\text{ V}$$
$$v_L = 10\text{ V}$$

The inductor initially appears as an open, so all of the input voltage appears across *L*.

During the pulse,

$$i \text{ increases exponentially to } \frac{V_p}{R} = \frac{10\text{ V}}{1.5\text{ k}\Omega} = 6.67\text{ mA in } 16.7\ \mu\text{s}$$

v_R increases exponentially to 10 V in 16.7 μs

v_L decreases exponentially to zero in 16.7 μs

At the falling edge of the pulse,

$$i = 6.67\text{ mA}$$
$$v_R = 10\text{ V}$$
$$v_L = -10\text{ V}$$

After the pulse,

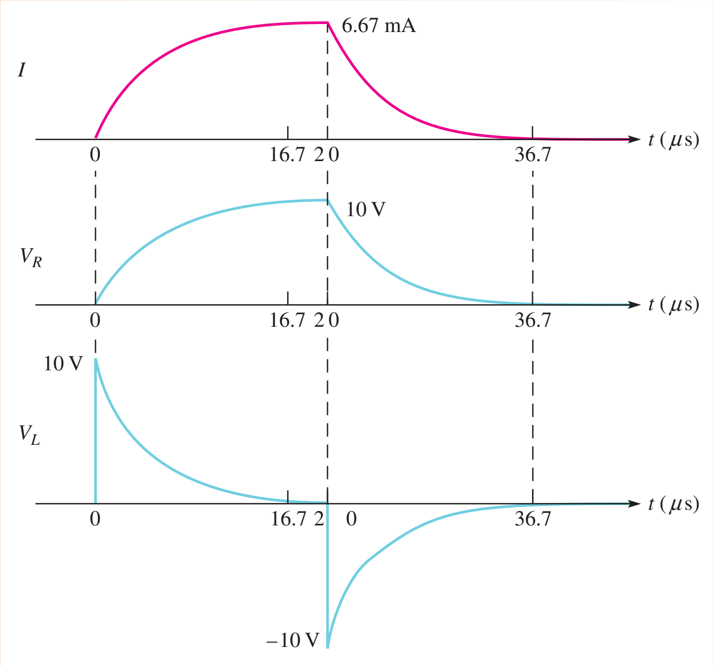
i decreases exponentially to zero in 16.7 μs

v_R decreases exponentially to zero in 16.7 μs

v_L increases exponentially to zero in 16.7 μs

The waveforms are shown in Figure 37.

▶ FIGURE 37

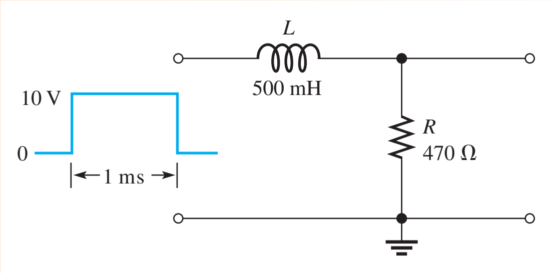


Related Problem What will be the maximum output voltage if the amplitude of the input pulse is increased to 20 V in Figure 36?

Use Multisim files E20-07A and E20-07B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

EXAMPLE 8

A 10 V pulse with a width of 1 ms is applied to the *RL* integrator in Figure 38. Determine the voltage level that the output will reach during the pulse. If the source has an internal resistance of 30 Ω, how long will it take the output to decay to zero? Draw the output voltage waveform.



▲ FIGURE 38

Solution The inductor charges through the 30 Ω source resistance plus the 470 Ω external resistor. The time constant is

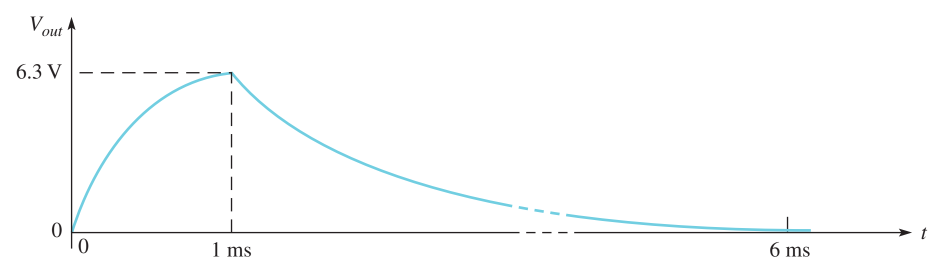
$$\tau = \frac{L}{R_{tot}} = \frac{500\text{ mH}}{470\ \Omega + 30\ \Omega} = \frac{500\text{ mH}}{500\ \Omega} = 1\text{ ms}$$

Notice that in this case the pulse width is exactly equal to τ. Thus, the output V_R will reach approximately 63% of the full input amplitude in 1τ. Therefore, the output voltage gets to **6.3 V** at the end of the pulse.

After the pulse is gone, the inductor discharges back through the 30 Ω source resistance and the 470 Ω resistor. The output voltage takes **5τ** to completely decay to zero.

$$5\tau = 5(1\text{ ms}) = 5\text{ ms}$$

The output voltage is shown in Figure 39.



▲ **FIGURE 39**

Related Problem To what value must R be changed to allow the output voltage to reach the input level during the pulse?

Use Multisim files E20-08A and E20-08B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 6 CHECKUP

1. In an *RL* integrator, across which component is the output voltage taken?
2. When a pulse is applied to an *RL* integrator, what condition must exist in order for the output voltage to reach the amplitude of the input?
3. Under what condition will the output voltage have the approximate shape of the input pulse?

7 RESPONSE OF *RL* DIFFERENTIATORS TO PULSE INPUTS

A series *RL* circuit in which the output voltage is taken across the inductor is known as a differentiator in terms of time response. Although only the response to a single pulse is discussed, it can be extended to repetitive pulses, as was described for the *RC* differentiator.

After completing this section, you should be able to

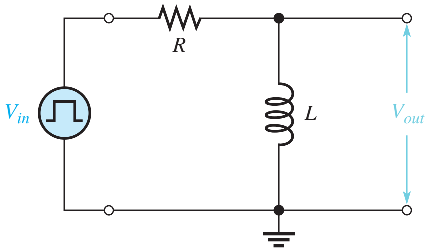
- **Analyze the operation of an *RL* differentiator**
  - Determine the response to a single input pulse

Response of the *RL* Differentiator to a Single Pulse

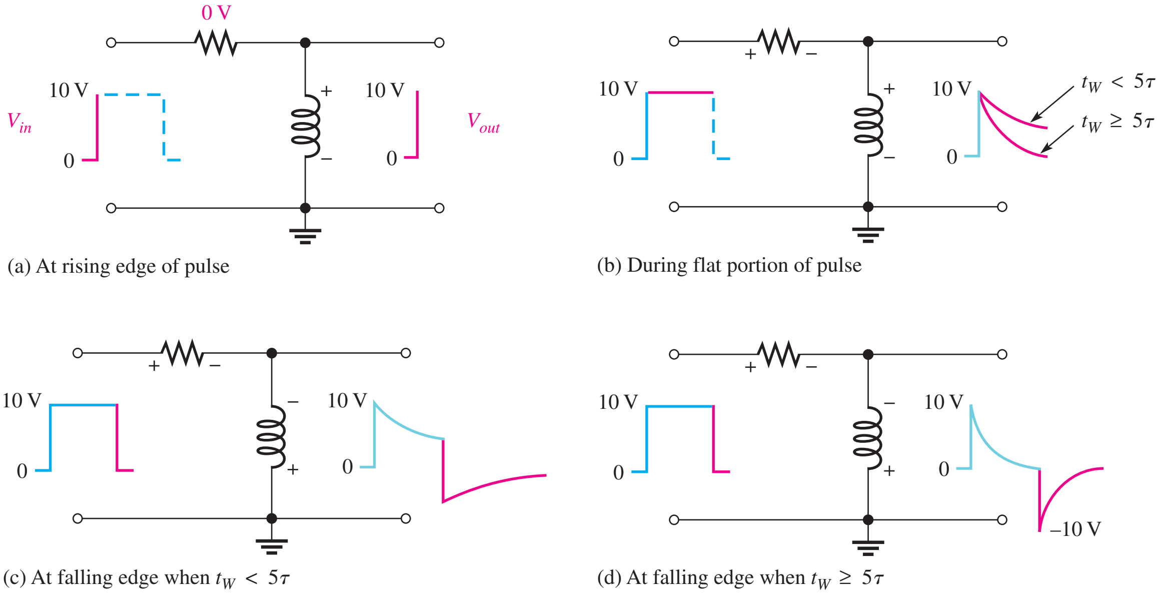
Figure 40 shows an *RL* differentiator with a pulse generator connected to the input.

▶ **FIGURE 40**

An *RL* differentiator with a pulse generator connected.



Initially, before the pulse, there is no current in the circuit. When the input pulse goes from its low level to its high level, the inductor prevents a sudden change in current. It does so, as you know, with an induced voltage equal and opposite to the input. As a result, *L* looks like an open, and all of the input voltage appears across it at the instant of the rising edge, as shown in Figure 41(a) with a 10 V pulse.



(a) At rising edge of pulse

(b) During flat portion of pulse

(c) At falling edge when $t_W < 5\tau$

(d) At falling edge when $t_W \geq 5\tau$

▲ **FIGURE 41**

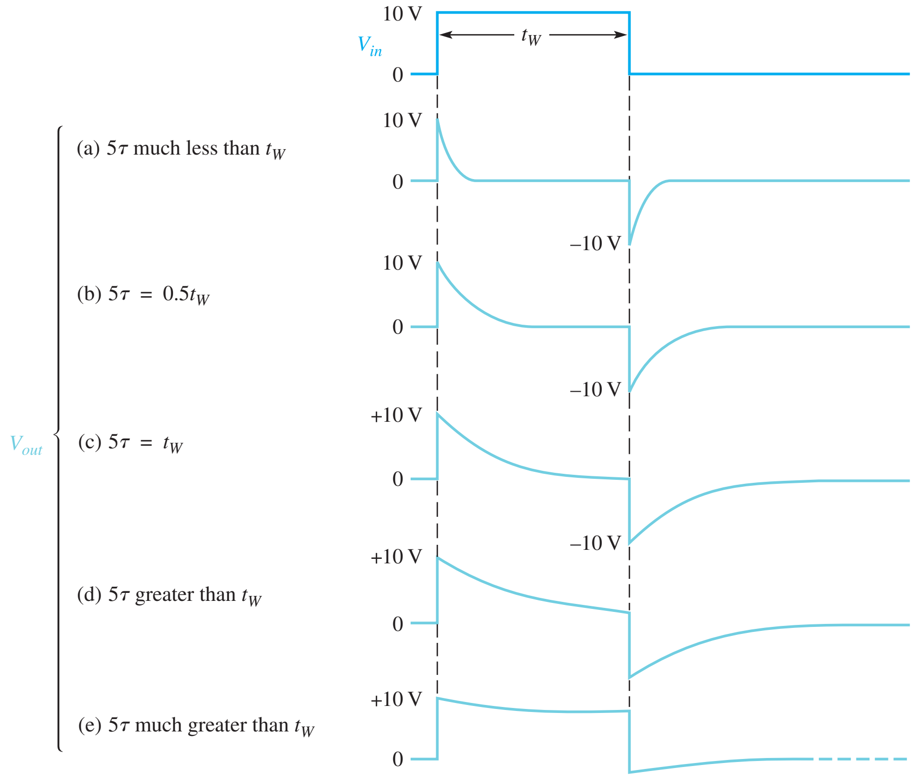
Illustration of the response of an *RL* differentiator for both time constant conditions.

During the pulse, the current exponentially builds up. As a result, the inductor voltage decreases, as shown in Figure 41(b). The rate of decrease, as you know, depends on the *L*/*R* time constant. When the falling edge of the input occurs, the inductor reacts to keep the current as is, by creating an induced voltage in a direction as indicated in Figure 41(c). This reaction is seen as a sudden negative-going transition of the inductor voltage, as indicated in Figure 41(c) and (d).

Two conditions are possible, as indicated in Figure 41(c) and (d). In part (c), 5τ is greater than the input pulse width, and the output voltage does not have time to decay to zero. In part (d), 5τ is less than or equal to the pulse width, and so the output decays to zero before the end of the pulse. In this case a −10 V transition occurs at the trailing edge.

Keep in mind that as far as the input and output waveforms are concerned, the *RL* integrator and differentiator perform the same as their *RC* counterparts.

A summary of the *RL* differentiator response for relationships of various time constants and pulse widths is shown in Figure 42.



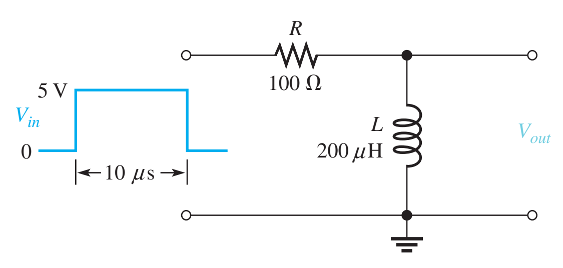
▲ **FIGURE 42**

Illustration of the variation in output pulse shape with the *RL* time constant.

EXAMPLE 9

Draw the output voltage for the *RL* differentiator in Figure 43.

▶ **FIGURE 43**



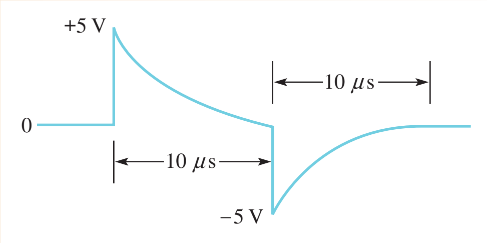
Solution First, calculate the time constant.

$$\tau = \frac{L}{R} = \frac{200\ \mu\text{H}}{100\ \Omega} = 2\ \mu\text{s}$$

In this case, $t_W = 5\tau$, so the output will decay to zero at the end of the pulse.

On the rising edge, the inductor voltage jumps to +5 V and then decays exponentially to zero. It reaches approximately zero at the instant of the falling edge. On the falling edge of the input, the inductor voltage jumps to −5 V and then goes back to zero. The output waveform is shown in Figure 44.

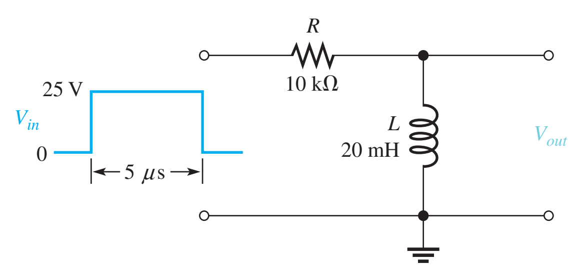
▶ **FIGURE 44**



Related Problem Draw the output voltage if the pulse width is reduced to 5 μs in Figure 43.

EXAMPLE 10

Determine the output voltage waveform for the *RL* differentiator in Figure 45.



▲ **FIGURE 45**

Solution First, calculate the time constant.

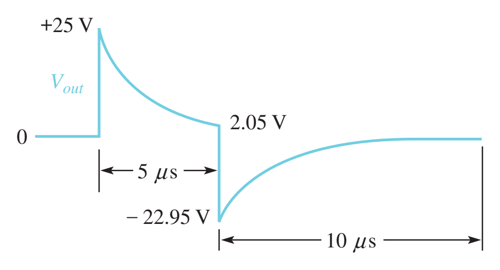
$$\tau = \frac{L}{R} = \frac{20\text{ mH}}{10\text{ k}\Omega} = 2\ \mu\text{s}$$

On the rising edge, the inductor voltage immediately jumps to +25 V. Because the pulse width is 5 μs, the inductor charges for only 2.5τ, so you must use the formula for a decreasing exponential with $V_F = 0$ and $\tau = L/R$.

$$v_L = V_i e^{-t/\tau} = 25e^{-5\mu\text{s}/2\mu\text{s}} = 25e^{-2.5} = 25(0.082) = 2.05\text{ V}$$

This result is the inductor voltage at the end of the 5 μs input pulse.

On the falling edge, the output immediately jumps from +2.05 V down to −22.95 V (a 25 V negative-going transition). The complete output waveform is shown in Figure 46.



▲ **FIGURE 46**

Related Problem What must be the value of R for the output voltage to reach zero by the end of the pulse in Figure 45?

Use Multisim files E20-10A and E20-10B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 7 CHECKUP

1. In an RL differentiator, across which component is the output taken?
2. Under what condition does the output pulse shape most closely resemble the input pulse?
3. If the inductor voltage in an RL differentiator is down to $+2$ V at the end of a $+10$ V input pulse, to what negative voltage will the output go in response to the falling edge of the input?

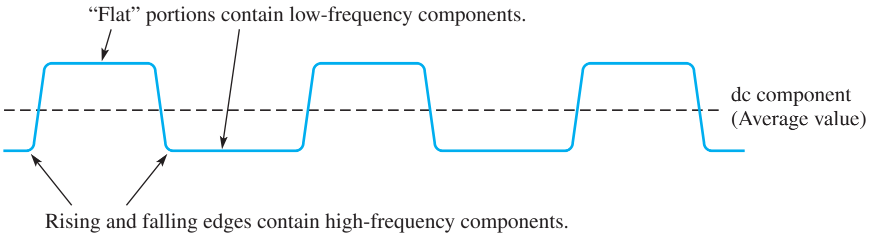
8 RELATIONSHIP OF TIME RESPONSE TO FREQUENCY RESPONSE

A definite relationship exists between time (pulse) response and frequency response. The fast rising and falling edges of a pulse waveform contain the higher frequency components. The flatter portions of the pulse waveform, which are the tops and the baseline of the pulses, represent slow changes or lower frequency components. The average value of the pulse waveform is its dc component.

After completing this section, you should be able to

- **Explain the relationship of time response to frequency response**
    - Describe a pulse waveform in terms of its frequency components
    - Explain how RC and RL integrators act as filters
    - Explain how RC and RL differentiators act as filters
    - State the formulas that relate rise and fall times to frequency

The relationships of pulse characteristics and frequency content of pulse waveforms are indicated in Figure 47.

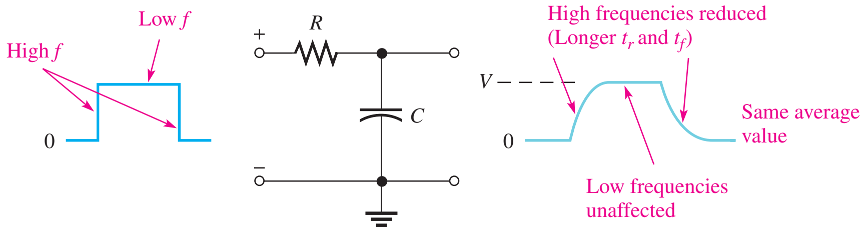

"Flat" portions contain low-frequency components.
dc component
(Average value)
Rising and falling edges contain high-frequency components.

▲ FIGURE 47

Frequency content of a pulse waveform.

The Integrator

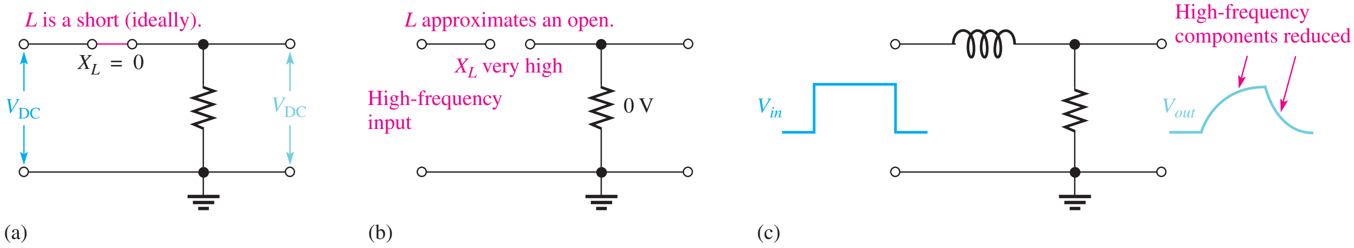
RC Integrator In terms of frequency response, the *RC* integrator acts as a low-pass filter. As you learned, the *RC* integrator tends to exponentially "round off" the edges of the applied pulses. This rounding off occurs to varying degrees, depending on the relationship of the time constant to the pulse width and period. The rounding off of the edges indicates that the integrator tends to reduce the higher frequency components of the pulse waveform, as illustrated in Figure 48.


Low *f*
High *f*
0
+
R
C
–
High frequencies reduced
(Longer t_r and t_f)
V
0
Same average value
Low frequencies unaffected

▲ FIGURE 48

Time and frequency response relationship in an *RC* integrator (one pulse in a repetitive waveform shown).

RL Integrator Like the *RC* integrator, the *RL* integrator also acts as a basic low-pass filter because *L* is in series between the input and output. The inductive reactance, X_L, is small for low frequencies and offers little opposition. It increases with frequency, so at higher frequencies most of the total voltage is dropped across *L* and very little across *R*, the output. If the input is dc, *L* is like a short ($X_L = 0$). At high frequencies, *L* becomes like an open, as illustrated in Figure 49.


L is a short (ideally).
$X_L = 0$
V_{DC}
V_{DC}
(a)
L approximates an open.
X_L very high
High-frequency input
0 V
(b)
V_{in}
V_{out}
High-frequency components reduced
(c)

▲ FIGURE 49

Low-pass filtering action.

The Differentiator

RC Differentiator In terms of frequency response, the *RC* differentiator acts as a high-pass filter. As you know, the differentiator tends to introduce tilt to the flat portion of a pulse. That is, it tends to reduce the lower frequency components of a pulse waveform. Also, it completely eliminates the dc component of the input and produces a zero average-value output. This action is illustrated in Figure 50.

RL Differentiator Again like the *RC* differentiator, the *RL* differentiator also acts as a basic high-pass filter. Because *L* is connected across the output, less voltage is developed

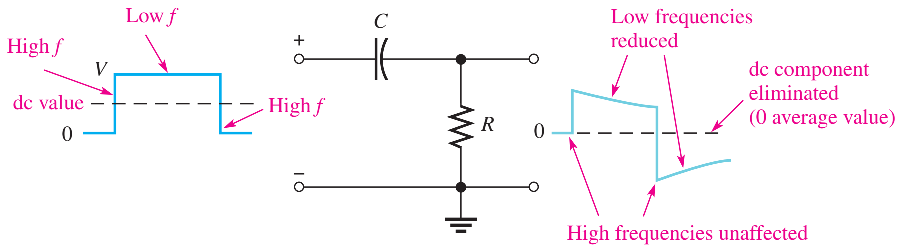
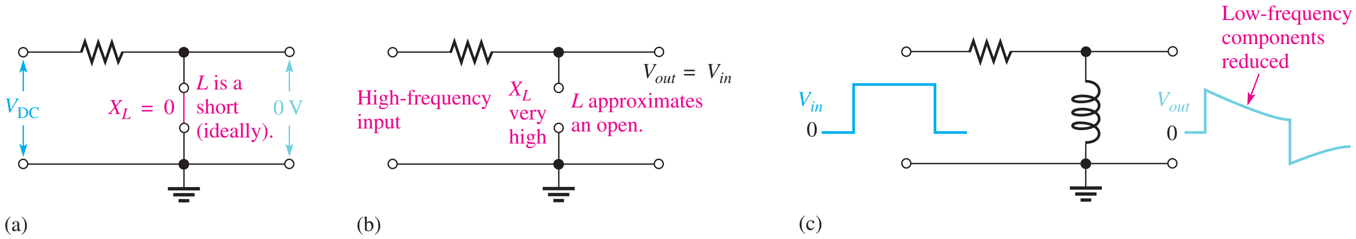

Low f
High f
V
dc value
0
High f
C
+
R
−
0
Low frequencies reduced
dc component eliminated (0 average value)
High frequencies unaffected

FIGURE 50

Time and frequency response relationship in an RC differentiator (one pulse in a repetitive waveform shown).

across it at lower frequencies than at higher ones. There are zero volts across the output for dc (ignoring winding resistance). For high frequencies, most of the input voltage is dropped across the output coil ($X_L = 0$ for dc; $X_L \cong$ open for high frequencies). Figure 51 shows high-pass filter action.


V_{DC}
$X_L = 0$
L is a short (ideally).
0 V
High-frequency input
X_L very high
$V_{out} = V_{in}$
L approximates an open.
V_{in}
0
V_{out}
0
Low-frequency components reduced

(a) (b) (c)

FIGURE 51

High-pass filtering action.

Formulas Relating Time Response to Frequency Response

The fast transitions of a pulse (rise time, t_r, and fall time, t_f) are related to the highest frequency component, f_h, in that pulse by the following formula:

$$t_r = \frac{0.35}{f_h}$$

Equation 1

This formula also applies to fall time, and the fastest transition determines the highest frequency in the pulse waveform.

Equation 1 can be rearranged to give the highest frequency as follows:

$$f_h = \frac{0.35}{t_r}$$

Equation 2

also,

$$f_h = \frac{0.35}{t_f}$$

Equation 3

EXAMPLE 11

What is the highest frequency contained in a pulse that has rise and fall times equal to 10 nanoseconds (10 ns)?

Solution

$$f_h = \frac{0.35}{t_r} = \frac{0.35}{10 \times 10^{-9}\text{ s}} = 0.035 \times 10^{9}\text{ Hz}$$

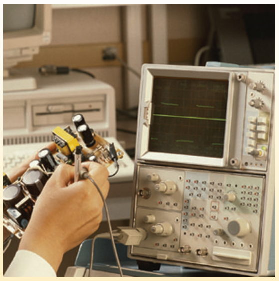
$$= 35 \times 10^{6}\text{ Hz} = \mathbf{35\text{ MHz}}$$

Related Problem What is the highest frequency in a pulse with $t_r = 20$ ns and $t_f = 15$ ns?

SECTION 8 CHECKUP

1. What type of filter is an integrator?
2. What type of filter is a differentiator?
3. What is the highest frequency component in a pulse waveform having t_r and t_f equal to 1 μs?

9 TROUBLESHOOTING



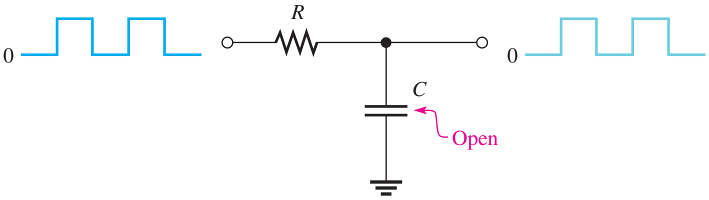
In this section, *RC* circuits with pulse inputs are used to demonstrate the effects of common component failures in selected cases. The concepts can then be easily related to *RL* circuits.

After completing this section, you should be able to

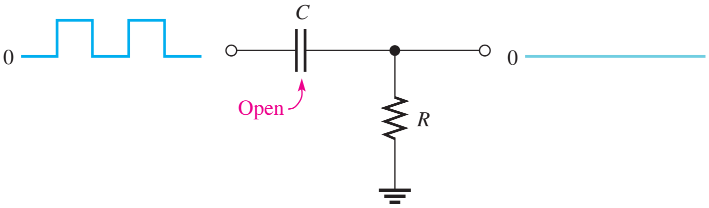
- **Troubleshoot *RC* integrators and *RC* differentiators**
  - Recognize the effect of an open capacitor
  - Recognize the effect of a leaky capacitor
  - Recognize the effect of a shorted capacitor
  - Recognize the effect of an open resistor

Open Capacitor

If the capacitor in an *RC* integrator opens, the output has the same waveshape as the input, as shown in Figure 52(a). If the capacitor in a differentiator opens, the output is zero because it is held at ground through the resistor, as illustrated in part (b).



(a) Integrator



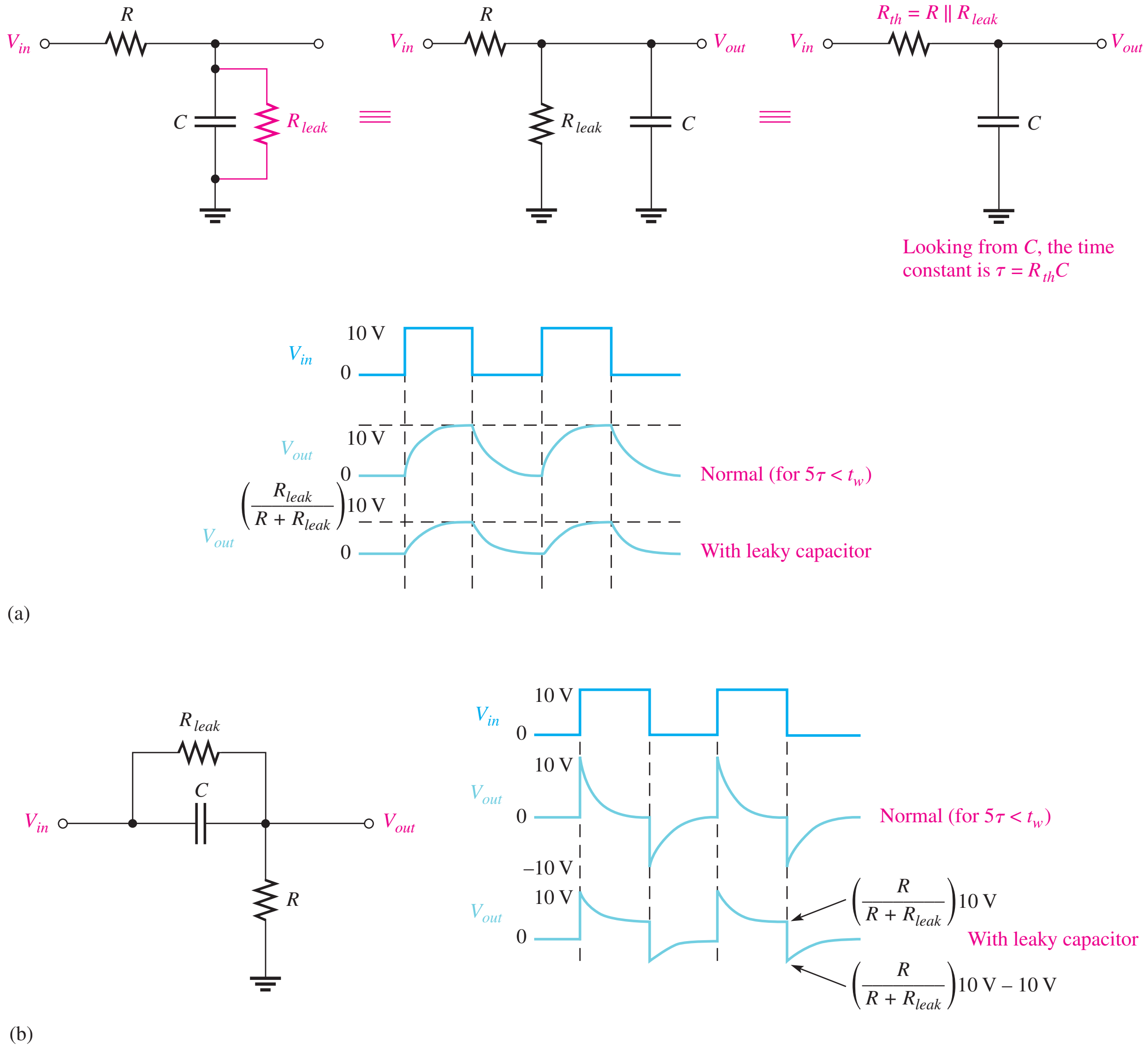
(b) Differentiator

FIGURE 52

Examples of the effect of an open capacitor.

Leaky Capacitor

If the capacitor in an *RC* integrator becomes leaky, three things happen: (a) the time constant will be effectively reduced by the leakage resistance (when thevenized, looking from *C* it appears in parallel with *R*); (b) the waveshape of the output voltage (across *C*) is altered from normal by a shorter charging time; and (c) the amplitude of the output is reduced because *R* and R_{leak} effectively act as a voltage divider. These effects are illustrated in Figure 53(a).


V_{in}
R
C
R_{leak}
$\equiv$
V_{out}
$R_{th} = R \parallel R_{leak}$
Looking from C, the time constant is $\tau = R_{th}C$
10 V
0
V_{out}
$\left(\frac{R_{leak}}{R + R_{leak}}\right)$10 V
Normal (for $5\tau < t_w$)
With leaky capacitor
(a)
-10 V
$\left(\frac{R}{R + R_{leak}}\right)$10 V
$\left(\frac{R}{R + R_{leak}}\right)$10 V – 10 V
(b)

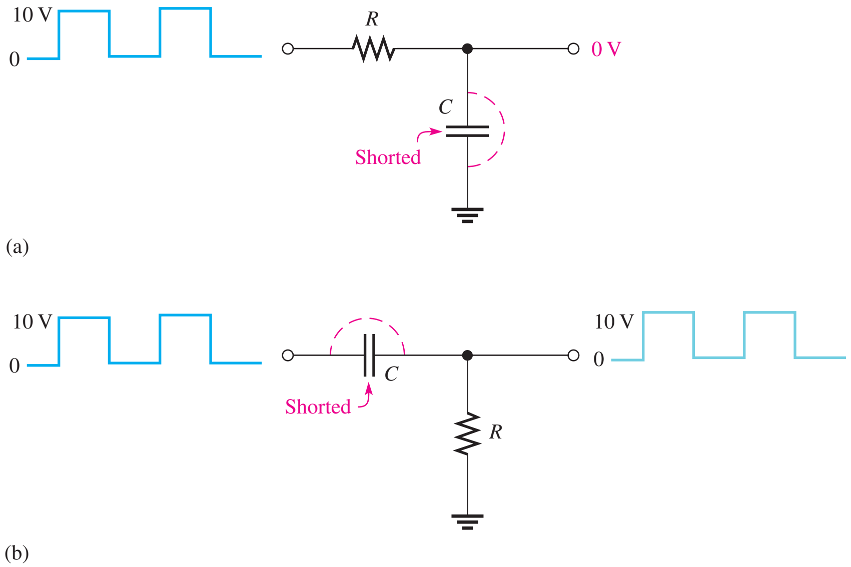
▲ **FIGURE 53**

Examples of the effect of a leaky capacitor.

If the capacitor in a differentiator becomes leaky, the time constant is reduced, just as in the integrator (they are both simply series *RC* circuits). When the capacitor reaches full charge, the output voltage (across *R*) is set by the effective voltage-divider action of *R* and R_{leak}, as shown in Figure 53(b).

Shorted Capacitor

If the capacitor in an *RC* integrator shorts, the output is at ground, as shown in Figure 54(a). If the capacitor in a *RC* differentiator shorts, the output is the same as the input, as shown in part (b).

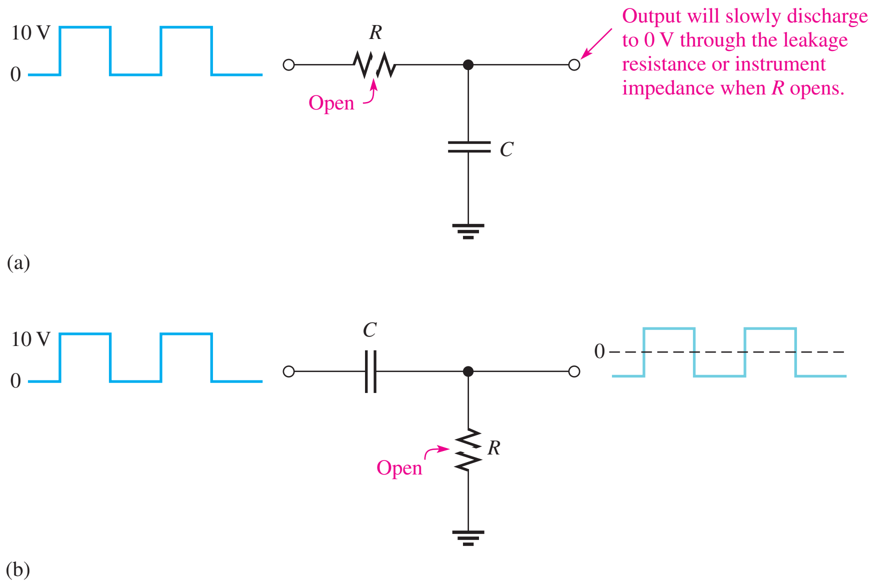


▲ FIGURE 54

Examples of the effect of a shorted capacitor.

Open Resistor

If the resistor in an *RC* integrator opens, the capacitor has no discharge path, and, ideally, it will hold its charge. In an actual situation, the charge will gradually leak off or the capacitor will discharge slowly through a measuring instrument connected to the output. This is illustrated in Figure 55(a).



Output will slowly discharge to 0 V through the leakage resistance or instrument impedance when *R* opens.

▲ FIGURE 55

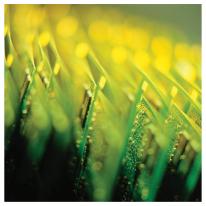
Examples of the effects of an open resistor.

If the resistor in a differentiator opens, the output looks like the input except for the dc level because the capacitor now must charge and discharge through the extremely high resistance of the oscilloscope, as shown in Figure 55(b).

SECTION 9 CHECKUP

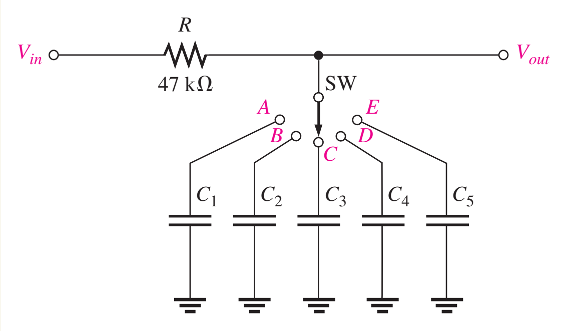
1. An *RC* integrator has a zero output with a square wave input. What are the possible causes of this problem?
2. If the capacitor in a differentiator is shorted, what is the output for a square wave input?

Application Activity



In this application activity, you are asked to build and test a time-delay circuit that will provide five switch-selectable delay times. An *RC* integrator is selected for this application. The input is a 5 V pulse of long duration, and the output goes to a threshold trigger circuit that is used to turn the power on to a portion of a system at any of the five selected time intervals after the occurrence of the original pulse.

A schematic of the selectable time-delay integrating circuit is shown in Figure 56. The *RC* integrator is driven by a pulse input; and the output is an exponentially increasing voltage that is used to trigger a threshold circuit at the 3.5 V level, which then turns power on to part of a system. The basic concept is shown in Figure 57. In this application, the delay time of the integrator is specified to be the time from the rising edge of the input pulse to the point where the output voltage reaches 3.5 V. The specified delay times are as listed in Table 1.

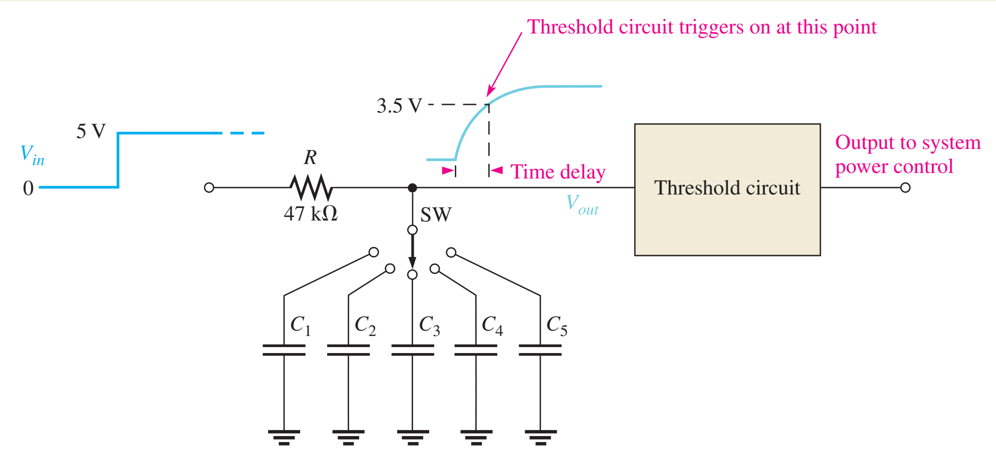


▲ FIGURE 56

Integrator delay circuit.

Capacitor Values

1. Determine a value for each capacitor that will provide the specified delay times within 10%. Select from the following list of standard values (all are in μF): 0.1, 0.12, 0.15, 0.18,



▲ FIGURE 57

Illustration of the time-delay application.

TABLE 1

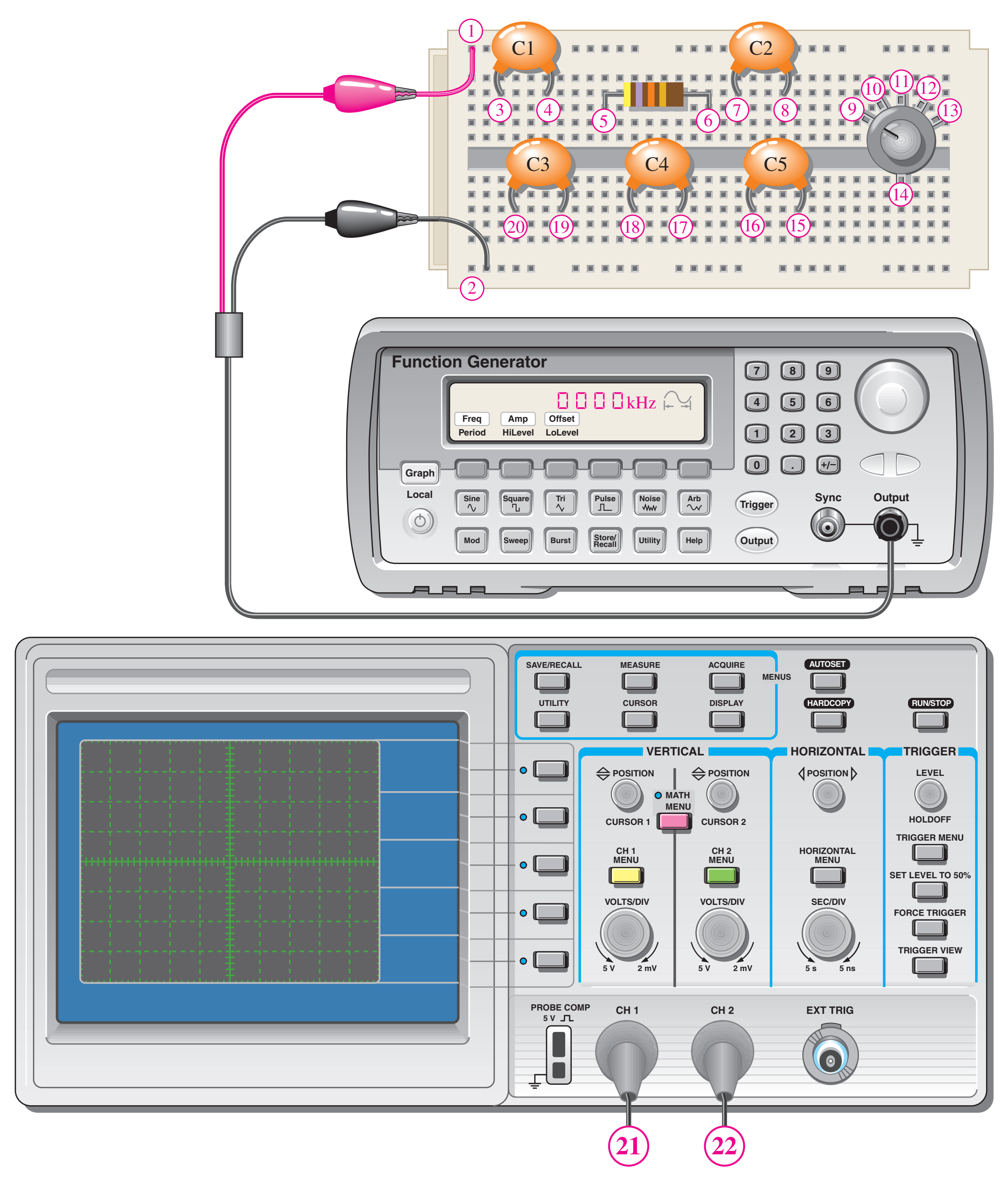
SWITCH POSITION	DELAY TIME
A	10 ms
B	25 ms
C	40 ms
D	65 ms
E	85 MS

0.22, 0.27, 0.33, 0.39, 0.47, 0.56, 0.68, 0.82, 1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9, 4.7, 5.6, 6.8, 8.2.

Circuit Connections

Refer to Figure 58. The components for the *RC* integrator in Figure 56 are assembled, but not interconnected, on the circuit board.

2. Using the circled numbers, develop a point-to-point wiring list to properly connect the circuit on the board.


①
C1
C2
③
④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
⑫
⑬
C3
C4
C5
⑭
⑳
⑲
⑱
⑰
⑯
⑮
②
Function Generator
0000kHz
Freq
Period
Amp
HiLevel
Offset
LoLevel
7
8
9
4
5
6
1
2
3
0
.
+/−
Graph
Local
Sine
Square
Tri
Pulse
Noise
Arb
Trigger
Sync
Output
Mod
Sweep
Burst
Store/Recall
Utility
Help
Output
SAVE/RECALL
MEASURE
ACQUIRE
UTILITY
CURSOR
DISPLAY
MENUS
AUTOSET
HARDCOPY
RUN/STOP
VERTICAL
HORIZONTAL
TRIGGER
POSITION
POSITION
POSITION
LEVEL
MATH MENU
CURSOR 1
CURSOR 2
HOLDOFF
TRIGGER MENU
CH 1 MENU
CH 2 MENU
HORIZONTAL MENU
SET LEVEL TO 50%
VOLTS/DIV
VOLTS/DIV
SEC/DIV
FORCE TRIGGER
TRIGGER VIEW
5 V
2 mV
5 V
2 mV
5 s
5 ns
PROBE COMP
5 V
CH 1
CH 2
EXT TRIG
㉑
㉒

▲ FIGURE 58

3. Indicate, using the appropriate circled numbers, how you would connect the instruments to test the circuit.

Test Procedure

4. Specify the function, amplitude, and minimum frequency settings for the function generator in order to test all output delay times in Figure 58.
5. Specify the minimum oscilloscope settings for measuring each of the specified delay times in Figure 58.

Review

6. To add an additional time delay to the circuit of Figure 57, what changes must be made?
7. An additional time delay of 100 ms is required for the time-delay circuit. Determine the capacitor value that should be added.

SUMMARY

- In an *RC* integrating circuit, the output voltage is taken across the capacitor.
- In an *RC* differentiating circuit, the output voltage is taken across the resistor.
- In an *RL* integrating circuit, the output voltage is taken across the resistor.
- In an *RL* differentiating circuit, the output voltage is taken across the inductor.
- In an integrator, when the pulse width (t_W) of the input is much less than the transient time, the output voltage approaches a constant level equal to the average value of the input.
- In an integrator, when the pulse width of the input is much greater than the transient time, the output voltage approaches the shape of the input.
- In a differentiator, when the pulse width of the input is much less than the transient time, the output voltage approaches the shape of the input but with an average value of zero.
- In a differentiator, when the pulse width of the input is much greater than the transient time, the output voltage consists of narrow, positive-going and negative-going spikes occurring on the leading and trailing edges of the input pulses.
- The rising and falling edges of a pulse waveform contain the higher frequency components.
- The flat portion of the pulse contains the lower frequency components.

KEY TERMS

DC component	The average value of a pulse waveform.
Differentiator	A circuit producing an output that approaches the mathematical derivative of the input.
Integrator	A circuit producing an output that approaches the mathematical integral of the input.
Steady state	The equilibrium condition of a circuit that occurs after an initial transient time.
Transient time	An interval equal to approximately five time constants.

FORMULAS

1	$t_r = \dfrac{0.35}{f_h}$	Rise time
2	$f_h = \dfrac{0.35}{t_r}$	Highest frequency in relation to rise time
3	$f_h = \dfrac{0.35}{t_f}$	Highest frequency in relation to fall time

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The output of an *RC* integrator is taken across the capacitor.
2. The capacitor voltage follows an exponential curve in response to a step input.
3. The time constant of a circuit is the time it takes for the input voltage to get to the output.
4. For the output of a differentiator to reach the amplitude of the input pulse, the time constant must be very large compared to the pulse width.
5. The transient time of an *RC* circuit is the time for the capacitor to fully charge or discharge when there is a pulse input.
6. It takes five transient times to make up one time constant.
7. The output of an *RC* differentiator is taken across the resistor.
8. For the output of an integrator to approximate an input pulse, the time constant must be very small compared to the pulse width.
9. The output of an *RL* integrator is taken across the inductor.
10. In terms of frequency response, an integrator acts as a low-pass filter and a differentiator acts as a high-pass filter.

SELF-TEST

Answers are at the end of the chapter.

1. The output of an *RC* integrator is taken across the
   (a) resistor **(b)** capacitor **(c)** source **(d)** coil
2. When a 10 V input pulse with a width equal to one time constant is applied to an *RC* integrator, the capacitor charges to
   (a) 10 V **(b)** 5 V **(c)** 6.3 V **(d)** 3.7 V
3. When a 10 V pulse with a width equal to one time constant is applied to an *RC* differentiator, the capacitor charges to
   (a) 6.3 V **(b)** 10 V **(c)** 0 V **(d)** 3.7 V
4. In an *RC* integrator, the output pulse closely resembles the input pulse when
   (a) τ is much larger than the pulse width **(b)** τ is equal to the pulse width
   (c) τ is less than the pulse width **(d)** τ is much less than the pulse width
5. In an *RC* differentiator, the output pulse closely resembles the input pulse when
   (a) τ is much larger than the pulse width **(b)** τ is equal to the pulse width
   (c) τ is less than the pulse width **(d)** τ is much less than the pulse width
6. The positive and negative portions of a differentiator's output voltage are equal when
   (a) $5\tau < t_W$ **(b)** $5\tau > t_W$ **(c)** $5\tau = t_W$
   (d) $5\tau > 0$ **(e)** both (a) and (c) **(f)** both (b) and (d)
7. The output of an *RL* integrator is taken across the
   (a) resistor **(b)** coil **(c)** source **(d)** capacitor
8. The maximum possible current in an *RL* integrator is
   (a) $I = V_p/X_L$ **(b)** $I = V_p/Z$ **(c)** $I = V_p/R$
9. The current in an *RL* differentiator reaches its maximum possible value when
   (a) $5\tau = t_W$ **(b)** $5\tau < t_W$ **(c)** $5\tau > t_W$ **(d)** $\tau = 0.5t_W$
10. If you have an *RC* and an *RL* differentiator with equal time constants sitting side-by-side and you apply the same input pulse to both,
    (a) the *RC* has the widest output pulse
    (b) the *RL* has the most narrow spikes on the output
    (c) the output of one is an increasing exponential and the output of the other is a decreasing exponential
    (d) you can't tell the difference by observing the output waveforms

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 60.

1. If R_2 opens, the amplitude of the output voltage
   (a) increases **(b)** decreases **(c)** stays the same
2. If C doubled in value, the time constant
   (a) increases **(b)** decreases **(c)** stays the same
3. If R_1 is reduced in value, the output voltage amplitude
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 63.

4. If R_3 opens, the amplitude of the output voltage
   (a) increases **(b)** decreases **(c)** stays the same
5. If a constant dc voltage is applied to the input, the output voltage
   (a) increases **(b)** decreases **(c)** stays the same
6. If R_1 is 3.3 kΩ instead of 2.2 kΩ, the time constant
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 66.

7. If L is increased, the rise time of the output
   (a) increases **(b)** decreases **(c)** stays the same
8. If the width of the input pulse is increased to 5 ms, the amplitude of the output pulse
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 68.

9. If R_1 opens, the maximum amplitude of the output
   (a) increases **(b)** decreases **(c)** stays the same
10. If R_2 is shorted, the maximum amplitude of the output
    (a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

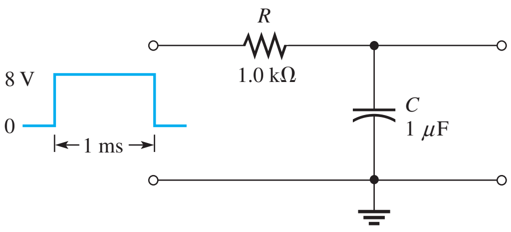
Answers to odd-numbered problems are at the end of the chapter.

SECTION 1 The *RC* Integrator

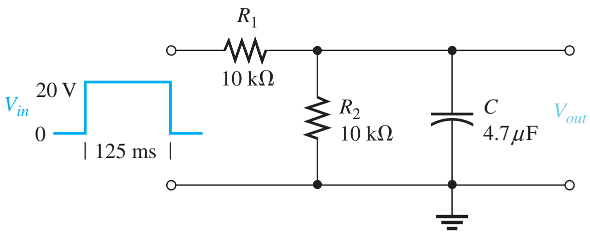
1. An integrating circuit has $R = 2.2\ \text{k}\Omega$ in series with $C = 0.047\ \mu\text{F}$. What is the time constant?
2. Determine how long it takes the capacitor in an integrating circuit to reach full charge for each of the following series *RC* combinations:
   (a) $R = 56\ \Omega, C = 47\ \mu\text{F}$ **(b)** $R = 3300\ \Omega, C = 0.015\ \mu\text{F}$
   (c) $R = 22\ \text{k}\Omega, C = 100\ \text{pF}$ **(d)** $R = 5.6\ \text{M}\Omega, C = 10\ \text{pF}$

SECTION 2 Response of an *RC* Integrator to a Single Pulse

3. A 20 V pulse is applied to an *RC* integrator. The pulse width equals one time constant. To what voltage does the capacitor charge during the pulse? Assume that it is initially uncharged.
4. Repeat Problem 3 for the following values of t_W:
   (a) 2τ **(b)** 3τ **(c)** 4τ **(d)** 5τ
5. Draw the approximate shape of an integrator output voltage where 5τ is much less than the pulse width of a 10 V square-wave input. Repeat for the case in which 5τ is much larger than the pulse width.
6. Determine the output voltage for an *RC* integrator with a single input pulse, as shown in Figure 59. For repetitive pulses, how long will it take this circuit to reach steady state?


R
1.0 kΩ
8 V
0
1 ms
C
1 μF

▲ FIGURE 59

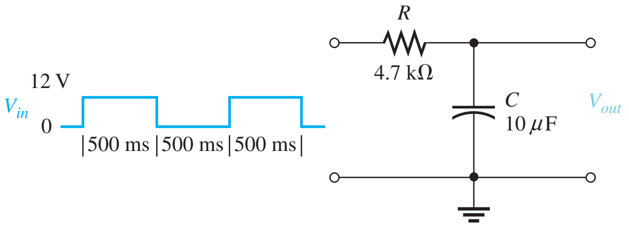

R1
10 kΩ
20 V
Vin
0
125 ms
R2
10 kΩ
C
4.7 μF
Vout

▲ FIGURE 60

7. (a) What is τ in Figure 60?
   (b) Draw the output voltage.
8. Sketch the output voltage in Figure 60 if the pulse width is increased to 1.25 s.
9. Repeat Problem 8 if the pulse width is reduced to 23.5 ms.

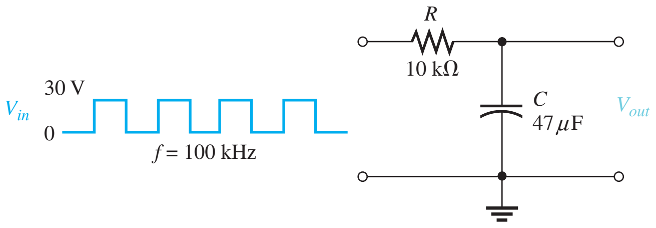
SECTION 3 Response of *RC* Integrators to Repetitive Pulses

10. Determine the transient time for Figure 61.
11. Draw the integrator output voltage in Figure 61, showing maximum voltages.
12. Sketch the output voltage if the pulse width of V_{in} in Figure 60 is changed to 47 ms and the frequency is the same.


R
4.7 kΩ
12 V
Vin
0
500 ms
500 ms
500 ms
C
10 μF
Vout

▲ FIGURE 61

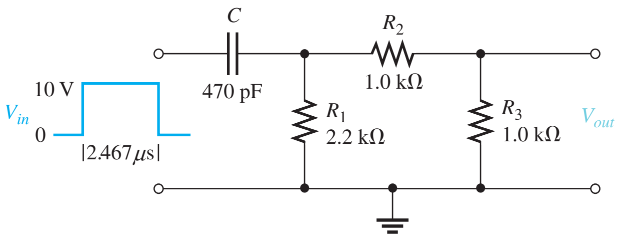
13. A 1 V, 10 kHz pulse waveform with a duty cycle of 25% is applied to an integrator with $\tau = 25\ \mu$s. Graph the output voltage for three initial pulses. *C* is initially uncharged.
14. What is the steady-state output voltage of the *RC* integrator with a square-wave input shown in Figure 62?


R
10 kΩ
30 V
Vin
0
f = 100 kHz
C
47 μF
Vout

▲ FIGURE 62

SECTION 4 Response of an *RC* Differentiator to a Single Pulse

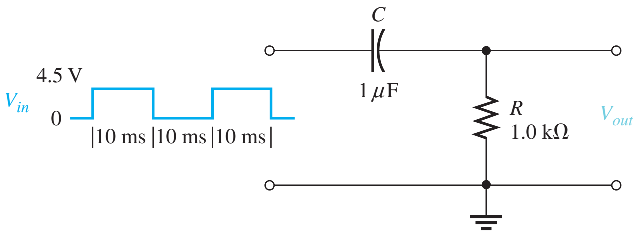
15. Repeat Problem 5 for an *RC* differentiator.
16. Redraw the circuit in Figure 59 to make it a differentiator, and repeat Problem 6.
17. (a) What is τ in Figure 63?
    (b) Draw the output voltage.


C
470 pF
R_2
1.0 kΩ
10 V
V_{in}
0
|2.467 μs|
R_1
2.2 kΩ
R_3
1.0 kΩ
V_{out}

▲ **FIGURE 63**

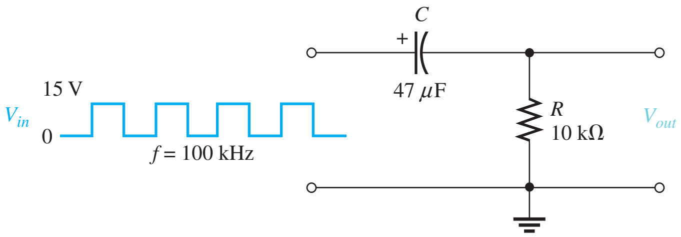
SECTION 5 Response of *RC* Differentiators to Repetitive Pulses

18. Draw the differentiator output in Figure 64, showing maximum voltages.


C
1 μF
4.5 V
V_{in}
0
|10 ms |10 ms |10 ms |
R
1.0 kΩ
V_{out}

▲ **FIGURE 64**

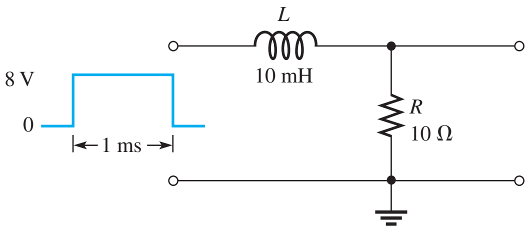
19. What is the steady-state output voltage of the differentiator with the square-wave input shown in Figure 65?


C
+
47 μF
15 V
V_{in}
0
f = 100 kHz
R
10 kΩ
V_{out}

▲ **FIGURE 65**

SECTION 6 Response of *RL* Integrators to Pulse Inputs

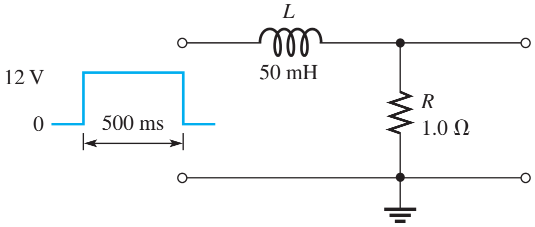
20. Determine the output voltage for the circuit in Figure 66. A single input pulse is applied as shown.


L
10 mH
8 V
0
1 ms
R
10 Ω

▲ **FIGURE 66**

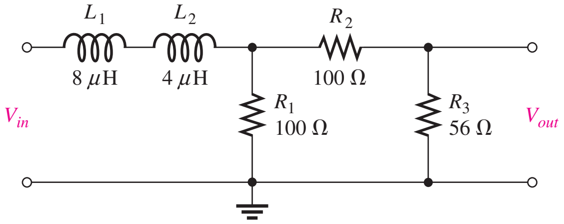
21. Draw the integrator output voltage in Figure 67, showing maximum voltages.

▶ FIGURE 67



22. Determine the time constant in Figure 68. Is this circuit an integrator or a differentiator?

▶ FIGURE 68

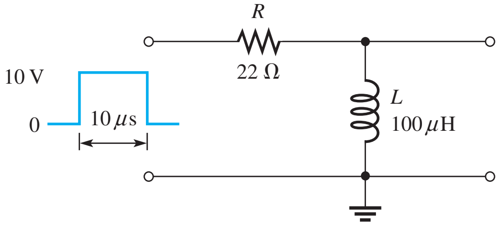


SECTION 7 Response of *RL* Differentiators to Pulse Inputs

23. **(a)** What is τ in Figure 69?

(b) Draw the output voltage.

▶ FIGURE 69



24. Draw the output waveform if a periodic pulse waveform with $t_W = 25\ \mu s$ and $T = 60\ \mu s$ is applied to the circuit in Figure 69.

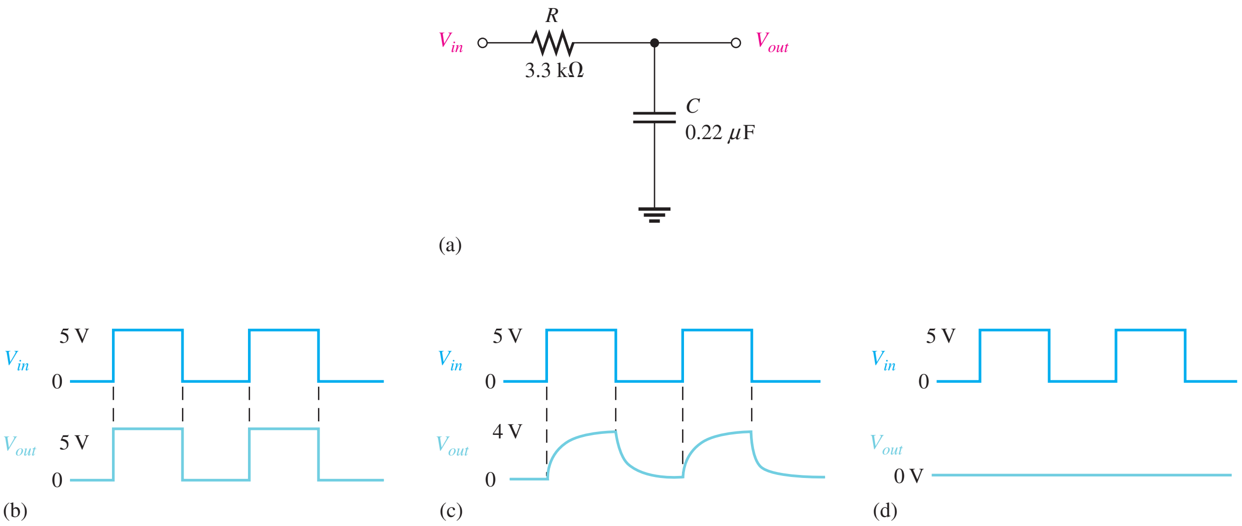
SECTION 8 Relationship of Time Response to Frequency Response

25. What is the highest frequency component in the output of an integrator with $\tau = 10\ \mu s$? Assume that $5\tau < t_W$.

26. A certain pulse waveform has a rise time of 55 ns and a fall time of 42 ns. What is the highest frequency component in the waveform?

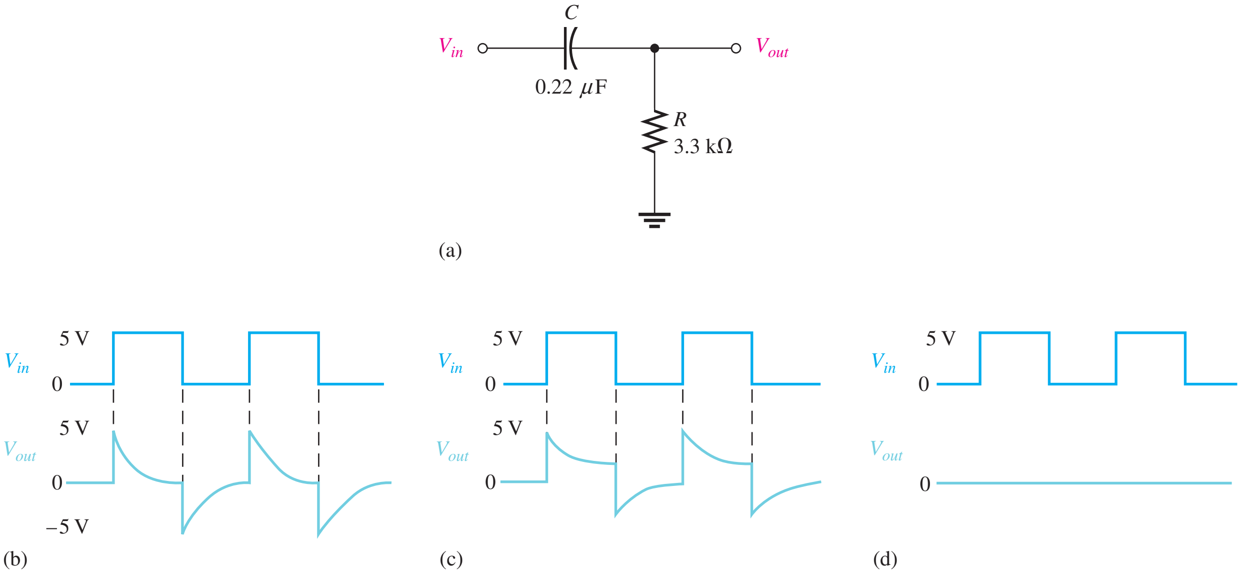
SECTION 9 Troubleshooting

27. Determine the most likely fault(s) in the circuit of Figure 70(a) for each set of waveforms in parts (b) through (d). V_{in} is a square wave with a period of 8 ms.


V_{in}
R
3.3 kΩ
V_{out}
C
0.22 μF
(a)
V_{in}
5 V
0
V_{out}
5 V
0
(b)
V_{in}
5 V
0
V_{out}
4 V
0
(c)
V_{in}
5 V
0
V_{out}
0 V
(d)

▲ **FIGURE 70**

28. Determine the most likely fault(s), if any, in the circuit of Figure 71(a) for each set of waveforms in parts (b) through (d). V_{in} is a square wave with a period of 8 ms.


C
V_{in}
0.22 μF
V_{out}
R
3.3 kΩ
(a)
V_{in}
5 V
0
V_{out}
5 V
0
−5 V
(b)
V_{in}
5 V
0
V_{out}
5 V
0
(c)
V_{in}
5 V
0
V_{out}
0
(d)

▲ **FIGURE 71**

Multisim Troubleshooting and Analysis

These problems require Multisim.

29. Open file P20-29 and determine if there is a fault. If so, find the fault.

30. Open file P20-30 and determine if there is a fault. If so, find the fault.

31. Open file P20-31 and determine if there is a fault. If so, find the fault.

32. Open file P20-32 and determine if there is a fault. If so, find the fault.

ANSWERS

SECTION CHECKUPS

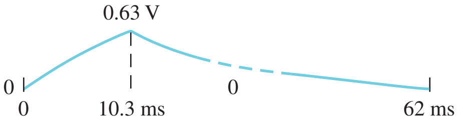
SECTION 1 The *RC* Integrator

1. An integrator is a series *RC* circuit in which the output is across the capacitor.
2. A voltage applied to the input causes the capacitor to charge. A short across the input causes the capacitor to discharge.

SECTION 2 Response of an *RC* Integrator to a Single Pulse

1. For the output of an integrator to reach amplitude, $5\tau \leq t_W$.
2. $V_{out(max)} = 630\text{ mV}$; $t_{disch} = 51.7\text{ ms}$
3. See Figure 72.

FIGURE 72



4. No, *C* will not fully charge.
5. The output has approximately the shape of the input when $5\tau << t_W$ (5τ much less than t_W).

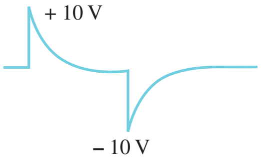
SECTION 3 Response of *RC* Integrators to Repetitive Pulses

1. *C* will fully charge and discharge when $5\tau \leq t_W$ and $5\tau \leq$ time between pulses.
2. When $\tau << t_W$, the output is approximately like the input.
3. Transient time
4. Steady-state response is the response after the transient time has passed.
5. An approximate dc voltage that is the average value of the input

SECTION 4 Response of an *RC* Differentiator to a Single Pulse

1. See Figure 73.

FIGURE 73



2. The output resembles the input when $5\tau >> t_W$.
3. The output appears to be positive and negative spikes.
4. V_R will go to -10 V.

SECTION 5 Response of *RC* Differentiators to Repetitive Pulses

1. *C* will fully charge and discharge when $5\tau \leq t_W$ and $5\tau \leq$ time between pulses.
2. The output appears to be positive and negative spikes.
3. The average value is 0 V.

SECTION 6 Response of *RL* Integrators to Pulse Inputs

1. The output is taken across the resistor.
2. The output reaches the input amplitude when $5\tau \leq t_W$.
3. The output has the approximate shape of the input when $5\tau << t_W$.

SECTION 7 Response of *RL* Differentiators to Pulse Inputs

1. The output is taken across the inductor.
2. The output has the approximate shape of the input when $5\tau >> t_W$.
3. V_L will go to -8 V.

SECTION 8 Relationship of Time Response to Frequency Response

1. An integrator is a low-pass filter.
2. A differentiator is a high-pass filter.
3. $f_{max} = 350$ kHz

SECTION 9 Troubleshooting

1. A 0 V output may be caused by an open resistor or shorted capacitor.
2. If C is shorted, the output is the same as the input.

RELATED PROBLEMS FOR EXAMPLES

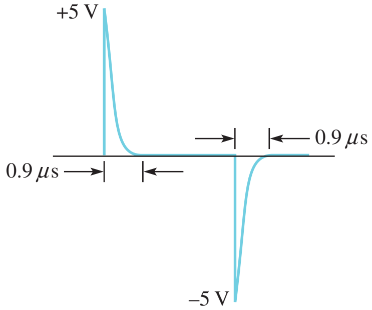
1 8.65 V

2 24.7 V

3 1.08 V

4 See Figure 74.

▶ FIGURE 74



5 892 mV

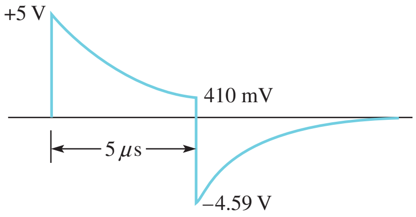
6 Impossible with a 50 Ω rheostat

7 20 V

8 2.5 kΩ

9 See Figure 75.

▶ FIGURE 75



10 20 kΩ

11 23.3 MHz

TRUE/FALSE QUIZ

1. T **2.** T **3.** F **4.** F **5.** T
6. F **7.** T **8.** T **9.** F **10.** T

SELF-TEST

1. (b) **2.** (c) **3.** (a) **4.** (d) **5.** (a)
6. (e) **7.** (a) **8.** (c) **9.** (b) **10.** (d)

CIRCUIT DYNAMICS QUIZ

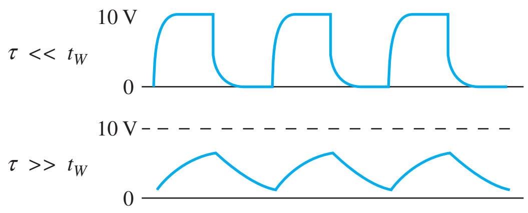
1. (b) **2.** (a) **3.** (c) **4.** (a) **5.** (b)
6. (a) **7.** (a) **8.** (a) **9.** (c) **10.** (a)

ANSWERS TO ODD-NUMBERED PROBLEMS

1. 103 μs

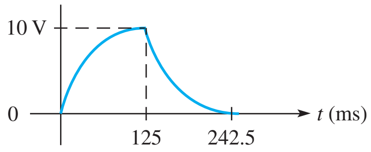
3. 12.6 V

5. See Figure 76.



▲ **FIGURE 76**

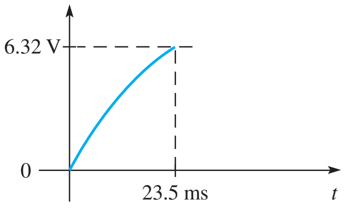
7. **(a)** 23.5 ms **(b)** See Figure 77.



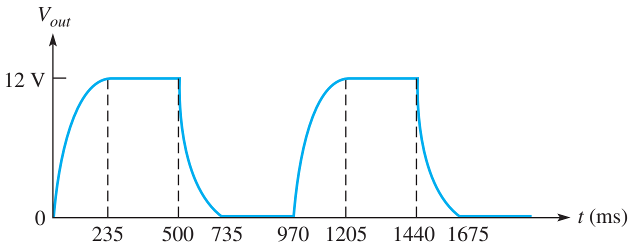
▲ **FIGURE 77**

9. See Figure 78.

▶ **FIGURE 78**

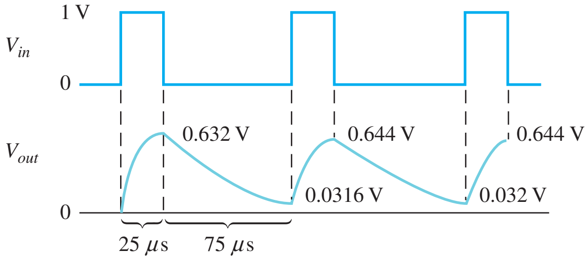


11. See Figure 79.



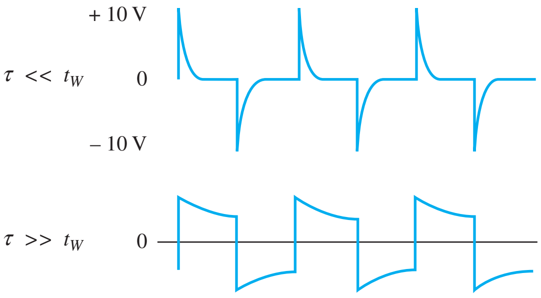
▲ **FIGURE 79**

13. See Figure 80.



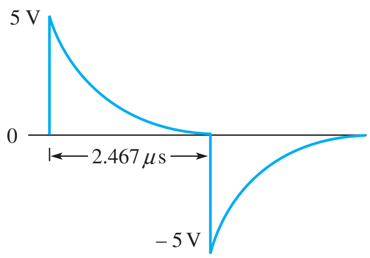
▲ **FIGURE 80**

15. See Figure 81.



▲ **FIGURE 81**

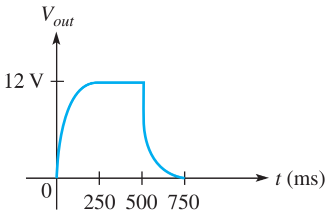
17. **(a)** 493.5 ns **(b)** See Figure 82.



▲ **FIGURE 82**

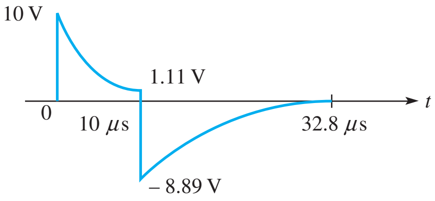
19. An approximate square wave with an average value of zero.

21. See Figure 83.



▲ **FIGURE 83**

23. **(a)** 4.55 μs **(b)** See Figure 84.



▲ **FIGURE 84**

25. 15.9 kHz

27. **(a)** Capacitor open or R shorted.

(b) C leaky or $R > 3.3\ \text{k}\Omega$ or $C > 0.22\ \mu\text{F}$

(c) Resistor open or capacitor shorted

29. C_1 open or R_1 shorted

31. R_1 or R_2 open

THREE-PHASE SYSTEMS IN POWER APPLICATIONS

CHAPTER OUTLINE

1 Generators in Power Applications
2 Types of Three-Phase Generators
3 Three-Phase Source/Load Analysis
4 Three-Phase Power

CHAPTER OBJECTIVES

- Discuss the advantages of three-phase generators in power applications
- Analyze three-phase generator connections
- Analyze three-phase generators with three-phase loads
- Discuss power measurements in three-phase systems

KEY TERMS

- Balanced load
- Phase voltage (V_θ)
- Phase current (I_θ)
- Line current (I_L)
- Line voltage (V_L)

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/floyd

INTRODUCTION

In this chapter, the basic generation of three-phase sinusoidal waveforms is examined in depth. The advantages of three-phase systems in power applications are covered, and various types of three-phase connections and power measurement are introduced.

From Chapter 21 of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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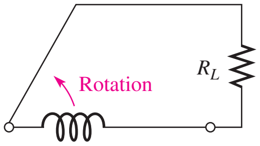
1 GENERATORS IN POWER APPLICATIONS

The advantages of using three-phase generators (alternators) to deliver power to a load over using a single-phase machine are discussed in this section.

After completing this section, you should be able to

- **Discuss the advantages of three-phase generators in power applications**
    - Explain the copper advantage
    - Compare single-phase and three-phase systems in terms of the copper advantage
    - Explain the advantage of constant power
    - Explain the advantage of a constant rotating magnetic field



▲ FIGURE 1

Simplified representation of a single-phase generator connected to a resistive load.

The size of the copper wire required to carry current from a generator to a load can be reduced when a three-phase rather than a single-phase generator is used.

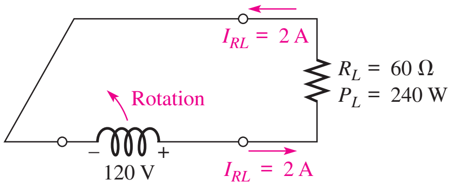
Figure is a simplified representation of a single-phase generator connected to a resistive load. The coil symbol represents the generator winding.

For example, a single-phase sinusoidal voltage is induced in the winding and applied to a 60 Ω load, as indicated in Figure 2. The resulting load current is

$$\mathbf{I}_{RL} = \frac{120\angle 0^\circ\text{ V}}{60\angle 0^\circ\ \Omega} = 2\angle 0^\circ\text{ A}$$

► FIGURE 2

Single-phase example.



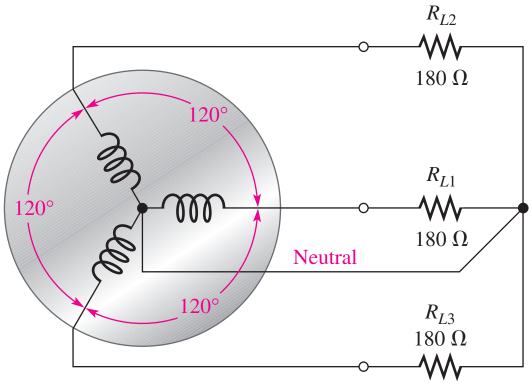
The total current that must be delivered by the generator to the load is $2\angle 0^\circ$A. This means that the two conductors carrying current to and from the load must each be capable of handling 2 A; thus, the total copper cross section must handle 4 A. (The copper cross section is a measure of the total amount of wire required based on its physical size as related to its diameter.) The total load power is

$$P_{L(tot)} = I_{RL}^2 R_L = 240\text{ W}$$

Figure 3 shows a simplified representation of a three-phase generator connected to three 180 Ω resistive loads. An equivalent single-phase system would be required to feed

► FIGURE 3

A simplified representation of a three-phase generator with each phase connected to a 180 Ω load.



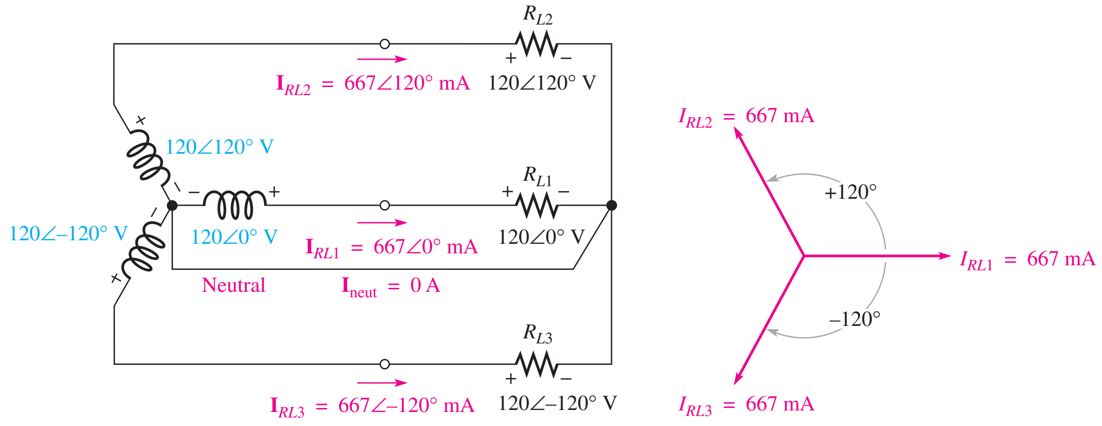
three 180 Ω resistors in parallel, thus creating an effective load resistance of 60 Ω. The coils represent the generator windings separated by 120°.

The voltage across R_{L1} is $120\angle 0°$ V, the voltage across R_{L2} is $120\angle 120°$ V, and the voltage across R_{L3} is $120\angle -120°$ V, as indicated in Figure 4(a). The current from each winding to its respective load is as follows:

$$\mathbf{I}_{RL1} = \frac{120\angle 0°\ \text{V}}{180\angle 0°\ \Omega} = 667\angle 0°\ \text{mA}$$

$$\mathbf{I}_{RL2} = \frac{120\angle 120°\ \text{V}}{180\angle 0°\ \Omega} = 667\angle 120°\ \text{mA}$$

$$\mathbf{I}_{RL3} = \frac{120\angle -120°\ \text{V}}{180\angle 0°\ \Omega} = 667\angle -120°\ \text{mA}$$



(a)

(b)

▲ **FIGURE 4**

Three-phase example.

The total load power is

$$P_{L(tot)} = I^2_{RL1}R_{L1} + I^2_{RL2}R_{L2} + I^2_{RL3}R_{L3} = 240\ \text{W}$$

This is the same total load power as delivered by the single-phase system previously discussed.

Notice that four conductors, including the neutral, are required to carry the currents to and from the loads. The current in each of the three conductors is 667 mA, as indicated in Figure 4(a). The current in the neutral conductor is the phasor sum of the three load currents and is equal to zero, as shown in the following equation, with reference to the phasor diagram in Figure 4(b).

$$\begin{aligned}\mathbf{I}_{RL1} + \mathbf{I}_{RL2} + \mathbf{I}_{RL3} &= 667\angle 0°\ \text{mA} + 667\angle 120°\ \text{mA} + 667\angle -120°\ \text{mA}\\ &= 667\ \text{mA} - 333.5\ \text{mA} + j578\ \text{mA} - 333.5\ \text{mA} - j578\ \text{mA}\\ &= 667\ \text{mA} - 667\ \text{mA} = 0\ \text{A}\end{aligned}$$

This condition, where all load currents are equal and the neutral current is zero, is called a **balanced load** condition.

The total copper cross section must handle 667 mA + 667 mA + 667 mA + 0 mA = 2 A. This result shows that considerably less copper is required to deliver the same load power with a three-phase system than is required for the single-phase system. The amount of copper is an important consideration in power distribution systems.

EXAMPLE 1 Compare the total copper cross sections in terms of current-carrying capacity for single-phase and three-phase 120 V systems with effective load resistances of 12 Ω.

Solution *Single-phase system:* The load current is

$$I_{RL} = \frac{120\text{ V}}{12\ \Omega} = 10\text{ A}$$

The conductor to the load must carry 10 A, and the conductor from the load must also carry 10 A.

The total copper cross section, therefore, must be sufficient to handle $2 \times 10\text{ A} = 20\text{ A}$.

Three-phase system: For an effective load resistance of 12 Ω, the three-phase generator feeds three load resistors of 36 Ω each. The current in each load resistor is

$$I_{RL} = \frac{120\text{ V}}{36\ \Omega} = 3.33\text{ A}$$

Each of the three conductors feeding the balanced load must carry 3.33 A, and the neutral current is zero.

Therefore, the total copper cross section must be sufficient to handle $3 \times 3.33\text{ A} \cong 10\text{ A}$. This is significantly less than for the single-phase system with an equivalent load.

Related Problem* Compare the total copper cross sections in terms of current-carrying capacity for single-phase and three-phase 240 V systems with effective load resistances of 100 Ω.

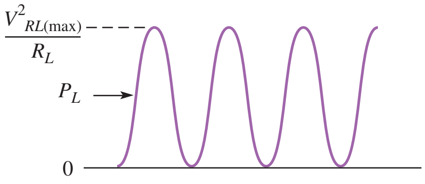
---

*Answers are at the end of the chapter.

A second advantage of three-phase systems over a single-phase system is that three-phase systems produce a constant amount of power in the load. As shown in Figure 5, the load power fluctuates as the square of the sinusoidal voltage divided by the resistance. It changes from a maximum of $V^2_{RL(\max)}/R_L$ to a minimum of zero at a frequency equal to twice that of the voltage.

▶ **FIGURE 5**

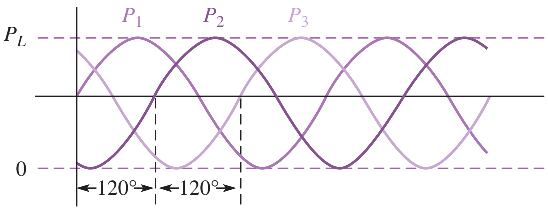
Single-phase load power ($\sin^2$ curve).



The power waveform across one of the load resistors in a three-phase system is 120° out of phase with the power waveforms across the other loads, as shown in Figure 6. Examination of the power waveforms shows that when three instantaneous values are added, the sum is always constant and equal to $V^2_{RL(\max)}/R_L$. A constant load power means

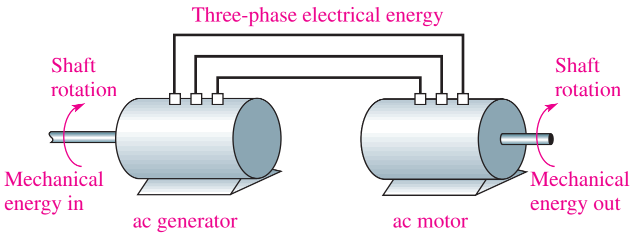
▶ **FIGURE 6**

Three-phase power ($P_L = V^2_{RL(\max)}/R_L$).



a uniform conversion of mechanical to electrical energy, which is an important consideration in many power applications.

In many applications, ac generators are used to drive ac motors for conversion of electrical energy to mechanical energy in the form of shaft rotation in the motor. The original energy for operation of the generator can come from any of several sources, such as hydroelectric or steam. Figure 7 illustrates the basic concept.



◀ **FIGURE 7**

Simple example of mechanical-to-electrical-to-mechanical energy conversion.

When a three-phase generator is connected to the motor windings, a magnetic field is created within the motor that has a constant flux density and that rotates at the frequency of the three-phase sine wave. The motor's rotor is pulled around at a constant rotational velocity by the rotating magnetic field, producing a constant shaft rotation, which is an advantage of three-phase systems.

A single-phase system is unsuitable for many applications because it produces a magnetic field that fluctuates in flux density and reverses direction during each cycle without providing the advantage of constant rotation.

SECTION 1 CHECKUP

Answers are at the end of the chapter.

1. List three advantages of three-phase systems over single-phase systems.
2. Which advantage is most important in mechanical-to-electrical energy conversions?
3. Which advantage is most important in electrical-to-mechanical energy conversions?

2 TYPES OF THREE-PHASE GENERATORS

In the previous sections, the Y-connection was used for illustration. In this section, the Y-connection is examined further and a second type, the Δ-connection, is introduced.

After completing this section, you should be able to

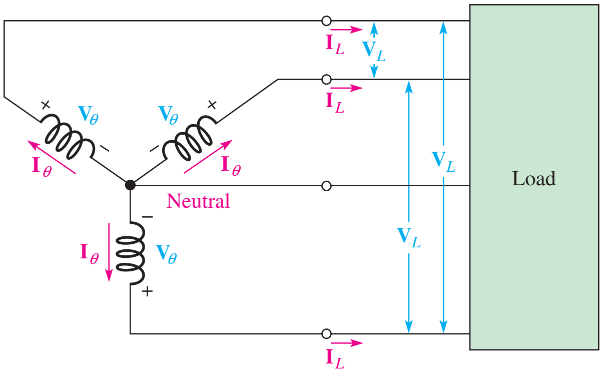
- **Analyze three-phase generator connections**
  - Analyze the Y-connected generator
  - Analyze the Δ-connected generator

The Y-Connected Generator

A Y-connected system can be either a three-wire or, when the neutral is used, a four-wire system, as shown in Figure 8, connected to a generalized load, which is indicated by the green block. Recall that when the loads are perfectly balanced, the neutral current is zero; therefore, the neutral conductor is unnecessary. However, in cases where the loads are not equal (unbalanced), a neutral wire is essential to provide a return current path because the neutral current has a nonzero value.

FIGURE 8

Y-connected generator.


$\mathbf{I}_L$
$\mathbf{V}_L$
$\mathbf{I}_L$
$\mathbf{V}_\theta$
$\mathbf{V}_\theta$
$\mathbf{I}_\theta$
$\mathbf{I}_\theta$
$\mathbf{V}_L$
Load
Neutral
$\mathbf{V}_L$
$\mathbf{I}_\theta$
$\mathbf{V}_\theta$
$\mathbf{I}_L$

The voltages across the generator windings are called **phase voltages (V_θ)**, and the currents through the windings are called **phase currents (I_θ)**. Also, the currents in the lines connecting the generator windings to the load are called **line currents (I_L)**, and the voltages across the lines are called the **line voltages (V_L)**. Note that the magnitude of each line current is equal to the corresponding phase current in the Y-connected circuit.

Equation 1

$$I_L = I_\theta$$

In Figure 9, the line terminations of the windings are designated *a*, *b*, and *c*, and the neutral point is designated *n*. These letters are added as subscripts to the phase and line currents to indicate the phase with which each is associated. The phase voltages are also designated in the same manner. Notice that the phase voltages are always positive at the terminal end of the winding and are negative at the neutral point. The line voltages are from one winding terminal to another, as indicated by the double-letter subscripts. For example, $\mathbf{V}_{L(ba)}$ is the line voltage from *b* to *a*.

FIGURE 9

Phase voltages and line voltages in a Y-connected system.

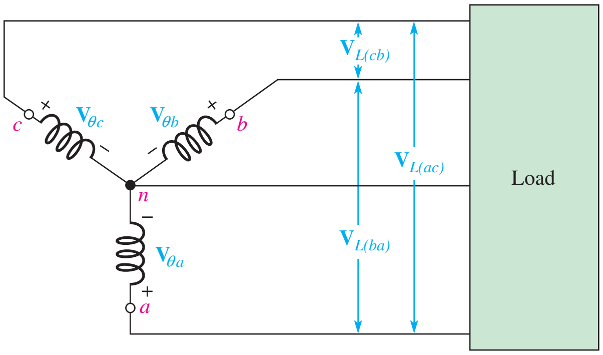

$\mathbf{V}_{L(cb)}$
c
$\mathbf{V}_{\theta c}$
$\mathbf{V}_{\theta b}$
b
$\mathbf{V}_{L(ac)}$
Load
n
$\mathbf{V}_{\theta a}$
$\mathbf{V}_{L(ba)}$
a

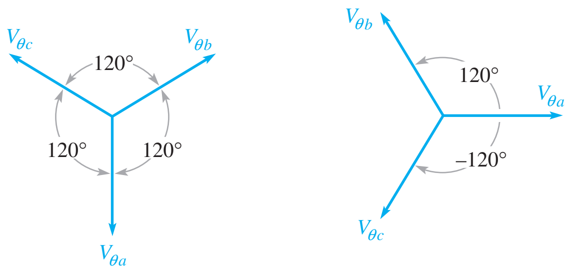
Figure 10(a) shows a phasor diagram for the phase voltages. By rotation of the phasors, as shown in part (b), $V_{\theta a}$ is given a reference angle of zero, and the polar expressions for the phasor voltages are as follows:

$$\mathbf{V}_{\theta a} = V_{\theta a}\angle 0^\circ$$
$$\mathbf{V}_{\theta b} = V_{\theta b}\angle 120^\circ$$
$$\mathbf{V}_{\theta c} = V_{\theta c}\angle -120^\circ$$

There are three line voltages: one between *a* and *b*, one between *a* and *c*, and another between *b* and *c*. It can be shown that the magnitude of each line voltage is equal to $\sqrt{3}$ times the magnitude of the phase voltage and that there is a phase angle of 30° between each line voltage and the nearest phase voltage.

Equation 2

$$V_L = \sqrt{3}V_\theta$$


$V_{\theta c}$
$V_{\theta b}$
120°
120°
120°
$V_{\theta a}$
$V_{\theta b}$
120°
$V_{\theta a}$
–120°
$V_{\theta c}$

(a) (b)

FIGURE 10

Phase voltage diagram.

Since all phase voltages are equal in magnitude,

$$\mathbf{V}_{L(ba)} = \sqrt{3}V_\theta \angle 150°$$
$$\mathbf{V}_{L(ac)} = \sqrt{3}V_\theta \angle 30°$$
$$\mathbf{V}_{L(cb)} = \sqrt{3}V_\theta \angle -90°$$

The line voltage phasor diagram is shown in Figure 11 superimposed on the phasor diagram for the phase voltages. Notice that there is a phase angle of 30° between each line voltage and the nearest phase voltage and that the line voltages are 120° apart.

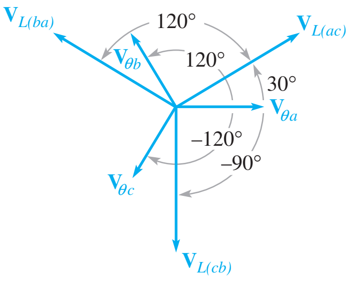

$\mathbf{V}_{L(ba)}$
120°
$\mathbf{V}_{L(ac)}$
$\mathbf{V}_{\theta b}$
120°
30°
$\mathbf{V}_{\theta a}$
–120°
–90°
$\mathbf{V}_{\theta c}$
$\mathbf{V}_{L(cb)}$

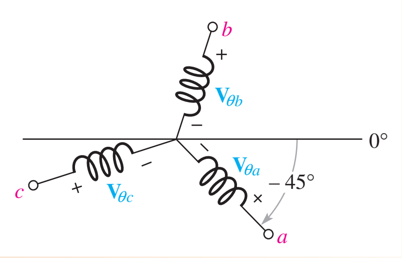
FIGURE 11

Phase diagram for the phase voltages and line voltages in a Y-connected, three-phase system.

EXAMPLE 2

The instantaneous position of a certain Y-connected ac generator is shown in Figure 12. If each phase voltage has a magnitude of 120 V rms, determine the magnitude of each line voltage, and draw the phasor diagram.

FIGURE 12

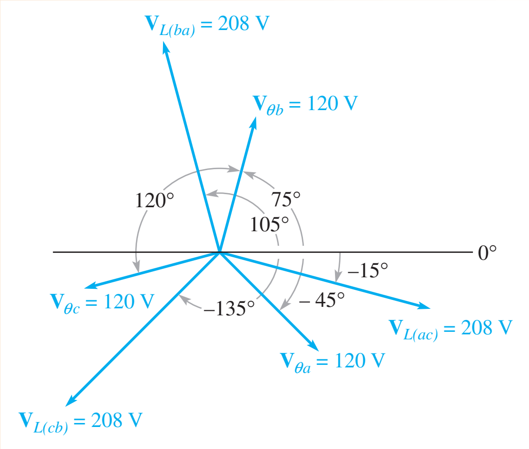

b
+
$\mathbf{V}_{\theta b}$
–
0°
–
$\mathbf{V}_{\theta a}$
– 45°
c
+
$\mathbf{V}_{\theta c}$
–
+
a

Solution The magnitude of each line voltage is

$$V_L = \sqrt{3}V_\theta = \sqrt{3}(120\text{ V}) = \mathbf{208\text{ V}}$$

The phasor diagram for the given instantaneous generator position is shown in Figure 13.

▶ FIGURE 13


$V_{L(ba)} = 208$ V
$V_{\theta b} = 120$ V
120°
75°
105°
0°
−15°
− 45°
−135°
$V_{\theta c} = 120$ V
$V_{L(ac)} = 208$ V
$V_{\theta a} = 120$ V
$V_{L(cb)} = 208$ V

Related Problem Determine the line voltage magnitude if the generator position indicated in Figure 13 is rotated another 45° clockwise.

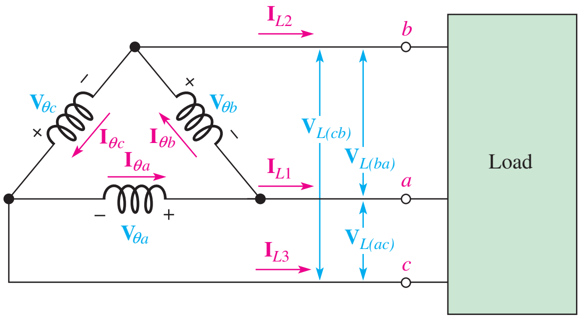
The Δ-Connected Generator

In the Y-connected generator, two voltage magnitudes are available at the terminals in the four-wire system: the phase voltage and the line voltage. Also, in the Y-connected generator, the line current is equal to the phase current. Keep these characteristics in mind as you examine the Δ-connected generator.

The windings of a three-phase generator can be rearranged to form a Δ-connected generator, as shown in Figure 14. By examination of this diagram, you can see that the magnitudes of the line voltages and phase voltages are equal, but the line currents do not equal the phase currents.

▶ FIGURE 14

Δ-connected generator.


I_{L2}
b
$V_{\theta c}$
$V_{\theta b}$
$I_{\theta c}$
$I_{\theta b}$
$I_{\theta a}$
$V_{L(cb)}$
$V_{L(ba)}$
I_{L1}
a
Load
−
+
$V_{\theta a}$
I_{L3}
$V_{L(ac)}$
c

Since this is a three-wire system, only a single voltage magnitude is available, expressed as

Equation 3

$$V_L = V_\theta$$

All of the phase voltages are equal in magnitude; thus, the line voltages are expressed in polar form as follows:

$$\mathbf{V}_{L(ac)} = V_\theta\angle 0^\circ$$
$$\mathbf{V}_{L(ba)} = V_\theta\angle 120^\circ$$
$$\mathbf{V}_{L(cb)} = V_\theta\angle -120^\circ$$

The phasor diagram for the phase currents is shown in Figure 15, and the polar expressions for each current are as follows:

$$\begin{aligned}\mathbf{I}_{\theta a} &= I_{\theta a}\angle 0^\circ\\ \mathbf{I}_{\theta b} &= I_{\theta b}\angle 120^\circ\\ \mathbf{I}_{\theta c} &= I_{\theta c}\angle -120^\circ\end{aligned}$$

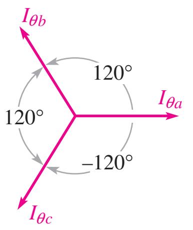

$I_{\theta b}$
120°
$I_{\theta a}$
120°
−120°
$I_{\theta c}$

FIGURE 15

Phase current diagram for the Δ-connected system.

It can be shown that the magnitude of each line current is equal to $\sqrt{3}$ times the magnitude of the phase current and that there is a phase angle of 30° between each line current and the nearest phase current.

$$I_L = \sqrt{3}I_\theta$$

Equation 4

Since all phase currents are equal in magnitude,

$$\begin{aligned}\mathbf{I}_{L1} &= \sqrt{3}I_\theta\angle -30^\circ\\ \mathbf{I}_{L2} &= \sqrt{3}I_\theta\angle 90^\circ\\ \mathbf{I}_{L3} &= \sqrt{3}I_\theta\angle -150^\circ\end{aligned}$$

The current phasor diagram is shown in Figure 16.

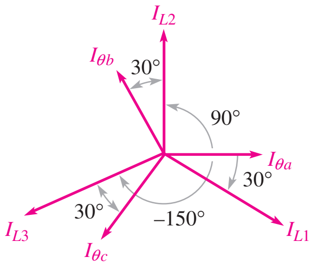

I_{L2}
$I_{\theta b}$
30°
90°
$I_{\theta a}$
30°
30°
−150°
I_{L3}
$I_{\theta c}$
I_{L1}

FIGURE 16

Phasor diagram of phase currents and line currents.

EXAMPLE 3

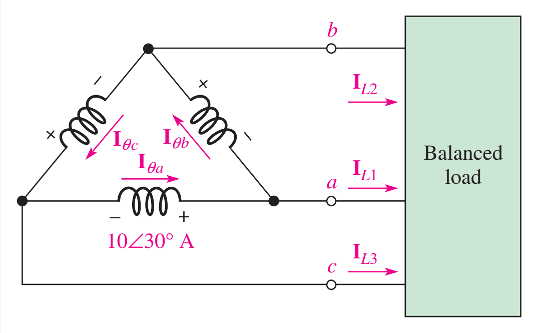
The three-phase Δ-connected generator represented in Figure 17 is driving a balanced load such that each phase current is 10 A in magnitude. When $\mathbf{I}_{\theta a} = 10\angle 30^\circ$ A, determine the following:

(a) The polar expressions for the other phase currents

(b) The polar expressions for each of the line currents

(c) The complete current phasor diagram

FIGURE 17


b
$\mathbf{I}_{L2}$
$\mathbf{I}_{\theta c}$
$\mathbf{I}_{\theta b}$
$\mathbf{I}_{\theta a}$
Balanced load
a
$\mathbf{I}_{L1}$
−
+
10∠30° A
c
$\mathbf{I}_{L3}$

Solution **(a)** The phase currents are separated by 120°; therefore,

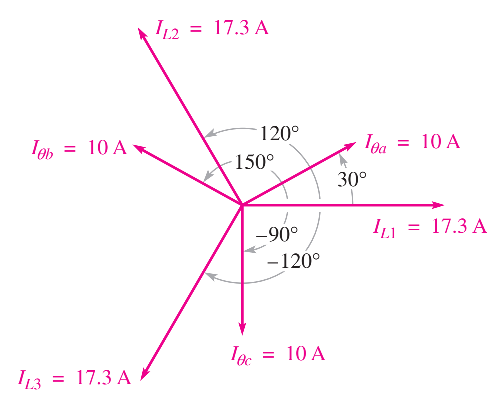
$$\begin{aligned}\mathbf{I}_{\theta b} &= 10\angle(30^\circ + 120^\circ) = \mathbf{10\angle 150^\circ\ A}\\ \mathbf{I}_{\theta c} &= 10\angle(30^\circ - 120^\circ) = \mathbf{10\angle -90^\circ\ A}\end{aligned}$$

(b) The line currents are separated from the nearest phase current by 30°; therefore,

$$\mathbf{I}_{L1} = \sqrt{3}I_{\theta a}\angle(30° - 30°) = \mathbf{17.3\angle 0°\ A}$$
$$\mathbf{I}_{L2} = \sqrt{3}I_{\theta b}\angle(150° - 30°) = \mathbf{17.3\angle 120°\ A}$$
$$\mathbf{I}_{L3} = \sqrt{3}I_{\theta c}\angle(-90° - 30°) = \mathbf{17.3\angle -120°\ A}$$

(c) The phasor diagram is shown in Figure 18.

FIGURE 18



Related Problem Repeat parts (a) and (b) of the example if $\mathbf{I}_{\theta a} = 8\angle 60°$ A.

SECTION 2 CHECKUP

1. In a certain three-wire, Y-connected generator, the phase voltages are 1 kV. Determine the magnitude of the line voltages.
2. In the Y-connected generator mentioned in Question 1, all the phase currents are 5 A. What are the line current magnitudes?
3. In a Δ-connected generator, the phase voltages are 240 V. What are the line voltages?
4. In a Δ-connected generator, a phase current is 2 A. Determine the magnitude of the line current.

3 THREE-PHASE SOURCE/LOAD ANALYSIS

In this section, we look at four basic types of source/load configurations. As with the generator connections, a load can be either a Y or a Δ configuration.

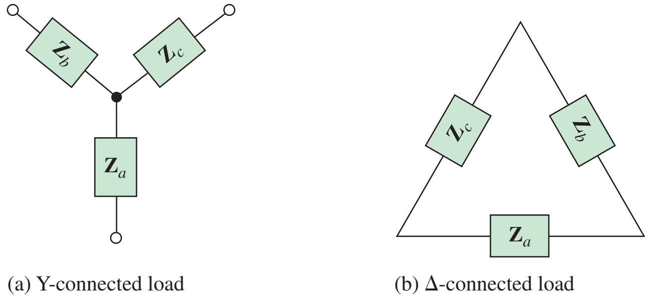
After completing this section, you should be able to

- **Analyze three-phase generators with three-phase loads**
  - Analyze the Y-Y source/load configuration
  - Analyze the Y-Δ source/load configuration
  - Analyze the Δ-Y source/load configuration
  - Analyze the Δ-Δ source/load configuration

A Y-connected load is shown in Figure 19(a), and a Δ-connected load is shown in part (b). The blocks $\mathbf{Z}_a$, $\mathbf{Z}_b$, and $\mathbf{Z}_c$ represent the load impedances, which can be resistive, reactive, or both.

The four source/load configurations are:

1. Y-connected source driving a Y-connected load (Y-Y system)
2. Y-connected source driving a Δ-connected load (Y-Δ system)
3. Δ-connected source driving a Y-connected load (Δ-Y system)
4. Δ-connected source driving a Δ-connected load (Δ-Δ system)



$\mathbf{Z}_b$
$\mathbf{Z}_c$
$\mathbf{Z}_a$
$\mathbf{Z}_c$
$\mathbf{Z}_b$
$\mathbf{Z}_a$

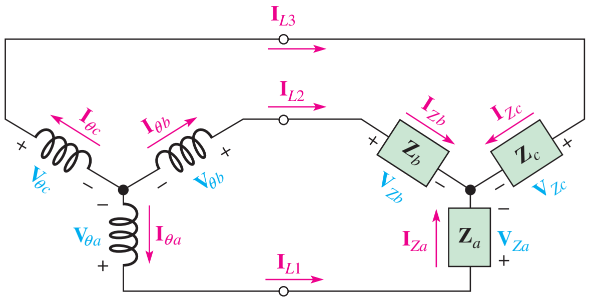
(a) Y-connected load (b) Δ-connected load

FIGURE 19

Three-phase loads.

The Y-Y System

Figure 20 shows a Y-connected source driving a Y-connected load. The load can be a balanced load, such as a three-phase motor where $\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c$, or it can be three independent single-phase loads where, for example, $\mathbf{Z}_a$ is a lighting circuit, $\mathbf{Z}_b$ is a heater, and $\mathbf{Z}_c$ is an air-conditioning compressor.



$\mathbf{I}_{L3}$
$\mathbf{I}_{L2}$
$\mathbf{I}_{\theta c}$
$\mathbf{I}_{\theta b}$
$\mathbf{I}_{Zb}$
$\mathbf{I}_{Zc}$
$\mathbf{V}_{\theta c}$
$\mathbf{V}_{\theta b}$
$\mathbf{Z}_b$
$\mathbf{Z}_c$
$\mathbf{V}_{Zb}$
$\mathbf{V}_{Zc}$
$\mathbf{V}_{\theta a}$
$\mathbf{I}_{\theta a}$
$\mathbf{I}_{Za}$
$\mathbf{Z}_a$
$\mathbf{V}_{Za}$
$\mathbf{I}_{L1}$
+
−

FIGURE 20

A Y-connected source driving a Y-connected load.

An important feature of a Y-connected source is that two different values of three-phase voltage are available: the phase voltage and the line voltage. For example, in the standard power distribution system, a three-phase transformer can be considered a source of three-phase voltage supplying 120 V and 208 V. In order to utilize a phase voltage of 120 V, the loads are connected in the Y configuration. A Δ-connected load is used for the 208 V line voltages.

Notice in the Y-Y system in Figure 20 that the phase current, the line current, and the load current are all equal in each phase. Also, each load voltage equals the corresponding phase voltage. These relationships are expressed as follows and are true for either a balanced or an unbalanced load.

$$I_\theta = I_L = I_Z$$ Equation 5

$$V_\theta = V_Z$$ Equation 6

where V_Z and I_Z are the load voltage and current, respectively.

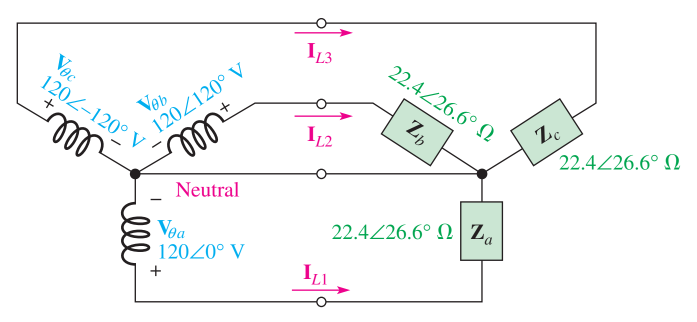
For a balanced load, all the phase currents are equal, and the neutral current is zero. For an unbalanced load, each phase current is different, and the neutral current is, therefore, nonzero.

EXAMPLE 4

In the Y-Y system of Figure 21, determine the following:

(a) Each load current **(b)** Each line current **(c)** Each phase current

(d) Neutral current **(e)** Each load voltage


$\mathbf{V}_{\theta c}$
$120\angle -120°$ V
$\mathbf{V}_{\theta b}$
$120\angle 120°$ V
+
−
$\mathbf{I}_{L3}$
$\mathbf{I}_{L2}$
$22.4\angle 26.6°\ \Omega$
$\mathbf{Z}_b$
$\mathbf{Z}_c$
$22.4\angle 26.6°\ \Omega$
Neutral
$\mathbf{V}_{\theta a}$
$120\angle 0°$ V
$22.4\angle 26.6°\ \Omega$
$\mathbf{Z}_a$
$\mathbf{I}_{L1}$

▲ **FIGURE 21**

Solution This system has a balanced load. $\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = 22.4\angle 26.6°\ \Omega$.

(a) The load currents are

$$\mathbf{I}_{Za} = \frac{\mathbf{V}_{\theta a}}{\mathbf{Z}_a} = \frac{120\angle 0°\text{ V}}{22.4\angle 26.6°\ \Omega} = \mathbf{5.36\angle -26.6°\ A}$$

$$\mathbf{I}_{Zb} = \frac{\mathbf{V}_{\theta b}}{\mathbf{Z}_b} = \frac{120\angle 120°\text{ V}}{22.4\angle 26.6°\ \Omega} = \mathbf{5.36\angle 93.4°\ A}$$

$$\mathbf{I}_{Zc} = \frac{\mathbf{V}_{\theta c}}{\mathbf{Z}_c} = \frac{120\angle -120°\text{ V}}{22.4\angle 26.6°\ \Omega} = \mathbf{5.36\angle -147°\ A}$$

(b) The line currents are

$$\mathbf{I}_{L1} = \mathbf{5.36\angle -26.6°\ A}$$
$$\mathbf{I}_{L2} = \mathbf{5.36\angle 93.4°\ A}$$
$$\mathbf{I}_{L3} = \mathbf{5.36\angle -147°\ A}$$

(c) The phase currents are

$$\mathbf{I}_{\theta a} = \mathbf{5.36\angle -26.6°\ A}$$
$$\mathbf{I}_{\theta b} = \mathbf{5.36\angle 93.4°\ A}$$
$$\mathbf{I}_{\theta c} = \mathbf{5.36\angle -147°\ A}$$

(d)
$$\begin{aligned}\mathbf{I}_{\text{neut}} &= \mathbf{I}_{Za} + \mathbf{I}_{Zb} + \mathbf{I}_{Zc}\\ &= 5.36\angle -26.6°\text{ A} + 5.36\angle 93.4°\text{ A} + 5.36\angle -147°\text{ A}\\ &= (4.80\text{ A} - j2.40\text{ A}) + (-0.33\text{ A} + j5.35\text{ A}) + (-4.47\text{ A} - j2.95\text{ A}) = \mathbf{0\ A}\end{aligned}$$

If the load impedances were not equal (unbalanced load), the neutral current would have a nonzero value.

(e) The load voltages are equal to the corresponding source phase voltages.

$$\mathbf{V}_{Za} = \mathbf{120\angle 0° \; V}$$
$$\mathbf{V}_{Zb} = \mathbf{120\angle 120° \; V}$$
$$\mathbf{V}_{Zc} = \mathbf{120\angle -120° \; V}$$

Related Problem Determine the neutral current if $\mathbf{Z}_a$ and $\mathbf{Z}_b$ are the same as in Figure 21, but $\mathbf{Z}_c = 50\angle 26.6° \; \Omega$.

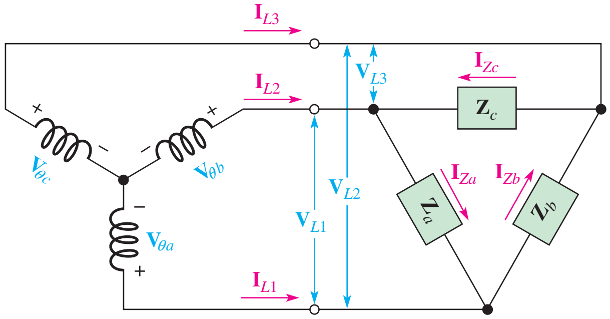
The Y-Δ System

Figure 22 shows a Y-connected source driving a Δ-connected load. An important feature of this configuration is that each phase of the load has the full line voltage across it.

$$V_Z = V_L \qquad \textbf{Equation 7}$$

The line currents equal the corresponding phase currents, and each line current divides into two load currents, as indicated. For a balanced load ($\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c$), the expression for the current in each load is

$$I_L = \sqrt{3}I_Z \qquad \textbf{Equation 8}$$

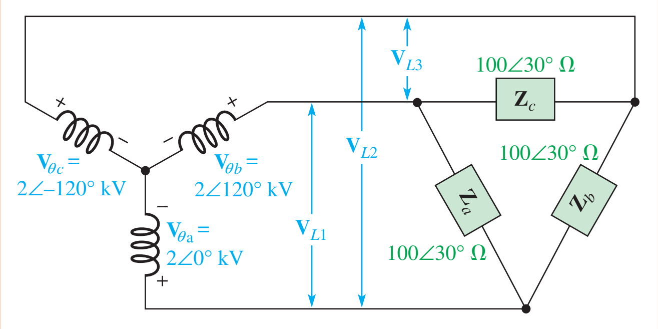

$\mathbf{I}_{L3}$
$\mathbf{I}_{L2}$
$\mathbf{I}_{L1}$
$\mathbf{V}_{L3}$
$\mathbf{V}_{L2}$
$\mathbf{V}_{L1}$
$\mathbf{I}_{Zc}$
$\mathbf{Z}_c$
$\mathbf{I}_{Za}$
$\mathbf{I}_{Zb}$
$\mathbf{Z}_a$
$\mathbf{Z}_b$
$\mathbf{V}_{\theta c}$
$\mathbf{V}_{\theta b}$
$\mathbf{V}_{\theta a}$
+
−
−
+
−
+

◀ **FIGURE 22**

A Y-connected source driving a Δ-connected load.

EXAMPLE 5

Determine the load voltages and load currents in Figure 23, and show their relationship in a phasor diagram.


$\mathbf{V}_{L3}$
$\mathbf{V}_{L2}$
$\mathbf{V}_{L1}$
100∠30° Ω
$\mathbf{Z}_c$
100∠30° Ω
$\mathbf{Z}_a$
$\mathbf{Z}_b$
100∠30° Ω
$\mathbf{V}_{\theta c}$ =
2∠−120° kV
$\mathbf{V}_{\theta b}$ =
2∠120° kV
$\mathbf{V}_{\theta a}$ =
2∠0° kV
+
−
−
+
−
+

▲ **FIGURE 23**

Solution Using $V_L = \sqrt{3}V_\theta$ (Equation 2) and the fact that there is 30° between each line voltage and the nearest phase voltage, the load voltages are

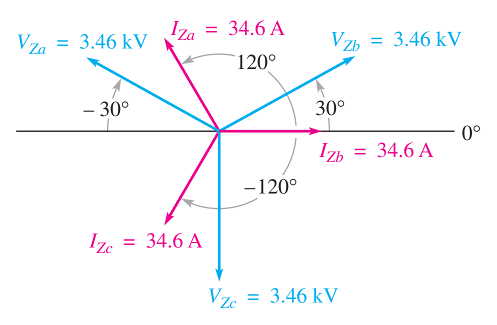
$$\mathbf{V}_{Za} = \mathbf{V}_{L1} = 2\sqrt{3}\angle 150° \text{ kV} = \mathbf{3.46\angle 150° \text{ kV}}$$
$$\mathbf{V}_{Zb} = \mathbf{V}_{L2} = 2\sqrt{3}\angle 30° \text{ kV} = \mathbf{3.46\angle 30° \text{ kV}}$$
$$\mathbf{V}_{Zc} = \mathbf{V}_{L3} = 2\sqrt{3}\angle -90° \text{ kV} = \mathbf{3.46\angle -90° \text{ kV}}$$

The load currents are

$$\mathbf{I}_{Za} = \frac{\mathbf{V}_{Za}}{\mathbf{Z}_a} = \frac{3.46\angle 150° \text{ kV}}{100\angle 30° \ \Omega} = \mathbf{34.6\angle 120° \text{ A}}$$
$$\mathbf{I}_{Zb} = \frac{\mathbf{V}_{Zb}}{\mathbf{Z}_b} = \frac{3.46\angle 30° \text{ kV}}{100\angle 30° \ \Omega} = \mathbf{34.6\angle 0° \text{ A}}$$
$$\mathbf{I}_{Zc} = \frac{\mathbf{V}_{Zc}}{\mathbf{Z}_c} = \frac{3.46\angle -90° \text{ kV}}{100\angle 30° \ \Omega} = \mathbf{34.6\angle -120° \text{ A}}$$

The phasor diagram is shown in Figure 24.

▶ FIGURE 24


V_{Za} = 3.46 kV
I_{Za} = 34.6 A
V_{Zb} = 3.46 kV
120°
– 30°
30°
0°
I_{Zb} = 34.6 A
–120°
I_{Zc} = 34.6 A
V_{Zc} = 3.46 kV

Related Problem Determine the load currents in Figure 23 if the phase voltages have a magnitude of 240 V.

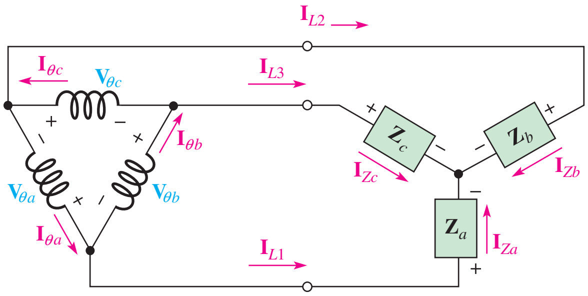
The Δ-Y System

Figure 25 shows a Δ-connected source driving a Y-connected balanced load. By examination of the figure, you can see that the line voltages are equal to the corresponding phase voltages of the source. Also, each phase voltage equals the difference of the corresponding load voltages, as you can see by the polarities.

Each load current equals the corresponding line current. The sum of the load currents is zero because the load is balanced; thus, there is no need for a neutral return.

▶ FIGURE 25

A Δ-connected source driving a Y-connected load.


$\mathbf{I}_{L2}$
$\mathbf{I}_{\theta c}$
$\mathbf{V}_{\theta c}$
$\mathbf{I}_{L3}$
+
–
+
–
$\mathbf{I}_{\theta b}$
$\mathbf{V}_{\theta a}$
$\mathbf{V}_{\theta b}$
$\mathbf{I}_{\theta a}$
$\mathbf{I}_{L1}$
$\mathbf{Z}_c$
$\mathbf{Z}_b$
$\mathbf{Z}_a$
$\mathbf{I}_{Zc}$
$\mathbf{I}_{Zb}$
$\mathbf{I}_{Za}$
+
–
+
–
–
+

The relationship between the load voltages and the corresponding phase voltages (and line voltages) is

$$V_\theta = \sqrt{3}V_Z$$

Equation 9

The line currents and corresponding load currents are equal, and for a balanced load, the sum of the load currents is zero.

$$\mathbf{I}_L = \mathbf{I}_Z$$

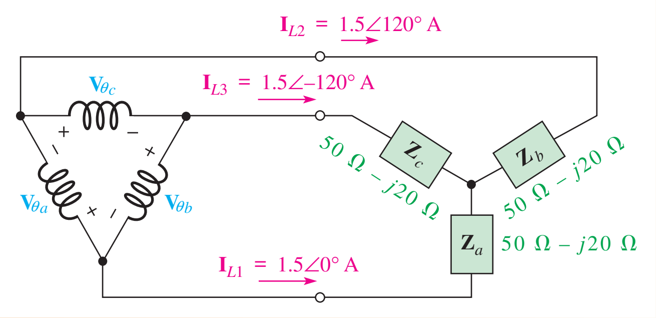
Equation 10

As you can see in Figure 25, each line current is the difference of two phase currents.

$$\begin{aligned}\mathbf{I}_{L1} &= \mathbf{I}_{\theta a} - \mathbf{I}_{\theta b}\\ \mathbf{I}_{L2} &= \mathbf{I}_{\theta c} - \mathbf{I}_{\theta a}\\ \mathbf{I}_{L3} &= \mathbf{I}_{\theta b} - \mathbf{I}_{\theta c}\end{aligned}$$

EXAMPLE 6

Determine the currents and voltages in the balanced load and the magnitude of the line voltages in Figure 26.



▲ FIGURE 26

Solution The load currents equal the specified line currents.

$$\begin{aligned}\mathbf{I}_{Za} &= \mathbf{I}_{L1} = \mathbf{1.5\angle 0^\circ\ A}\\ \mathbf{I}_{Zb} &= \mathbf{I}_{L2} = \mathbf{1.5\angle 120^\circ\ A}\\ \mathbf{I}_{Zc} &= \mathbf{I}_{L3} = \mathbf{1.5\angle -120^\circ\ A}\end{aligned}$$

The load voltages are

$$\begin{aligned}\mathbf{V}_{Za} &= \mathbf{I}_{Za}\mathbf{Z}_a\\ &= (1.5\angle 0^\circ\ \text{A})(50\ \Omega - j20\ \Omega)\\ &= (1.5\angle 0^\circ\ \text{A})(53.9\angle -21.8^\circ\ \Omega) = \mathbf{80.9\angle -21.8^\circ\ V}\\ \mathbf{V}_{Zb} &= \mathbf{I}_{Zb}\mathbf{Z}_b\\ &= (1.5\angle 120^\circ\ \text{A})(53.9\angle -21.8^\circ\ \Omega) = \mathbf{80.9\angle 98.2^\circ\ V}\\ \mathbf{V}_{Zc} &= \mathbf{I}_{Zc}\mathbf{Z}_c\\ &= (1.5\angle -120^\circ\ \text{A})(53.9\angle -21.8^\circ\ \Omega) = \mathbf{80.9\angle -142^\circ\ V}\end{aligned}$$

The magnitude of the line voltages is

$$V_L = V_\theta = \sqrt{3}V_Z = \sqrt{3}(80.9\ \text{V}) = \mathbf{140\ V}$$

Related Problem If the magnitudes of the line currents are 1 A, what are the load currents?

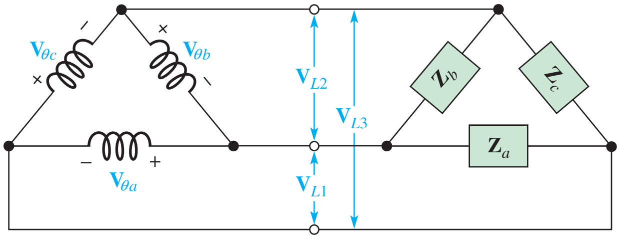
The Δ-Δ System

Figure 27 shows a Δ-connected source driving a Δ-connected load. Notice that the load voltage, line voltage, and source phase voltage are all equal for a given phase.

$$V_{\theta a} = V_{L1} = V_{Za}$$
$$V_{\theta b} = V_{L2} = V_{Zb}$$
$$V_{\theta c} = V_{L3} = V_{Zc}$$

FIGURE 27

A Δ-connected source driving a Δ-connected load.



Of course, when the load is balanced, all the voltages are equal, and a general expression can be written

Equation 11

$$V_{\theta} = V_L = V_Z$$

For a balanced load and equal source phase voltages, it can be shown that

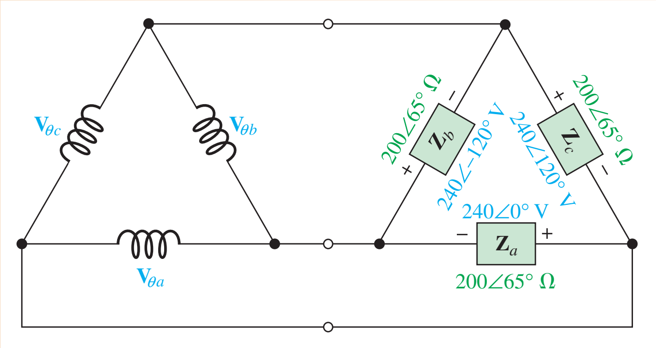
Equation 12

$$I_L = \sqrt{3}I_Z$$

EXAMPLE 7

Determine the magnitude of the load currents and the line currents in Figure 28.

FIGURE 28



Solution

$$V_{Za} = V_{Zb} = V_{Zc} = 240\text{ V}$$

The magnitude of the load currents is

$$I_{Za} = I_{Zb} = I_{Zc} = \frac{V_{Za}}{Z_a} = \frac{240\text{ V}}{200\ \Omega} = \mathbf{1.20\ A}$$

The magnitude of the line currents is

$$I_L = \sqrt{3}I_Z = \sqrt{3}(1.20\text{ A}) = \mathbf{2.08\ A}$$

Related Problem Determine the magnitude of the load and line currents in Figure 28 if the magnitude of the load voltages is 120 V and the impedances are 600 Ω.

SECTION 3 CHECKUP

1. List the four types of three-phase source/load configurations.
2. In a certain Y-Y system, the source phase currents each have a magnitude of 3.5 A. What is the magnitude of each load current for a balanced load condition?
3. In a given Y-Δ system, $V_L = 220$ V. Determine V_Z.
4. Determine the line voltages in a balanced Δ-Y system when the magnitude of the source phase voltages is 60 V.
5. Determine the magnitude of the load currents in a balanced Δ-Δ system having a line current magnitude of 3.2 A.

4 THREE-PHASE POWER

In this section, power in three-phase systems is studied and methods of power measurement are introduced.

After completing this section, you should be able to

- **Discuss power measurements in three-phase systems**
  - Describe the three-wattmeter method
  - Describe the two-wattmeter method

Each phase of a balanced three-phase load has an equal amount of power. Therefore, the total true load power is three times the power in each phase of the load.

$$P_{L(tot)} = 3V_Z I_Z \cos\theta$$

Equation 13

where V_Z and I_Z are the voltage and current associated with each phase of the load, and $\cos\theta$ is the power factor.

Recall that in a balanced Y-connected system, the line voltage and line current were

$$V_L = \sqrt{3}V_Z \qquad \text{and} \qquad I_L = I_Z$$

and in a balanced Δ-connected system, the line voltage and line current were

$$V_L = V_Z \qquad \text{and} \qquad I_L = \sqrt{3}I_Z$$

When either of these relationships is substituted into Equation 13, the total true power for both Y- and Δ-connected systems is

$$P_{L(tot)} = \sqrt{3}V_L I_L \cos\theta$$

Equation 14

EXAMPLE 8

In a certain Δ-connected balanced load, the line voltages are 250 V and the impedances are $50\angle 30^\circ\ \Omega$. Determine the total load power.

Solution In a Δ-connected system, $V_Z = V_L$ and $I_L = \sqrt{3}I_Z$. The load current magnitudes are

$$I_Z = \frac{V_Z}{Z} = \frac{250\text{ V}}{50\ \Omega} = 5\text{ A}$$

and

$$I_L = \sqrt{3}I_Z = \sqrt{3}(5\text{ A}) = 8.66\text{ A}$$

The power factor is

$$\cos\theta = \cos 30° = 0.866$$

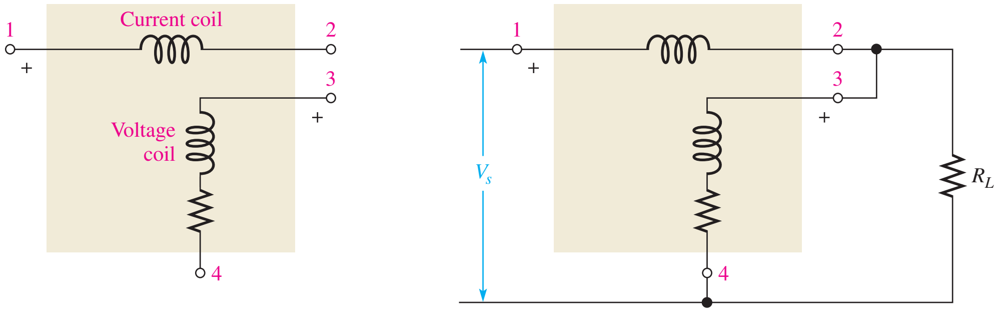
The total load power is

$$P_{L(tot)} = \sqrt{3}V_L I_L \cos\theta = \sqrt{3}(250\text{ V})(8.66\text{ A})(0.866) = \mathbf{3.25\text{ kW}}$$

Related Problem Determine the total load power if $V_L = 120$ V and $\mathbf{Z} = 100\angle 30°\ \Omega$.

Power Measurement

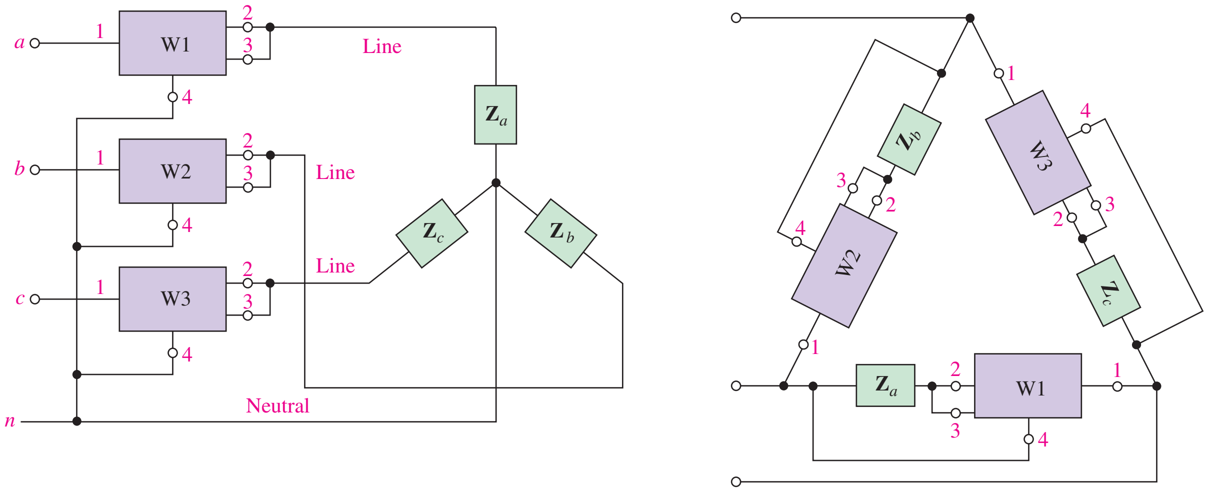
Power is measured in three-phase systems using wattmeters. The wattmeter uses a basic electrodynamometer-type movement consisting of two coils. One coil is used to measure the current, and the other is used to measure the voltage. Figure 29 shows a basic wattmeter schematic and the connections for measuring power in a load. The resistor in series with the voltage coil limits the current through the coil to a small amount proportional to the voltage across the coil.


Current coil
1
2
3
+
+
Voltage coil
4
1
2
3
+
+
V_s
R_L
4

(a) Wattmeter schematic (b) Wattmeter connected to measure load power

▲ FIGURE 29

Three-Wattmeter Method Power can be measured easily in a balanced or unbalanced three-phase load of either the Y or the Δ type by using three wattmeters connected as shown in Figure 30. This is sometimes known as the *three-wattmeter method.*


a
b
c
n
W1
W2
W3
1
2
3
4
Line
Line
Line
Neutral
$\mathbf{Z}_a$
$\mathbf{Z}_b$
$\mathbf{Z}_c$
W1
W2
W3
$\mathbf{Z}_a$
$\mathbf{Z}_b$
$\mathbf{Z}_c$

(a) Y-connected load (b) Δ-connected load

▲ FIGURE 30

Three-wattmeter method of power measurement.

The total power is determined by summing the three wattmeter readings.

$$P_{tot} = P_1 + P_2 + P_3$$

Equation 15

If the load is balanced, the total power is simply three times the reading on any one wattmeter.

In many three-phase loads, particularly the Δ configuration, it is difficult to connect a wattmeter such that the voltage coil is across the load or such that the current coil is in series with the load because of inaccessibility of points within the load.

Two-Wattmeter Method Another method of three-phase power measurement uses only two wattmeters. The connections for this two-wattmeter method are shown in Figure 31. Notice that the voltage coil of each wattmeter is connected across a line voltage and that the current coil has a line current through it. It can be shown that the algebraic sum of the two wattmeter readings equals the total power in the Y- or Δ-connected load.

$$P_{tot} = P_1 \pm P_2$$

Equation 16

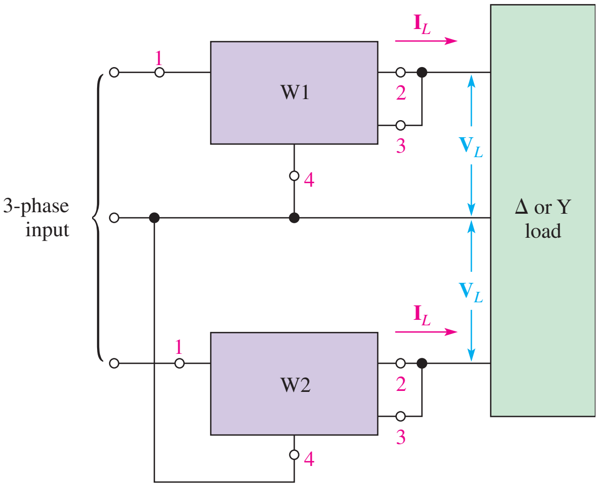


FIGURE 31

Two-wattmeter method.

SECTION 4 CHECKUP

1. $V_L = 30$ V, $I_L = 1.2$ A, and the power factor is 0.257. What is the total power in a balanced Y-connected load? In a balanced Δ-connected load?
2. Three wattmeters connected to measure the power in a certain balanced load indicate a total of 2678 W. How much power does each meter measure?

SUMMARY

- A simple three-phase generator consists of three conductive loops separated by 120°.
- Three advantages of three-phase systems over single-phase systems are a smaller copper cross section for the same power delivered to the load, constant power delivered to the load, and a constant, rotating magnetic field.
- In a Y-connected generator, $I_L = I_\theta$ and $V_L = \sqrt{3}V_\theta$.
- In a Y-connected generator, there is a 30° difference between each line voltage and the nearest phase voltage.
- In a Δ-connected generator, $V_L = V_\theta$ and $I_L = \sqrt{3}I_\theta$.
- In a Δ-connected generator, there is a 30° difference between each line current and the nearest phase current.
- A balanced load is one in which all the impedances are equal.
- Power is measured in a three-phase load using either the three-wattmeter method or the two-wattmeter method.

KEY TERMS

Balanced load	A condition where all the load currents are equal and the neutral current is zero.
Line current (I_L)	The current through a line feeding a load.
Line voltage (V_L)	The voltage between lines feeding a load.
Phase current (I_θ)	The current through a generator winding.
Phase voltage (V_θ)	The voltage across a generator winding.

FORMULAS

Y Generator

1 $I_L = I_\theta$

2 $V_L = \sqrt{3}V_\theta$

Δ Generator

3 $V_L = V_\theta$

4 $I_L = \sqrt{3}I_\theta$

Y-Y System

5 $I_\theta = I_L = I_Z$

6 $V_\theta = V_Z$

Y-Δ System

7 $V_Z = V_L$

8 $I_L = \sqrt{3}I_Z$

Δ-to-Y System

9 $V_\theta = \sqrt{3}V_Z$

10 $I_L = I_Z$

Δ-Δ System

11 $V_\theta = V_L = V_Z$

12 $I_L = \sqrt{3}I_Z$

Three-Phase Power

13 $P_{L(tot)} = 3V_Z I_Z \cos\theta$

14 $P_{L(tot)} = \sqrt{3}V_L I_L \cos\theta$

Three-Wattmeter Method

15 $P_{tot} = P_1 + P_2 + P_3$

Two-Wattmeter Method

16 $P_{tot} = P_1 \pm P_2$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The copper wire in a three-phase system can be smaller than in a single-phase system.
2. A balanced load condition occurs when all load currents and the neutral current are equal.
3. A three-phase system produces a constant amount of power to the load.

4. A Y-connected system can have either three or four wires.
5. Voltages across the generator windings are called line voltages.
6. In a Δ-connected generator the line currents and the phase currents are equal.
7. Source-load connections can be Y-Y, Δ-Y, Y-Δ, or Δ-Δ.
8. The three-wattmeter method and the two-wattmeter method are two ways of measuring three-phase power.
9. A wattmeter uses a basic electrodynameter-type movement consisting of three coils.
10. If a three-phase load is balanced, the total power is indicated by any one of the three meters.

SELF-TEST

Answers are at the end of the chapter.

1. In a three-phase system, the voltages are separated by

   (a) 90° **(b)** 30° **(c)** 180° **(d)** 120°
2. Two major parts of an ac generator are

   (a) rotor and stator **(b)** rotor and stabilizer

   (c) regulator and slip-ring **(d)** magnets and brushes
3. Advantages of a three-phase system over a single-phase system are

   (a) smaller cross-sectional area for the copper conductors

   (b) slower rotor speed

   (c) constant power

   (d) smaller chance of overheating

   (e) both (a) and (c)

   (f) both (b) and (c)
4. The phase current produced by a certain 240 V, Y-connected generator is 12 A. The corresponding line current is

   (a) 36 A **(b)** 4 A **(c)** 12 A **(d)** 6 A
5. A certain Δ-connected generator produces phase voltages of 30 V. The magnitude of the line voltages are

   (a) 10 V **(b)** 30 V **(c)** 90 V **(d)** none of these
6. A certain Δ-Δ system produces phase currents of 5 A. The line currents are

   (a) 5 A **(b)** 15 A **(c)** 8.66 A **(d)** 2.87 A
7. A certain Y-Y system produces phase currents of 15 A. Each line and load current is

   (a) 26 A **(b)** 8.66 A **(c)** 5 A **(d)** 15 A
8. If the source phase voltages of a Δ-Y system are 220 V, the magnitude of the load voltages is

   (a) 220 V **(b)** 381 V **(c)** 127 V **(d)** 73.3 V

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the chapter.

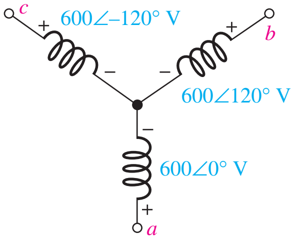
SECTION 1 Generators in Power Applications

1. A single-phase generator drives a load consisting of a 200 Ω resistor and a capacitor with a reactance of 175 Ω. The generator produces a voltage of 100 V. Determine the magnitude of the load current.
2. Determine the phase of the load current with respect to the generator voltage in Problem 1.
3. A certain three-phase unbalanced load in a four-wire system has currents of $2\angle 20^\circ$ A, $3\angle 140^\circ$ A, and $1.5\angle -100^\circ$ A. Determine the current in the neutral line.

SECTION 2 Types of Three-Phase Generators

4. Determine the line voltages in Figure 32.

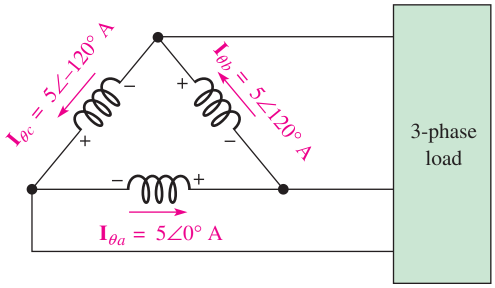
▶ **FIGURE 32**


c
600∠–120° V
b
600∠120° V
600∠0° V
a

5. Determine the line currents in Figure 33.

6. Develop a complete current phasor diagram for Figure 33.

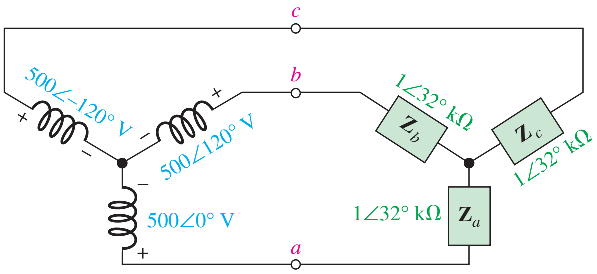
▶ **FIGURE 33**


$I_{\theta c}$ = 5∠–120° A
$I_{\theta b}$ = 5∠120° A
$I_{\theta a}$ = 5∠0° A
3-phase load

SECTION 3 Three-Phase Source/Load Analysis

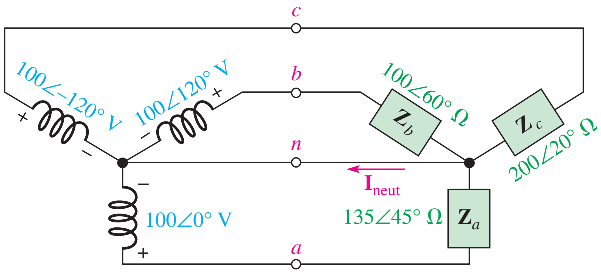
7. Determine the following quantities for the Y-Y system in Figure 34:

(a) Line voltages **(b)** Phase currents **(c)** Line currents
(d) Load currents **(e)** Load voltages


c
b
a
500∠–120° V
500∠120° V
500∠0° V
1∠32° kΩ
Z_b
Z_c
1∠32° kΩ
1∠32° kΩ
Z_a

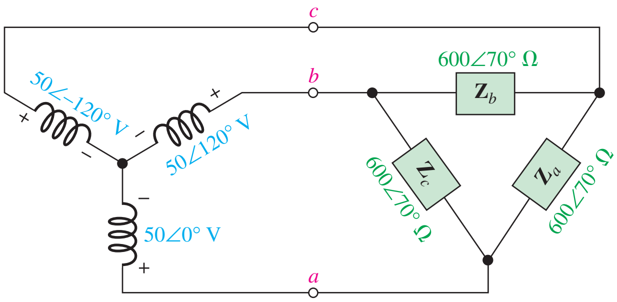
▲ **FIGURE 34**

8. Repeat Problem 7 for the system in Figure 35, and also find the neutral current.


c
b
n
a
100∠–120° V
100∠120° V
100∠0° V
100∠60° Ω
Z_b
Z_c
200∠20° Ω
I_{neut}
135∠45° Ω
Z_a

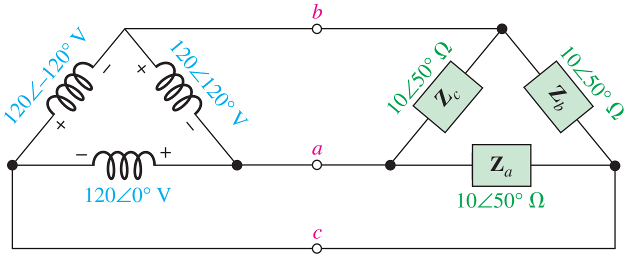
▲ **FIGURE 35**

9. Repeat Problem 7 for the system in Figure 36.


c
b
a
50∠−120° V
50∠120° V
50∠0° V
+
−
600∠70° Ω
Z_b
Z_c
Z_a
600∠70° Ω
600∠70° Ω

▲ FIGURE 36

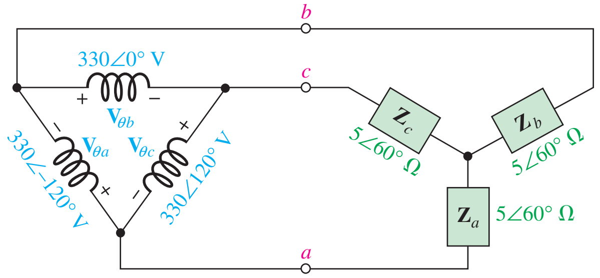
10. Repeat Problem 7 for the system in Figure 37.


b
a
c
120∠−120° V
120∠120° V
120∠0° V
+
−
10∠50° Ω
Z_c
Z_b
Z_a
10∠50° Ω
10∠50° Ω

▲ FIGURE 37

11. Determine the line voltages and load currents for the system in Figure 38.

▶ FIGURE 38


b
c
a
330∠0° V
V_θb
V_θa
V_θc
330∠−120° V
330∠120° V
+
−
Z_c
5∠60° Ω
Z_b
5∠60° Ω
Z_a
5∠60° Ω

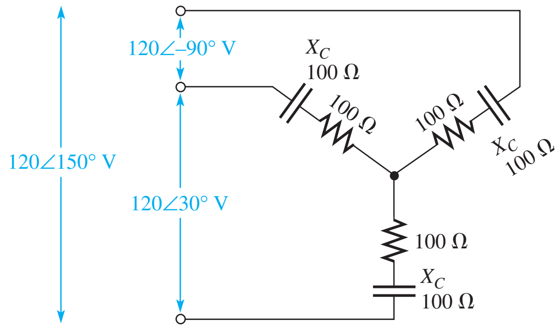
SECTION 4 Three-Phase Power

12. The power in each phase of a balanced three-phase system is 1200 W. What is the total power?

13. Determine the load power in Figures 34 through 38.

14. Find the total load power in Figure 39.

▶ FIGURE 39


120∠−90° V
120∠150° V
120∠30° V
X_C
100 Ω
100 Ω
100 Ω
X_C
100 Ω
100 Ω
X_C
100 Ω

***15.** Using the three-wattmeter method for the system in Figure 39, how much power does each wattmeter indicate?

***16.** Repeat Problem 15 using the two-wattmeter method.

ANSWERS

SECTION CHECKUPS

SECTION 1 Generators in Power Applications

1. The advantages of polyphase systems are less copper cross section to conduct current; constant power to load; and constant, rotating magnetic field.
2. Constant power
3. Constant magnetic field

SECTION 2 Types of Three-Phase Generators

1. $V_L = 1.73\text{ kV}$
2. $I_L = 5\text{ A}$
3. $V_L = 240\text{ V}$
4. $I_L = 3.46\text{ A}$

SECTION 3 Three-Phase Source/Load Analysis

1. The source/load configurations are Y-Y, Y-Δ, Δ-Y, and Δ-Δ.
2. $I_L = 3.5\text{ A}$
3. $V_Z = 220\text{ V}$
4. $V_L = 60\text{ V}$
5. $I_Z = 1.85\text{ A}$

SECTION 4 Three-Phase Power

1. $P_{\text{Y}} = 16.0\text{ W}$; $P_{\Delta} = 16.0\text{ W}$
2. $P = 893\text{ W}$

RELATED PROBLEMS FOR EXAMPLES

1 4.8 A total for single phase; 2.4 A total for three-phase

2 208 V

3 **(a)** $\mathbf{I}_{\theta b} = 8\angle 180^\circ\text{ A}, \mathbf{I}_{\theta c} = 8\angle -60^\circ\text{ A}$

  (b) $\mathbf{I}_{L1} = 13.9\angle 30^\circ\text{ A}, \mathbf{I}_{L2} = 13.9\angle 150^\circ\text{ A}, \mathbf{I}_{L3} = 13.9\angle -90^\circ\text{ A}$

4 $2.96\angle 33.4^\circ\text{ A}$

5 $\mathbf{I}_{Za} = 4.16\angle 120^\circ\text{ A}, \mathbf{I}_{Zb} = 4.16\angle 0^\circ, \mathbf{I}_{Zc} = 4.16\angle -120^\circ\text{ A}$

6 $\mathbf{I}_{L1} = \mathbf{I}_{Za} = 1\angle 0^\circ\text{ A}, \mathbf{I}_{L2} = \mathbf{I}_{Zb} = 1\angle 120^\circ\text{ A}, \mathbf{I}_{L3} = \mathbf{I}_{Zc} = 1\angle -120^\circ\text{ A}$

7 $I_Z = 200\text{ mA}, I_L = 346\text{ mA}$

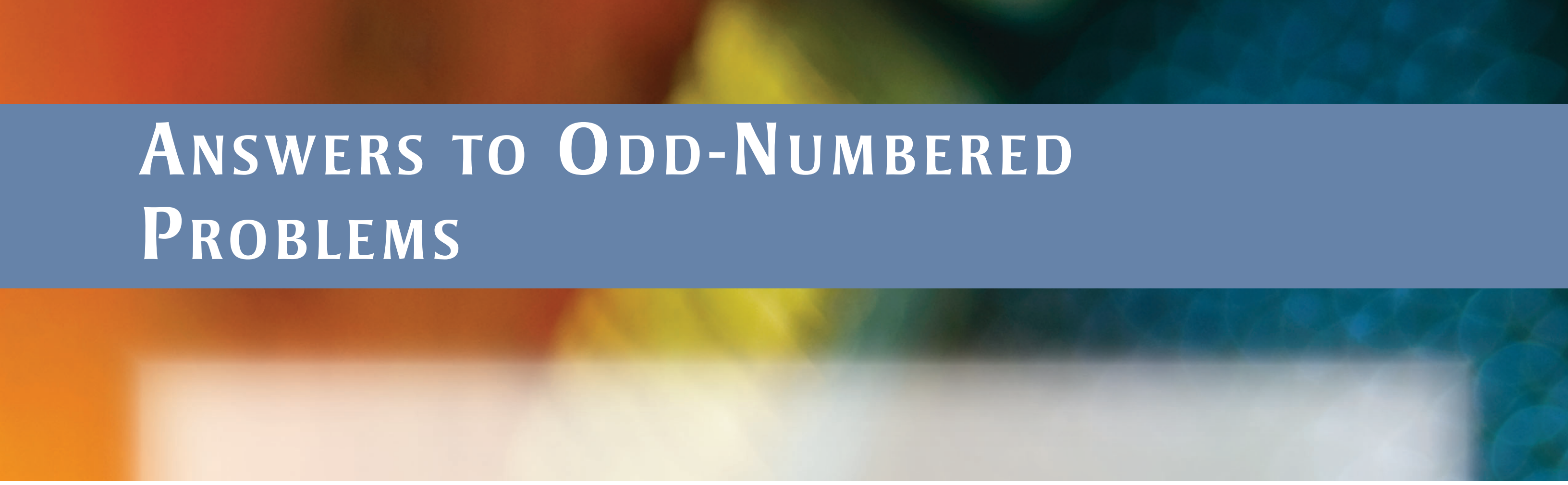
8 374 W

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F **6.** F **7.** T **8.** T **9.** F **10.** F

SELF-TEST

1. (d) **2.** (a) **3.** (e) **4.** (c) **5.** (b) **6.** (c) **7.** (d) **8.** (c)



ANSWERS TO ODD-NUMBERED PROBLEMS

1. 376 mA

3. $1.32\angle 121^\circ$ A

5. $\mathbf{I}_{La} = 8.66\angle -30^\circ$ A

$\mathbf{I}_{Lb} = 8.66\angle 90^\circ$ A

$\mathbf{I}_{Le} = 8.66\angle -150^\circ$ A

7. **(a)** $\mathbf{V}_{L(ab)} = 866\angle -30^\circ$ V

$\mathbf{V}_{L(ca)} = 866\angle -150^\circ$ V

$\mathbf{V}_{L(bc)} = 866\angle 90^\circ$ V

(b) $\mathbf{I}_{\theta a} = 500\angle -32^\circ$ mA

$\mathbf{I}_{\theta b} = 500\angle 88^\circ$ mA

$\mathbf{I}_{\theta c} = 500\angle -152^\circ$ mA

(c) $\mathbf{I}_{La} = 500\angle -32^\circ$ mA

$\mathbf{I}_{Lb} = 500\angle 88^\circ$ mA

$\mathbf{I}_{Lc} = 500\angle -152^\circ$ mA

(d) $\mathbf{I}_{Za} = 500\angle -32^\circ$ mA

$\mathbf{I}_{Zb} = 500\angle 88^\circ$ mA

$\mathbf{I}_{Zc} = 500\angle -152^\circ$ mA

(e) $\mathbf{V}_{Za} = 500\angle 0^\circ$ V

$\mathbf{V}_{Zb} = 500\angle 120^\circ$ V

$\mathbf{V}_{Zc} = 500\angle -120^\circ$ V

9. **(a)** $\mathbf{V}_{L(ab)} = 86.6\angle -30^\circ$ V

$\mathbf{V}_{L(ca)} = 86.6\angle -150^\circ$ V

$\mathbf{V}_{L(bc)} = 86.6\angle 90^\circ$ V

(b) $\mathbf{I}_{\theta a} = 250\angle 110^\circ$ mA

$\mathbf{I}_{\theta b} = 250\angle -130^\circ$ mA

$\mathbf{I}_{\theta c} = 250\angle -10^\circ$ mA

(c) $\mathbf{I}_{La} = 250\angle 110^\circ$ mA

$\mathbf{I}_{Lb} = 250\angle -130^\circ$ mA

$\mathbf{I}_{Lc} = 250\angle -10^\circ$ mA

(d) $\mathbf{I}_{Za} = 144\angle 140^\circ$ mA

$\mathbf{I}_{Zb} = 144\angle 20^\circ$ mA

$\mathbf{I}_{Zc} = 144\angle -100^\circ$ mA

(e) $\mathbf{V}_{Za} = 86.6\angle -150^\circ$ V

$\mathbf{V}_{Zb} = 86.6\angle 90^\circ$ V

$\mathbf{V}_{Zc} = 86.6\angle -30^\circ$ V

11. $\mathbf{V}_{L(ab)} = 330\angle -120^\circ$ V

$\mathbf{V}_{L(ca)} = 330\angle 120^\circ$ V

$\mathbf{V}_{L(bc)} = 330\angle 0^\circ$ V

$\mathbf{I}_{Za} = 38.2\angle -150^\circ$ A

$\mathbf{I}_{Zb} = 38.2\angle -30^\circ$ A

$\mathbf{I}_{Zc} = 38.2\angle 90^\circ$ A

13. Figure 34: 636 W

Figure 35: 149 W

Figure 36: 12.8 W

Figure 37: 2.78 kW

Figure 38: 10.9 kW

15. 24.2 W

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Appendix

TABLE OF STANDARD RESISTOR VALUES

Resistance Tolerance (±%)

0.1% 0.25% 0.5%	1%	2% 5%	10%	0.1% 0.25% 0.5%	1%	2% 5%	10%	0.1% 0.25% 0.5%	1%	2% 5%	10%	0.1% 0.25% 0.5%	1%	2% 5%	10%	0.1% 0.25% 0.5%	1%	2% 5%	10%	0.1% 0.25% 0.5%	1%	2% 5%	10%
10.0	10.0	10	10	14.7	14.7	—	—	21.5	21.5	—	—	31.6	31.6	—	—	46.4	46.4	—	—	68.1	68.1	68	68
10.1	—	—	—	14.9	—	—	—	21.8	—	—	—	32.0	—	—	—	47.0	—	47	47	69.0	—	—	—
10.2	10.2	—	—	15.0	15.0	15	15	22.1	22.1	22	22	32.4	32.4	—	—	47.5	47.5	—	—	69.8	69.8	—	—
10.4	—	—	—	15.2	—	—	—	22.3	—	—	—	32.8	—	—	—	48.1	—	—	—	70.6	—	—	—
10.5	10.5	—	—	15.4	15.4	—	—	22.6	22.6	—	—	33.2	33.2	33	33	48.7	48.7	—	—	71.5	71.5	—	—
10.6	—	—	—	15.6	—	—	—	22.9	—	—	—	33.6	—	—	—	49.3	—	—	—	72.3	—	—	—
10.7	10.7	—	—	15.8	15.8	—	—	23.2	23.2	—	—	34.0	34.0	—	—	49.9	49.9	—	—	73.2	73.2	—	—
10.9	—	—	—	16.0	—	16	—	23.4	—	—	—	34.4	—	—	—	50.5	—	—	—	74.1	—	—	—
11.0	11.0	11	—	16.2	16.2	—	—	23.7	23.7	—	—	34.8	34.8	—	—	51.1	51.1	51	—	75.0	75.0	75	—
11.1	—	—	—	16.4	—	—	—	24.0	—	24	—	35.2	—	—	—	51.7	—	—	—	75.9	—	—	—
11.3	11.3	—	—	16.5	16.5	—	—	24.3	24.3	—	—	35.7	35.7	—	—	52.3	52.3	—	—	76.8	76.8	—	—
11.4	—	—	—	16.7	—	—	—	24.6	—	—	—	36.1	—	36	—	53.0	—	—	—	77.7	—	—	—
11.5	11.5	—	—	16.9	16.9	—	—	24.9	24.9	—	—	36.5	36.5	—	—	53.6	53.6	—	—	78.7	78.7	—	—
11.7	—	—	—	17.2	—	—	—	25.2	—	—	—	37.0	—	—	—	54.2	—	—	—	79.6	—	—	—
11.8	11.8	—	—	17.4	17.4	—	—	25.5	25.5	—	—	37.4	37.4	—	—	54.9	54.9	—	—	80.6	80.6	—	—
12.0	—	12	12	17.6	—	—	—	25.8	—	—	—	37.9	—	—	—	56.2	—	—	—	81.6	—	—	—
12.1	12.1	—	—	17.8	17.8	—	—	26.1	26.1	—	—	38.3	38.3	—	—	56.6	56.6	56	56	82.5	82.5	82	82
12.3	—	—	—	18.0	—	18	18	26.4	—	—	—	38.8	—	—	—	56.9	—	—	—	83.5	—	—	—
12.4	12.4	—	—	18.2	18.2	—	—	26.7	26.7	—	—	39.2	39.2	39	39	57.6	57.6	—	—	84.5	84.5	—	—
12.6	—	—	—	18.4	—	—	—	27.1	—	27	27	39.7	—	—	—	58.3	—	—	—	85.6	—	—	—
12.7	12.7	—	—	18.7	18.7	—	—	27.4	27.4	—	—	40.2	40.2	—	—	59.0	59.0	—	—	86.6	86.6	—	—
12.9	—	—	—	18.9	—	—	—	27.7	—	—	—	40.7	—	—	—	59.7	—	—	—	87.6	—	—	—
13.0	13.0	13	—	19.1	19.1	—	—	28.0	28.0	—	—	41.2	41.2	—	—	60.4	60.4	—	—	88.7	88.7	—	—
13.2	—	—	—	19.3	—	—	—	28.4	—	—	—	41.7	—	—	—	61.2	—	—	—	89.8	—	—	—
13.3	13.3	—	—	19.6	19.6	—	—	28.7	28.7	—	—	42.2	42.2	—	—	61.9	61.9	62	—	90.9	90.9	91	—
13.5	—	—	—	19.8	—	—	—	29.1	—	—	—	42.7	—	—	—	62.6	—	—	—	92.0	—	—	—
13.7	13.7	—	—	20.0	20.0	20	—	29.4	29.4	—	—	43.2	43.2	43	—	63.4	63.4	—	—	93.1	93.1	—	—
13.8	—	—	—	20.3	—	—	—	29.8	—	—	—	43.7	—	—	—	64.2	—	—	—	94.2	—	—	—
14.0	14.0	—	—	20.5	20.5	—	—	30.1	30.1	30	—	44.2	44.2	—	—	64.9	64.9	—	—	95.3	95.3	—	—
14.2	—	—	—	20.8	—	—	—	30.5	—	—	—	44.8	—	—	—	65.7	—	—	—	96.5	—	—	—
14.3	14.3	—	—	21.0	21.0	—	—	30.9	30.9	—	—	45.3	45.3	—	—	66.5	66.5	—	—	97.6	97.6	—	—
14.5	—	—	—	21.3	—	—	—	31.2	—	—	—	45.9	—	—	—	67.3	—	—	—	98.8	—	—	—

NOTE: These values are generally available in multiples of 0.1, 1, 10, 100, 1 k, and 1 M.

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DERIVATIONS

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Appendix

DERIVATIONS

Output Voltage of Temperature Measuring Bridge

At balance $V_{\text{OUT}} = 0$ and all resistances have the value R. For a small imbalance:

$$V_B = \frac{V_S}{2} \text{ and } V_A = \left(\frac{R}{2R + \Delta R_{\text{THERM}}}\right)V_S$$

$$\begin{aligned}
\Delta V_{\text{OUT}} = V_B - V_A &= \frac{V_S}{2} - \left(\frac{R}{2R + \Delta R_{\text{THERM}}}\right)V_S \\
&= \left(\frac{1}{2} - \frac{R}{2R + \Delta R_{\text{THERM}}}\right)V_S \\
&= \left(\frac{2R + \Delta R_{\text{THERM}} - 2R}{2(2R + \Delta R_{\text{THERM}})}\right)V_S \\
&= \left(\frac{\Delta R_{\text{THERM}}}{4R + 2\Delta R_{\text{THERM}}}\right)V_S
\end{aligned}$$

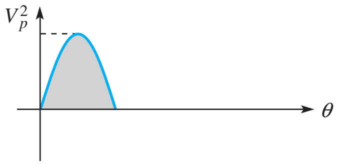
Assume $2\Delta R_{\text{THERM}} << 4R$, then

$$\Delta V_{\text{OUT}} \cong \left(\frac{\Delta R_{\text{THERM}}}{4R}\right)V_S = \Delta R_{\text{THERM}}\left(\frac{V_S}{4R}\right)$$

RMS (Effective) Value of a Sine Wave

The abbreviation “rms” stands for the root mean square process by which this value is derived. In the process, we first square the equation of a sine wave.

$$v^2 = V_p^2 \sin^2\theta$$



▲ FIGURE 1

Next, we obtain the mean or average value of v^2 by dividing the area under a half-cycle of the curve by π (see Figure 1). The area is found by integration and trigonometric identities.

$$\begin{aligned}
V_{\text{avg}}^2 = \frac{\text{area}}{\pi} &= \frac{1}{\pi}\int_0^\pi V_p^2 \sin^2\theta \, d\theta \\
&= \frac{V_p^2}{2\pi}\int_0^\pi (1 - \cos 2\theta) d\theta = \frac{V_p^2}{2\pi}\int_0^\pi 1 \, d\theta - \frac{V_p^2}{2\pi}\int_0^\pi (-\cos 2\theta)\, d\theta \\
&= \frac{V_p^2}{2\pi}\left(\theta - \tfrac{1}{2}\sin 2\theta\right)_0^\pi = \frac{V_p^2}{2\pi}(\pi - 0) = \frac{V_p^2}{2}
\end{aligned}$$

Finally, the square root of V_{avg}^2 is V_{rms}.

$$V_{\text{rms}} = \sqrt{V_{\text{avg}}^2} = \sqrt{V_p^2/2} = \frac{V_p}{\sqrt{2}} = 0.707V_p$$

Average Value of a Half-Cycle Sine Wave

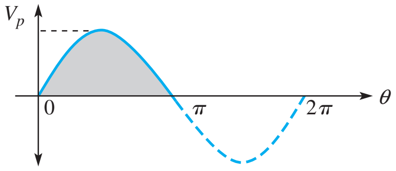
The average value of a sine wave is determined for a half-cycle because the average over a full cycle is zero.

The equation for a sine wave is

$$v = V_p \sin\theta$$

The average value of the half-cycle is the area under the curve divided by the distance of the curve along the horizontal axis (see Figure 2).

$$V_{\text{avg}} = \frac{\text{area}}{\pi}$$



▲ FIGURE 2

To find the area, we use integral calculus.

$$\begin{aligned}
V_{\text{avg}} &= \frac{1}{\pi}\int_0^{\pi} V_p \sin\theta\, d\theta = \frac{V_p}{\pi}(-\cos\theta)\Big|_0^{\pi} \\
&= \frac{V_p}{\pi}[-\cos\pi - (-\cos 0)] = \frac{V_p}{\pi}[-(-1) - (-1)] \\
&= \frac{V_p}{\pi}(2) = \frac{2}{\pi}V_p = 0.637V_p
\end{aligned}$$

Reactance Derivations

Derivation of Capacitive Reactance

$$\begin{aligned}
\theta &= 2\pi ft = \omega t \\
i &= C\frac{dv}{dt} = C\frac{d(V_p \sin\theta)}{dt} = C\frac{d(V_p \sin\omega t)}{dt} = \omega C(V_p \cos\omega t) \\
I_{\text{rms}} &= \omega C V_{\text{rms}} \\
X_C &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\text{rms}}}{\omega C V_{\text{rms}}} = \frac{1}{\omega C} = \frac{1}{2\pi fC}
\end{aligned}$$

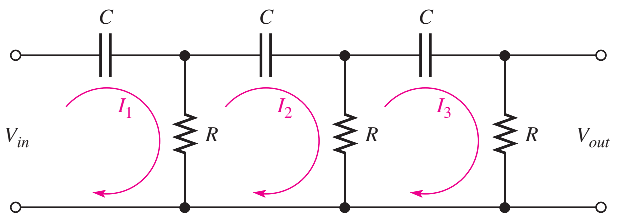
Derivation of Inductive Reactance

$$\begin{aligned}
v &= L\frac{di}{dt} = L\frac{d(I_p \sin\omega t)}{dt} = \omega L(I_p \cos\omega t) \\
V_{\text{rms}} &= \omega L I_{\text{rms}} \\
X_L &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{\omega L I_{\text{rms}}}{I_{\text{rms}}} = \omega L = 2\pi fL
\end{aligned}$$

Phase-Shift Oscillator Frequency

The feedback circuit in the phase-shift oscillator consists of three *RC* stages, as shown in Figure 3. An expression for the attenuation is derived using the loop analysis method for the loop assignment shown. All *R*s are equal in value, and all *C*s are equal in value.

$$
\begin{aligned}
(R - j1/2\pi fC)I_1 - RI_2 + 0I_3 &= V_{in} \\
-RI_1 + (2R - j1/2\pi fC)I_2 - RI_3 &= 0 \\
0I_1 - RI_2 + (2R - j1/2\pi fC)I_3 &= 0
\end{aligned}
$$



▲ **FIGURE 3**

In order to get V_{out}, we must solve for I_3 using determinants:

$$
I_3 = \frac{\begin{vmatrix} (R - j1/2\pi fC) & -R & V_{in} \\ -R & (2R - j1/2\pi fC) & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} (R - j1/2\pi fC) & -R & 0 \\ -R & (2R - j1/2\pi fC) & -R \\ 0 & -R & (2R - j1/2\pi fC) \end{vmatrix}}
$$

$$
I_3 = \frac{R^2 V_{in}}{(R - j1/2\pi fC)(2R - j1/2\pi fC)^2 - R^2(2R - j1/2\pi fC) - R^2(R - 1/2\pi fC)}
$$

$$
\begin{aligned}
\frac{V_{out}}{V_{in}} &= \frac{RI_3}{V_{in}} \\
&= \frac{R^3}{(R - j1/2\pi fC)(2R - j1/2\pi fC)^2 - R^3(2 - j1/2\pi fRC) - R^3(1 - 1/2\pi fRC)} \\
&= \frac{R^3}{R^3(1 - j1/2\pi fRC)(2 - j1/2\pi fRC)^2 - R^3[(2 - j1/2\pi fRC) - (1 - j1/2\pi fRC)]} \\
&= \frac{R^3}{R^3(1 - j1/2\pi fRC)(2 - j1/2\pi fRC)^2 - R^3(3 - j1/2\pi fRC)}
\end{aligned}
$$

$$
\frac{V_{out}}{V_{in}} = \frac{1}{(1 - j1/2\pi fRC)(2 - j1/2\pi fRC)^2 - (3 - j1/2\pi fRC)}
$$

Expanding and combining the real terms and the *j* terms separately,

$$
\frac{V_{out}}{V_{in}} = \frac{1}{\left(1 - \dfrac{5}{4\pi^2 f^2 R^2 C^2}\right) - j\left(\dfrac{6}{2\pi fRC} - \dfrac{1}{(2\pi f)^3 R^3 C^3}\right)}
$$

For oscillation in the phase-shift amplifier, the phase shift through the *RC* circuit must equal 180°. For this condition to exist, the *j* term must be 0 at the frequency of oscillation f_r.

$$\frac{6}{2\pi f_r RC} - \frac{1}{(2\pi f_r)^3 R^3 C^3} = 0$$

$$\frac{6(2\pi)^2 f_r^2 R^2 C^2 - 1}{(2\pi)^3 f_r^3 R^3 C^3} = 0$$

$$6(2\pi)^2 f_r^2 R^2 C^2 - 1 = 0$$

$$f_r^2 = \frac{1}{6(2\pi)^2 R^2 C^2}$$

$$f_r = \frac{1}{2\pi\sqrt{6}RC}$$

Resonant Frequency for a Nonideal Parallel Resonant Circuit

$$\frac{1}{\mathbf{Z}} = \frac{1}{-jX_C} + \frac{1}{R_W + jX_L}$$

$$= j\left(\frac{1}{X_C}\right) + \frac{R_W - jX_L}{(R_W + jX_L)(R_W - jX_L)} = j\left(\frac{1}{X_C}\right) + \frac{R_W - jX_L}{R_W^2 + X_L^2}$$

The first term plus splitting the numerator of the second term yields

$$\frac{1}{\mathbf{Z}} = j\left(\frac{1}{X_C}\right) - j\left(\frac{X_L}{R_W^2 + X_L^2}\right) + \frac{R_W}{R_W^2 + X_L^2}$$

The *j* terms are equal.

$$\frac{1}{X_C} = \frac{X_L}{R_W^2 + X_L^2}$$

Thus,

$$R_W^2 = X_L^2 = X_L X_C$$

$$R_W^2 + (2\pi f_r L)^2 = \frac{2\pi f_r L}{2\pi f_r C}$$

$$R_W^2 + 4\pi^2 f_r^2 L^2 = \frac{L}{C}$$

$$4\pi^2 f_r^2 L^2 = \frac{L}{C} - R_W^2$$

Solving for f_r^2,

$$f_r^2 = \frac{\left(\frac{L}{C}\right) - R_W^2}{4\pi^2 L^2}$$

Multiplying both numerator and denominator by *C*,

$$f_r^2 = \frac{L - R_W^2 C}{4\pi^2 L^2 C} = \frac{L - R_W^2 C}{L(4\pi^2 LC)}$$

Factoring an *L* out of the numerator and canceling gives

$$f_r^2 = \frac{1 - (R_W^2 C/L)}{4\pi^2 LC}$$

Taking the square root of both sides yields

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}}$$

Impedance of Nonideal Tank Circuit at Resonance

Begin with the following expression for $1/\mathbf{Z}$ that was developed in the derivation for the previous equation.

$$\frac{1}{\mathbf{Z}} = j\left(\frac{1}{X_C}\right) - j\left(\frac{X_L}{R_W^2 + X_L^2}\right) + \frac{R_W}{R_W^2 + X_L^2}$$

At resonance, $\mathbf{Z}$ is purely resistive; so it has no j part (the j terms in the last expression cancel). Thus, only the real part is left, as stated in the following equation for Z at resonance:

$$Z_r = \frac{R_W^2 + X_L^2}{R_W}$$

Splitting the denominator, we get

$$Z_r = \frac{R_W^2}{R_W} + \frac{X_L^2}{R_W} = R_W + \frac{X_L^2}{R_W}$$

Factoring out R_W gives

$$Z_r = R_W\left(1 + \frac{X_L^2}{R_W^2}\right)$$

Since $X_L^2/R_W^2 = Q^2$, then

$$Z_R = R_W(Q^2 + 1)$$

Appendix

CAPACITOR LABEL CODING

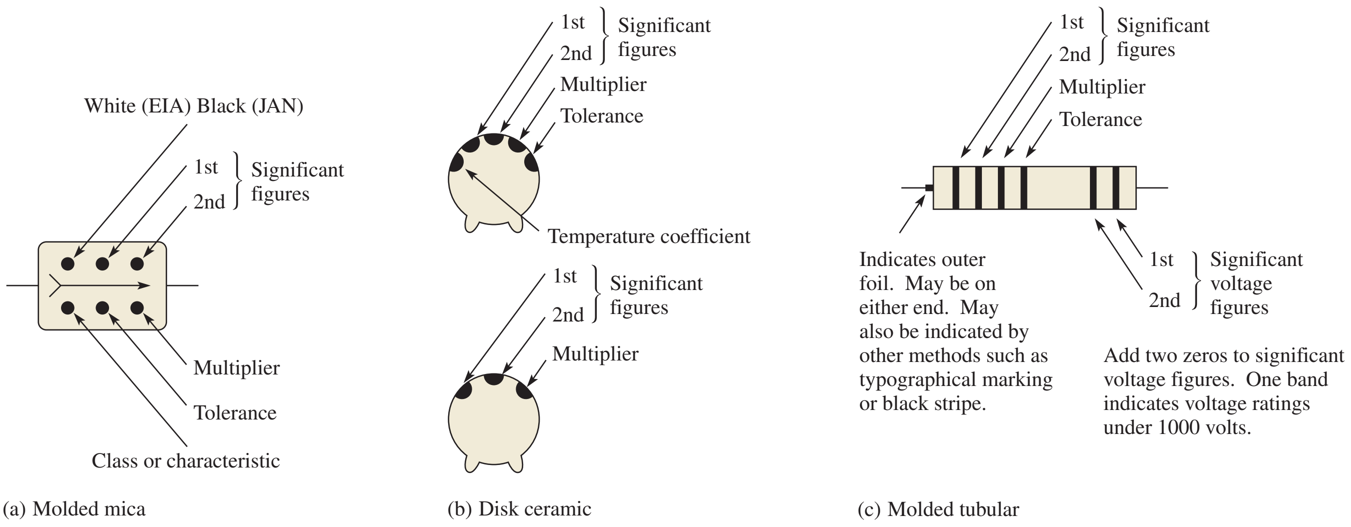
Some capacitors have color-coded designations. The color code used for capacitors is basically the same as that used for resistors. Some variations occur in tolerance designation. The basic color codes are shown in Table 1, and some typical color-coded capacitors are illustrated in Figure 1.

TABLE 1

Typical composite color codes for capacitors (picofarads).

COLOR	DIGIT	MULTIPLIER	TOLERANCE
Black	0	1	20%
Brown	1	10	1%
Red	2	100	2%
Orange	3	1000	3%
Yellow	4	10000	
Green	5	100000	5% (EIA)
Blue	6	1000000	
Violet	7		
Gray	8		
White	9		
Gold		0.1	5% (JAN)
Silver		0.01	10%

NOTE: EIA stands for Electronic Industries Association, and JAN stands for Joint Army-Navy, a military standard.


White (EIA) Black (JAN)
1st
2nd
Significant figures
Multiplier
Tolerance
Class or characteristic
1st
2nd
Significant figures
Multiplier
Tolerance
Temperature coefficient
1st
2nd
Significant figures
Multiplier
1st
2nd
Significant figures
Multiplier
Tolerance
Indicates outer foil. May be on either end. May also be indicated by other methods such as typographical marking or black stripe.
1st
2nd
Significant voltage figures
Add two zeros to significant voltage figures. One band indicates voltage ratings under 1000 volts.

(a) Molded mica (b) Disk ceramic (c) Molded tubular

FIGURE 1

Typical color-coded capacitors.

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Marking Systems

A capacitor, as shown in Figure 2, has certain identifying features.

- Body of one solid color (off-white, beige, gray, tan or brown).
- End electrodes completely enclose ends of part.
- Many different sizes:
  1. Type 1206: 0.125 inch long by 0.063 inch wide (3.2 mm × 1.6 mm) with variable thickness and color.
  2. Type 0805: 0.080 inch long by 0.050 inch wide (2.0 mm × 1.25 mm) with variable thickness and color.
  3. Variably sized with a single color (usually translucent tan or brown). Sizes range from 0.059 inch (1.5 mm) to 0.220 inch (5.6 mm) in length and in width from 0.032 inch (0.8 mm) to 0.197 inch (5.0 mm).
- Three different marking systems:
  1. Two place (letter and number only).
  2. Two place (letter and number or two numbers).
  3. One place (letter of varying color).

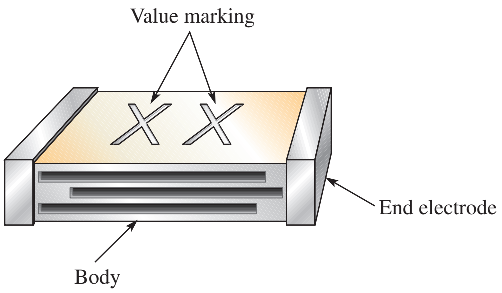
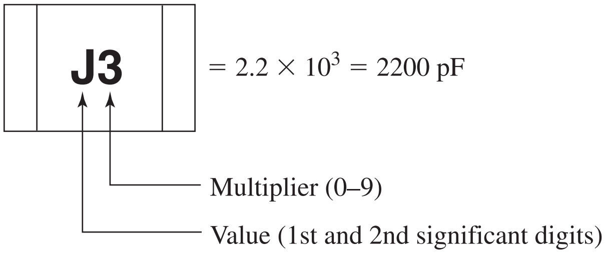

Value marking
XX
End electrode
Body

FIGURE 2

Capacitor marking.

Standard Two-Place Code

Refer to Table 2.


J3
$= 2.2 \times 10^3 = 2200$ pF
Multiplier (0–9)
Value (1st and 2nd significant digits)

Examples: S2 = 4.7 × 100 = 470 pF
b0 = 3.5 × 1.0 = 3.5 pF

TABLE 2

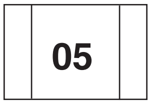
VALUE*						MULTIPLIER
A	1.0	L	2.7	T	5.1	0 = ×1.0
B	1.1	M	3.0	U	5.6	1 = ×10
C	1.2	N	3.3	m	6.0	2 = ×100
D	1.3	b	3.5	V	6.2	3 = ×1000
E	1.5	P	3.6	W	6.8	4 = ×10000
F	1.6	Q	3.9	n	7.0	5 = ×100000
G	1.8	d	4.0	X	7.5	etc.
H	2.0	R	4.3	t	8.0	
J	2.2	e	4.5	Y	8.2	
K	2.4	S	4.7	y	9.0	
a	2.5	f	5.0	Z	9.1	

*Note uppercase and lowercase letters.

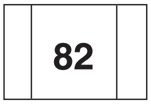
Alternate Two-Place Code

Refer to Table 3.

- Values below 100 pF—Value read directly

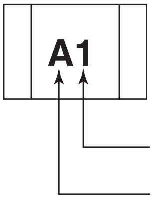


= 5 pF

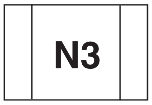


= 82 pF

- Values 100 pF and above—Letter/Number code



= 10 × 10 = 100 pF



= 33 × 1000
= 33000 pF = .033 μF

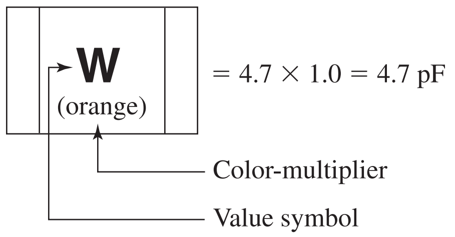
TABLE 3

VALUE*						MULTIPLIER
A	10	J	22	S	47	1 = ×10
B	11	K	24	T	51	2 = ×100
C	12	L	27	U	56	3 = ×1000
D	13	M	30	V	62	4 = ×10000
E	15	N	33	W	68	5 = ×100000
F	16	P	36	X	75	etc.
G	18	Q	39	Y	82	
H	20	R	43	Z	91	

*Note uppercase letters only.

Standard Single-Place Code

Refer to Table 4.


W
(orange)
= 4.7 × 1.0 = 4.7 pF
Color-multiplier
Value symbol

Examples: R (Green) = 3.3 × 100 = 330 pF
7 (Blue) = 8.2 × 1000 = 8200 pF

TABLE 4

VALUE						MULTIPLIER (COLOR)
A	1.0	K	2.2	W	4.7	Orange = ×1.0
B	1.1	L	2.4	X	5.1	Black = ×10
C	1.2	N	2.7	Y	5.6	Green = ×100
D	1.3	O	3.0	Z	6.2	Blue = ×1000
E	1.5	R	3.3	3	6.8	Violet = ×10000
H	1.6	S	3.6	4	7.5	Red = ×100000
I	1.8	T	3.9	7	8.2	
J	2.0	V	4.3	9	9.1	

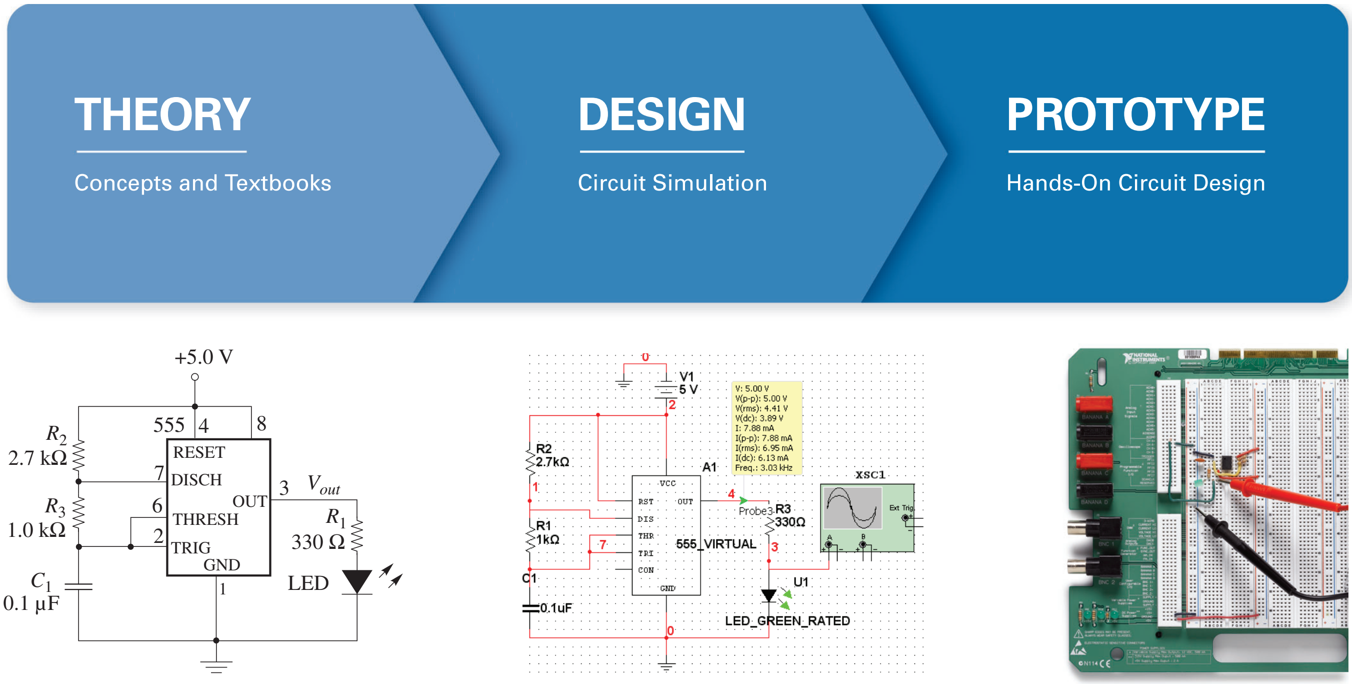
NI MULTISIM FOR CIRCUIT SIMULATION

Appendix

Simulate, Prototype, and Test Circuits

Theory, Design, and Prototype

As electronic circuits and systems become more advanced, circuit designers rely on computers in the design process. It is essential for engineers and technicians to systematically design, simulate, prototype, and lay out the circuit. Students can also use the engineering and design process to reinforce concepts and theory in the classroom. The three stages of the student design process are illustrated in Figure 1.



THEORY
Concepts and Textbooks
DESIGN
Circuit Simulation
PROTOTYPE
Hands-On Circuit Design
+5.0 V
R_2
2.7 kΩ
555
4
8
RESET
7
DISCH
OUT
3
V_{out}
R_3
1.0 kΩ
6
THRESH
R_1
330 Ω
2
TRIG
GND
1
C_1
0.1 μF
LED
0
V1
5 V
2
V: 5.00 V
V(p-p): 5.00 V
V(rms): 4.41 V
V(dc): 3.89 V
I: 7.88 mA
I(p-p): 7.88 mA
I(rms): 6.95 mA
I(dc): 6.13 mA
Freq.: 3.03 kHz
R2
2.7kΩ
A1
XSC1
1
VCC
RST
OUT
4
Probe3
R3
330Ω
Ext Trig
DIS
R1
1kΩ
THR
7
555_VIRTUAL
TRI
3
A
B
CON
C1
GND
U1
0.1uF
LED_GREEN_RATED
0

▲ FIGURE 1

Three steps of the student design process for a practical circuit.

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Notice: This appendix was written for use with the Multisim Version 10.0 software. Some or all of the information in this Appendix may not apply to other versions of the software.

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The National Instruments Electronics Education Platform is an end-to-end tool-chain designed to meet student and educator needs. The platform consists of NI Multisim simulation software, the NI Educational Laboratory Virtual Instrumentation Suite (NI ELVIS) prototyping workstation, and the NI LabVIEW graphical programming environment. NI Multisim provides intuitive schematic capture, SPICE simulation, and integration with the NI ELVIS to help students explore circuit theory and design circuits to investigate behavior. NI ELVIS is a prototyping platform that allows students to quickly and easily create their circuits. With NI LabVIEW, students can measure real-world signals and compare simulation results.

1. Investigate Theory Learn the fundamental theory of circuit design through the *Principles of Electric Circuits* textbook and course lectures. Reinforce important concepts in the easy-to-use Multisim environment by downloading the Multisim circuit files for the textbook. The Multisim circuit files help build the foundation for an in-depth understanding of circuit behavior. Using the prebuilt circuit files, simulate and analyze the circuit behavior of examples and problems in each chapter.

2. Design and Simulate Circuit simulation provides an interactive view into a circuit. Build a circuit from scratch and learn about the design performance using built-in circuit instruments and probes in an ideal pre-laboratory environment. With the 3D breadboard in Multisim, take the leap from circuit diagram to real-world physical implementation.

3. Prototype, Measure, and Compare Hands-on experience building physical circuitry is essential. Move from the 3D breadboard in Multisim to a real-world breadboard on NI ELVIS to seamlessly complete the entire design process by prototyping the circuit. Within the LabVIEW environment, compare real-world measurements to the simulation values to reinforce theory, fully understand circuit behavior, and build the foundations of professional engineering analysis.

NI Multisim

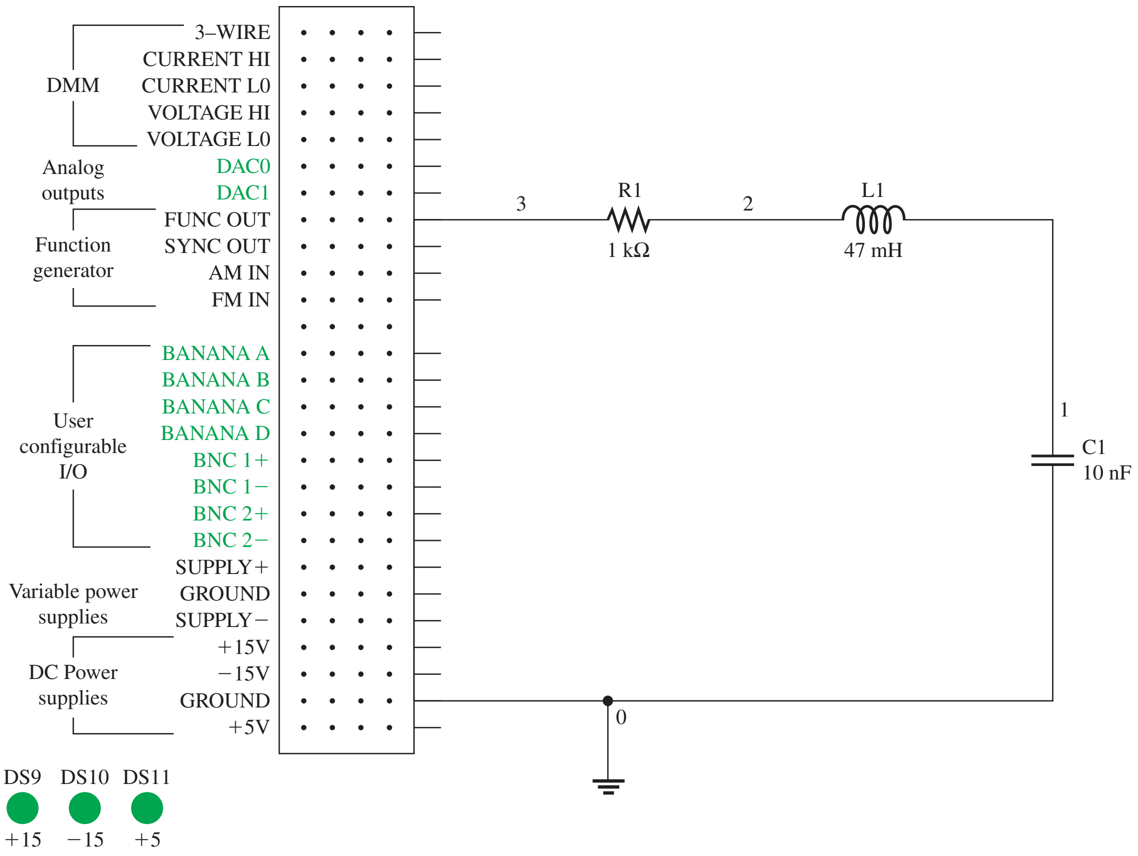
Multisim software integrates powerful SPICE simulation and schematic capture into a highly intuitive electronics lab on the computer. Use the Multisim Student Edition to enhance learning by:

Building circuits in a simple, easy-to-learn environment
Simulating and analyzing circuits for homework and prelab assignments
Breadboarding in 3D at home before lab sessions
Creating designs of up to 50 components using a library of nearly 4,000 devices
Integrating with the NI ELVIS prototyping workstation

While using industry-standard SPICE in the background, you can take advantage of the Multisim drag-and-drop interface to make circuit drawing, wiring, and analysis simple and easy to use. You can build circuits from scratch and learn about design performances using built-in virtual instruments and probes in an ideal environment for experimentation. With the 3D breadboard, you can easily take the leap from circuit diagram to real-world implementation.

Using Multisim with Principles of Electric Circuits Create an example from the textbook to get familiar with the Multisim environment. First, launch Multisim and open a new schematic window (**File >> New >> Schematic Capture**). Design circuits in the circuit window by placing components from the component toolbar. Clicking on the component toolbar opens the component browser. Choose the family of components and select an individual component to place on the circuit window by double-clicking on it.

Once you have selected a component, it attaches itself and "ghosts" the mouse cursor. Place the component by clicking again on the desired location in the schematic. If you are


DMM
3–WIRE
CURRENT HI
CURRENT L0
VOLTAGE HI
VOLTAGE L0
Analog outputs
DAC0
DAC1
Function generator
FUNC OUT
SYNC OUT
AM IN
FM IN
User configurable I/O
BANANA A
BANANA B
BANANA C
BANANA D
BNC 1+
BNC 1−
BNC 2+
BNC 2−
Variable power supplies
SUPPLY+
GROUND
SUPPLY−
DC Power supplies
+15V
−15V
GROUND
+5V
DS9
DS10
DS11
+15
−15
+5
3
R1
1 kΩ
2
L1
47 mH
1
C1
10 nF
0

▲ FIGURE 2

Multisim schematic of *RLC* circuit.

new to Multisim, you should use the **BASIC_VIRTUAL** family of components, which you can assign any arbitrary value. As an example of a simple Multisim circuit, an *RLC* circuit is shown in Figure 2. The circuit connects a 1 kΩ resistor, a 47 mH inductor, and a 10 nF capacitor. All these components are found in the Basic Group of the Component Database.

The next step is to wire the components together. Simply left-click on the source terminal and left-click on the destination terminal. Multisim automatically chooses the best path for the virtual wire between the two terminals. Always be sure you have fully captured your circuit; then you can simulate it. You can use your simulation results as a comparison with the physical circuit.

To analyze the circuit, use a measurement probe to measure the voltages and other characteristics from the circuit while the simulation is running. Use the virtual oscilloscope to analyze the output signal from our sample *RLC* circuit.

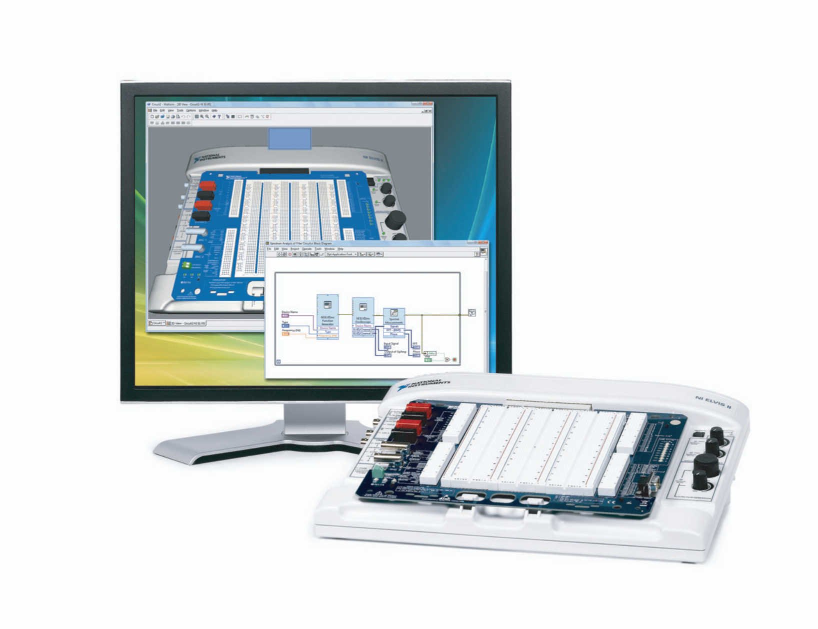
NI ELVIS

NI ELVIS is a LabVIEW-based design and prototyping environment based on LabVIEW for university science and engineering laboratories. NI ELVIS consists of virtual instruments, or functions, based on LabVIEW; a multifunction data acquisition device; and a custom-designed benchtop workstation and prototyping board. This combination provides a ready-to-use suite of instruments for educational laboratories. The NI ELVIS workstation is shown in Figure 3.

NI ELVIS is an ideal companion to any electronics lab that uses Multisim. It features USB connectivity for easy setup, maintenance, and portability, as well as a portable breadboard for real-world circuit prototyping.

▶ FIGURE 3

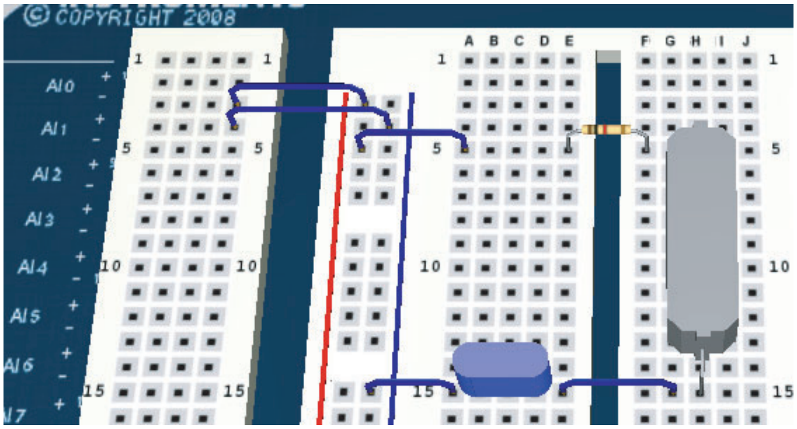
The NI ELVIS workstation.



NI ELVIS features a breadboard prototyping environment with built-in instruments including a function generator, digital multimeter (DMM), oscilloscope, and variable power supplies. The breadboard is detachable, so you can work on your projects and labs independently of the NI ELVIS unit. NI ELVIS provides software based on LabVIEW for interacting with virtual instruments.

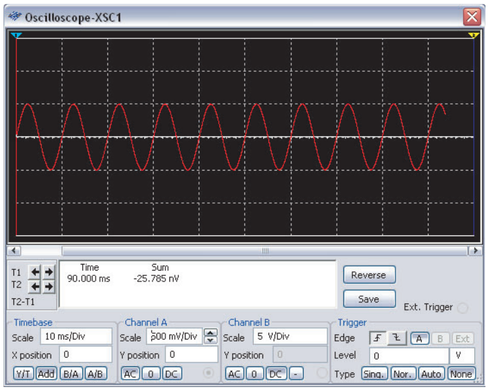
Using NI ELVIS with Principles of Electric Circuits Returning to the *RLC* circuit introduced with Multisim as an example, you can continue the prototyping step. With prototyping hardware, you can quickly construct a circuit on a standard protoboard and test it with the laboratory instruments to complete the design.

Alternatively, you can repeat the simulation step but this time as a prototype using the Virtual 3D NI ELVIS from within Multisim. To construct a 3D NI ELVIS prototype, open the 3D breadboard by clicking **Tools >> Show Breadboard**. Place components and wires to build up your circuit. The corresponding connection points and symbols on the NI ELVIS schematic turn green, indicating the 3D connections are correct. If you create a traditional schematic, you see a standard breadboard. Figure 4 shows the circuit on 3D Virtual



▲ FIGURE 4

Prototyped circuit in multisim.

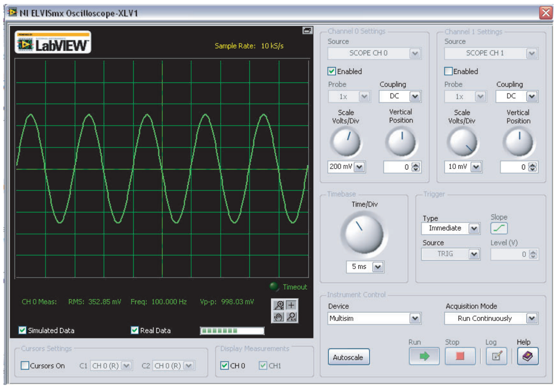

Oscilloscope-XSC1
T1
T2
T2-T1
Reverse
Save
Ext. Trigger
Timebase
Channel A
Channel B
Trigger
Edge
A
B
Ext
Y/T
Add
B/A
A/B
AC
0
DC
-
Type
Sing.
Nor.
Auto
None

Time 90.000 ms
Sum -25.785 nV

Timebase: Scale 10 ms/Div; X position 0
Channel A: Scale 500 mV/Div; Y position 0
Channel B: Scale 5 V/Div; Y position 0
Trigger: Level 0 V

▲ **FIGURE 5**

Multisim simulated signal.


NI ELVISmx Oscilloscope-XLV1
LabVIEW
Channel 0 Settings
Channel 1 Settings
Timebase
Trigger
Instrument Control
Cursors Settings
Display Measurements
Timeout
Autoscale
Run
Stop
Log
Help

Sample Rate: 10 kS/s

CH 0 Meas: RMS: 352.85 mV Freq: 100.000 Hz Vp-p: 998.03 mV

☑ Simulated Data ☑ Real Data

Channel 0 Settings: Source SCOPE CH 0; ☑ Enabled; Probe 1x; Coupling DC; Scale Volts/Div 200 mV; Vertical Position 0

Channel 1 Settings: Source SCOPE CH 1; ☐ Enabled; Probe 1x; Coupling DC; Scale Volts/Div 10 mV; Vertical Position 0

Timebase: Time/Div 5 ms

Trigger: Type Immediate; Source TRIG; Level (V) 0

Device: Multisim; Acquisition Mode: Run Continuously

☐ Cursors On C1 CH 0 (R) C2 CH 0 (R)

☑ CH 0 ☑ CH1

▲ **FIGURE 6**

NI ELVIS measured signal.

NI ELVIS prototyped in Multisim and ready for construction. Once you have verified the layout of your circuit using the 3D virtual environment, you can physically build it on NI ELVIS.

After constructing the circuit, you can use the NI ELVIS instrument launcher to measure the output signal of this circuit. Figure 5 shows the simulated signal of the *RLC* circuit on the Multisim oscilloscope. Figure 6 shows the measured signal using the ELVIS II Oscilloscope. The most important step in the process is to compare the measurement of the prototyped circuit to the simulation. This helps you determine where potential errors exist in the design. After comparing the measurement with the theoretical values, you can revisit your design to improve it or prepare it for layout on a PCB board, using software such as NI Ultiboard.

NI Multisim Circuit Files Download the Multisim circuit files to develop in-depth understanding of circuit behavior. To download the pre-built circuit files and Multisim resources, visit: ni.com/academic/floyd

NI Multisim Resources The link provides references to the following resources to help you get started with NI Multisim:

Download the free Multisim 30-day evaluation version
View the Getting Started Guide to Multisim
Learn Multisim in 3-Hours tutorial
Discuss Multisim in an online discussion forum

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GLOSSARY

accuracy The degree to which a measured value represents the true or accepted value of a quantity.

admittance (*Y*) A measure of the ability of a reactive circuit to permit current; the reciprocal of impedance. The unit is the siemens (S).

alternator An ac generator that converts energy of motion into electrical energy.

ammeter An electrical instrument used to measure current.

ampere (A) The unit of electrical current.

ampere-hour (Ah) rating A capacity rating for batteries determined by multiplying the current (A) times the length of time (h) a battery can deliver that current to a load.

ampere-turn The current in a single loop (turn) of wire.

amplitude The maximum value of a voltage or current.

angular velocity The rotational rate of a phasor which is related to the frequency of the sine wave that the phasor represents.

anode The terminal of a polarized device that electrons flow from. For a battery that is supplying current, this is the negative terminal; for a diode, it is the positive terminal.

apparent power The phasor combination of resistive power (true power) and reactive power. The unit is the volt-ampere (VA).

apparent power rating The method of rating transformers in which the power capability is expressed in volt-amperes (VA).

armature The power-producing coil in an alternator, generator, or motor. In dc generators, the armature is the rotor, but in alternators it can be either the rotor or the stator. In motors, the armature is the rotor.

atom The smallest particle of an element possessing the unique characteristics of that element.

atomic number The number of protons in a nucleus.

attenuation A reduction of the output signal compared to the input signal, resulting in a ratio with a value of less than 1 for the output voltage to the input voltage of a circuit.

autotransformer A transformer in which the primary and secondary are in a single winding.

average value The average of a sine wave over one half-cycle. It is 0.637 times the peak value.

AWG American wire gauge; a standardization based on wire diameter.

back emf (electromotive force) A voltage developed across a spinning armature that opposes the original applied voltage.

balun A type of transformer used to convert a balanced line (such as twisted pair wiring) to an unbalanced line (such as coaxial cable) or vice-versa.

balanced bridge A bridge circuit that is in the balanced state as indicated by 0 V across the output.

balanced load A condition where all the load currents are equal and the neutral current is zero.

band-pass filter A filter that passes a range of frequencies lying between two critical frequencies and rejects frequencies above and below that range.

band-stop filter A filter that rejects a range of frequencies lying between two critical frequencies and passes frequencies above and below that range.

bandwidth The range of frequencies for which the current (or output voltage) is equal to or greater than 70.7% of its value at the resonant frequency that is considered to be passed by a filter.

baseline The normal level of a pulse waveform; the voltage level in the absence of a pulse.

battery An energy source that uses a chemical reaction to convert chemical energy into electrical energy.

bias The application of a dc voltage to an electronic device to produce a desired mode of operation.

bleeder current The current left after the total load current is subtracted from the total current into the circuit.

Bode plot The graph of a filter's frequency response showing the change in the output voltage to input voltage ratio expressed in dB as a function of frequency for a constant input voltage.

branch One current path in a parallel circuit; a current path that connects two nodes.

branch current The actual current in a branch.

capacitance The ability of a capacitor to store electrical charge.

capacitive reactance The opposition of a capacitor to sinusoidal current. The unit is the ohm (Ω).

capacitive susceptance (B_C) The ability of a capacitor to permit current; the reciprocal of capacitive reactance. The unit is the siemens (S).

capacitor An electrical device consisting of two conductive plates separated by an insulating material and possessing the property of capacitance.

cathode The terminal of a polarized device that electrons flow to. For a battery that is supplying current, this is the positive terminal; for a diode, it is the negative terminal.

center frequency (f_0) The resonant frequency of a bandpass or band-stop filter.

center tap (CT) A connection at the midpoint of a winding in a transformer.

charge An electrical property of matter that exists because of an excess or a deficiency of electrons. Charge can be either positive or negative.

choke A type of inductor used to block or choke off high frequencies.

From Glossary of *Principles of Electric Circuits: Conventional Current Version*, Ninth Edition, Thomas L. Floyd.
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circuit An interconnection of electrical components designed to produce a desired result. A basic circuit consists of a source, a load, and an interconnecting current path.

circuit breaker A resettable protective device used for interrupting excessive current in an electric circuit.

circular mil (CM) A unit of the cross-sectional area of a wire.

closed circuit A circuit with a complete current path.

coefficient The constant number that appears in front of a variable.

coefficient of coupling (k) A constant associated with transformers that is the ratio of secondary magnetic flux to primary magnetic flux. The ideal value of 1 indicates that all the flux in the primary winding is coupled into the secondary winding.

common Reference ground.

complex conjugate A complex number having the same real part and an oppositely signed imaginary part; an impedance containing the same resistance and a reactance opposite in phase but equal in magnitude to that of a given impedance.

complex plane An area consisting of four quadrants on which a quantity containing both magnitude and direction can be represented.

conductance (G) The ability of a circuit to allow current; the reciprocal of resistance. The unit is the siemens (S).

conductor A material in which electric current is easily established. An example is copper.

core The physical structure around which the winding of an inductor is formed. The core material influences the electromagnetic characteristics of the inductor. Also, the inner shells of an atom.

coulomb (C) The unit of electrical charge; the total charge possessed by 6.25×10^{18} electrons.

Coulomb's law A law that states a force exists between two charged bodies that is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

critical frequency (f_c) The frequency at which a filter's output voltage is 70.7% of the maximum.

current The rate of flow of charge (electrons).

current divider A parallel circuit in which the currents divide inversely proportional to the parallel branch resistances.

current source A device that provides a constant current for a varying load.

cutoff frequency (f_c) The frequency at which the output voltage of a filter is 70.7% of the maximum output voltage; another term for critical frequency.

cycle One repetition of a periodic waveform.

DC component The average value of a pulse waveform.

decade A tenfold change in frequency or other parameter.

decibel A logarithmic measurement of the ratio of one power to another or one voltage to another, which can be used to express the input-to-output relationship of a filter.

degree The unit of angular measure corresponding to 1/360 of a complete revolution.

determinant The solution of a matrix consisting of an array of coefficients and constants for a set of simultaneous equations.

dielectric The insulating material between the plates of a capacitor.

dielectric constant A measure of the ability of a dielectric material to establish an electric field.

dielectric strength A measure of the ability of a dielectric material to withstand voltage without breaking down.

differentiator A circuit producing an output that approaches the mathematical derivative of the input.

digital multimeter An electronic instrument that combines meters for the measurement of voltage, current, and resistance.

DMM Digital multimeter; an electronic instrument that combines meters for measurement of voltage, current, and resistance.

duty cycle A characteristic of a pulse waveform that indicates the percentage of time that a pulse is present during a cycle; the ratio of pulse width to period, expressed as either a fraction or as a percentage.

effective value A measure of the heating effect of a sine wave; also known as the rms (root mean square) value.

efficiency The ratio of the output power delivered to a load to the input power to a circuit, usually expressed as a percentage.

electrical Related to the use of electrical voltage and current to achieve desired results.

electrical isolation The condition in which two circuits have no common conductive path between them.

electrical shock The physical sensation resulting from electrical current through the body.

electromagnetic field A formation of a group of magnetic lines of force surrounding a conductor created by electrical current in the conductor.

electromagnetism The production of a magnetic field by current in a conductor.

electromagnetic induction The phenomenon or process by which a voltage is produced in a conductor when there is relative motion between the conductor and a magnetic or electromagnetic field.

electron A basic particle of electrical charge in matter. The electron possesses negative charge.

electronic Related to the movement and control of free electrons in semiconductors or vacuum devices.

electronic power supply A voltage source that converts the ac voltage from a wall outlet to a constant (dc) voltage at a level suitable for electronic components.

element One of the unique substances that make up the known universe. Each element is characterized by a unique atomic structure.

energy The ability to do work.

engineering notation A system for representing any number as a one-, two-, or three-digit number times a power of ten with an exponent that is a multiple of 3.

equivalent circuit A circuit that produces the same voltage and current to a given load as the original circuit that it replaces.

error The difference between the true or best-accepted value of some quantity and the measured value.

exciter A separate dc generator used to supply current to the field coils of a large generator or alternator. Normally, excitation current is controlled automatically.

exponent The number to which a base number is raised.

falling edge The negative-going transition of a pulse.

fall time (t_f) The time interval required for a pulse to change from 90% to 10% of its amplitude.

farad (F) The unit of capacitance.

Faraday's law A law stating that the voltage induced across a coil of wire equals the number of turns in the coil times the rate of change of the magnetic flux.

field winding The winding on the rotor of an ac generator.

filter A type of circuit that passes certain frequencies and rejects all others.

free electron A valence electron that has broken away from its parent atom and is free to move from atom to atom within the atomic structure of a material.

frequency A measure of the rate of change of a periodic function; the number of cycles completed in 1 s. The unit of frequency is the hertz.

frequency response In electric circuits, the variation in the output voltage (or current) over a specified range of frequencies.

fuel cell A device that converts electrochemical energy into dc voltage directly.

function generator An electronic instrument that produces electrical signals in the form of sine waves, triangular waves, and pulses.

fundamental frequency The repetition rate of a waveform.

fuse A protective device that burns open when there is excessive current in a circuit.

gauss (G) A CGS unit of flux density.

generator An energy source that produces electrical signals.

ground In electric circuits, the common or reference point.

ground plane A conducting surface used as a reference point for circuit returns; a conducting surface used in antenna systems to image a radiating structure.

half-power frequency The frequency at which the output power of a resonant circuit is 50% of the maximum (the output voltage is 70.7% of maximum); another name for critical or cutoff frequency.

half-splitting A troubleshooting procedure where one starts in the middle of a circuit or system and, depending on the first measurement, works toward the output or toward the input to find the fault.

Hall effect A change in current across a conductor or semiconductor when current in the material is perpendicular to a magnetic field. The change in current produces a small transverse voltage in the material, called the Hall voltage.

harmonics The frequencies contained in a composite waveform, which are integer multiples of the repetition frequency (fundamental).

henry (H) The unit of inductance.

hertz (Hz) The unit of frequency. One hertz equals one cycle per second.

high-pass filter A type of filter that passes all frequencies above a critical frequency and rejects all frequencies below that critical frequency.

hysteresis A characteristic of a magnetic material whereby a change in magnetization lags the application of magnetic field intensity.

imaginary number A number that exists on the vertical axis of the complex plane.

impedance The total opposition to sinusoidal current expressed in ohms.

impedance matching A technique used to match a load resistance to a source resistance in order to achieve maximum transfer of power.

induced current (i_{ind}) A current induced in a conductor when the conductor moves through a magnetic field.

induced voltage (v_{ind}) Voltage produced as a result of a changing magnetic field.

inductance The property of an inductor whereby a change in current causes the inductor to produce a voltage that opposes the change in current.

induction motor An ac motor that achieves excitation to the rotor by transformer action.

inductive reactance The opposition of an inductor to sinusoidal current. The unit is the ohm (Ω).

inductive susceptance The ability of an inductor to permit current; the reciprocal of inductive reactance. The unit is the siemens (S).

inductor An electrical device formed by a coil of wire a having the property of inductance; also known as *coil.*

instantaneous power The value of power in a circuit at any given instant of time.

instantaneous value The voltage or current value of a waveform at a given instant in time.

insulator A material that does not allow current under normal conditions.

integrator A circuit producing an output that approaches the mathematical integral of the input.

ion An atom that has a net positive or negative charge.

joule (J) The SI unit of energy.

junction A point at which two or more components are connected.

kilowatt-hour (kWh) A large unit of energy used mainly by utility companies.

Kirchhoff's current law A law stating that the total current into a node equals the total current out of the node. Equivalently, the algebraic sum of all the currents entering and leaving a node is zero.

Kirchhoff's voltage law A law stating that (1) the sum of the voltage drops around a single closed path equals the source voltage in that loop or (2) the algebraic sum of all the voltages around any closed path in a circuit is zero.

lag Refers to a condition of the phase or time relationship of waveforms in which one waveform is behind the other in phase or time.

lead Refers to a condition of the phase or time relationship of waveforms in which one waveform is ahead of the other in phase or time; also, a wire or cable connection to a device or instrument.

leading edge The first step or transition of a pulse.

Lenz's law A law that states when the current through a coil changes, the polarity of the induced voltage created by the changing magnetic field is such that it always opposes the change in current that caused it. The current cannot change instantaneously.

linear Characterized by a straight-line relationship.

line current The current through a line feeding a load.

lines of force Magnetic flux lines in a magnetic field radiating from the north pole to the south pole.

line voltage The voltage between lines feeding a load.

load An element (resistor or other component) connected across the output terminals of a circuit that draws current from the source; an element in a circuit upon which work is done.

load cell A transducer that uses strain gauges to convert mechanical force into an electrical signal.

loop A closed current path in a circuit.

loop current A current assigned to a circuit purely for the purpose of mathematical analysis and not normally representing the actual physical current.

low-pass filter A type of filter that passes all frequencies below a critical frequency and rejects all frequencies above that critical frequency.

magnetic coupling The magnetic connection between two coils as a result of the changing magnetic flux lines of one coil cutting through the second coil.

magnetic field A force field radiating from the north pole to the south pole of a magnet.

magnetic field intensity The amount of mmf per unit length of magnetic material; also called *magnetizing force*.

magnetic flux The lines of force between the north and south poles of a permanent magnet or an electromagnet.

magnetic flux density The amount of flux per unit area perpendicular to the magnetic field.

magnetomotive force (mmf) The cause of a magnetic field, measured in ampere-turns.

magnitude The value of a quantity, such as the number of volts of voltage or the number of amperes of current.

matrix An array of numbers.

maximum power transfer A transfer of maximum power from a source to a load when the load resistance equals the internal source resistance.

metric prefix An affix that represents a power-of-ten number expressed in engineering notation.

multimeter An instrument that measures voltage, current, and resistance.

mutual inductance The inductance between two separate coils, such as in a transformer.

neutron An atomic particle having no electrical charge.

node A point in a circuit where two or more components are connected; also known as a *junction*.

Norton's theorem A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistance or impedance.

nucleus The central part of an atom containing protons and neutrons.

ohm (Ω) The unit of resistance.

ohmmeter An instrument for measuring resistance.

Ohm's law A law stating that current is directly proportional to voltage and inversely proportional to resistance.

open A circuit condition in which there is not a complete current path.

open circuit A circuit in which there is not a complete current path.

oscillator An electronic circuit that produces a time-varying signal without an external input signal using positive feedback.

oscilloscope A measurement instrument that displays signal waveforms on a screen.

parallel The relationship between two circuit components that exists when they are connected between the same pair of nodes.

parallel resonance A condition in a parallel *RLC* circuit in which the reactances ideally are equal and the impedance is maximum.

passband The range of frequencies passed by a filter.

peak-to-peak value The voltage or current value of a waveform measured from its minimum to its maximum points.

peak value The voltage or current value of a waveform at its maximum positive or negative points.

period (T) The time interval of one complete cycle of a periodic waveform.

periodic Characterized by a repetition at fixed-time intervals.

permeability The measure of ease with which a magnetic field can be established in a material.

phase The relative angular displacement of a time-varying quantity with respect to a given reference.

phase current (I_0) The current through a generator winding.

phase voltage (V_0) The voltage across a generator winding.

phasor A representation of a sine wave in terms of its magnitude (amplitude) and direction (phase angle).

photoconductive cell A type of variable resistor that is light-sensitive.

photovoltaic effect The process whereby light energy is converted directly into electrical energy.

plezoelectric effect The property of a crystal whereby a changing mechanical stress produces a voltage across the crystal.

polar form One form of a complex number made up of a magnitude and an angle.

potentiometer A three-terminal variable resistor.

power The rate of energy usage. The unit is the watt.

power factor The relationship between volt-amperes and true power or watts. Volt-amperes multiplied by the power factor equals true power.

power of ten A numerical representation consisting of a base of 10 and an exponent; the number 10 raised to a power.

power rating The maximum amount of power that a resistor can dissipate without being damaged by excessive heat buildup.

power supply An electronic device that converts ac voltage to dc voltage.

primary winding The input winding of a transformer; also called *primary.*

proton A positively charged atomic particle.

pulse A type of waveform that consists of two equal and opposite steps in voltage or current separated by a time interval.

pulse repetition frequency The fundamental frequency of a repetitive pulse waveform; the rate at which the pulses repeat expressed in either hertz or pulses per second.

pulse width (t_W) For a nonideal pulse, the time between the 50% points of the leading and trailing edges; the time interval between the opposite steps of an ideal pulse.

quality factor (Q) The ratio of true power to reactive power in a resonant circuit or the ratio of inductive reactance to winding resistance in an inductor.

radian A unit of angular measurement. There are 2π radians in one complete 360° revolution. One radian equals 57.3°.

ramp A type of waveform characterized by a linear increase or decrease in voltage or current.

***RC* lag circuit** A phase shift circuit in which the output voltage, taken across the capacitor, lags the input voltage by a specified angle.

***RC* lead circuit** A phase shift circuit in which the output voltage, taken across the resistor, leads the input voltage by a specified angle.

***RC* time constant** A fixed time interval set by the values of *R* and *C* that determines the time response of a series *RC* circuit. It equals the product of the resistance and the capacitance.

reactive power The rate at which energy is alternately stored and returned to the source by a capacitor or inductor. The unit is the VAR.

real number A number that exists on the horizontal axis of the complex plane.

rectangular form One form of a complex number made up of a real part and an imaginary part.

rectifier An electronic circuit that converts ac into pulsating dc; one part of a power supply.

reference ground A method of grounding whereby a large conducive area on a printed circuit board or the metal chassis that houses the assembly is used as the common or reference point.

reflected load The load as it appears to the source in the primary of a transformer.

reflected resistance The resistance in the secondary circuit reflected into the primary circuit.

regulate To constantly sense and automatically adjust an output if it tries to change because of a change in the line voltage of the load.

relay An electromagnetically controlled mechanical device in which electrical contacts are opened or closed by a magnetizing current.

reluctance The opposition to the establishment of a magnetic field in a material.

resistance Opposition to current. The unit is the ohm (Ω).

resistor An electrical component specifically designed to have a certain amount of resistance.

resolution The smallest increment of a quantity that a DMM can measure.

resonance A condition in a series *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude; thus, they cancel each other and result in a purely resistive impedance.

resonant frequency The frequency at which resonance occurs; also known as *center frequency.*

retentivity The ability of a material, once magnetized, to maintain a magnetized state without the presence of a magnetizing force.

rheostat A two-terminal variable resistor.

ripple voltage The variation in the dc voltage on the output of a filtered rectifier caused by the slight charging and discharging action of the filter capacitor.

rise time (t_r) The time interval required for a pulse to change from 10% to 90% of its amplitude.

rising edge The positive-going transition of a pulse.

***RL* lag circuit** A phase shift circuit in which the output voltage, taken across the resistor, lags the input voltage by a specified angle.

***RL* lead circuit** A phase shift circuit in which the output voltage, taken across the inductor, leads the input voltage by a specified angle.

***RL* time constant** A fixed time interval set by the values of *R* and *L* that determines the time response of a circuit and is equal to *L/R*.

roll-off The rate of decrease of a filter's frequency response.

rms value The value of a sinusoidal voltage that indicates its heating effect, also known as the effective value. It is equal to 0.707 times the peak value. *rms* stands for root mean square.

rotor The rotating assembly in a generator or motor.

sawtooth waveform A type of electrical waveform composed of ramps; a special case of a triangular waveform in which one ramp is much shorter than the other.

schematic A symbolized diagram of an electrical or electronic circuit.

scientific notation A system for representing any number as a number between 1 and 10 times an appropriate power of ten.

secondary winding The output winding of a transformer; also called *secondary.*

Seebeck effect The generation of a voltage at the junction of two different materials that have a temperature difference between them.

selectivity A measure of how effectively a resonant circuit passes certain desired frequencies and rejects all others. Generally, the narrower the bandwidth, the greater the selectivity.

self-excited generator A generator in which the field windings derive their current from the output.

semiconductor A material that has a conductance value between that of a conductor and an insulator. Silicon and germanium are examples.

series In an electric circuit, a relationship of components in which the components are connected such that they provide a single current path between two points.

series resonance A condition in a series *RLC* circuit in which the reactances ideally cancel and the impedance is minimum.

shell The orbit in which an electron revolves.

short A circuit condition in which there is a zero or abnormally low resistance path between two points; usually an inadvertent condition.

SI Standardized international system of units used for all engineering and scientific work; abbreviation for French *Le Système International d'Unités*.

siemens (S) The unit of conductance.

simultaneous equations A set of *n* equations containing *n* unknowns, where *n* is a number with a value of 2 or more.

sine wave A type of waveform that follows a cyclic sinusoidal pattern defined by the formula $y = A \sin \theta$.

slip The difference between the synchronous speed of the stator field and the rotor speed in an induction motor.

solenoid An electromagnetically controlled device in which the mechanical movement of a shaft or plunger is activated by a magnetizing current.

solenoid valve An electrically controlled valve for control of air, water, steam, oils, refrigerants, and other fluids.

source A device that produces electrical energy.

speaker An electromagnetic device that converts electrical signals to sound waves.

squirrel cage An aluminum frame within the rotor of an induction motor that forms the electrical conductors for a rotating current.

stator The stationary outer part of a generator or motor.

steady state The equilibrium condition of a circuit that occurs after an initial transient time.

step-down transformer A transformer in which the secondary voltage is less than the primary voltage.

step-up transformer A transformer in which the secondary voltage is greater than the primary voltage.

stiff voltage Voltage in which the load resistor is at least ten times larger than the divider resistors.

strain gauge A type of variable resistor that changes resistance when force is applied.

superposition theorem A method for the analysis of circuits with more than one source.

switch An electrical device for opening and closing a current path.

synchronous motor An ac motor in which the rotor moves at the same rate as the rotating magnetic field of the stator.

tank circuit A parallel resonant circuit.

tapered Nonlinear, such as a tapered potentiometer.

temperature coefficient A constant specifying the amount of change in the value of a quantity for a given change in temperature.

terminal equivalency The concept that when any given load resistance is connected to two sources, the same load voltage and load current are produced by both sources.

tesla (T) The SI unit of flux density.

thermistor A type of variable resistor that is temperature-sensitive.

thermocouple A thermoelectric type of voltage source commonly used to sense temperature.

Thevenin's theorem A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a voltage source in series with a resistance or impedance.

time constant A fixed-time interval, set by R and C, or R and L values, that determines the time response of a circuit.

tolerance The limits of variation in the value of a component.

trailing edge The second step or transition of a pulse.

transducer A device that senses a change in a physical parameter and converts that change into an electrical quantity, such as a change in resistance.

transformer An electrical device constructed of two or more coils (windings) that are electromagnetically coupled to each other to provide a transfer of power from one coil to another.

transient time An interval equal to approximately five time constants.

triangular waveform A type of electrical waveform that consists of two ramps.

trigger The activating signal for some electronic devices or instruments.

trimmer A small variable capacitor.

troubleshooting A systematic process of isolating, identifying, and correcting a fault in a circuit or system.

true power The power that is dissipated in a circuit, usually in the form of heat.

turns ratio (*n*) The ratio of turns in the secondary winding to turns in the primary winding.

unbalanced bridge A bridge circuit that is in the unbalanced state as indicated by a voltage across the bridge that is proportional to the amount of deviation from the balanced state.

valance Related to the outer shell or orbit of an atom.

valence electron An electron that is present in the outermost shell of an atom.

VAR (volt-ampere reactive) The unit of reactive power.

varactor A semiconductor device that exhibits a capacitance characteristic that is varied by changing the voltage across its terminals.

volt The unit of voltage or electromotive force.

voltage The amount of energy per charge available to move electrons from one point to another in an electric circuit.

voltage divider A circuit consisting of series resistors across which one or more output voltages are taken.

voltage drop The decrease in voltage across a resistor due to a loss of energy.

voltage source A device that provides a constant voltage for a varying load.

voltmeter An instrument used to measure voltage.

Watt (W) The unit of power. One watt is the power when 1 J of energy is used in 1 s.

Watt's law A law that states the relationships of power to current, voltage, and resistance.

waveform The pattern of variations of a voltage or current showing how the quantity changes with time.

weber The SI unit of magnetic flux, which represents 10^8 lines.

Wheatstone bridge A 4-legged type of bridge circuit with which an unknown resistance can be accurately measured using the balanced state of the bridge. Deviations in resistance can be measured using the unbalanced state.

winding The loops or turns of wire in an inductor.

wiper The sliding contact in a potentiometer.

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